

Heating and Kinetic Energy Dissipation in the NCAR Community Atmosphere Model

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ABSTRACT

Conservation of energy and the incorporation of parameterized heating in an atmospheric model are discussed. Energy conservation is used to unify the treatment of heating and kinetic energy dissipation within the Community Atmosphere Model, version 2 (CAM2). Dry static energy is predicted within the individual physical parameterizations and updated following each parameterization. Hydrostatic balance leads to an efficient method for determining the temperature and geopotential from the updated dry static energy. A consistent formulation for the heating due to kinetic energy dissipation associated with the vertical diffusion of momentum is also derived. Both continuous and discrete forms are presented. Tests of the new formulation verify that the impact on the simulated climate is very small.

1. Introduction

The solution of the equations of motion in an atmospheric model requires the parameterization of processes that occur on scales smaller than those explicitly represented by the model. Parameterizations are typically included for several processes (e.g., radiation, convection, vertical diffusion), which are treated as physically distinct, even though they are intimately coupled. Because of the difference between horizontal and vertical length scales, the parameterized processes typically operate entirely in the vertical direction, treating the atmosphere as a set of distinct columns, which can be treated independently. This paper discusses conservation of energy in the Community Atmosphere Model, version 2 (CAM2; more information available online at <http://www.cesm.ucar.edu/models/atm-cam/>), particularly within the parameterization suite, and a unification of the application of parameterized thermodynamic processes in terms of dry static energy.

CAM2 is the successor to the National Center for Atmospheric Research (NCAR) Community Climate Model, Version 3 (CCM3; Kiehl et al. 1998). In common with many atmospheric models, the column parameter-

izations in CAM2 are clearly separated from the solution of the resolved adiabatic dynamics. In fact, CAM2 supports three “dynamical cores”: an Eulerian spectral core, as in CCM0 through CCM3 (CCM0–3); a semi-Lagrangian spectral core based on Williamson and Olson (1994); and a finite volume core based on Lin and Rood (1997). Horizontal diffusion is the only parameterized process that acts across columns in CAM2 and it is treated within the dynamical cores. Conservation of energy within the resolved dynamics is the responsibility of the dynamical core and the latter two (which contain implicit diffusion) include ad hoc fixers to enforce conservation.

Energy conservation was not a significant consideration in developing the original CCM0, which contained an energy imbalance of $\sim 10 \text{ W m}^{-2}$ (Williamson 1988), primarily due to inconsistencies in the vertical numerical approximations. Energy conservation was taken more seriously in CCM1–3, which conserved energy to $\sim 0.4 \text{ W m}^{-2}$ (e.g., Boville and Gent 1998). This level of conservation was partly due to a cancellation of errors since individual processes conserved energy only to $\sim 1 \text{ W m}^{-2}$. Energy conservation is now a serious issue because of the application of CAM2 in multicentury coupled ocean–atmosphere simulations in which even imbalances of 0.4 W m^{-2} can cause spurious long-term trends.

For historical reasons, the parameterizations of thermodynamic processes in CCM0–3 have involved evaluating tendencies for a mixture of temperature, potential temperature and dry static energy. Both computational

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constraints and the evolution of the conceptual framework for the individual parameterizations have contributed to this diversity. In addition, the forcing terms have been applied using a mixture of time and process splitting (see below). Recently, Williamson (2002) has reformulated the CCM3 parameterization suite entirely in time-split form. In this formalism, the parameterizations operate sequentially, on profiles modified by the previous parameterization.

In CCM0–3, heating was considered equivalent to temperature tendency within the parameterization suite, so that energy was not conserved for the intermediate states which follow time split parameterizations. The total heating due to all parameterizations was correctly reported to the dynamics and applied in a nearly conservative fashion. This manuscript describes a method for applying heating to the dry static energy so that the energy is locally conserved at all stages within the parameterization suite, regardless of the splitting method. The methods are equivalent when height is the vertical coordinate but the distinction becomes important for pressure-based vertical coordinates, as used in most global models. The meaning of energy conservation is stated precisely, including the effects of variable gas mixtures, although CAM2 is still somewhat inconsistent in its treatment of variations in water vapor.

The vertical diffusion in CAM2 has been reformulated using dry static energy and both the kinetic energy flux and the heating due to kinetic energy dissipation are derived. These terms depend correctly on the stress tensor as discussed by Fiedler (2000) and Becker (2001). In CCM1–3, the kinetic energy fluxes were ignored.

We begin by reviewing the total energy equation in section 2 and its application to the physical parameterizations, together with the hydrostatic approximation in section 3. The splitting procedures used for parameterizations are summarized in section 4, the procedure for deriving temperature and geopotential from dry static energy is discussed in section 5, and vertical diffusion and dissipation of kinetic energy are discussed in section 6. Energy conservation considerations for the dynamical cores are commented on briefly in section 7, the impact of the changes on the CAM2 simulation is discussed in section 8, and conclusions appear in section 9.

2. Total energy conservation

Energy conservation is an important property of the equations of motion, which should be maintained when approximations are made in order to solve them. Energy conservation can be guaranteed by forming an evolution equation for total energy and verifying that it is satisfied by the approximate equations. The total energy equation is obtained by adding the kinetic energy equation to the first law of thermodynamics. The derivations of these equations may be found in Gill (1982), among other sources, and will only be summarized here.

A fundamental statement of the first law for a fluid

is $dI/dt + dW/dt = Q$, where I is the internal energy per unit mass, W is the work done by the fluid through expansion, Q is the heating rate, and d/dt represents a substantive derivative. For a variable mixture of ideal gases, each with a fixed specific heat at constant volume c_v , $p\alpha = RT$, and $I \equiv c_v T$, where R and c_v are the apparent (mass weighted) constants for the mixture, α is specific volume, p is pressure, and T is temperature. Noting that the work term is $dW/dt \equiv p(d\alpha/dt) = (d/dt)p\alpha - \alpha(dp/dt)$, $\omega \equiv dp/dt$ and $c_p \equiv c_v + R$, the first law is

$$\frac{d(c_v T)}{dt} + p \frac{d\alpha}{dt} = Q, \quad (1)$$

or, equivalently,

$$\frac{d(c_p T)}{dt} - \frac{RT\omega}{p} = Q. \quad (2)$$

Here Q may have contributions from the divergence of radiative fluxes, molecular conduction, phase change and precipitation processes, chemical reactions, and viscous dissipation. In the lower atmosphere, molecular conduction is usually replaced by turbulent transport.

For a variable mixture of gases, c_v , R , and c_p are determined by their mass-weighted average over the gases in the mixture. In CAM2, we make the common approximation that dry air is well mixed and that water vapor is the only additional substance present in sufficient abundance to affect the total pressure. Let superscripts ω and d denote water vapor and dry air, respectively. Then the apparent constants are

$$c_p = c_p^d + (c_p^w - c_p^d)q, \quad (3)$$

$$R = R^d + (R^w - R^d)q, \quad (4)$$

where q is specific humidity. Water vapor is sufficiently abundant in the Tropics so that vertical variations in R may change the sign of the static stability, while horizontal variations may produce pressure gradients even with uniform temperatures. Variations in c_p are often considered to be less important, since they affect the response to heating, but do not directly affect the density and pressure gradients. CAM2 uses $c_p = c_p^d$, so that (2) becomes $dT/dt - RT\omega/p c_p^d = Q/c_p^d$. This is the thermodynamic equation solved by both the spectral (Eulerian and semi-Lagrangian) cores, although c_p is used in the second term due to an oversight. The finite volume core converts (2) to a virtual potential temperature equation. We retain a variable c_p in the derivations below, but properly accounting for variations in c_p in a model will probably require predicting an enthalpy ($c_p T$) related variable. This will become increasingly important as the thermosphere is incorporated into CAM2, since diffusive separation of constituents results in large variations of c_p and R above ~ 120 km.

The kinetic energy equation is derived from the momentum equation,

$$\frac{d\mathbf{V}}{dt} + 2\boldsymbol{\Omega} \times \mathbf{V} = -\alpha\nabla p - \mathbf{g} - \alpha\nabla \cdot \boldsymbol{\tau}, \quad (5)$$

where $\boldsymbol{\tau}$ is the stress tensor due to molecular or turbulent viscosity and other notation is standard. Defining kinetic energy $K \equiv \mathbf{V} \cdot \mathbf{V}/2$, and taking the scalar product of \mathbf{V} with (5) we obtain

$$\frac{dK}{dt} = -\alpha\nabla \cdot \nabla p - wg - \alpha\nabla \cdot \mathbf{F}_K - D, \quad (6)$$

where the last two terms are the divergence of the diffusive flux ($\mathbf{F}_K = \mathbf{V} \cdot \boldsymbol{\tau}$) and the dissipation ($D = -\alpha\nabla \mathbf{V} \cdot \boldsymbol{\tau}$) of kinetic energy, as in Fiedler (2000).

We obtain the total energy equation by adding (6) and (1), using mass continuity ($\nabla \cdot \mathbf{V} = \alpha^{-1}d\alpha/dt$), and noting that $wg \equiv d\Phi/dt$, where $\Phi \equiv gz$ is the geopotential:

$$\frac{d}{dt}(c_v T + \Phi + K) = -\alpha\nabla \cdot (p\mathbf{V} + \mathbf{F}_K) + Q_0. \quad (7)$$

The heating $Q_0 = Q - D$ in (7) excludes kinetic energy dissipation, which merely converts one form of energy to another. Energy conservation requires that Q in (2) include all kinetic energy dissipation, whether it arises explicitly from viscous processes in (5), or implicitly from imperfections in the numerical approximations of the dynamical core. The local value of the implicit dissipation is usually difficult to determine, but the globally averaged value may be determined by integrating (7).

Converting (7) to fixed volume elements and assuming that $p\mathbf{V}$ and \mathbf{F}_K vanish on the top and bottom boundaries, the global integral of (7) is

$$\begin{aligned} \frac{d}{dt} \int_A \int_{z=0}^{z_t} (c_v T + \Phi + K) \rho \, dz \, dA \\ = \int_A \int_{z=0}^{z_t} Q_0 \rho \, dz \, dA, \end{aligned} \quad (8)$$

where dA is an area element and z_t is the height of the model top. The globally integrated total energy may change only due to the globally integrated nondissipative heating, which can be expressed as the net boundary fluxes of sensible, latent, and radiative energy. Implicit dissipation due to the numerical approximations will result in an imbalance in (8), with the left-hand side usually being less than the right-hand side. In order to maintain energy balance, the globally averaged implicit dissipative heating, determined by this imbalance, can be added to (2) as either a globally uniform heating, or other ad hoc function. The implicit heating for the CAM2 dynamical cores is discussed in section 7.

The form of the total energy equation used to construct conservative parameterizations is a variation of (7). Applying the product rule for a divergence and using mass continuity, $\alpha\nabla \cdot (p\mathbf{V}) = d(\alpha p)/dt - \alpha\partial p/\partial t$, which is substituted into (7) to obtain

$$\frac{d}{dt}(s + K) = \alpha \frac{\partial p}{\partial t} - \alpha\nabla \cdot \mathbf{F}_K + Q_0, \quad (9)$$

where $s \equiv c_p T + \Phi$ is the dry static energy. The dry static energy includes the internal energy ($c_v T$), the potential energy (Φ), and the work done ($RT = p\alpha$) to raise the volume of a fluid element from 0 to α at constant p .

3. Hydrostatic approximation

Up to this point, we have not made the hydrostatic approximation:

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{g}{\alpha}. \quad (10)$$

However, CAM2 is a hydrostatic model and we require that the state be in hydrostatic balance in addition to conserving energy. Furthermore, each atmospheric column is treated independently within the physical parameterization suite and the parameterizations are time split. A sequence of intermediate states is constructed and we require both energy conservation and hydrostatic balance for each state. In fact, within the CAM2 parameterization suite, we require local conservation for both energy and constituents within each layer of every column.

Satisfying these requirements begins by forming a dry static energy equation from (2). Expanding the substantive derivative in the second term of (2) and applying (10),

$$\frac{ds}{dt} = Q + \alpha \left(\frac{\partial p}{\partial t} + \mathbf{V}_h \cdot \nabla_h p \right), \quad (11)$$

where \mathbf{V}_h is the horizontal velocity. We emphasize that (11), which is adopted in CAM2, is an exact statement of the first law of thermodynamics in a hydrostatic system. A similar dry static energy equation, in which the heating Q_0 does not include D , may be obtained from (9) by neglecting K , as discussed in Gill (1982). This approximation is not made in CAM2.

From the hydrostatic form of (5), we obtain the hydrostatic kinetic energy equation,

$$\frac{dK}{dt} = -\alpha\nabla_h \cdot \nabla_h p - \alpha\nabla \cdot \mathbf{F}_K - D, \quad (12)$$

and the total energy equation is still (9). Note that in the hydrostatic approximation, the kinetic energy is redefined to include only the horizontal wind, $K = \mathbf{V}_h \cdot \mathbf{V}_h/2$. The pressure tendency and all advective and horizontal gradient terms are treated within the dynamical core, so that within the parameterizations (11) and (12) are just

$$(\partial s/\partial t)_c = Q_c, \quad (13)$$

$$(\partial K/\partial t)_c = -\alpha\partial F_K/\partial z - D_c, \quad (14)$$

where the subscript c refers to the column parameterizations and F_K is the vertical flux of kinetic energy due to parameterized stresses. We distinguish between $Q_c = Q_0 + D_c$ and $Q = Q_0 + D$, because kinetic energy dissipation within the dynamical core, whether implicit or explicit, is treated separately. Within the parameterizations, the total energy is defined as $s + K$ and its tendency is given by (13) + (14). Total energy is locally conserved by using (13) to add all parameterized heating, including D_c . Hydrostatic balance is maintained by using (10) to obtain T and Φ from s .

In the CAM2 column parameterizations, only vertical diffusion and orographic gravity wave drag dissipate kinetic energy by acting on momentum. The dissipation was computed in CCM2-3 by neglecting F_K in (14) so $D = -\partial K/\partial t = -\mathbf{V} \cdot \partial \mathbf{V}/\partial t$ for each process. This is actually the correct heating for orographic (zero phase speed) gravity waves, which have no vertical energy flux. The correct vertical integral is still obtained for molecular and turbulent diffusion processes, despite neglecting F_K . The correct forms of F_K and D for the vertical diffusion parameterization in CAM2 are derived below.

Latent energy is conserved by conserving constituents individually and computing the energy associated with phase changes. As in previous versions of the CCM, the latent heat of fusion is neglected in CAM2, but not in the land and sea ice models that determine the surface fluxes. This inconsistency results in a globally and annually averaged conservation error of $\sim 0.2 \text{ W m}^{-2}$, as mentioned in Boville and Gent (1998). Of course, the error is systematic and is much greater over ice surfaces. The latent heat of fusion has been implemented in CAM2 and will be discussed in a separate manuscript.

A statement of vertically integrated energy conservation within the physical parameterizations acting on each column can be obtained by adding the vertical integrals of (13) and (14) and writing the heating Q_c as the net latent heating due to moist processes plus the convergence of an upward total sensible heat flux F_s due to radiation and vertical transport:

$$\left[\frac{\partial}{\partial t} \int_{z_s}^{z_t} (s + K)\rho dz \right]_c = F_s|_{z_s}^{z_t} + L_v C, \quad (15)$$

where L_v is the latent heat of vaporization, z_s and z_t are the surface and model top heights, F_s is the heat flux, and C is the vertically integrated net condensation rate. In CCM0-3, C was just the precipitation rate. CAM2 includes a prognostic cloud water that acts as a reservoir of liquid water, so that C is not balanced instantaneously by precipitation. We have assumed that the stress vanishes at z_t and that $\mathbf{V}(z_s) = 0$, so $F_K(z_s) = F_K(z_t) = 0$. Even if the motion of the sea and ice surfaces is accounted for when computing the surface stress, one must still assume that $F_K(z_s) = 0$ unless the kinetic energy flux is also accounted for in the ocean and sea ice. For

most applications, $F_s(z_t)$ is just the net radiative flux, while $F_s(z_s)$ also includes the sensible heat flux. In the thermosphere, emission of longwave radiation is very inefficient and absorption of solar radiation is largely balanced by downward molecular diffusion of heat. Therefore, in upper-atmosphere applications $F_s(z_t)$ may include a downward diffusive flux of heat due to solar heating above z_t .

The hydrostatic approximation is used to pose the parameterizations in pressure coordinates in CAM2. Since the pressure tendency is treated within the dynamical core, the pressure is assumed to be invariant over the parameterization step. This procedure is not strictly correct in the presence of parameterized water fluxes, which should change the local pressure. Omitting this effect implies a compensating flux of dry air. The net surface flux of water (precipitation minus evaporation) should change the surface pressure, but instead implies a surface flux of dry air. For the finite volume core, the dry pressure in each layer is corrected after the parameterizations, while conserving the masses of all constituents. An energy and momentum conserving form has now been tested, but is not in CAM2. The Eulerian and semi-Lagrangian cores require a fixed relationship between the layer pressures and the surface pressure, so a simple correction to the dry mass does not work and only a global correction to the dry air mass is applied.

4. Process and time splitting

The set of parameterizations in a model may be logically conceived of as operating simultaneously (process splitting) or sequentially (time splitting) within a time step. In process splitting, each parameterization computes a heating based on the same input state,

$$Q = \sum_i Q_i(s, T, \Phi, \dots), \quad (16)$$

where Q_i is the heating due to a particular parameterization. Although the same states enter each parameterization, the parameterizations are not independent. For example, radiative heating depends on cloud properties, which depend on convection and large-scale condensation. Therefore, the solution may depend on the order of the parameterizations. In time splitting, the order dependence of the parameterizations is made explicit. The partial time derivative in (13) is evaluated after each parameterization and the state is updated sequentially:

$$s_i = s_{i-1} + \delta t Q_i(s_{i-1}, T_{i-1}, \Phi_{i-1}, \dots). \quad (17)$$

After the application of all I parameterizations, the net heating is calculated as

$$Q = (s_I - s)/\delta t. \quad (18)$$

In CCM3, the first three parameterizations (penetrative convection, shallow convection, large-scale con-

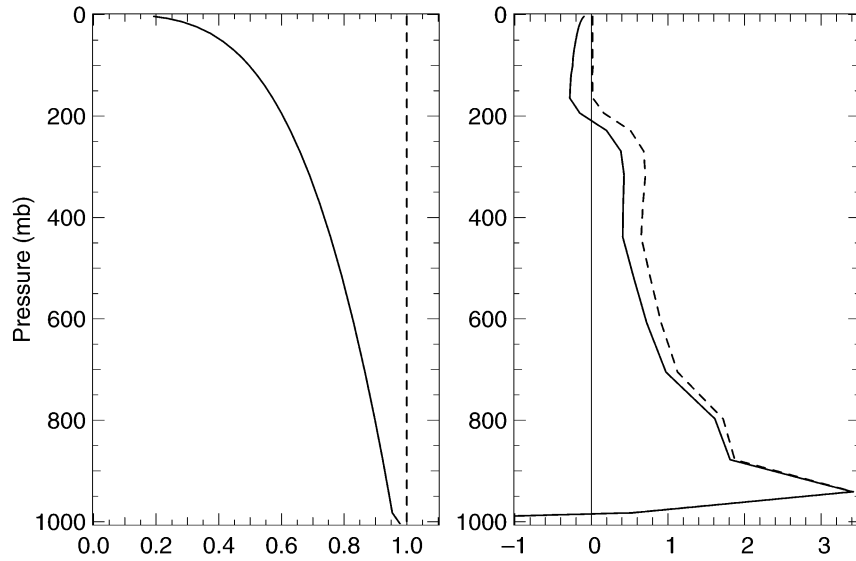


FIG. 1. Heating rate Q/c_p (dash, K day^{-1}) and temperature tendency $\partial T/\partial t$ (solid), (left) for an idealized heating $Q/c_p = 1$ and (right) for the actual net tropical convective heating from CAM2. Both solutions for $\partial T/\partial t$ use the average tropical profile of T and q from CAM2.

denensation), which involve condensation and latent heat release, were time split while the last three parameterizations (radiation, vertical diffusion, and gravity wave drag) were process split. Time splitting the moist processes allows precise enforcement of the constraint that relative humidity $\leq 100\%$. In CAM2, all parameterizations are time split. The surface turbulent fluxes are computed after radiation and are applied as boundary conditions in vertical diffusion.

Application of (17) is straightforward if z is the vertical coordinate, since Φ is invariant. However, CAM2 actually solves the parameterizations in pressure coordinates and the sequence defined by (17) requires that T_i and Φ_i be derived from s_i after each parameterization, using (10). The time-split parameterizations in CCM3 (and earlier versions) were not actually posed as in (17). Instead, heating was equated with temperature tendency, so that

$$T_i = T_{i-1} + \delta t Q_i/c_p, \quad (19)$$

and hydrostatic balance was imposed by integrating (10) to determine Φ_i . While this procedure is apparently more straightforward, it does not conserve energy within the parameterization suite.

Energy was conserved outside of the parameterizations by reporting the net heating from the time-split parameterization as

$$\frac{Q}{c_p} = \sum_{i=1}^l \frac{Q_i}{c_p} + \frac{T_l - T}{\delta t}, \quad (20)$$

with $Q_i = 0$ for time-split parameterizations. Williamson (2002) time split all parameterizations in CCM3 using this method.

An efficient method to solve for T_i and Φ_i from s_i is

given below, allowing (17) to be used at the same cost as (19). The difference is shown in Fig. 1 for a uniform heating of $Q/c_p = 1 \text{ K day}^{-1}$ and for the net convective heating from CAM2 over the western equatorial Pacific Ocean. Applying a uniform heating in the vertical results in a temperature tendency that decreases with height. Heating increases both the temperature and the thickness of lower layers, therefore increasing Φ in layers above. In the absence of heating at upper levels, conservation of s would require $\partial T/\partial t < 0$ as Φ increases. This can be clearly seen for the convective heating, which is >0 below 200 mb (except in the surface layer) and ~ 0 above. The resulting temperature tendency is smaller than Q/c_p at all levels and is negative above 200 mb.

We note that the time-splitting method does not allow all properties of the original equations to be preserved for intermediate states. We have chosen energy conservation and hydrostatic balance as the properties to preserve, while potential temperature conservation has not been preserved within the parameterization suite.

5. Discrete equations for s , T , and Φ

We define a compact notation for use in the discrete equations below. For an arbitrary variable ψ , let a subscript denote a discrete time level, with current step ψ_n and next step ψ_{n+1} . The model has L layers in the vertical, with indices running from top to bottom. Let ψ^k denote a layer midpoint quantity and let ψ^{k-} denote the value on the upper interface of layer k while ψ^{k+} denotes the value on the lower interface. The relevant quantities are then

$$\begin{aligned}\psi^{k+} &= (\psi^k + \psi^{k+1})/2, & k \in (1, 2, 3, \dots, L-1) \\ \psi^{k-} &= (\psi^{k-1} + \psi^k)/2, & k \in (2, 3, 4, \dots, L) \\ \delta^k \psi &= \psi^{k+} - \psi^{k-}, & \delta^{k+} \psi = \psi^{k+1} - \psi^k, \\ \delta^{k-} \psi &= \psi^k - \psi^{k-1}, & \psi_{n+} = (\psi_n + \psi_{n+1})/2, \\ \delta_n \psi &= \psi_{n+1} - \psi_n, & \delta t = t_{n+1} - t_n.\end{aligned}$$

Using the above notation, the dry static energy at step n and level k is

$$s_n^k = c_p^k T_n^k + \Phi^k, \quad (21)$$

which can be calculated from T_n by integrating (10):

$$\Phi = \Phi_s - \int_{p_s}^p RT \, d \ln p', \quad (22)$$

where Φ_s is the geopotential at the earth's surface and p_s is the surface pressure. A fairly arbitrary discretization of (22) can be represented using a triangular hydrostatic matrix \mathbf{H}^{kl} :

$$\Phi^k = \Phi_s + \sum_{l=L}^k R^l T^l \mathbf{H}^{kl}. \quad (23)$$

Note that (23) is often written in terms of the virtual temperature $T_v = TR/R^d$. Using (23) in (21),

$$s_n^k = c_p^k T_n^k + \Phi_s + \sum_{l=L}^k R^l T_n^l \mathbf{H}^{kl}, \quad (24)$$

$$= (c_p^k + R^k \mathbf{H}^{kk}) T_n^k + \Phi_n^{k+}. \quad (25)$$

The interface geopotential in (25) is defined as

$$\Phi^{k+} = \Phi_s + \sum_{l=L}^{k+1} R^l T^l \mathbf{H}^{kl}, \quad (26)$$

and c_p and R are evaluated from (3) and (4), using q_n .

The definition of the hydrostatic matrix \mathbf{H} depends on the numerical method used in the dynamics and is subject to constraints from energy and mass conservation (see, e.g., Williamson and Olson 1994). The definitions of \mathbf{H} for the three dynamical methods used in CAM2 are given in appendix A.

If s_n is modified by diabatic heating in a time-split process, then the new $s_{n+1} = s_n + Q_n \delta t$ can be converted into T_{n+1} and Φ_{n+1} using (25):

$$T_{n+1}^k = (s_{n+1}^k - \Phi_{n+1}^{k+}) (c_p^k + R^k \mathbf{H}^{kk})^{-1}, \quad (27)$$

with c_p^k and R^k evaluated from (3) and (4) using q_{n+1}^k (recall that $c_p^k \equiv c_p^d$ in CAM2). Once \mathbf{H} is defined, (26) and (27) can be solved for T_{n+1} and Φ_{n+1} . Calculating T and Φ from s involves the same amount of computation as calculating Φ and s from T .

The only significant constraint imposed on the numerical algorithm to obtain (27) is that \mathbf{H} be triangular, which implies that $\Phi(p)$ depends only on $T(p' \geq p)$. This constraint is a property of the continuous system represented by (22) and is obeyed by most numerical

algorithms in use today. In fact, for “well-behaved” algorithms (including those in CAM2), \mathbf{H} is an upper-triangular matrix with the special property that all above-diagonal entries in each column are identical. That is, the thickness of a layer l in (23) is independent of the level k to which the sum is taken. Therefore, the solution of (27) can proceed from the bottom up and is much cheaper than the matrix notation might lead one to believe.

6. Vertical diffusion

A general vertical diffusion parameterization for a hydrostatic model can be written in terms of the divergence of diffusive fluxes:

$$\frac{\partial}{\partial t}(u, v, q) = -\frac{1}{\rho} \frac{\partial}{\partial z}(F_u, F_v, F_q) \quad (28)$$

$$\frac{\partial}{\partial t}s = -\frac{1}{\rho} \frac{\partial}{\partial z} F_s + D, \quad (29)$$

where D is the heating rate due to the dissipation of resolved kinetic energy in the diffusion process. In the notation of section 2, (F_u, F_v) are the stress tensor components τ_{31}, τ_{32} corresponding to horizontal stresses on interfaces normal to the vertical direction. These are the only stresses relevant for hydrostatic column processes. We note that the vertical diffusion of heat is written here in terms of s , which is the variable diffused in CAM2. The vertical diffusion in CCM3 operated on θ , which is very similar to operating on s , although it is easier to write formally conservative operators for s .

Following Holtslag and Boville (1993), the diffusive fluxes are defined as

$$F_{u,v} = -\rho K_m \frac{\partial}{\partial z}(u, v), \quad (30)$$

$$F_{q,s} = -\rho K_{q,s} \frac{\partial}{\partial z}(q, s) + \rho K_{q,s}^t \gamma_{q,s}. \quad (31)$$

The viscosity K_m and diffusivities $K_{q,s}$ are the sums of turbulent components $K_{m,q,s}^t$, which dominate below the mesopause; and molecular components $K_{m,q,s}^m$, which dominate above ~ 120 km. The nonlocal transport terms $\gamma_{q,s}$ apply in the boundary layer and represent the effects of horizontally subgrid-scale eddies with vertical scale comparable to the boundary layer depth.

We form the equation for total energy, equivalent to (9), from (28)–(29):

$$\frac{\partial E}{\partial t} = u \frac{\partial u}{\partial t} + v \frac{\partial v}{\partial t} + \frac{\partial s}{\partial t} \quad (32)$$

$$= -\frac{1}{\rho} \left(u \frac{\partial F_u}{\partial z} + v \frac{\partial F_v}{\partial z} + \frac{\partial F_s}{\partial z} \right) + D \quad (33)$$

$$= -\frac{1}{\rho} \left(\frac{\partial F_{KE}}{\partial z} + \frac{\partial F_s}{\partial z} \right). \quad (34)$$

The diffusive kinetic energy flux in (34) is

takes $T_{n+1} = T_n + Q/c_p$ as the input state, leading to a small energy imbalance. This issue will be addressed in a future model revision.

The dissipation of kinetic energy into heat D must be calculated explicitly and included in the heating Q in the first law of thermodynamics (2), in order to conserve energy. In Newtonian fluids, D is a positive definite quantity given by the product of the stress tensor and the velocity gradient, as discussed by Fiedler (2000) and Becker (2001). The spectral Eulerian core in CAM2 includes a biharmonic horizontal diffusion operator that cannot be represented by a symmetric stress tensor and therefore the kinetic energy dissipation cannot be correctly defined, as noted by Becker (2001). Instead, \mathbf{F}_K is ignored and $D = -\partial K/\partial t = -\mathbf{V} \cdot (\partial \mathbf{V}/\partial t)_d$, where $(\partial \mathbf{V}/\partial t)_d$ is the specific force from the diffusion process. Note that $D < 0$ (cooling) if $\partial(|\mathbf{V}|/\partial t)_d > 0$.

Both the finite volume and semi-Lagrangian dynamical cores in CAM2 have some diffusion implicit in their numerical approximations, for which D is unknown. In principle, D could be determined as a residual in (6), by predicting K in addition to \mathbf{V} . In practice, (8) is used to define the global integral of D , which is then applied in (2) as a uniform heating. For all three dynamical cores, the globally averaged value of D is $\sim 2 \text{ W m}^{-2}$ (D. Williamson 2002, personal communication).

Conservation of water vapor within the transport component of the dynamical core is also required in order to conserve the latent energy included in Q . The finite-volume core conserves constituents explicitly while the semi-Lagrangian transport algorithm, used by both the spectral Eulerian and semi-Lagrangian cores, contains an ad hoc mass fixer to ensure global conservation, as discussed in Rasch and Williamson (1990). Sources in the constituent equations are always time split in CAM2.

8. CAM2 results

The formalism discussed above is implemented in CAM2. The impact of the changes compared to CCM3 have been evaluated with a series of 5-yr simulations using the standard CAM2 configuration of spectral Eulerian dynamics with a triangular truncation at total wavenumber 42 (T42) and 26 layers in the vertical, extending from the surface to a top interface at ~ 2 mb. (The tests actually used a pre-release version of CAM2 that had slightly different adjustable constants for some parameterizations.) All of the simulations used climatological sea surface temperatures. The comparison to ‘‘CCM3’’ has been made by returning specific processes to the CCM3 form while retaining the CAM2 form elsewhere. The CCM3 form requires updating T instead of s after parameterizations, computing D from $\partial K/\partial t$ instead of (37), mixing θ instead of s , and all of the above changes together. In addition to the time-splitting method and vertical diffusion parameterization discussed above, the primary differences between CAM2 and

CCM3 are the inclusion of: the prognostic cloud water variable of Rasch and Kristjánsson (1998), the generalized cloud overlap treatment in the radiative transfer of Collins (2001), the updated treatment of the infrared radiative effect of water vapor of Collins et al. (2002), and a simple parameterization for evaporation of convective rainfall.

Changing between updating T and updating s had no significant impact on the simulation. The change does alter the conceptual framework for incorporating parameterizations and energy conservation within the parameterization suite can be verified. In fact, a few minor conservation errors in CAM2 were identified during this work that will be fixed in the next version. Most notably, the radiative heating rates (flux divergences) are computed hourly, saved, and applied to multiple time steps. The fluxes should be saved instead to allow for pressure changes. On an annual average basis, the standard configuration of CAM2 conserves energy to $< 0.3 \text{ W m}^{-2}$, of which $\sim 0.2 \text{ W m}^{-2}$ is due to the neglect of the latent heat of fusion within the atmosphere. The remainder is due to lack of conservation within the spectral dynamical core, including the use of c_p from (3) instead of c_p^d .

For a model that does not extend at least into the mesosphere, such as the standard version of CAM2, almost all of the vertical diffusion takes place in the boundary layer. Figure 2 shows the annually averaged heating from vertical diffusion over the bottom 100 mb of the atmosphere. Note that 1 m W kg^{-1} corresponds to a temperature tendency of $0.0864 \text{ K day}^{-1}$. The diffusion heats throughout the boundary layer on average, except in polar regions. Turbulent mixing of s will cool the upper part of the boundary layer, since s increases upward in a stably stratified atmosphere. However, the input of sensible heat at the surface dominates the turbulent cooling over most of the globe. The mixing in polar regions is primarily mechanically driven and the surface sensible heat flux is small, resulting in cooling in the upper part of the boundary layer.

The difference in vertical diffusion heating between the CAM2 and CCM3 forms is very small, as shown in Fig. 2, where the contour interval for the difference is 0.1 of that for the heating itself. The difference due to diffusing s rather than θ is negligible. Most of the change comes from using the correct definition (36) for kinetic energy dissipation. The dissipative heating for the two definitions is shown in Fig. 3. Kinetic energy dissipation is distributed through the boundary layer in the CCM3 form, while it is mostly confined in the surface layer in the CAM2 form. The turbulent mixing of momentum in the boundary layer results in a relatively smooth decrease of kinetic energy in the vertical. However, most of the kinetic energy change results from the kinetic energy flux in (34), rather than dissipation. The CCM3 form ignores this distinction. The vertical integral of the heating is identical for the two definitions, given the same input profile, and is indistinguishable in climate simulations.

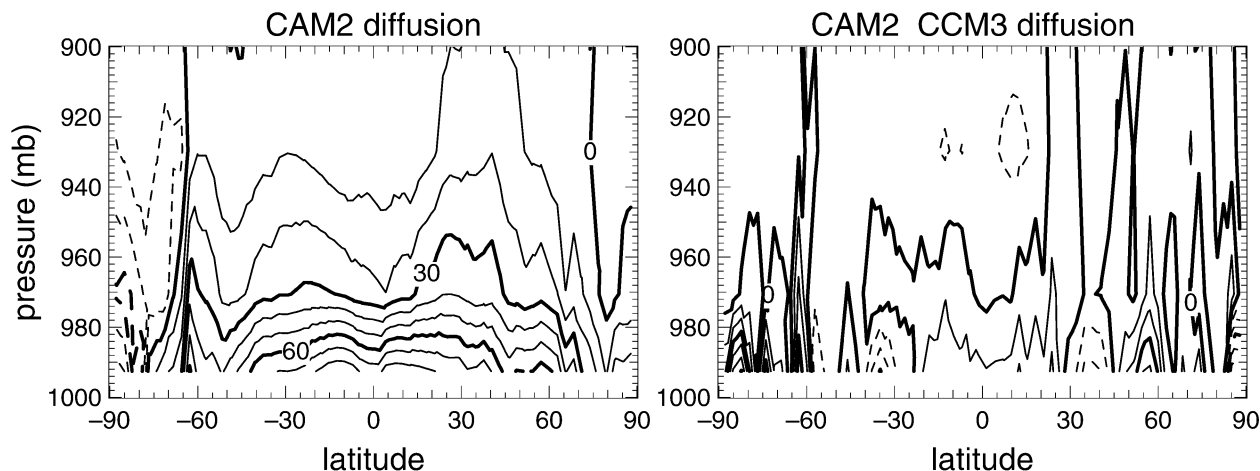


FIG. 2. Annual and zonal average of the heating from vertical diffusion (mW kg^{-1}) for (left, contour interval 10) the CAM2 diffusion of s and (right, contour interval 1) the difference between CAM2 and CCM3 diffusion. Negative contours are dashed.

The global and annual average of the explicit dissipative heating due to surface stress and vertical diffusion is 2.0 W m^{-2} in CAM2. The surface layer accounts for 1.65 W m^{-2} of the global mean, while the rest of the atmosphere accounts for only 0.35 W m^{-2} . Although the contributions of $d_{u,v}^{L+}$ and $d_{u,v}^{L-}$ were not separated in the CAM2 output, the dissipative heating in the surface layer results almost entirely from the surface stress terms. In fact, the heating is mostly due to surface stress in the oceanic storm tracks with somewhat more dissipation in the Southern Hemisphere than in the Northern Hemisphere (Fig. 3). There is a distinct seasonal cycle, maximizing in winter in each hemisphere, when the dissipation in the storm track exceeds 8 W m^{-2} (not shown).

9. Conclusions

The parameterized heating in CAM2 has been reformulated around the unifying concept of conservation of

energy. Vertical diffusion and gravity wave drag, which alter kinetic energy, conserve total energy by including heating due to kinetic energy dissipation. This heating was included in an ad hoc form in previous versions (CCM1–3), rather than depending correctly on the stress tensor and velocity gradient.

Within the parameterization suite, heating is always applied to the dry static energy s in CAM2, rather than being applied to T , as in CCM3 and earlier versions. Therefore, CAM2 conserves energy at each stage within a time step, except for any nonconservation within the dynamical core. Earlier versions approximately conserved energy in aggregate, but not within the parameterization suite.

For time-split parameterizations, applying heating to s requires inverting the hydrostatic equation to determine T and Φ from the modified s . The inversion method developed for CAM2 is actually as efficient as evaluating the hydrostatic equation to determine Φ and s from T . We note that time splitting is not necessarily the best

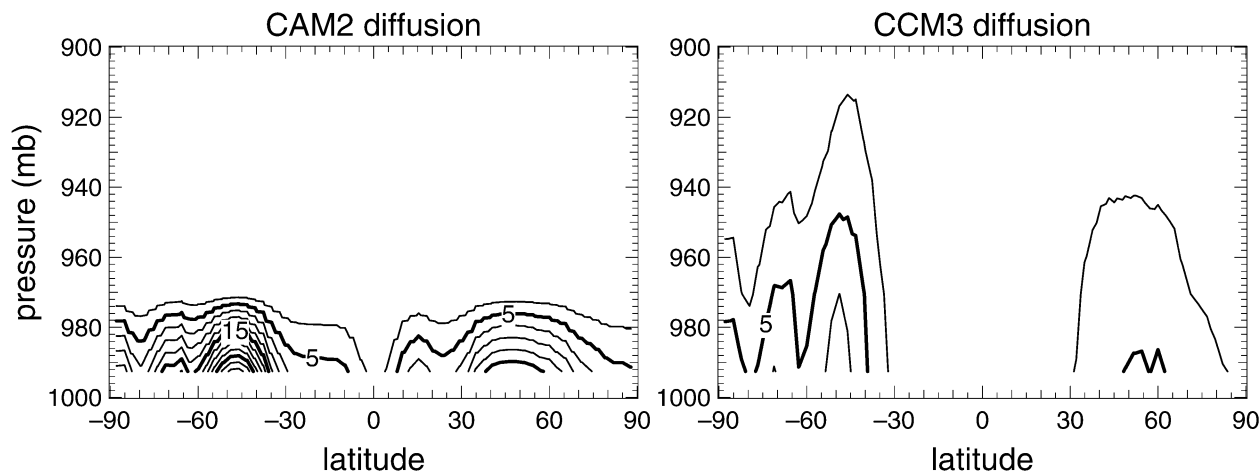


FIG. 3. Annual and zonal average of the heating from kinetic energy dissipation (mW kg^{-1}) for (left) the CAM2 diffusion of s and (right) the CCM3 diffusion. The contour interval is 1 mW kg^{-1} and negative contours are dashed.

The terms E^k and F^k can be determined upward from $k = L$, using the boundary conditions:

$$E^{L+1} = F^{L+1} = A^L = 0. \quad (\text{B10})$$

Finally, (B7) can be solved downward for ψ_{n+1}^k , using the boundary condition:

$$C^1 = 0 \Rightarrow E^1 = 0. \quad (\text{B11})$$

CCM1–3 used the same solution method, but with the order of the solution reversed, which merely requires writing (B6) for ψ_{n+1}^{k-1} instead of ψ_{n+1}^{k+1} . The order used here is particularly convenient because the turbulent diffusivities for heat and all constituents are the same but their molecular diffusivities are not. Since the terms in (B8)–(B9) are determined from the bottom upward, it is only necessary to recalculate A^k , C^k , E^k , and $1/(B^k - A^k E^{k+1})$ for each constituent within the region where molecular diffusion is important.

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