Characterization of Aircraft Wake Vortices by 2-μm Pulsed Doppler Lidar

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ABSTRACT

The 2-μm pulsed Doppler lidar, already successfully used for wind and turbulence measurements, has been modified for long-range wake-vortex characterization. In particular, a four-stage data processing algorithm has been developed to achieve precise profiles of tangential velocities from which the vortex parameters such as trajectories, core separation, tilt angle, and circulation can be derived. The main advantage of the pulsed lidar is its long-range capability of more than 1 km. This allows for observations over long periods from the moment of wake generation to a progressed state of vortex decay. With the field experiment at Tarbes airfield the potential of the 2-μm pulsed Doppler lidar for full-scale wake-vortex characterization has been demonstrated. Two examples showing the parameters of wake vortices generated by large transport aircraft (LTA)-type aircraft will be presented.

1. Introduction

Strong vortices generated by heavy aircraft present a potential hazard to other aircraft following close behind. In times of increasing air traffic and aircraft dimensions, the wake-vortex issue is becoming more and more important for flight safety, airport capacity, and aircraft design. Several research programs based on simulations, model calculations, wind channel and water tank measurements, catapult tests, and so on have been carried out. For experimental investigations of wake vortices in the atmospheric environment, the coherent lidar has proved to be the most effective and flexible tool. This technique involves transmitting a laser beam into the atmosphere and then detecting the radiation scattered back by the aerosol particles. The backscattered signal is analyzed with respect to the resulting Doppler-frequency shift to give the line-of-sight (LOS) velocity component of the aerosols and, hence, the air motion along the laser beam.

Extended field experiments for wake-vortex characterization have been carried out using the continuous-wave (CW) version of the coherent lidar (e.g., Köpp 1994, 1999; Vaughan et al. 1996; Campbell et al. 1997; Harris et al. 2000, 2002; Gerz 2001). These CW measurements are characterized by precise and highly resolved distributions of tangential velocities. On the other hand, the detection range is limited to a few hundred meters, since the range setting is achieved by focusing the transceiver telescope to the distance of interest. While the accurate angular location of the vortex cores is generally easy to achieve, the range (i.e., the distance between the lidar and the core) is usually subject to much greater uncertainty, so that the calculated trajectories contain larger positional errors along the laser beam direction. This problem can be overcome by positioning two or more CW lidar systems in one line and by applying a new triangulation procedure for determination of the core positions (Köpp et al. 2003).

Another approach is to use the pulsed version of the coherent lidar for wake-vortex measurements (Brockman et al. 1999; Hannon and Thomson 1997). The present paper describes a similar approach that uses the Deutsches Zentrum für Luft- und Raumfahrt (DLR) 2-μm pulsed Doppler lidar and special data processing algorithms for full-scale wake-vortex characterization. The sensor hardware is based on the pulsed lidar developed as part of the European Community (EC) project Multifunction Future Laser Atmospheric Measurement Equipment (MFLAME) for the detection of wake vortices and atmospheric hazards (Combe et al. 2000; Keane et al. 2002). The data processing procedure, now adapted to the new application, includes four estimation stages: Doppler spectra, radial velocity with velocity envelopes, core position, and vortex circulation. During the EC project Wake Vortex Characterization and Control (C-Wake), the modified lidar was successfully tested at the airfield in Tarbes, France. Two examples of wake vortices generated by large transport aircraft (LTA) are presented.
2. Experiment

a. The 2-µm pulsed Doppler lidar

The requirements of a ground-based sensor for wake-vortex characterization are a range resolution of several meters matching the dimensions of the phenomenon, a field of view (FOV) and a measurement range covering the expected vertical and horizontal transport of the vortices, and the speed and attitude variations of the wake-generating aircraft. This defines, in our case, an elevation sector of approximately 20° and a detection range from several 100 m to more than 1 km. To provide sufficient updates, this area has to be scanned within a period of a few seconds.

The 2-µm pulsed Doppler lidar is based on the transceiver unit of a MAG-1 instrument from CLR Photonics (Henderson et al. 1993). It comprises the CW master laser for injection seeding of the slave laser and for acting as the local oscillator for the coherent detection, and the slave laser for transmission of laser pulses into the atmosphere. The master laser is characterized by single-frequency operation and low bandwidth to provide high heterodyne efficiency. The slave laser has special features necessary for wake-vortex characterization, for example, a pulse repetition frequency of 500 Hz allowing for scanning of the measurement plane within a few seconds. Both lasers are diode-pumped Tm:LuAG lasers with optimum matching to the atmospheric window at 2022-nm wavelength and allowing eye-safe operation of the lidar. The transceiver unit also contains the optical components for coherent detection and the transceiver telescope.

The measurement plane is linearly scanned by an oscillating mirror of 200-mm diameter with a scan speed of 2° s⁻¹ and a fly-back time of 0.5 s, which allows for an elevation sector of 22° to be scanned within 11.5 s. The analog signal from the signal detector is amplified by a 1-GHz amplifier with an adjustable gain of 30–70 dB. This amplified signal and the reference signal from the pulse-monitor detector are fed into the data acquisition and recording unit. Links with the scanner unit enable the localization of each LOS within the elevation sector. For postprocessing of the acquired data, the four-stage approach described in section 3 has been developed. This algorithm has been applied to data covering the range from 500 to 1100 m. The main parameters of the 2-µm pulsed Doppler lidar are listed in Table 1.

b. Measurement strategy

Within the framework of the EC project C-Wake, an extended field campaign was carried out in June 2002 at the airfield in Tarbes, France. There, a large number of different sensors was deployed for simultaneous investigation of the wake vortices and the atmospheric environment (Gerz 2001). This campaign for the first time offered the possibility of using the 2-µm pulsed lidar for wake-vortex characterization.

As shown in Fig. 1, the measurements were performed in a vertical plane adjusted perpendicularly to the direction of the approaching aircraft. In this plane, the pulsed lidar (L1) continuously scanned the elevation sector of 22°. In addition to the 2-µm pulsed lidar, two 10-µm CW lidars (L2) from QinetiQ, Great Malvern, Worcestershire, United Kingdom, and (L3) from the Of-

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### Table 1. Main parameters of the 2-µm pulsed Doppler lidar.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slave laser (Tm:LuAG)</td>
<td></td>
</tr>
<tr>
<td>Wavelength</td>
<td>2022.54 nm</td>
</tr>
<tr>
<td>Pulse energy</td>
<td>2.0 mJ</td>
</tr>
<tr>
<td>Pulse length</td>
<td>400 ± 40 ns (FWHM)</td>
</tr>
<tr>
<td>Pulse repetition rate</td>
<td>500 Hz</td>
</tr>
<tr>
<td>LO/SO frequency offset</td>
<td>102 ± 3 MHz</td>
</tr>
<tr>
<td>Telescope</td>
<td></td>
</tr>
<tr>
<td>Aperture</td>
<td>108 mm</td>
</tr>
<tr>
<td>Scanner</td>
<td></td>
</tr>
<tr>
<td>Oscillating mirror</td>
<td></td>
</tr>
<tr>
<td>Vertical scan range</td>
<td>≥20°</td>
</tr>
<tr>
<td>Scan duration</td>
<td>=11 s</td>
</tr>
<tr>
<td>Fly-back time</td>
<td>=0.5 s</td>
</tr>
<tr>
<td>Data acquisition</td>
<td></td>
</tr>
<tr>
<td>Concept of early digitizing</td>
<td></td>
</tr>
<tr>
<td>Sampling rate</td>
<td>500 MHz</td>
</tr>
<tr>
<td>Sample length</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Measurement range</td>
<td></td>
</tr>
<tr>
<td>Processed data</td>
<td>500–1100 m</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td></td>
</tr>
<tr>
<td>Along LOS</td>
<td>3 m</td>
</tr>
<tr>
<td>Perpendicular to LOS</td>
<td>0.9–1.9 m</td>
</tr>
</tbody>
</table>

**Fig. 1.** Strategy of the lidar measurements at the Tarbes airfield: L1, 2-µm pulsed lidar; L2 and L3, 10-µm CW lidars. The flight direction is into the drawing plane.
Fig. 2. Example of raw data measured with the DLR 2-μm pulsed Doppler lidar: (a) signal vs sample number (time, range), (b) 256 samples of the signal used for the backscatter signal power spectrum estimation, and (c) signal power spectrum (Doppler spectrum) estimate with velocity resolution $\Delta V = 0.494$ m s$^{-1}$.

Before each aircraft overflight, the LOS velocity components measured by L1 were used to calculate the vertical profiles of the cross wind and turbulence (turbulent energy dissipation rate, $e$), important parameters for understanding vortex behavior. Several seconds after an overflight, pronounced wake-vortex signatures appear in the LOS velocity fields. They can be observed during several scan periods on the online monitor. In this manner, the wake vortices generated by an LTA-type test aircraft were measured during more than 30 overflights within 4 days of the field experiment. Two examples of these measurements are presented in section 4. The validation of the pulsed-lidar results by comparison with CW-lidar results will be the subject of a following paper.

3. Data processing

The processing of the data measured by the 2-μm pulsed Doppler lidar includes four main stages: the estimation of 1) the Doppler spectra (spectra of the power of coherently detected backscatter signals), 2) the radial
of the intermediate frequency (first momentum or "instantaneous" sensing volume).

The Doppler spectra are estimated every 3 m along the beam axis from $R = 500$ m up to $R = 1100$ m ($R = R_1 + \Delta R n'$, $R_1 = 500$ m, $\Delta R = 3$ m, $n' = 1, 2, \ldots, 200$). Although there is a strong overlapping of the sensing volumes ($L_i = 88$ m $\gg \Delta R = 3$ m), the approach with $\Delta R = 3$ m allows us to find the vortex core position more accurately.

The scattering particles in the sensing volume have
a certain velocity distribution. When the signal-to-noise ratio (SNR) is high (as in the case of the data shown in Fig. 2), it is easy to estimate the mean radial velocity and the velocity variance (spectrum width) from the Doppler spectrum estimate. However, the evaluation of the maximal velocity (e.g., in the case of aircraft wake vortex) with acceptable accuracy is not possible, because of the strong fluctuations of the signal and noise components in the spectrum estimate. In order to reduce these fluctuations, the method of spectral accumulation is used, in our case accumulating 25 shots. Then taking into account that the angular speed of scanning is $2^\circ$ s$^{-1}$ and that the pulse repetition frequency is 500 Hz, the elevation angle resolution $\Delta \varphi = 0.1^\circ$ and, for example, at a range of $R = 600$ m the vertical resolution is approximately 1 m.

For each scan, the 3D array of the Doppler spectra $W(V_0 + \Delta V k', R_0 + \Delta R m', \varphi_0 + \Delta \varphi l')$ is obtained, where $V_0 = -25$ m s$^{-1}$; $\Delta V = 0.494$ m s$^{-1}$; $k' = 0, 1, 2, \ldots, 102$; $R_0 = 500$ m; $\Delta R = 3$ m; $m' = 0, 1, 2, \ldots, 200$; $\varphi_0 = 3^\circ$ (in the case of the measurements on 13 June 2002); $\Delta \varphi = 0.1^\circ$; and $l' = 0, 1, 2, \ldots, 218$.

Figure 3 shows an example of the Doppler spectra versus the velocity and range at two different elevation angles, when the beam axis is below (Fig. 3a) and above (Fig. 3b) the core of the left vortex. These data were measured in Tarbes on 12 June 2002, approximately 7 s after the LTA overflight. The aircraft height was 180 m, and the distance between the lidar and the vertical plane of the flight was 830 m. The spectra measured at a range of approximately 850 m show the broadening due to the wake vortex. The effective size (along the beam axis) of the spectral broadening area is close to the sensing volume size, $L_v = 88$ m.

Figure 4 shows an example of the Doppler spectra versus the velocity and elevation angle at a fixed range equal to the distance from the lidar to the right vortex core. These spectra were measured at the same time as the data in Fig. 3. They show the strong spectral broadening due to the velocity distribution of the right vortex. The effect of the left vortex can also be seen, but it is weaker than the right one, because the distance between the lidar and the left vortex is different to the range selected here.

The spectra shown in Fig. 5 are taken from the data in Fig. 4 at various elevation angles, labeled 1, 2, and 3. Graph 1 of the figure shows a spectrum measured outside the vortex area. Graphs 2 and 3 show spectra measured inside the right vortex below and above, respectively, the vortex center, where the strongest spectral broadening takes place.
b. Velocity envelopes

In the Doppler spectra, one can select a threshold (horizontal lines in Fig. 5) and find two points of intersection with the spectral curve to the right and to the left of the main spectral peak. These points correspond to the positive and negative velocity envelopes (vertical lines in Fig. 5). Since only the first intersecting point on each side of the main peak is chosen, intersections with possible noise peaks are suppressed. In the case of a successful threshold selection, the positive (negative) envelope is the maximal (minimal) value of the radial velocity distribution in the sensing volume. If the vortex

Fig. 5. Spectra taken from Fig. 3 for the cases when data were measured 1) outside of the area of vortex localization, and 2) below and 3) above the right vortex core.

Fig. 6. Example of the radial velocity (mean velocity of the spectral peak), the positive velocity envelope (envelope 1), and the negative velocity envelope (envelope 2) as a function of the elevation angle and range. These characteristics were obtained from the data measured in Tarbes on 13 Jun 2002 approximately 72 s after LTA overflight.

Fig. 7. Radial velocity vs elevation angle in front of ($V_a$) and behind ($V_b$) the wake vortices and the mean radial velocity (dark solid curve) used for taking into account the background wind in the measured velocity envelopes (see Fig. 5).
core is close to the sensing volume, the envelopes are representing the tangential velocity of the vortex plus the radial velocity of the background wind.

To obtain the envelope representing the sum of the tangential velocity of the vortex and the radial wind velocity, the threshold has to be chosen in accordance with the measured Doppler spectra. These spectra depend on the SNR, the lidar pulse profile, and the radial velocity profile in the sensing volume. On the basis of Doppler lidar signal simulation [the simulation algorithm is given by Banakh and Smalikho (1997), and the wake vortex model is given in the appendix], the optimal thresholds as functions of the SNR and the vortex circulation $\Gamma$ can be found. During the experiments at the Tarbes airfield (4 days of measurements), the SNR values were between 0.5 and 2, where the SNR is the ratio of the mean signal power to the mean noise power within the bandwidth of 50 MHz. (For the data shown in Figs. 10–13 the SNR = 1.) Since the threshold depends on the circulation $\Gamma$, an iterative procedure has been applied.

Figure 6 shows an example of the measured radial velocity, the positive velocity envelope (envelope 1), and the negative velocity envelope (envelope 2). This measurement was carried out in Tarbes on 13 June 2002, approximately 72 s after the LTA overflight at an elevation of 332 m and a lateral distance to the lidar of 414 m. The wind direction was pointing from the lidar to the measurement area. In the figure, the velocity and envelopes are represented as 2D functions of the elevation angle $\varphi$ and the range $R$.

The envelopes include the radial wind velocity. In order to take into account this background wind we chose the measured radial velocities in front ($V_a$) and behind ($V_b$) the wake vortices. The horizontal lines shown in Fig. 6 indicate the ranges at which these radial velocities were chosen. From the velocities $V_a$ and $V_b$, shown in Fig. 7, the mean radial velocity (dark solid line in Fig. 7) is derived and then subtracted from the positive and negative envelopes.

c. Wake-vortex core position

Figures 8a and 8b show the envelopes obtained after subtraction of the mean (background) radial velocity. Here, the signature of the two vortices can be clearly seen. For each separate vortex we can identify the elevation angle and range, where the positive envelope $V^n_1(R_0 + \Delta Rm', \varphi_0 + \Delta \varphi l')$ has its maximum, and where the negative envelope $V^n_2(R_0 + \Delta Rm', \varphi_0 + \Delta \varphi l')$ has its minimum (the index $n'$ indicates the number of the vortex; that is, $n' = 1$ for the right vortex and $n' = 2$ for the left one). In order to estimate these elevations and ranges with better precision we searched for the maximum (minimum) of the sums $\sum_{m' = -M/2}^{M/2} V^n_1[R_0 + \Delta R(m' + m), \varphi_0 + \Delta \varphi l']$, where $\Delta R \approx L_1 = 88$ m and $l' = 1, 2$.

In Figs. 8a and 8b the points of these maxima (for the positive envelope) and minima (for the negative envelope) are shown by the intersections of the horizontal and vertical lines. Then, we determine the core position of the right ($R_{C1}$, $\varphi_{C1}$) and the left ($R_{C2}$, $\varphi_{C2}$) vortices as the mean between the coordinates of the maximum of the positive envelope and the minimum of the negative envelope. The points connected by the dashed lines indicate the resulting core positions in Figs. 8a and 8b.

Figures 8c and 8d show the velocity envelopes measured at ranges $R_{C1}$ and $R_{C2}$, respectively. The vertical core is close to the sensing volume, the envelopes are representing the tangential velocity of the vortex plus the radial velocity of the background wind.
solid lines correspond to the elevation angles $\varphi_{c1}$ and $\varphi_{c2}$, respectively.

d. Wake-vortex circulation

In order to estimate the vortex circulation from the measured lidar data it is assumed that there exist parts in the velocity envelopes where the tangential velocity of one separate vortex has potential dependence on the distance between the vortex center and an arbitrary point. In the case of LTA-type aircraft, a minimal distance of 5 m and a maximal distance of 15 m are optimal for vortex measurements with a Doppler lidar. For our measurement geometry such parts can be the outer ones (relative to the vortex cores), except in the case of very strong tilt of the vortex pair, when the inner part (between the vortex cores) can also be used. In Fig. 9, the chosen parts for $V_1(\varphi_i)$ and $V_2(\varphi_i)$ are represented by the gray bands.

For estimation of the circulation of the right ($\Gamma_1$) and left ($\Gamma_2$) vortices we use the following formulas (see the appendix):

$$
\Gamma_1 = \frac{2\pi}{nm} \sum_i \sum_k \frac{V_1(\varphi_i)\mu_2(\varphi_i) - V_2(\varphi_i)\eta_1(\varphi_i)}{\mu_1(\varphi_i)\mu_2(\varphi_i) - \eta_1(\varphi_i)\eta_2(\varphi_i)},
$$

$$
\Gamma_2 = \frac{2\pi}{nm} \sum_i \sum_k \frac{V_2(\varphi_i)\mu_1(\varphi_i) - V_1(\varphi_i)\eta_2(\varphi_i)}{\mu_1(\varphi_i)\mu_2(\varphi_i) - \eta_1(\varphi_i)\eta_2(\varphi_i)},
$$

where

$$
\mu_i(\varphi_i) = \frac{1}{R_{c1} \sin(\varphi_i - \varphi_{c1})}, \quad \mu_2(\varphi_i) = \frac{1}{R_{c2} \sin(\varphi_{c2} - \varphi_i)};
$$

$$
\eta_1(\varphi_i) = \frac{R_{c2} \sin(\varphi_{c2} - \varphi_i)}{[R_{c2} \sin(\varphi_{c2} - \varphi_i)]^2 + [R_{c1} \cos(\varphi_i - \varphi_{c1}) - R_{c2} \cos(\varphi_i - \varphi_{c2})]^2},
$$

$$
\eta_2(\varphi_i) = \frac{R_{c1} \sin(\varphi_{c1} - \varphi_i)}{[R_{c1} \sin(\varphi_{c1} - \varphi_i)]^2 + [R_{c1} \cos(\varphi_i - \varphi_{c1}) - R_{c2} \cos(\varphi_i - \varphi_{c2})]^2},
$$

$i = i_0, \ i_0 + 1, \ldots, i_0 + n - 1$ and $k = k_0, \ k_0 + 1, \ldots, k_0 + m - 1$. 

Fig. 9. Example of the chosen parts in the velocity envelopes (gray bands) for estimation of the vortex circulation.
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Fig. 11. Time dependence of the wake-vortex parameters for the example shown in Fig. 10. (top) Separation between the vortex cores, (middle) tilt angle of the vortex pair, and (bottom) vortex circulation.

Fig. 10. LTA wake vortices measured on 17 Jun 2002. Trajectories of the port vortex (open circles) and starboard vortex (filled circles) observed during 11 scans.

If \( \varphi_{C2} > \varphi_{C1} \), then \( V_1(\varphi_i) < 0, V_2(\varphi_i) < 0, \) and \( R_{C1} \sin(\varphi_{C1} - \varphi_i) = R_{C2} \sin(\varphi_{C2} - \varphi_i) = 5 \text{ m; } R_{C1} \sin(\varphi_{C1} - \varphi_i) = R_{C2} \sin(\varphi_{C2} - \varphi_i) = 15 \text{ m, where } \alpha = i_0 + n - 1, \beta = k_0, \gamma = i_0, \chi = k_0 + n - 1. \) Otherwise, when \( \varphi_{C2} < \varphi_{C1} \), the positive envelopes have to be used:

\[
V_1(\varphi_i) > 0, V_2(\varphi_i) > 0, \text{ and } R_{C1} \sin(\varphi_{C1} - \varphi_i) = R_{C2} \sin(\varphi_{C2} - \varphi_i) = 5 \text{ m; } R_{C1} \sin(\varphi_{C1} - \varphi_i) = R_{C2} \sin(\varphi_{C2} - \varphi_i) = 15 \text{ m. When } \varphi_{C2} = \varphi_{C1}, \text{ it is possible to use both negative and positive envelopes for estimation of the circulation as in the case } \varphi_{C2} > \varphi_{C1} \text{ or in the case } \varphi_{C2} < \varphi_{C1}.
\]

4. Experimental results

The processing algorithms described in section 3 have been applied to the wake-vortex data acquired by the 2-\( \mu \)m pulsed lidar during the field experiment at the Tarbes airfield. In the following, two examples of LTA overflights are presented in Figs. 10–13.

The first example was measured during the evening of 17 June, during low crosswinds (2 m s\(^{-1}\) from port side) and very low turbulence (\( \epsilon_c < 7 \times 10^{-5} \text{ m}^2 \text{s}^{-3} \)). The turbulent energy dissipation rate is \( \epsilon_c \), as estimated from the lidar data using the technique described in Banakh and Smalikho (1997). A section of the measurement plane sketched in Fig. 1 is drawn in Fig. 10. The dashed line shows the upper limit of the elevation scan (21°) and the maximal range of data processing at 1100 m. The aircraft overflight at a lateral distance of 610 m from the pulsed lidar and an altitude of 283 m is directed into the drawing plane. Approximately 30 s after overflight, the wake-vortex pair reaches the measurement region. The positions of the port (open circles) and starboard vortices (full circles) can be observed during 11 scans. The resulting wake trajectory shows the self-induced vertical descent and the crosswind-induced horizontal displacement.

The time dependence of the vortex parameters for the example in Fig. 10 is shown in Fig. 11. In the top panel, the separation between the vortex cores is drawn. For the first six scans, we find full agreement with the theoretical value of 46 m (dashed line) calculated for the aircraft weight and flight configuration for this overflight. The strong increase in core separation after scan 7, leading to a reduction in self-induced downward motion, is in good agreement with the deceleration observed in the trajectory plot. The tilt angle of the vortex pair (middle panel) shows moderate values between +2° and +14°. The most important parameter is the vortex circulation (bottom panel), since it is a measure of the vortex strength and hence of the potential hazard for an aircraft following behind.

Fig. 12. LTA wake vortices measured on 13 Jun 2002: trajectories of the port vortex (open circles) and starboard vortex (filled circles) observed during nine scans.
The circulation curves of the port (open circles) and starboard vortices (full circles) show a moderate linear decrease until approximately 100 s, which then turns into a steeper slope characterizing the phase of rapid decay. In this plot, the dashed line indicates the theoretical circulation value of 428 m s\(^{-1}\) calculated for the aircraft weight and configuration.

The second example was measured during the evening of 13 June, which had a stronger crosswind (4.3 m s\(^{-1}\) from the port side) and moderate turbulence (\(\varepsilon = 5 \times 10^{-4} \text{ m}^2 \text{ s}^{-2}\)). On this day, the elevation scan covers the range from 3° to 25°. The aircraft overflight at a lateral distance of 340 m and an altitude of 334 m lies outside the drawing. From this position of overflight it takes approximately 45 s until the vortex pair is entering the measurement region. The vortices can be observed during nine scans before they decay.

The time dependence of the vortex parameters for the example in Fig. 12 is shown in Fig. 13. The separation between the vortex cores (top panel) again is in good agreement with the theoretical value of 46 m (dashed line) for scans 2–7. In this case, the strong increase in core separation after scan 7 is not coupled with a remarkable deceleration in the downward motion (Fig. 12). The tilt angle of the vortex pair (middle panel) shows values close to the horizontal direction. The circulation values are decreasing from close to the theoretical value of 490 m s\(^{-1}\) (dashed line) to half the value after 130 s. In this example, the transition from moderate decrease to the phase of rapid decay is less pronounced.

After a certain period of the vortex’s life span, the vortex decay has reached a state where it is difficult to identify the characteristic pattern generated by two separate and well-rolled-up vortices. At which point, it is not longer possible to define the positions of the core locations or to estimate the vortex parameters, like core separation, tilt angle, and circulation, in the way described in this paper. Nevertheless, clear wake signatures can be observed in the LOS velocity fields measured during one or several of the following scans. This indicates the presence of vorticity beyond the time threshold given by the processing algorithms. For the atmospheric conditions experienced during the examples presented, this threshold lies between 130 and 140 s.

5. Conclusions

Using data gathered during a field experiment at the Tarbes airfield, the potential of the 2-µm pulsed Doppler lidar for full-scale wake-vortex characterization has been demonstrated. The good system performance, together with the four-stage data processing, allows us to derive precise vortex parameters, such as trajectory, core separation, tilt angle, and circulation. The main advantage of using the pulsed lidar is its long-range capability of more than 1 km. This allows for observations over long periods, from the moment of wake generation to a progressed state of vortex decay. During the experiment at the Tarbes airfield, the SNR (as defined in section 3b) was always larger than 0.5. The minimal SNR values where the described data processing method can still be applied have to be derived by simulation.

The applicability of the method presented herein has been demonstrated for the strong wake vortices of LTA-type aircraft. From this experience it can be concluded that the characterization of vortices generated by medium-size aircraft is also possible. The measurement of small vortices will be difficult, since the relationship of laser pulse length to phenomenon size is becoming more unfavorable. By pointing the scanning area more steeply in the upward direction, wake vortices can also be investigated outside the ground influence. The operation on board an aircraft for the detection and characterization of vortices generated by a preceding aircraft is under consideration.

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is the distance from the center of the $l$th vortex to the point $(x', z')$.

Taking into account that the vortices are rotating in opposite directions, on the basis of Eqs. (A1)–(A3), the complex velocity can be represented in the form

$$V_{ci} = \frac{(-1)^{l-1} \Gamma_i}{2\pi} \frac{1}{x' + (-1)^l S/2 + jz'}.$$  \hfill (A4)

Using the superposition principle for the fields of the right and left vortices, that is, the resulting complex velocity is determined as $v' + ju' = V_c = V_{c1} + V_{c2}$, from Eq. (A4), we have

$$v' + ju' = \frac{\Gamma_i}{2\pi} \left( \frac{1}{x' - S/2 + jz'} - \frac{1}{x' + S/2 + jz'} \right).$$  \hfill (A5)

A complete review of wake-vortex generation, structure, transport, and decay, including vortex geometry and typical profiles of tangential velocity, is given by, for example, Hallock and Eberle (1977).

The relations between the point coordinates and the velocity vector coordinates in different coordinate systems are determined, respectively, by the formulas (see Fig. A1)

$$x' = (x - x_0) \cos \theta - (z - z_0) \sin \theta,$$

$$z' = (z - z_0) \cos \theta + (x - x_0) \sin \theta,$$

$$u = u' \cos \theta + v' \sin \theta,$$

$$v = u' \cos \theta - v' \sin \theta.$$  \hfill (A7)

For the points $(x, z)$ and $(x_0, z_0)$ in the polar system of coordinates we have

$$x = R \cos \varphi, \quad x_0 = R_0 \cos \varphi_0,$$

$$z = R \sin \varphi, \quad z_0 = R_0 \sin \varphi_0.$$  \hfill (A8)

A Doppler lidar measures the radial velocity $V_r$ (the projection of the velocity vector on the beam axis) where the motion of the scattering particles toward the lidar is defined as the positive velocity. Then we can write

$$V_r = -u \cos \varphi - v \sin \varphi.$$  \hfill (A9)

The use of Eqs. (A5)–(A9) allows us to obtain the equation for the radial velocity $V_r$ at an arbitrary point C (see Fig. A1) with coordinates $(R, \varphi)$ in the form

$$V_r(R, \varphi) = \frac{\Gamma_i}{2\pi} F_i(R, \varphi) - \frac{\Gamma_j}{2\pi} F_j(R, \varphi),$$  \hfill (A10)

where

$$F_i(R, \varphi) = \frac{r_i(\varphi)}{[X(R, \varphi) + (-1)^l S/2]^2 + Z'(R, \varphi)},$$  \hfill (A11)

$$r_i(\varphi) = R_0 \sin(\varphi - \varphi_0) - (-1)^l(S/2) \sin(\varphi + \theta),$$  \hfill (A12)
\[ X(R, \varphi) = R \cos(\varphi + \theta) - R_0 \cos(\varphi_0 + \theta), \quad \text{(A13)} \]

and

\[ Z(R, \varphi) = R \sin(\varphi + \theta) - R_0 \sin(\varphi_0 + \theta). \quad \text{(A14)} \]

The measured envelopes are the maximal positive and negative radial velocities in the sensing volume. From these data we can find the range, where at a fixed elevation angle \( \varphi \) the absolute value of the radial velocity \( |V(R)| \) has its maximum. If the minimal distance from the core of the \( l \)th vortex is less than the corresponding distance for the other vortex (the beam axis does not cross the circles shown in Fig. A1), then we can approximately find the range \( R_m = \max |V(R)| = |V(R_m)| \) from the equation

\[ \frac{\partial F(R, \varphi)}{\partial R} \bigg|_{R=R_m} = 0, \quad \text{(A15)} \]

where \( l = 1, 2 \). The solution of Eq. (A15) is

\[ R_m = R_0 \cos(\varphi - \varphi_0) - (-1)^l (S/2) \cos(\varphi + \theta). \quad \text{(A16)} \]

As a result of Eqs. (A10)–(A16), we obtain the equations for the radial velocities (envelopes) \( V_i(\varphi) = V_i(R_m, \varphi) \) and \( V_2(\varphi) = V_2(R_m, \varphi) \) in the form

\[
V_i(\varphi) = \frac{\Gamma_i}{2\pi r_i(\varphi)} \left( \frac{1}{2} - \frac{r_i(\varphi)}{2\pi r_i(\varphi) + d^2(\varphi)} \right) \quad \text{and} \\
V_2(\varphi) = \frac{\Gamma_2}{2\pi r_i(\varphi)} \left( \frac{1}{2} - \frac{r_i(\varphi)}{2\pi r_i(\varphi) + d^2(\varphi)} \right), \quad \text{(A17)}
\]

where

\[ d(\varphi) = S \cos(\varphi + \theta) \quad \text{(A18)} \]

and \( r_i(\varphi) \) (\( i = 1, 2 \)) is described by Eq. (A12).

The ranges \( R_{C1}, R_{C2} \) and the elevation angles \( \varphi_{C1}, \varphi_{C2} \) shown in Fig. A1 can be found from the measured data. These parameters are related to \( R_0, \varphi_0, S, \) and \( \theta \) by the equations

\[ R_0 \exp(j\varphi_0) = [R_{C1} \exp(j\varphi_{C1}) + R_{C2} \exp(j\varphi_{C2})] / 2 \quad \text{(A19)} \]

and

\[ S \exp(j\theta) = R_{C1} \exp(-j\varphi_{C1}) - R_{C2} \exp(-j\varphi_{C2}). \quad \text{(A20)} \]

From Eqs. (A12) and (A18)–(A20) we obtain

\[ r_i(\varphi) = R_{C1} \sin(\varphi - \varphi_{C1}), \quad \text{(A21)} \]

\[ r_i(\varphi) = R_{C2} \sin(\varphi - \varphi_{C2}), \quad \text{and} \quad \text{(A22)} \]

\[ d(\varphi) = R_{C1} \cos(\varphi - \varphi_{C1}) - R_{C2} \cos(\varphi - \varphi_{C2}). \quad \text{(A23)} \]

In particular, from Eqs. (A16)–(A18) and Eqs. (A21) and (A22) it follows that in the limit, when the vortex core separation is very large (i.e., \( S \to \infty \)) and when the mutual effects of the vortices are neglected, the velocities \( V_i(\varphi) \) and \( V_2(\varphi) \) can be represented as

\[ V_i(\varphi) = (-1)^l \frac{1}{2\pi} \sin(\varphi - \varphi_{C}) V_{i0}(\varphi), \quad \text{(A24)} \]

where \( V_{i0}(\varphi) = \Gamma_i [2\pi r_i(\varphi)]^{-1} \) is the tangential velocity of the \( l \)th vortex.

In Eq. (A17) we will assume that the envelopes \( V_i(\varphi) \) and \( V_2(\varphi) \) are measured in the outside parts. Then at \( \varphi_{C1} > \varphi_{C1} \) (as is shown in Fig. A1), the angle \( \varphi \) has to meet the following conditions: \( \varphi < \varphi_{C1} \) for \( V_i(\varphi) \) and \( \varphi > \varphi_{C2} \) for \( V_2(\varphi), \) where \( \varphi_{C1} \) is the angle between the line OM and the x axis and \( \varphi_{C2} \) is the angle between the line ON and the x axis (see Fig. A1). In this case \( V_1(\varphi) < 0 \) and \( V_2(\varphi) > 0. \) If \( \varphi_{C2} < \varphi_{C1} \), then \( \varphi > \varphi_{C1} \) for \( V_i(\varphi) \) and \( \varphi < \varphi_{C2} \) for \( V_2(\varphi), \) where \( \varphi_{C1} \) is the angle between the line OF and the x axis and \( \varphi_{C2} \) is the angle between the line OD and the x axis. When \( \varphi_{C2} < \varphi_{C1}, \) the envelopes are positive \( [V_1(\varphi) > 0 \) and \( V_2(\varphi) > 0]. \) If \( \varphi_{C2} = \varphi_{C1}, \) then either of the two conditions, 1) \( \varphi < \varphi_{C1} \) for \( V_i(\varphi) \) and \( \varphi > \varphi_{C2} \) for \( V_2(\varphi) \) or 2) \( \varphi > \varphi_{C1} \) for \( V_i(\varphi) \) and \( \varphi < \varphi_{C2} \) for \( V_2(\varphi), \) can be used in Eq. (A17).

Thus, the angles \( \varphi_i(V_i(\varphi)), i = i_0, i_0 + 1, \ldots, i_0 + n - 1 \) and \( \varphi_i(V_i(\varphi), k = k_0, k_0 + 1, \ldots, k_0 + m - 1 \) are not in the overlapping parts \( \varphi_i \neq \varphi_i \) determined above.

From Eq. (A17) we have the following system of linear equations:

\[ \Gamma_{ik} \mu_i(\varphi_i) + \Gamma_{2ik} \eta_i(\varphi_i) = 2\pi V_i(\varphi_i), \]

\[ \Gamma_{ik} \eta_i(\varphi_i) + \Gamma_{2ik} \mu_i(\varphi_i) = 2\pi V_2(\varphi_i), \quad \text{(A25)} \]

where \( \mu_i(\varphi_i) = 1/r_i(\varphi_i)), \eta_i(\varphi_i) = -r_i(\varphi_i)/[r_i(\varphi_i) + d^2(\varphi_i)], \) \( \mu_i(\varphi_i) = -1/r_i(\varphi_i)), \eta_i(\varphi_i) = r_i(\varphi_i)/[r_i(\varphi_i) + d^2(\varphi_i)], \) and \( \Gamma_{ik} \) and \( \Gamma_{2ik} \) are estimates obtained at \( \varphi_i \) and \( \varphi_f \) for the circulation of the right and left vortex, respectively.

Let us assume that we have measured \( n \) envelopes \( V_i(\varphi) \) and \( m \) envelopes \( V_2(\varphi) \). Then, using the solution of Eq. (A25) we can obtain \( nm \) estimates for the circulations \( \Gamma_{1ik} \) and \( \Gamma_{2ik} \). These estimates can be averaged.

To obtain the mean estimates of \( \Gamma_i \), and \( \Gamma_2 \), we use the formula

\[ \Gamma_i = n^{-1} m^{-1} \sum_{k} \sum_{l} \Gamma_{ik}, \quad \text{(A26)} \]

where \( i = i_0, i_0 + 1, \ldots, i_0 + n - 1 \); \( k = k_0, k_0 + 1, \ldots, k_0 + m - 1 \); \( l = 1, 2 \).

As result from Eqs. (A25) and (A26) we obtain

\[ \Gamma_1 = \frac{2\pi}{nm} \sum_{i} \sum_{k} \frac{V_i(\varphi_i) \mu_i(\varphi_i) - V_2(\varphi_i) \eta_i(\varphi_i)}{\mu_i(\varphi_i) \mu_i(\varphi_i) - \eta_i(\varphi_i) \eta_i(\varphi_i)}, \quad \text{(A27)} \]

and

\[ \Gamma_2 = \frac{2\pi}{nm} \sum_{i} \sum_{k} \frac{V_i(\varphi_i) \mu_i(\varphi_i) - V_2(\varphi_i) \eta_i(\varphi_i)}{\mu_i(\varphi_i) \mu_i(\varphi_i) - \eta_i(\varphi_i) \eta_i(\varphi_i)}, \quad \text{(A28)} \]
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