Standardization of Gustiness Values from Aircraft

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ABSTRACT

A universal turbulence standardization technique is described which is based quantitatively on the atmospheric turbulence itself rather than on the effects it produces on an aircraft. It provides a single turbulence intensity number which may be measured continuously in flight in a variety of ways, and, with knowledge of the aircraft type and speed, can be linearly related to the rms value of vertical accelerations of the aircraft. The universal intensity number concept as applied here is an extension for aircraft use of the inertial subrange concept of atmospheric turbulence. In the inertial subrange all the statistical properties of turbulence of wavelengths less than a few hundred meters (the wavelength range which dominates gust loading for most aircraft) can be simply related to a single number. The intensity measurement also has value as a basic meteorological parameter, and has proven to be particularly useful in diffusion studies.

The intensity scale is derived from various turbulence power spectrum studies. A larger background of data on turbulent distributions is also available because existing statistical discrete gust and continuous gust data can be fitted approximately to this standardization procedure, and meteorological studies have also given estimates of the distribution of this intensity number. The intensity number can be found from virtually any device moving through the air, as well as by some indirect meteorological means. A particular version of a universal turbulence indicator is described.

The universal turbulence intensity rating provides a standard for communication about turbulence and for the collection of statistical turbulence information. It also provides a quantitative method of relating aircraft response and pilot "feel" to the turbulence, so that the pilot can select or preselect the appropriate speed for specific missions.

1. Introduction

Turbulence refers to motions at various intensities and scales in three dimensions and so a complete statistical description of it would be expected to be complex. However, over a certain range of eddy sizes, the inertial subrange defined in the similarity hypothesis of Kolmogoroff (1941), all the statistical properties of atmospheric turbulence can be related to one parameter, ϵ, a dissipation rate of turbulent energy (the rate at which the turbulent energy is converted into heat for steady turbulence). It fortunately turns out that this inertial subrange includes those gusts which are of primary importance for many aircraft gust loads, for aircraft fatigue problems, and for the qualitative human "feel" of turbulence in flight. Thus, the simplifications inherent in the inertial subrange concept can be applied to aircraft problems to provide quantitative statistical relationships between turbulence and aircraft response. The simplifications also make feasible techniques for the measurement of the basic quantity ϵ.

Any turbulence measurements, no matter how accurate, can only be related to other conditions, forecasts, or designs in a somewhat qualitative sense because of the complex distributions and random nature of turbulence. Within this fundamental restriction, the relations are made most quantitative by the broad use of a simple turbulence concept. The simplicity of the suggested standardization principle is perhaps its greatest virtue; the technique is simple enough to permit routine handling of varying complex data.

This paper examines the validity of the inertial subrange concept as it relates to aircraft problems and shows that σ_g, the rms value of aircraft vertical loads, is linearly related to $\epsilon^{1/3}$ for a given aircraft type, speed, and loading; suggests a standardization scale for turbulence intensity treating $\epsilon^{1/3}$ as the basic intensity parameter; indicates how ϵ can be measured independent of airplane type or speed; and discusses some operational features and limitations of this standardization approach.

2. The inertial subrange concept and its application

Kolmogoroff's similarity hypothesis presents a picture of turbulence for which it is assumed that the turbulence enters the steady state system at large eddy sizes and the eddy energy "flows" from larger to smaller eddies. The eddies which are far smaller than the size at which the primary turbulent energy is put into the atmosphere have "forgotten their ancestry" and so can have no preferred direction; statistically they are isotropic.

Excluding the tiny eddies one centimeter or smaller which are affected by viscosity, the turbulence characteristic in the isotropic range of eddy sizes, the inertial subrange, can only depend on one parameter, ϵ, the equilibrium rate at which energy enters the system, at which energy flows to smaller eddies, and at which energy leaves the system through viscous heating. The simplicity of this picture permits simple formulas for correlation coefficients, power spectra, diffusion, etc., to be derived readily by dimensional analysis; all statistical quantities which depend on the inertial subrange eddies can be uniquely related to ϵ or to each other.

The formulas seem to be good first approximations to the experimental data over broad ranges of sizes and conditions—even to some wavelengths where the inertial subrange concept would not be expected to apply, such as where isotropy is not possible, or in non-steady turbulence. The inertial subrange concept as applied to meteorology is reviewed by Batchelor (1950), and, with examples, by MacCready (1962a, b). Excellent verification of the inertial subrange concept is provided by Grant, Stewart and Moilliet (1962) for ocean turbulence and by Pond, Stewart and Burling (1963) for the wind over waves. In the atmosphere, above a certain height which is of the order of several hundred meters, the spectrum law based on inertial subrange concepts appears to be a good approximation to the observed data at least to wavelengths up to several hundred meters. Rhyne and Steiner[1] present spectra from flights in severe thunderstorms which show that in these extreme turbulence cases the data are consistent with the inertial subrange spectrum law and isotropy for 1000-meter wavelengths. Shur (1962) shows spectra from jet stream turbulence which agree with inertial subrange predictions for wavelengths to 600 meters.

Fig. 1 gives a resume of the pertinent ranges and spectra for turbulence at heights well above the ground. k refers to wavenumber, the reciprocal of wavelength. $E(k)$ is the one-dimensional power spectral density for longitudinal velocity fluctuations, twice the energy per unit mass of air per unit wavenumber. $G(k)$ is the corresponding energy for lateral velocity fluctuations. It can be shown that $G(k)=4/3E(k)$ in the inertial subrange (see MacCready, 1962a); although the turbulence is isotropic, the velocity fluctuations measured from a moving vehicle are not true energies and thus differ slightly for different components. ϵ is the energy dissipation rate per unit mass and C_2 a constant which will be discussed later. It will be noted that the gust loads for slow aircraft come from eddies entirely within the inertial subrange, where the spectra are completely defined by ϵ. For fast, high-altitude aircraft some eddies

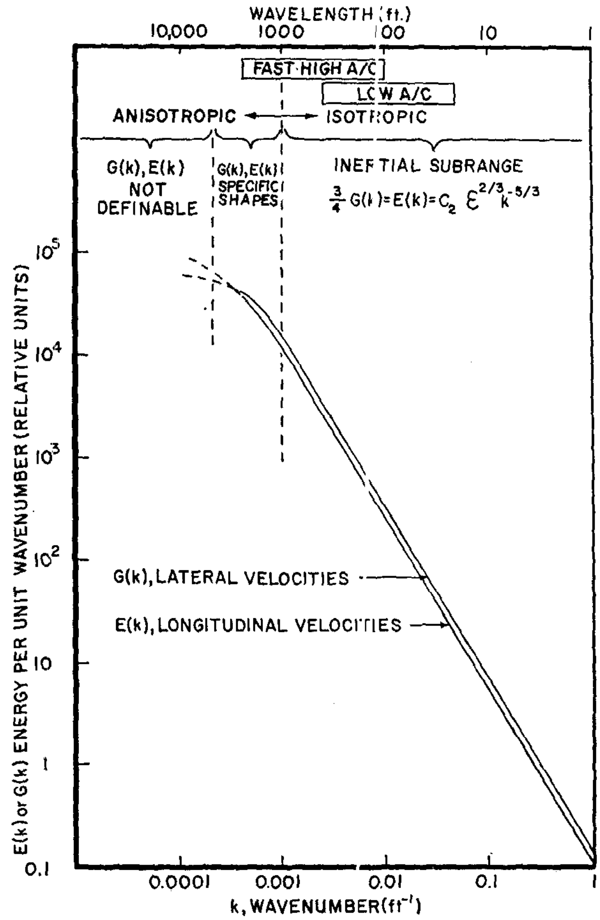

FIG. 1. Atmospheric turbulence power spectra and the wavelength ranges of importance for aircraft.

beyond the inertial subrange contribute appreciably to the gust loads; even at these larger wavelengths the inertial subrange spectra can be fair first approximations to the atmospheric spectra, although it is best to employ an empirical correction. The correction need not be particularly accurate because the contribution of these larger eddies to σ_g is usually small. Where the curves are dotted, the shapes can vary appreciably with the meteorological situation. In the "not definable" region, the curves cannot be realistically derived for large wavelengths which are a sizeable fraction of the distance over which the turbulence is relatively constant. In summary, ϵ will define the spectra curves in the inertial subrange, and with an empirically derived correction (which need not be particularly accurate for this gust load work), define them as an approximation even for the largest wavelengths of significance for the gust loads of typical fast aircraft. Over these size ranges the spectrum shapes are relatively unaffected by the meteorological situation, at least at heights well above the ground. $\epsilon^{2/3}$ can be considered as the turbulence level.

[1] Rhyne, R. H., and R. Steiner, 1962: Turbulence and precipitation problems associated with operation of supersonic transports. NASA Langley Res. Center preprint of paper presented at Fourth Conference on Applied Meteorology, Hampton, Virginia, Sept.

In the inertial subrange the longitudinal one-dimensional turbulent power spectrum is $E(k)=C_2\epsilon^{2/3}k^{-5/3}$. C_2 is a dimensionless constant which has a value of order unity. Panofsky and Pasquill (1963) show the values for C_2 ascertained by different investigators tend to cluster near 0.15 when k is in cycles per unit length as is used here. This exact value of C_2 and even the exactness of the $-5/3$ law cannot be considered completely established, but they at present appear to yield such consistent results that they can safely be employed for the purposes described here. For the suggested turbulence standardization with respect to gust loads the value of C_2 is actually immaterial. The turbulent intensity is proportional to $(C_2\epsilon^{2/3})$, and the intensity scales were derived for this factor $(C_2\epsilon^{2/3})$. However, as a convenience, the scales are simply given in terms of ϵ, based on the $(C_2\epsilon^{2/3})$ factor with C_2 set at 0.15.

When this spectrum represents the longitudinal turbulence (airspeed fluctuations) measured from an aircraft moving at mean airspeed U, the spectrum is usually given in terms of f, frequency, rather than k. Thus

$$E(f)=C_2U^{2/3}\epsilon^{2/3}f^{-5/3}.$$

Fig. 2, taken from MacCready (1962b), shows this sort of spectrum as ascertained from airspeed fluctuations from a sailplane. Fig. 2 indicates the good agreement between theory and observations that has been demonstrated on various projects.

As noted before, the vertical transverse component, which is the turbulence primarily responsible for gust loads on an aircraft, is statistically identical to the longitudinal turbulence, except for the 4/3 factor. Hence

$$\frac{G(k)}{E(k)}=\frac{G(f)}{E(f)}=\frac{4}{3}.$$

Fig. 3, from MacCready (1962b), shows the comparison for many flights of the spectrum $G_a(f)$ of vertical aircraft velocity (found from accelerometer records) with the spectrum of longitudinal velocity fluctuations $E(f)$. If the aircraft responded perfectly to vertical turbulence, $G_a(f)$ and $G(f)$ would be identical and so $3G_a(f)/4E(f)$ would be unity in the isotropic turbulence of the inertial subrange. The difference from unity shows the response characteristics of the aircraft, $G_a(f)/G(f)$, assuming isotropic turbulence. The point

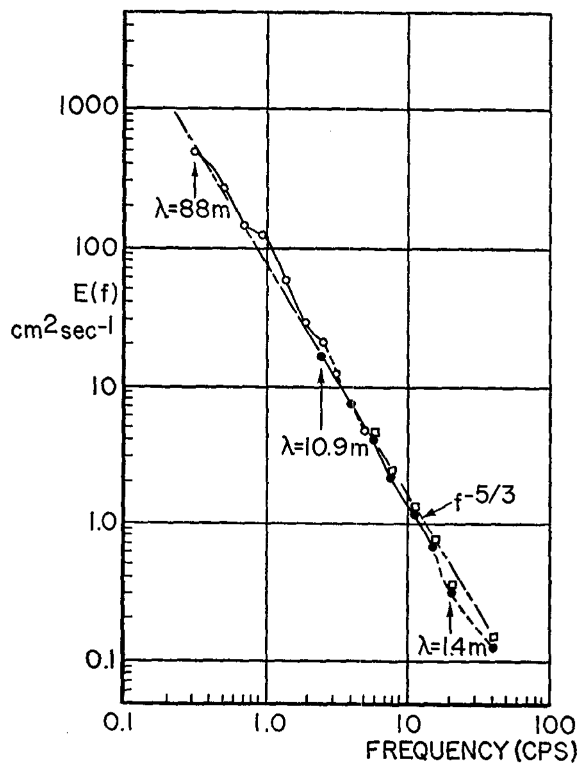

Fig. 2. Measured turbulence power spectrum compared to the $-5/3$ spectrum. Velocity fluctuations measured from a sailplane flying 27.5 m sec^{-1} in slope lift at 350 meters above the slope.

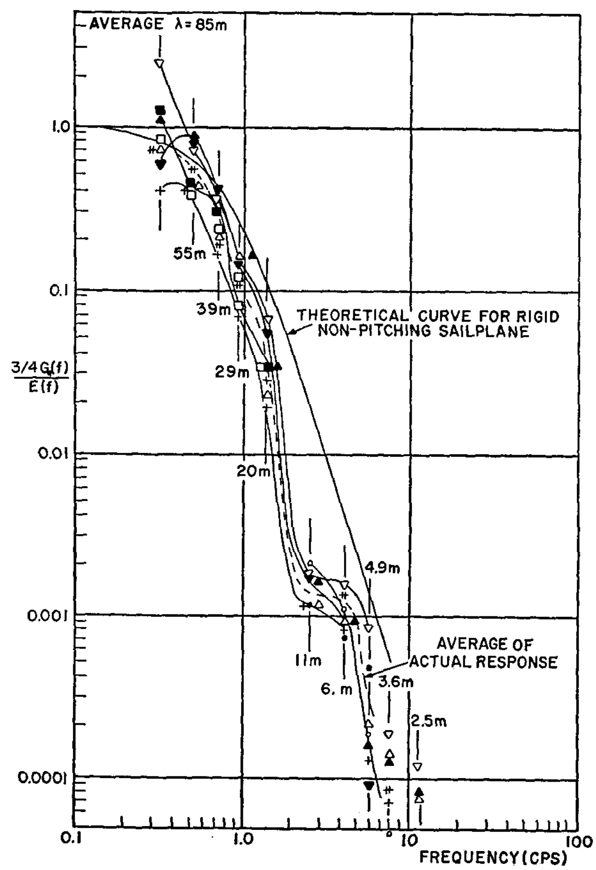

Fig. 3. Sailplane response characteristics. The plots of $3G_a(f)/4E(f)$ are shown for flights at a constant mean speed in a wide variety of turbulence conditions (severe wave turbulence, smooth wave, thermals, slope currents).

of Fig. 3 is that the longitudinal velocity fluctuations are uniquely and simply related to the vertical accelerations. The experimental scatter of points about the aircraft response line comes from statistical variations of the turbulence, instrument noise, and can also be partly due to nonlinear aircraft response.

3. Measurements

With C_2 known, ϵ can be computed from measurements of the energy at particular wavelengths or frequencies in the inertial subrange. From an airplane, ascending balloon, or rocket, the variable measured can relate to longitudinal or transverse turbulent velocity components. Thus ϵ can be derived by an appropriate energy meter from airspeed fluctuations from the spectrum of Fig. 2, from a fast response direction vane showing lateral velocities, or, with proper knowledge of aircraft response, from vertical accelerations of the aircraft, as implied by Fig. 3. As long as one is dealing with a linear system, the value of turbulent energy in the inertial subrange as measured by any means is proportional to that measured by any other means.

It is particularly easy to obtain ϵ from airspeed fluctuations. For at least a broad range of inertial subrange eddies the aircraft provides a stable measurement platform, with its true velocity unaffected by turbulence or control motions. Alternatively an air direction vane showing vertical or transverse airflow fluctuations can be employed, but the aircraft is not so stable a platform for this motion and only smaller eddies can be measured directly without complex instrumentation. The vertical velocity (or acceleration) of the whole aircraft can also be used, with suitable correction factors as previously discussed to relate the vertical air velocity to vertical aircraft velocity. All these techniques provide a signal for which the energy is measured by standard electronic means by passing through a band-pass filter and the rms filter output ascertained. The filter operates at a fixed frequency range, and so at higher airspeed is looking at longer wavelengths in which more energy resides. Thus the output must be corrected for mean airspeed, so ϵ can be found. The needed automatic correction is easy to derive because the turbulence spectrum shape is known. The Appendix gives further details on instrumentation principles.

One can work with the autocorrelation coefficient instead of the power spectrum to find ϵ. However, instrumentation for doing this needs to store data and so is more complex than that for dealing with the frequency spectrum curve.

Turbulence measurements were made by the Atmospheric Research Group (affiliated with Meteorology Research, Inc.) under NSF grant G11969 in connection with cumulus cloud studies at Flagstaff, Arizona, in July–August 1961. The instrument used senses vertical accelerations in a range within the inertial subrange and where the response characteristics of the aircraft can be simply estimated from standard computations based on the assumption that the aircraft is a rigid, non pitching vehicle. The instrument output voltage is proportional to $\epsilon^{1/3}$, at each particular airspeed. The instrument response time is on the order of one to two seconds. Although the calibration was somewhat crude, the data agree reasonably well with measurements made in convective situations with more refined apparatus.

The results of these measurements are shown diagrammatically on Fig. 4, together with values from a turbulence study by sailplane, MacCready (1962b), and severe storm investigations reported by Douglas Aircraft Co.[2] and Rhyne and Steiner.[1] The turbulence situations are plotted against ϵ and $\epsilon^{1/3}$, with the assumption that $C_2=0.15$. Vertical velocity variance values are available for some of these data, but are not presented here because of their strong dependence on large eddies beyond the inertial subrange. In the extreme case reported by Rhyne and Steiner the maximum equivalent gust velocity at 12,500 meters was about 14 m sec^{-1}.

4. Suggested turbulence magnitude scale

The four data columns of Fig. 4 show that the qualitative descriptions of the turbulence situation stratify reasonably well. On this basis, the Atmospheric Turbulence Intensity scales of Fig. 5 have been created to represent a reasonable summary. The suggested magnitude scale is:

Magnitude	Title	$C_2\epsilon^{2/3}$ (cm$^{4/3}$ sec^{-2})	ϵ (cm^2 sec^{-3})	$\epsilon^{1/3}$ (cm$^{2/3}$ sec^{-1})
0	Negligible	<0.055	<0.22	<0.605
1	Light	0.055–0.315	0.22– 3.0	0.605–1.45
2	Moderate	0.315–1.8	3.0 – 41.5	1.45 –3.46
3	Heavy	1.8 –10	41.5 –543	3.46 –8.2
4	Extreme	>10	>543	>8.2

As more evidence accumulates, a revision of the category limits suggested above may prove desirable.

5. Quantitative aspects

An important turbulence factor is the rms value of the vertical acceleration loads (gust loads) which are imposed on the aircraft. With σ_g being the rms value of the vertical accelerations, and $A(f)$ the amplitude response factor relating aircraft vertical accelerations to air vertical velocity,

$$\sigma_g^2=\int_0^\infty [A^2(f)E(f)]df.$$

For a given airspeed and a given airplane response characteristic $A(f)$ at that airspeed, since $E(f)=C_2U^{2/3}\epsilon^{2/3}f^{-5/3}$ wherever $A(f)\neq 0$, this becomes

[2] Douglas Aircraft Co., 1961: High altitude gust study progress report for period 1 August 1961 to 1 September 1961. Rpt. No. DEV-3460, Contract AF33(616)-7647, Wright-Patterson AFB, Ohio, 28 pp.

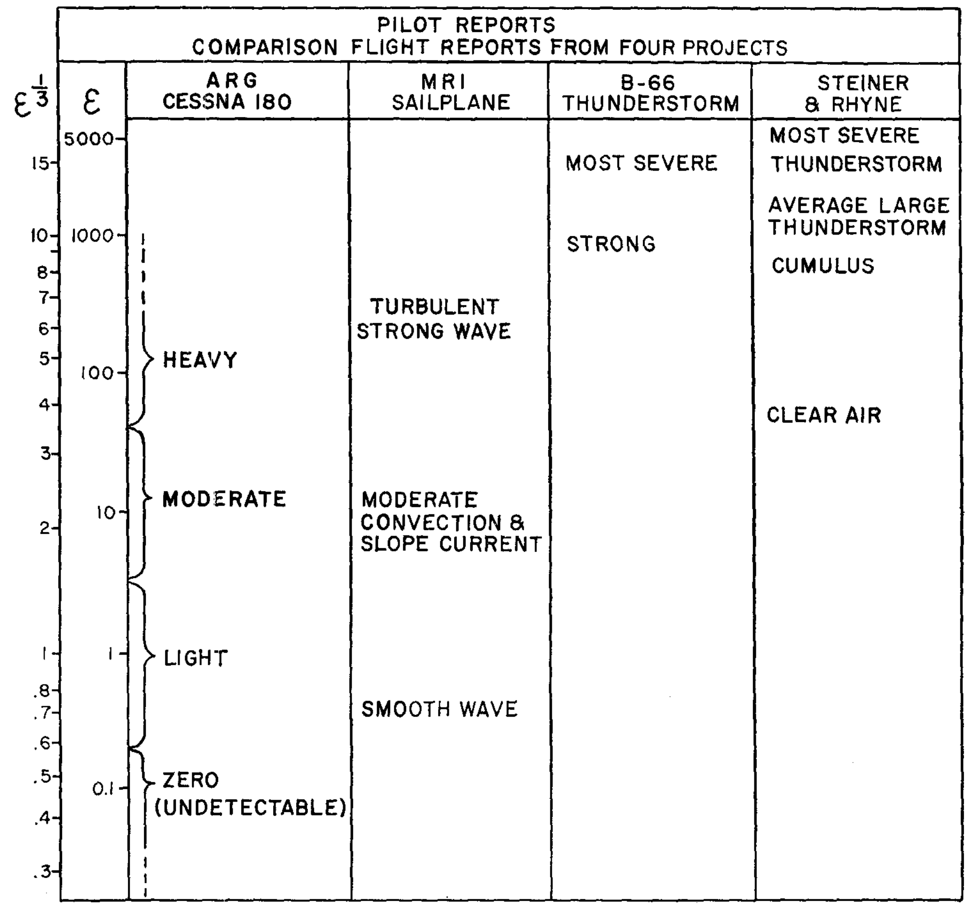

Fig. 4. Qualitative pilot reports compared to ϵ values in cm² sec⁻³ computed from measured spectra with C_2 set at 0.15.

$\sigma_g = K_1\epsilon^{1/3}$. Thus the rms value of vertical accelerations is proportional to $\epsilon^{1/3}$, at any given airspeed. K_1 is a function of the aircraft response characteristics, and thus varies somewhat with altitude, aircraft weight, and speed as well as aircraft type. K_1 increases for higher airspeeds.

The above analysis requires that $A(f)$ be small outside of the inertial subrange. This is generally valid except for very fast aircraft. An examination of the limits of the validity of this requirement is warranted here.

A plot of $KfA^2(f)E(f)$ vs. $\log f$ is instructive because it demonstrates the contribution to σ_g^2 which comes from each frequency range. The contribution is proportional to the area under the curve, and the total area under the curve represents σ_g^2.

Fig. 6, adapted from Fig. 7 of Zbrozek (1961), gives such a plot for a fighter type of airplane flying at 500 fps. Zbrozek assumed a turbulent spectrum having a shape suggested by Press (1957),

$$G(k) = 2L\sigma_w^2\left[\frac{1+12\pi^2k^2L^2}{(1+4\pi^2k^2L^2)^2}\right],$$

set $L = 1000$ ft and put $\sigma_w = 1$ fps. Also plotted on the same figure is the response resulting from the assumption of a $-5/3$ law spectrum throughout the wavelengths of interest, and having the same power at a 400-ft wavelength. With either spectrum, only a small contribution to σ_g comes from eddies larger than 1000 ft (an error of 2 in spectrum height at say 3000-ft wavelengths would be immaterial in the calculation). Thus σ_g derives primarily from eddies in the inertial subrange, and σ_g is proportional to $\epsilon^{1/3}$. A curve similar to Fig. 6 can be derived for the sailplane characteristics of Fig. 3; for this slower and lighter aircraft the dominant wavelengths for contributing to σ_g are those considerably less than 100 ft. Zbrozek summarizes such factors

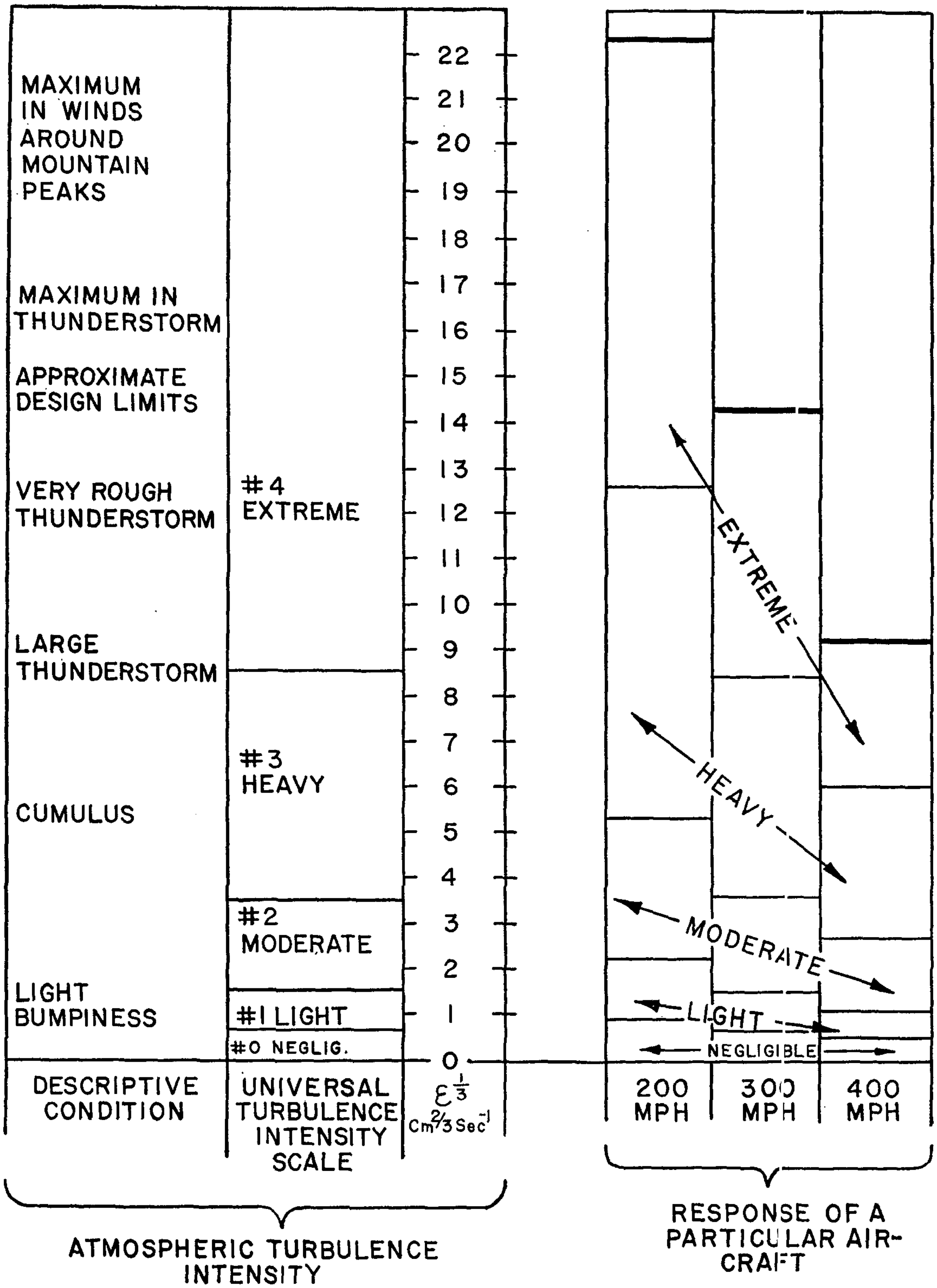

FIG. 5. Turbulence intensity scale and response of a particular aircraft.

in a plot of "wavelengths of interest to aircraft designer" vs. "altitude" [Zbrozek (1961) Fig. 10] and suggests that for the structural modes of operational aircraft the pertinent wavelengths are from perhaps 20 to 500 ft at sea level and 100 to 2500 ft at 50,000 ft. These whole ranges are pertinent to loads and fatigue parameters; stability and control factors are in the upper portions of the ranges. In summary, the 5/3 spectrum law of the inertial subrange extends over the main range of gust load producing eddies for most aircraft, and so the $\sigma_g = K_1\epsilon^{1/3}$ relationship is operationally useful. If at large wavelengths the spectrum shape does not follow the 5/3 law but does to a first approximation have a fixed shape with intensity proportional to $C_2\epsilon^{2/3}$, as suggested in Fig. 1, then $\sigma_g \sim \epsilon^{1/3}$ will still be valid for vehicles which are affected by large eddies, although the constant of proportionality must be recomputed. K_1 can be calculated from vehicle aerodynamic considerations. It would be found most easily empirically, by measuring σ_g and $\epsilon^{1/3}$ simultaneously.

Various spectra shapes have been proposed for aircraft gust load studies to represent the vertical component of atmospheric turbulence. The suggested spectra forms for altitudes well above the ground usually consider the spectra shapes to be invariant with intensity and thus the intensity at wavelengths causing gust loads to be a function of σ_w^2. See for example Press (1957) and Houbolt, Steiner and Pratt.[3] Near the ground the spectra shapes are given a dependence on height and sometimes also on roughness and stability. See for example Henry (1959), Etkin (1961), Lappe and Clodman[4] and Burns (1963). The turbulence scale decreases as height decreases and the eddy size range of validity of the inertial subrange concept also decreases. Thus the accuracy of the simple σ_g vs. $\epsilon^{1/3}$ relationship decreases as the ground is approached. The limit of validity of the inertial subrange concept near the ground is often taken to be wavelengths comparable to the height of measurement. Burns (1963) presents spectra consistent with this view below 300 meters. Some of the spectra representations approach k^{-2} at small wavenumbers, while others approach the inertial subrange prediction $k^{-5/3}$. The k^{-2} form is easier to handle in some analytical calculations. As noted above, the difference between the k^{-2} and $k^{-5/3}$ may not be important in calculating σ_g from the spectra, but the height of the spectrum in the range contributing

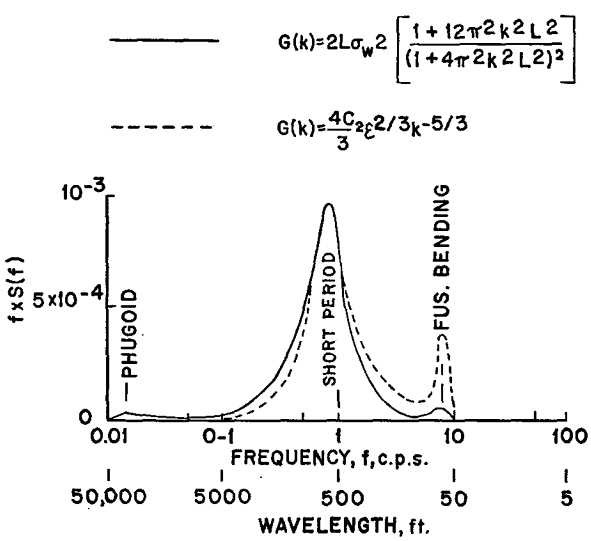

Fig. 6. Frequency times spectrum of aircraft response (cockpit acceleration).

most to the gust loads relates more closely to the inertial subrange and $\epsilon^{1/3}$ than to the large eddies and σ_w.

Statistical distributions of σ_w derived from vertical accelerometer data and an assumed spectra shape can be transformed into $\epsilon^{1/3}$ distributions. Even the statistical accelerometer count data of NACA can, with knowledge of the aircraft making the measurements, be reduced to distributions of σ_w (see Zbrozek, 1960) and hence distributions of ϵ.

When σ_w is actually derived from vertical velocity variance data, it turns out that the measured turbulence intensity depends most strongly on the large eddies, which are not the primary ones causing the gust loads. The same lack of correlation applies, even more strongly, when using horizontal velocity variance data; however, noting maximum airspeed excursions with existing airspeed equipment to provide a turbulence magnitude scale is so simple that it lends itself to certain operational uses where crude measurements will suffice. Measurement of the vertical turbulence of large eddies is difficult because of equipment requirements, and, more fundamentally for both vertical and longitudinal turbulence, because long averaging times (long distances) are required, at least an order of magnitude longer than the wavelengths of interest, and these long averaging times are inappropriate for inhomogeneous turbulence.

6. Human Factors

Roys[5] points out that there are many meteorological, personal, and operational factors which contribute to

[3] Houbolt, J. C., R. Steiner and K. G. Pratt, 1962: Flight data and considerations of the dynamic response of airplanes to atmospheric turbulence. NASA Langley Res. Center, paper presented at Structures and Materials Panel to Flight Mechanics Panel, Advisory Group for Aeronautical Research, and Development, Paris, July.

[4] Lappe, V. O., and J. Clodman, 1962: On developing a low altitude turbulence model for aircraft gust loads. NYU Final Report Contract No. AF 33(616)-7302 to Structures Branch, Flight Dynamics Lab, Wright-Patterson AFB, ASD-TDR-62-152.

[5] Roys, G. P., 1962: Penetrations of thunderstorms by an aircraft flying at supersonic speeds. Presented at AMS Conf. on Severe Storms, Norman, Okla.

a pilot's description of turbulence. For example, the rating a pilot gives to the observed turbulence intensity depends both on the number of gusts and their intensity, and also on qualitative factors not directly related to the gusts. The proposed scale can replace the qualitative pilot evaluation, but can still use the present conventional terms. With experience, the pilot can bring his qualitative evaluation toward agreement with an adopted scale. ϵ can be measured by instruments, but can also be estimated by a pilot flying a familiar type of aircraft without special instruments.

It seems logical that a quantitative scale of the "feel" of turbulence effects on the pilot or passenger of an aircraft can be related to the aircraft response over a certain frequency band. The "feel" quantity σ_h will be somewhat different from σ_g, the rms value of vertical accelerations, because the sensitivity of a human to vertical accelerations tends to peak at certain frequencies. Guignard and Irving (1960) show the transmissibility of the human body and demonstrate a resonance peak at 4.5 cps. Grande (1962), citing results by Parks and Snyder[6] shows curves of acceleration at discrete frequencies vs. frequency, labeled "alarming," "extremely annoying," and "mildly annoying." The curves tend to peak at about 3 cps and drop in the range below 2 cps, the bandwidth associated with motion sickness.

Considering typical aircraft response and the existing atmospheric turbulence spectra, it can be assumed from the human sensitivity to vertical accelerations that the qualitative human response feeling σ_h will depend primarily on turbulence in the 2 to 8 cps range, with some peaks around 3 to 5 cps. Thus σ_h vs. $\epsilon^{1/3}$ should have a unique relationship at any given mean airspeed, because σ_h derives from frequencies corresponding to wavelengths well within the inertial subrange.

Zbrozek (1961) provides the following qualitative table relating the pilot's description of turbulence severity to the rms value of the aircraft vertical acceleration (σ_g). The data were obtained with high speed aircraft, presumably of comparable types operating at comparable speeds.

Pilot's description	σ_g (rms of normal acceleration in "g")
negligible	<0.05
slight	0.05–0.10
moderate	0.10–0.15
moderately heavy	0.20–0.30
very heavy	>0.30

If all effects are within the inertial subrange, then at a particular airspeed $\sigma_g = K_1\epsilon^{1/3}$, $\sigma_h = K_2\epsilon^{1/3}$, and $\sigma_h = K_2/K_2\sigma_g = K_3\sigma_g$ where the constants K_1, K_2 and K_3 show proportionality.

The pilot's description pertains to σ_h, and so is proportional to σ_g. If one chooses appropriate specific values for the constants ($K_1 = 15$ to 20 for ϵ in cm^2 sec^{-3} and σ_g in "g" units), it will be observed that the scale values given by Zbrozek are reasonably proportional to the $\epsilon^{1/3}$ values proposed for the Universal Turbulence Scale. Note that "extreme" in the suggested magnitude scale is considered potentially catastrophic, and so is for stronger turbulence than the "very heavy" in the table above.

[6] Parks, D. L., and F. W. Snyder, 1961: Human reaction to low frequency vibration. The Boeing Co., Wichita Div., Document No. D3-3512-1, July.

7. Application

Fig. 5 shows the response of a particular aircraft in terms of σ_g at three airspeeds for various turbulence levels in terms of ϵ. A comparable figure can show the response in terms of σ_h, the human "feel," rather than σ_g which is the aircraft response. If the pilot has knowledge of ϵ from a forecast, from a pilot report, from a Universal Turbulence Indicator in his own aircraft (see the Appendix), or even from his own qualitative feeling interpreted with respect to airspeed, then he can determine the airspeed to fly to give a certain response. He thus can choose a speed at which the design load limits will not be exceeded, a speed at which the fatigue load limit will not unduly shorten the aircraft life, a speed for safe aerial refueling, a speed for serving meals to passengers, etc. The safe ϵ vs. speed relationships must first be established for these various criteria for his airplane at typical weights and altitudes. It might be particularly convenient to put the information of Fig. 5 on the indicator dial of an ϵ indicator, a different dial being required for each aircraft type.

It should be noted on Fig. 5 that, at a particular speed, the description of aircraft response exactly matches the description of the turbulence level. The same would be true, at a slightly different airspeed depending on definition, for the description of pilot gust load "feel" σ_h matching the description of the turbulence level. Thus each aircraft has a 'standard' airspeed where the σ_h scale is identical to the turbulence scale. If a variety of aircraft flew at their respective 'standard' airspeeds through an air volume having a given turbulence level ϵ, all the pilot descriptions of the turbulence would match (all σ_h would be the same).

8. Meteorological considerations

The process of turbulence generation and decay in the atmosphere tends to make the small inertial subrange eddies rather homogeneous in space. One can, of course, think of exceptions, such as strongly shearing zones and initial turbulence breakdown, but the widespread success of the inertial subrange concept shows that for small eddies the turbulence is often usably homogeneous. Spectra mentioned previously (MacCready, 1962b, Shur, 1962, and Rhyne and Steiner, 1962) do support the thesis that even in severe conditions of thunderstorm turbulence, jet stream turbulence, and the roll-area turbulence associated with mountain waves, the inertial subrange concept is usable for the aircraft gust load relationships. Conditions such as in flight across dust devils or in strong small mountain eddies will not fit the inertial subrange turbulence concept, although there would still be some correlation between gust loads and the indicated ϵ.

Some gust loads on aircraft arise from mean flows such as shears and waves, and are not related to turbulence in the ordinary sense. This becomes more important as aircraft get larger and faster. Reiter and Hayman (1962) feel that gravity waves on stable interfaces seem to be responsible for many cases of clear air turbulence in the upper troposphere, and for most cases in the stratosphere. A Universal Turbulence Indicator based on fast variations of airspeed or flow direction would correctly not show a high turbulence level in such cases; an accelerometer unit or pilot "feel" would indicate strong turbulence, though there is no way to interpret the information quantitatively for a different kind of aircraft. At least the dissimilarity between the gust load "feel" and the measured $\epsilon^{1/3}$ would show that shears or waves rather than turbulence were causing the loads. Severe small waves tend to be unstable and thus evolve into turbulence, and so cases of mixed turbulence and waviness are likely.

9. Conclusions

1) It is operationally desirable to have a measure of turbulence intensity which relates to the atmospheric turbulence itself instead of to its effect on aircraft.

2) The inertial subrange concept relates all the statistical quantities of eddies in its size range to a single quantity, ϵ, an "energy dissipation" factor. The power spectrum of turbulence in any direction is proportional to $\epsilon^{2/3}\lambda^{5/3}$, where λ is the wavelength. Thus ϵ, or some function of it, provides a logical intensity factor for a turbulence standardization system.

3) The inertial subrange concept typically covers turbulence wavelengths from a few centimeters to 200 or 300 meters, and its consequences are even often reasonably valid to considerably larger wavelengths. These wavelengths related to the inertial subrange are those primarily responsible for aircraft gust loads and for some aircraft stability factors.

4) ϵ can be measured in flight with special instrumentation. The measurement can be independent of the type of aircraft or its speed. Other methods are available for ascertaining ϵ, including devices on balloons and rockets. Meteorological investigations are providing data on the distribution of ϵ in the atmosphere. Data pertaining to ϵ can also be derived from classical discrete gust studies and continuous turbulence studies, with certain assumptions about spectra and response characteristics.

5) A specific atmospheric turbulence magnitude scale is suggested, giving ranges of ϵ values to five ranges labeled "negligible, light, moderate, heavy, extreme." The scale is based on various turbulence measurements from aircraft.

6) The rms value of aircraft vertical accelerations, σ_g, is proportional to $\epsilon^{1/3}$. The constant of proportionality is different for different aircraft, aircraft weights, airspeeds, and altitudes (the variation with airspeed is greatest), but it can be computed or measured. Thus a chart can be prepared for each aircraft, showing σ_g vs. $\epsilon^{1/3}$ for various speed ranges. Knowledge of ϵ will then permit the pilot to determine the airspeed to keep σ_g within desired limits. The chart can be incorporated onto the face of an ϵ-indicator. An aircraft response scale labeled "negligible, light, moderate, heavy, extreme" is given in terms of σ_g. A 'standard' speed can be derived for each aircraft at which the description of aircraft response in terms of σ_g is the same as the turbulence magnitude categories in terms of ϵ.

7) The feelings of a human in turbulence are assumed to relate to σ_h, his vertical acceleration in a particular frequency band of about 2 to 8 cps with a peak of sensitivity around 3 to 5 cps. σ_h thus depends on inertial subrange gusts. The relation between σ_h and σ_g depends on the type of aircraft and its speed, but in most cases the relation should be constant enough so that σ_g can logically be used for aircraft response intensity scales. An alternative response scale can be derived based on σ_h rather than σ_g, if desired.

8) The turbulence-gust load technique relates small-eddy measurements to the effects of small eddies, in contrast to the standard technique which characterizes turbulence intensity by the variance of vertical velocity, σ_w. σ_w is dominated by large eddies, which are not so well correlated with the gust-load producing eddies and which are hard to measure because distinct turbulence regimes are not much larger than the eddies which must be measured.

9) When acceleration loads or stability problems depend on wavelengths somewhat beyond the inertial subrange eddies the standardization technique is still valid as long as the spectrum shape over the wavelength range of interest is constant. In this case, although the spectrum shape is not proportional to $\lambda^{5/3}$ at the long wavelengths, the energies are still uniquely related to ϵ.

10) The standardization principle requires that the vertical velocity spectrum curve have a constant shape in the wavelength range which contributes most to gust loads; additional convenient measuring techniques are available if the turbulence is isotropic; the particular scale levels selected here for turbulence magnitude descriptions are derived using inertial subrange concepts.

11) One simple technique cannot perfectly cover all situations, but the simplicity of this technique permits better coverage of varied turbulence situations and thus actually results in more quantitative usefulness than would a more complex technique.

APPENDIX

Universal turbulence indicator

When atmospheric turbulence having a spectrum $E(k)=C_2\epsilon^{2/3}k^{-5/3}$ is measured in terms of frequency f by an airplane flying at mean airspeed U_0, the frequency spectrum is $E(f)=C_2U_0^{2/3}\epsilon^{2/3}f^{-5/3}$.

It is desirable to have an indicator which will show ϵ, or $(C_2^{1/2}\epsilon^{1/3})$, no matter what the speed or type is of the measuring aircraft. The following discussion shows methods of accomplishing this.

Assume the sensor provides a voltage fluctuation v corresponding to the small longitudinal (or lateral) velocity turbulent fluctuation u, such that $v=U_0^n u$.

With u having the spectrum $E(f)=C_2U_0^{2/3}\epsilon^{2/3}f^{-5/3}$, then v has the spectrum $J(f)=C_2U_0^{2n+2/3}\epsilon^{2/3}f^{-5/3}$. If the fluctuating voltage v is put into a filter, if p represents the voltage out of the filter, and if the filter has a frequency response function $B(f)$ which is non-zero only for frequencies (wavelengths) pertaining to the inertial subrange, then

$$\overline{p^2}=\int_0^\infty J(f)B^2(f)df=C_2\epsilon^{2/3}U_0^{2n+2/3}\left[\int_0^\infty B^2(f)f^{-5/3}df\right].$$

The term within brackets is a constant. The rms value of a fluctuating voltage can be obtained electronically (the simplest method for fluctuations with a Gaussian distribution is to take the average of the rectified signal), so q=indicator output$=\sqrt{\overline{p^2}}=K_2U_0^{n+1/3}\epsilon^{1/3}$ where all the constants of the system are lumped into K_2. Thus the indicator reads ϵ directly if $n=-1/3$. This is the case if the sensor measures velocity U in such a way that $V=U^{2/3}$, where V represents the sensor output voltage. For other cases, there must be an attenuation by a factor $U_0^{-n-1/3}$ put on the signal in the processing instrument, either on the mean signal entering the meter, or on the amplitude of the fluctuating signal before or after the filter. For example, if the sensor voltage is proportional to velocity squared, then the voltage fluctuations are related to turbulent velocity fluctuations by U_0 and so $n=1$, and the needed attenuation factor is $U_0^{-4/3}$. If the sensor is a direction vane, $v\sim u/U_0$ and so the needed attenuation factor is $U_0^{2/3}$. If the sensor is an accelerometer, say located near the center of gravity of the aircraft, the characteristics of the aircraft must be known for each speed; then the needed attenuation factor, a function of U_0, can be computed by the above concepts. Alternatively, the factor can be found experimentally by comparing an unattenuated indicator output to the $\epsilon^{1/3}$ value found simultaneously by the velocity or direction vane techniques described above.

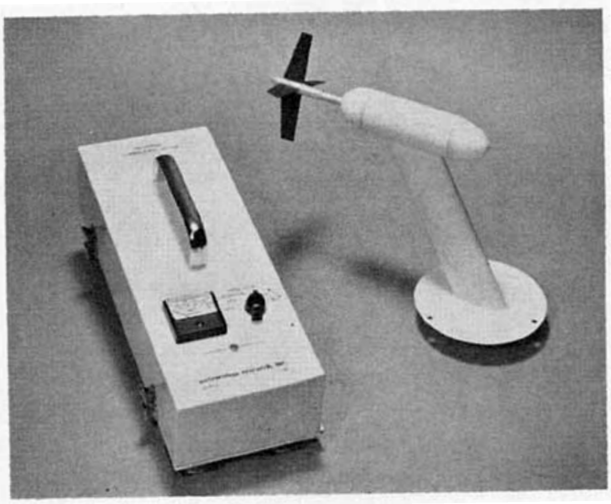

FIG. A-1. Universal turbulence indicator. The fast response 4 in. diameter propeller operates a light chopper from which is developed voltage directly proportional to true airspeed. The calibration includes consideration of the distortion of flow around the aircraft wherever the sensor is mounted. This sensor has been used at speeds to about 300 mph.
The electronics module has two voltage outputs, true airspeed and $\epsilon^{1/3}$.

Fig. A-1 shows a velocity-sensing unit used for turbulence measurements in connection with cloud physics and diffusion studies. This type of device, incorporating a nonlinear $U^{2/3}$ function, (or the equivalent, $v=U_0^{-1/3}u$) is perhaps the simplest possible version of a universal turbulence indicator. The aircraft provides a stable platform for measuring fairly long wavelengths of longitudinal velocity fluctuation, and the sensor can be fast response; thus the sensor system readily handles a broad band in the inertial subrange without introducing instrumental errors.

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