X-Band Polarimetric Radar Measurements of Rainfall

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ABSTRACT

A combined polarimetric estimator for rainfall rate (R) retrievals from polarimetric radar measurements at X band is proposed. This estimator uses the horizontal polarization radar reflectivity Z_e, differential reflectivity Z_{DR}, and specific differential phase shift K_{DP}, and it intrinsically accounts for changes in how drop oblateness increases with size. Because this estimator uses power measurements (i.e., Z_e and Z_{DR}), a procedure for correcting these measurements for effects of partial attenuation and differential attenuation using the differential phase measurement is suggested. An altitude correction for estimates of rainfall rates is also suggested. The proposed combined polarimetric estimator that uses K_{DP}, Z_{DR}, and Z_e, an estimator that uses K_{DP} alone for equilibrium drop shapes, and different Z_e−R relations were applied to the 15 rain events observed with the NOAA X-band transportable polarimetric radar during the eight-week field campaign at the NASA Wallops Island facility in Virginia. The observed rains ranged from very light stratiform events to very heavy convective ones with cells producing rainfall rates in excess of 100 mm h⁻¹. The three different ground validation sites were equipped with high-resolution (0.01 in.) tipping-bucket rain gauges. One of these sites also was equipped with disdrometers. In terms of the relative standard deviation, the combined polarimetric estimator provided the best overall agreement with gauge data (22%), closely followed by a case-tuned Z_e−R relation (23%) that was determined for each observational case from drop size distributions (DSD) measured in situ by a disdrometer and was available only a posteriori. The use of the K_{DP}-only estimator and a mean Z_e−R relation resulted in 30% and 32% relative standard deviations, correspondingly. The combined polarimetric estimator, the K_{DP}-only estimator, and the case-tuned Z_e−R relation estimator provided about a 6%−9% negative bias in comparison with the gauge data; the mean Z_e−R relation estimator provided a larger negative bias (18%).

1. Introduction

Improvement of rainfall parameter estimates from radar measurements has been one of the priorities of radar meteorology. It is generally accepted that the use of radar polarization parameters in the linear polarimetric basis improves quantitative estimates of rainfall rate. The radar polarization parameters of main interest for improving rainfall estimates are the specific differential propagational phase shift K_{DP} and differential reflectivity Z_{DR}.

Most research in the field of radar polarimetry as applied to rainfall parameter estimates has been performed for the radar wavelengths at S band (λ ~ 10–11 cm; e.g., Zrnić and Ryzhkov 1996; Chandrasekar et al. 1990) and C band (λ ~ 5–6 cm; e.g., May et al. 1999). These are the wavelengths of operational radars in many countries [e.g., the S-band Weather Surveillance Radar-1988 Doppler (WSR-88D) network in the United States]. Longer radar wavelengths (such as those at S band) are the obvious choice for measurements in moderate and heavy rain because of low attenuation and backscatter phase shifts effects. Partial attenuation of radar signals is already a problem at C-band frequencies. A number of studies have suggested and discussed several polarimetric and nonpolarimetric approaches for correcting partial attenuation at C band (e.g., Gorgucci et al. 1998; Carey et al. 2000).

Many research and some operational meteorological radars employ shorter wavelengths, such as those at X band (λ ~ 3 cm). The partial attenuation effects at X band are more severe when compared with those at C band, and accounting for these effects has been a significant problem for quantitative estimates of rainfall parameters based on reflectivity measurements at these
wavelengths. Introducing polarimetric diversity (viz., a differential phase shift capability) for X-band radars allows a robust way to account for attenuation effects, thus overcoming this drawback of X-band wavelengths in many situations. Moreover, the total attenuation constraint can be used for rain profiling similar to the spaceborne radar algorithms (e.g., Testud et al. 2000).

The use of shorter wavelengths, such as those at X band, has certain advantages over use of longer wavelengths with regard to polarimetric measurements in light and moderate rains (Matrosov et al. 1999). This includes a stronger, more readily detectable propagation differential phase shift that is proportional to the reciprocal of the wavelength for the Rayleigh conditions. At X band this amounts to about a factor-of-3 increase when compared with S band (a factor of about 1.7 in comparison with C band). Scattering and extinction of X-band wavelengths in rain has still largely the Rayleigh-type behavior except for possible backscatter phase shifts for larger rain drops.

X-band radars have additional advantages that make them a convenient tool and an appropriate choice for some practical applications. For a given transmitter power and antenna size, shorter wavelengths offer greatly increased sensitivity for detecting weak targets. As a result, these radars are generally relatively small and inexpensive and can be more easily transported to new locations than can S-band systems. Most X-band radars offer better spatial resolution and less problematic ground clutter than large S-band radars. As such, they are well suited to uses where transportability and fine-scale observations are important, such as in hydrometeorological studies across moderately sized complex terrain watersheds and urban basins. They may also be useful for filling critical gaps in the coverage of operational radar networks. These advantages and applications will increase since the partial attenuation problem that has limited the use of X band for quantitative rainfall estimations is being alleviated. Longer wavelength radars would still be the prime choice for wide-scale weather surveillance, but X-band systems would be able to contribute in important new ways to research and operations.

Over the last few years, the National Oceanic and Atmospheric Administration’s (NOAA) Environmental Technology Laboratory (ETL) has upgraded one of its X-band radars (Martner et al. 2001). This 3.2-cm-wavelength transportable radar (X pol) now is fully polarimetric and has full Doppler and scanning capabilities. Initial X-band differential phase shift measurement tests with the X-pol radar in moderate stratiform rains indicated no significant contributions from the backscatter phase shift (Matrosov et al. 1999). During these tests, $K_{DP}$-based rainfall rate estimators for X band provided a generally satisfactory agreement between rainfall accumulations derived from the radar data and accumulations measured by high-resolution rain gauges. These estimators are relatively insensitive to the details of drop size distributions (Zrnić and Ryzhkov 1996); however, they depend rather significantly on the model relating raindrop shape and drop size. Variability of this shape–size relation contributes a significant uncertainty of rainfall estimators that are based solely on the differential phase measurements. In this paper, we suggest and evaluate a combined rainfall rate X-band polarimetric estimator that uses measurements of differential phase shift, differential reflectivity, and the horizontal polarization reflectivity. This estimator intrinsically accounts for a changing degree of drop oblateness (i.e., drop shape) as a function of drop size.

2. Modeling of rainfall rate estimators at X band

It has been demonstrated for the S-band frequency radars that the addition of the polarimetric information usually improves accuracy of the rainfall rate $R$ and accumulation $A$ retrieval when compared with traditional approaches based only on measurements of equivalent radar reflectivity $Z$ (Ryzhkov and Zrnić 1995). More recent studies (e.g., Brandes et al. 2001) showed, however, that rainfall estimates obtained with a fixed coefficient $K_{DP}$–$R$ relation are similar to those obtained from the radar reflectivity only given that the radar is well calibrated. These results indicate that simple estimators relying on just one polarization parameter may not provide a sizable improvement over traditional approaches, and that combined polarimetric estimators may be needed.

a. $K_{DP}$–$R$ relations at X band

Most theoretical studies of $K_{DP}$–$R$ relations were performed assuming the equilibrium drop shape model, which predicts an almost linear decrease of the spheroidal raindrop aspect ratio $r$ as a function of $D_r$: $r = 1.03 - 0.62D_r$. This equation gives aspect ratios close to those in data of Pruppacher and Pitter (1971). Drops less than about 0.05 cm were usually assumed to be spherical in shape.

A number of recent studies (e.g., Kubesh and Beard 1993; Keenan et al. 1997; Andsager et al. 1999; Gor­guc­ci et al. 2000, 2001) indicate that the equilibrium drop shape is not unique and the variability in drop aspect ratio–diameter relations can be significant. Assuming a mostly linear trend of aspect ratio decrease with size, the following generalized $r$–$D_r$ relation can be suggested:

$$r = (1.0 + 0.05b) - b D_r \quad (D_r > 0.05 \text{ cm}),$$  

(1)

where $b$ is the shape factor in inverse centimeters. Note that aspect ratio model (1) differs from the one suggested earlier by Gorgucci et al. (2000) by assuming that for all values of $b$, drops smaller than 0.05 cm are spherical. This assumption is substantiated by various microphysical studies (e.g., Pruppacher and Klett 1978).

Figure 1 shows the results of modeling $K_{DP}$–$R$ relations for a radar wavelength of 3.2 cm for eight different values...
of the shape factor $b$ in the range from 0.4 to 0.8 cm$^{-1}$, which covers most of the dynamic range of natural changes of this parameter according to the case study of Gorgucci et al. (2000). The T-matrix approach for spheroidal shape drops was used for calculations (Barber and Yeh 1975). In order not to clutter the graph, the individual modeling results in Fig. 1 are depicted for $b = 0.6$ cm$^{-1}$, which approximately corresponds to the equilibrium drop shape while the power-law best-fit regressions are drawn for all considered values of $b$. The corresponding regressions are also shown in Fig. 1.

Experimental size spectra rather than model drop size distributions (DSD) were used to derive $K_{\text{DP}} - R$ relations in Fig. 1. These DSD spectra were 1-min averages recorded using an impact Joss–Waldvogel disdrometer during the two-month-long field observation campaign held at Wallops Island, Virginia, from 21 February to 19 April 2001. The observed rainfalls ranged from very light (about 1 mm h$^{-1}$) to very heavy rain (exceeding, at times, 100 mm h$^{-1}$). Both stratiform and convective rain types are present in the DSD dataset. The total number of DSD spectra used for modeling was about 3450. All measured DSD spectra were quality controlled. For most of the events considered here, the total rainfall accumulations estimated from disdrometer data usually were within 10% of measurements of collocated high-resolution (0.01 in.) tipping-bucket-type rain gauges.

As can be seen from modeling results in Fig. 1, the value of the exponent in $K_{\text{DP}} - R$ relations is fairly independent of the assumption about the shape factor $b$. The coefficient in these relations, however, depends on $b$ very strongly. A relation between rainfall rate and specific differential phase shift for changing value of $b$ can be expressed as

$$ R (\text{mm h}^{-1}) \approx 8.2b^{-0.82}K_{\text{DP}}^{0.81} (\text{o km}^{-1}) (b: \text{cm}^{-1}). $$

(2)

The natural variability of the shape factor $b$ contributes significantly to the uncertainty of $K_{\text{DP}} - R$ relations since the rainfall estimate almost inversely depends on $b$. Another source of the uncertainty in $K_{\text{DP}} - R$ relations is the variability in the DSD details (Keenan et al. 2001; Zrnić et al. 2000), although the influence of DSD is much smaller here in comparison with that for reflectivity–rain rate relations. For the Wallops experiment DSDs, the uncertainty in $K_{\text{DP}} - R$ relations due to DSD details (expressed in terms of the relative standard deviation of individual data points when a value of $b$ is fixed) is about 15%. It can be seen from equations in Fig. 1 that the similar uncertainty is caused by about 0.08–0.1 cm$^{-1}$ variability in $b$.

Gorgucci et al. (2001) showed that the shape factor $b$ can be estimated from combined measurements of the horizontal polarization reflectivity $Z_{\text{eh}}$, $Z_{\text{DR}}$, and $K_{\text{DP}}$. Applying their approach suggested for S band to X band yields the following multidimensional nonlinear regression:

$$ b (\text{cm}^{-1}) \approx 12Z_{\text{eh}}^{-0.36} (\text{mm}^6 \text{m}^{-3})K_{\text{DP}}^{0.40} (\text{o km}^{-1})Z_{\text{dr}}^{1.02}. $$

(3)

In (3) $Z_{\text{eh}}$ is expressed in linear units as opposed to the traditional definition of $Z_{\text{DR}}$ in decibels ($Z_{\text{DR}} = 10 \log_{10} Z_{\text{eh}}$). This distinction is denoted by the use of lowercase letters in the subscript rather than uppercase ones ($Z_{\text{eh}}$ vs $Z_{\text{DR}}$).

Figure 2 shows the scatterplot between the estimated value of the shape factor $b$, which was calculated using (3), and the true value of $b$ used for modeling of $Z_{\text{eh}}, K_{\text{DP}}$.  

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**Figure 1.** X-band $K_{\text{DP}} - R$ relations for different values of the shape factor $b$.

**Figure 2.** Scatterplot between true values of the shape factor $b$ and its estimates from Eq. (3).
and $Z_\omega$. The calculations were performed for all the 3450 DSDs from the Wallops experiment using values of $b$ from 0.4 to 0.75 cm$^{-1}$ with an increment of 0.05 cm$^{-1}$. The regression (3) provides no overall bias, though it is biased negatively for larger values of $b$ and positively for smaller values of $b$. The overall relative standard deviation is about 8%, which is a quantitative measure of quality of shape factor estimations using the regression (3). The 8% error in $b$ estimates is probably acceptable for practical reasons since it introduces less uncertainty in $K_{DP}$-$R$ relations than the effects of DSD details.

b. X-band combined polarimetric estimator of rainfall rate

Substituting estimates of the shape factor $b$ from (3) in (2) results in a rainfall rate estimator that uses three radar parameters:

$$R = c(h)1.06Z_{eh}^{0.35}K_{DP}^{0.50}Z_{d,eh}^{-0.84},$$

where $c$ is the altitude correction coefficient. This coefficient accounts for the drop fall velocity changes with altitude $h$ above sea level caused by change in the density of ambient air. It is usually ignored, though it may contribute to noticeable differences when comparing the results of radar measurements from near-sea level sites and ones that are relatively high above sea level. Since the drop terminal velocities are inversely proportional approximately to the 0.45th power of the air density $\rho$ (Beard 1985), the altitude correction coefficient $c(h)$ can be expressed as

$$c(h) \approx 1.1\rho(h)^{-0.45},$$

where the air density is in kilograms per cubic meter.

Though the combined X-band polarimetric estimator for instantaneous rainfall rates (4) was obtained using the experimental DSDs from a two-month-long period of rains on the Virginia coast, it might work for other geographical locations as well. This is supported by the fact that a procedure similar to the procedure described above for obtaining $Z_{eh}$-$K_{DP}$-$Z_\omega$ estimators based on 140 DSD disdrometer measurements from the Tropical Rainfall Measuring Mission Texas and Florida Underflights Phase A experiment near Houston, Texas, leads to a regression (Matrosov et al. 2001) that provides rainfall rates that are very close (within 10% typically) to those obtained using (4) for the most common rain reflectivity range 30–50 dBZ.

One of the reasons for this relative stability is the comparatively low sensitivity of (2) to the details of DSD (for a given value of $b$). Another factor contributing to it is the relatively low variability of the shape factor estimator (3) due to DSD changes. This estimator, obtained here for X band, is very close to the S-band relation suggested by Gorgucci et al. (2000) after the $K_{DP}$ wavelength dependence is accounted for. This fact is quite remarkable since results of this paper are based on experimental DSDs, while Gorgucci et al. (2000) used modeled drop spectra.

c. Accounting for partial attenuation

Attenuation of radar signals has been traditionally a significant limitation in using X-band radars for quantitative measurements of rainfall. The $K_{DP}$-only-based approaches for retrieving rainfall rates are immune to the partial attenuation of radar signals. The combined polarimetric estimator (4), however, also uses power measurements of $Z_{eh}$ and $Z_\omega$, which are subject to attenuation and differential attenuation, correspondingly. Different attenuation correction schemes, which adjust reflectivity measurements at a given range based on rainfall rates retrieved for closer ranges and empirical attenuation–rainfall rate relations, are often unstable, which results in the divergence of reflectivity estimates with range. Fortunately, the differential phase polarimetric capability provides a relatively robust way to overcome the partial attenuation problem.

Specific horizontal polarization attenuation $A_h$, specific differential attenuation $A_{DP}$, and specific propogational differential phase $K_{DP}$ are all related since they are obtained by integrating the forward scattering amplitudes on horizontal polarization, $f_{h,v}$, and vertical polarization, $f_{v,v}$. These amplitudes define the propagation parameter

$$K_{h,v} = k_v + (2\pi k_v^{-1})\int_0^{D_{\text{max}}} f_{h,v}(D_s)N(D_s)\,dD_s,$$

$$D_{\text{max}} \approx 6 \text{ mm},$$

where the integration is carried over the drop size distribution $N(D_s)$ and $k_v$ is the wavenumber.

The specific attenuation and the specific phase rotation are given by

$$A_{h,v} = 2\lambda\int_0^{D_{\text{max}}} \text{Im}[f_{h,v}(D_s)]N(D_s)\,dD_s,$$

$$\phi_{h,v} = \lambda\int_0^{D_{\text{max}}} \text{Re}[f_{h,v}(D_s)]N(D_s)\,dD_s,$$

where $\phi_{h,v}$ is in radians per unit length and $A_{h,v}$ is in the reciprocal of length. Note that $A_{h,v}$ represents the power attenuation, which explains a factor of 2 in (7).

The forward scattering amplitudes relate to the dimensionless amplitude matrix elements used in optics, $S_{h,v}$, (e.g., Bohren and Huffman 1983) as

$$S_{h,v} = -2\pi f_{h,v}\lambda^{-1}.$$
The assumed temperature is 5°C. The $A_\text{h}$–$K_{\text{dp}}$ relations are practically linear. As in Fig. 1, the symbols show results of modeling for the equilibrium drop shape ($b = 0.62\text{ cm}^{-1}$), but the best-fit linear regressions are depicted for eight values of $b$ in the range from 0.4 to 0.75 cm$^{-1}$. For the equilibrium drop shape the coefficient in the $A_\text{h} = a_1 K_{\text{dp}}$ relation is about 0.22 dB $\text{ cm}^{-1}$, which is quite close to the estimate of Bringi et al. (1990), $a_1 = 0.247 \text{ dB cm}^{-1}$, and Jameson (1991), $a_1 = 0.21 \text{ dB cm}^{-1}$, which were obtained using the simulated gamma function DSDs. This coefficient exhibits, however, a significant dependence on the shape parameter $b$, which can be approximated as

$$a_1 = 0.145 b^{-0.01}. \quad (12)$$

The total attenuation correction for horizontal polarization reflectivity measurements can be easily estimated from the accumulated propagation differential phase shift $\Phi_{\text{dp}}$:

$$\Delta Z_{\text{th}} (\text{dBZ}) = a_2 \Phi_{\text{dp}} (\text{°}). \quad (13)$$

Figure 4 shows the scatterplot between $A_{\text{dp}}$ and $K_{\text{dp}}$ as before, scatter points are shown only for the equilibrium drop shape assumption. Unlike for $A_\text{h}$–$K_{\text{dp}}$ relations discussed earlier, for $A_{\text{dp}}$–$K_{\text{dp}}$ relations there is a very little dependence on the assumption of the drop shape factor $b$. The corresponding best-fit curves differ by not more than about 5% for $b$ between 0.4 and 0.75 cm$^{-1}$ and $K_{\text{dp}} > 0.25\text{ cm}^{-1}$. This can be explained by the simultaneous and almost equal relative increase (decrease) of $A_{\text{dp}}$ and $K_{\text{dp}}$ as $b$ increases (decreases) for a given DSD. The variability of $A_{\text{dp}}$–$K_{\text{dp}}$ relations due to DSD details is higher than that for $A_\text{h}$–$K_{\text{dp}}$ relations.

As can be seen from data scatter in Fig. 4 there is some slight nonlinearity in $A_{\text{dp}}$–$K_{\text{dp}}$ relations. A nonlinear $A_{\text{dp}}$–$K_{\text{dp}}$ relation, however, would result in a relatively cumbersome accounting for differential attenuation requiring nonlinear integrating estimates of $K_{\text{dp}}$ along the radar beam. Given the fact that nonlinearity is not very significant and also because of relatively high variability due to DSD details (as compared with the variability of $A_\text{h}$–$K_{\text{dp}}$ relations), it can be assumed that $A_{\text{dp}} (\text{dB km}^{-1}) = 0.032 K_{\text{dp}} (\text{° km}^{-1})$. In this case the total differential attenuation correction for $Z_{\text{dr}}$ measurements can be given as

$$\Delta Z_{\text{dr}} (\text{dB}) = a_2 \Phi_{\text{dp}} (\text{°}). \quad (14)$$

Note that the coefficient obtained here ($a_2 = 0.032 \text{ dB km}^{-1}$) is smaller than the one suggested by Bringi et al. (1990). Their estimate, based on gamma function DSD modeling for X band, yielded $a_2 = 0.045 \text{ dB km}^{-1}$. Part of this difference, however, can be explained by the difference in considered wavelengths. Bringi et al. (1990) used $\lambda = 3\text{ cm}$, and here the NOAA X-band radar wavelength $\lambda = 3.2\text{ cm}$ was used. The coefficients $a_2$ and $a_1$ decrease when the wavelength increases. Bringi et al. (1990) give $a_2 = 0.014 \text{ dB km}^{-1}$ and $a_1 = 0.05 \text{ dB km}^{-1}$ for $\lambda = 5.5\text{ cm}$.

These coefficients are also temperature dependent. A 5°C temperature increase from 5°C to 10°C causes about a 4% increase in $a_2$ and about a 2.5% increase in $a_1$. It can be seen from Fig. 3 that the similar variability in $a_1$ can be caused either by a 5°C temperature uncertainty or by an about 5% uncertainty in $b$. Ignoring the temperature dependencies of $a_2$ and $a_1$ can cause biases in attenuation-corrected radar reflectivities and differential reflectivities for large values of $\Phi_{\text{dp}}$.

Since the combined polarimetric estimator (4) uses...
The horizontal polarization, i.e., $Z_e$, is listed in Table 1. The principal verification scheme was proposed by Sachidananda and Zrnic (1985), and it is being considered for the polarimetric scheme employed at the National Aeronautics and Space Administration (NASA) Wallops Island facility. Prior to rain measurements, the radar was calibrated in an absolute calibration site located at a range of 3.3 km along the 134° azimuthal direction from the radar site. This site was equipped with two high-resolution (0.01 in.) rain gauges of the tipping-bucket type (one belonging to NOAA ETL and the other to NASA), and several disdrometers. Two other NOAA ETL high-resolution rain gauges were located along the 69° azimuthal direction at ranges of 6.6 and 14.7 km, respectively. The typical scan procedure included low-elevation sector scans with intermittent range–height indicator (RHI) scans.

Figure 5 shows four examples of radar measurements in rain of different intensities. All these examples correspond to one long event of 21 March 2001, when the rain gradually intensified from very light to very heavy. The radar beam in these examples is pointed in the direction of the gauge site (134° azimuth). The elevation angle (1.8°) was the lowest for which the ground clutter from Wallops Island structures could be neglected. The measured (total) differential phase shift ($\phi_{DP}$) and reflectivity ($Z_{ee}$) scales for all the cases are the same for easier comparisons.

In a light rain (Fig. 5a), the trend of $\phi_{DP}$ is very small but still measurable. Typical standard deviation of $\phi_{DP}$ measurements due to noise was generally under 2°. The average $K_{DP}$ value is only about 0.1° km$^{-1}$. Note that the corresponding $K_{DP}$ value at S-band frequencies will be only about 0.03° km$^{-1}$, which may not reliably be measurable for reasonable range intervals. According to the ground-based gauge measurements at 3.3-km range, a rainfall rate near the time of this measurement (i.e., 1120:00 UTC) was about 2.4 mm h$^{-1}$. The radar reflectivity was less than and about 30 dBZ for all the ranges within this beam. The attenuation correction for this example is small. At the 20-km range it is only about 1 dB.

Figure 5b shows an example of radar measurements when rainfall intensity increased a little in comparison with the time in Fig. 5a. According to the gauges at 3.3 km, the rainfall rate for this example was around 4 mm h$^{-1}$ at the time of the measurements (1336:40 UTC). The $\phi_{DP}$ increase in the rain-filled area (up to about 20 km) is quite steady, though it is still quite small. The corresponding average $K_{DP}$ value is about 0.25° km$^{-1}$. The maximum value of reflectivity correction is about 2 dB.

At 1430:40 UTC the rain became heavier (Fig. 5c). The 3.3-km rain gauge was indicating a rainfall rate of about 10 mm h$^{-1}$ at the time of this measurement. The differential phase increase is very pronounced and steady as is the corrected value of the radar reflectivity of about 38 dBZ. The $K_{DP}$ is about 0.8° km$^{-1}$. The low variability of $K_{DP}$ indicates a rather uniform rain. At the longer ranges the attenuation correction for the reflectivity reaches about 10 dB. The steady values of the corrected reflectivity as a function of range in this uniform rain represent an independent qualitative check of the accounting for attenuation correction.

Figure 5d shows an example of measurements in very heavy rain. A cell with very high rainfall rates was observed near the radar at distances up to about 4 km. At
FIG. 5. Examples of radar polarimetric measurements in rains of different intensity: (a) very light rain, (b) light rain, (c) moderate rain, and (d) heavy rain. Solid and dashed reflectivity lines indicate measured values and those corrected for partial attenuation, respectively.

the time of these measurements, the high-resolution rain gauge was tipping once every 7 or 8 s, which corresponds to a rainfall rate of about 110–130 mm h\(^{-1}\). There were no indications of hail in this cell. The measured differential phase shift \(\phi_{DP}\) was increasing at a very high rate to about 4.5-km range with corresponding \(K_{DP}\) reaching up to 12–14° \(\text{km}^{-1}\). Very light rain was observed between 5 and 10 km, followed by an area of moderate rain beyond 10 km. All these changes in rain are nicely seen in the \(\Phi_{DP}\) pattern as a function of range. The attenuation in this heavy rain was significant. The received radar echoes decreased to the noise level beyond the range of about 20 km. The sensitivity of the radar at its current configuration is about 0 dBZ at 20-km range. For the Wallops experiment measurements, the transmitted power was about 4 dB down from its nominal value (about 16-kW peak power at each polarization) to avoid linear receiver saturations at very close ranges.

A remarkable fact about measurements in very heavy rains with X-band radar (such as in Fig. 5d) was the lack of the obvious manifestation of the backscatter phase shift \(\delta\) that would appear as a “bump” on otherwise steady and monotonic changes of \(\phi_{DP}\) (Hubert et al. 1993). It should be mentioned, however, that it is not the magnitude of \(\delta\) that matters but rather the difference of backscatter phase shifts \(\Delta \delta\) in the beginning and at the end of the interval used for estimation of \(K_{DP}\) as the range derivative of \(\Phi_{DP}\) (\(\Delta \phi_{DP} = \Delta \Phi_{DP} + \Delta \delta\), where \(\Delta \Phi_{DP}\) is the differential phase shift on propagation).

Model calculations of the backscatter differential phase shift \(\delta\) (for individual drops) are shown in Fig. 6 as a function of equal-volume drop diameter. At X band, \(\delta\) is very small for \(D_e < 3\) mm, but then it begins a monotonic increase at \(D_e = 3\) mm. Since this increase is rather gradual, one could expect that \(\Delta \delta\) will remain relatively small if the rainfall properties do not change very abruptly. The backscatter phase shift resonance at C band is more profound. It can be seen from Fig. 6 that if drops greater than about 5 mm are present, the backscatter phase shift at C band will be greater than at X band. It should be admitted that this is one possible explanation for the lack of significant backscatter differential phase shift effects in heavy rains observed at X band during the Wallops experiment. More heavy rain observations are needed to draw definitive conclusions about the wisdom of ignoring these effects. A filtering approach (e.g., Hubert and Bringi 1995) can be used to minimize effects of \(\delta\) if they are significant.

4. Comparisons of radar-derived rainfall rates with gauge and disdrometer data

The combined polarimetric estimator (4), the equilibrium drop shape \(K_{DP}-R\) relation, and the mean and
case-tuned $Z-R$ relations were applied to the 15 rains observed during the Wallops experiment. The attenuation-corrected values of $Z_{eb}$ were used in the $Z-R$ relations. Specific differential phase shift on propagation, $K_{DP}$, was estimated as the range derivative of measured differential phase shift, $f_{DP}$, using the least squares method applied for the sliding window range interval consisting of 25 range gates. The range gate resolution for the Wallops experiment was 150 m. The $f_{DP}$ measurements were filtered prior to estimating $K_{DP}$. The filtering procedure rejected all the data points that had low correlation between two consecutive pulses or were less than 3 dB above the noise floor of the radar ($-103$ dBm). Additional threshold applications, based on Doppler velocity measurements, rejected data points with suspected ground clutter contamination.

a. Differential reflectivity measurements used in the combined polarimetric estimator

As was mentioned above, the slant $45^\circ$ transmission scheme with two receivers was used with the NOAA ETL X-band radar. Since the combined polarimetric estimator of rainfall rates (4) uses the differential reflectivity measurements, an estimation of how well the slant–$45^\circ$ scheme measurements of this parameter approximate classical differential reflectivity is necessary. The slant–$45^\circ$ scheme offers some advantages such as the zero lag time between two copolar returns and a factor-of-2 increase of the number of samples for a given dwell. However, the differential reflectivity measurements, using this scheme, can be somewhat biased for canted drops through depolarization. It can be shown, however, that this bias is negligible for most practical cases. The true $Z_{DR}$ and the slant $45^\circ$ $Z_{DR}^{0\circ}$ can be expressed as

$$Z_{DR} = \frac{\langle|S_{hh}(D)|^2\rangle N(D) dD}{\langle|S_{hh}(D)|^2\rangle N(D) dD}$$

$$Z_{DR}^{0\circ} = \frac{\langle|S_{hh}(D) + S_{vv}(D)|^2\rangle N(D) dD}{\langle|S_{vv}(D) + S_{hh}(D)|^2\rangle N(D) dD}$$

where $S_{hh}, S_{vv}, S_{hv}$, and $S_{vh}$ are the elements of the amplitude backscatter matrix; the angular brackets denote averaging with respect to the canting angle $\alpha$; and the integration carried out from 0 to $D_{max} \approx 6$ mm. For the zero cant, $Z_{DR} = Z_{DR}^{0\circ}$ because nondiagonal elements $S_{hv}$ and $S_{vh}$ are proportional to $\sin(2\alpha)$. When drops are canted on average, differs from the true differential reflectivity. Since increasing (decreasing) of $\alpha$ from its mean value causes underestimation (overestimation) of $Z_{DR}$ in the slant–$45^\circ$ transmission mode (though not exactly balanced), the mean canting angle is more important than its standard deviation, given that this standard deviation is relatively small.

Figure 7 shows the scatterplot between $Z_{DR}$ and $Z_{DR}^{0\circ}$ assuming that the drops are canted at $5^\circ$ and $10^\circ$ of vertical. Typical mean canting angles are of an order of this value or smaller. Calculations of differential reflectivities in Fig. 7 were performed for experimental DSDs.
FIG. 8. Comparisons of rainfall rate estimates from the combined polarimetric estimator and from the high-resolution rain gauge for two rain events: (a) 25 Feb 2001 and (b) 21 Mar 2001.

and assuming the equilibrium drop shape model. It can be seen from this figure that the difference between $Z_{DR}$ and $\varphi_{Z_{DR}}$ does not usually exceed about 0.1 dB for a typical range of the variability of differential reflectivity in rains. Since this is less than the expected accuracy of $Z_{DR}$ measurements, it is assumed when interpreting the experimental data in this study that $Z_{DR} \approx \varphi_{Z_{DR}}$.

b. Examples of comparisons of instantaneous rainfall rates

Figure 8 shows examples of comparisons of instantaneous rainfall rate estimates from the radar measurements and from the data of ETL’s high-resolution rain gauge for two rain events: (a) 25 Feb 2001 and (b) 21 Mar 2001.

c. Comparisons of radar- and gauge-derived total accumulations

Figure 9 shows the total accumulations of rainfall as a function of time for the two rain events discussed above. Results of different radar estimates are depicted in this figure. These include accumulations derived from the combined polarimetric estimator that uses $K_{DP}$, $Z_{DR}$, and $Z_{eh}$ measurements; the equilibrium drop shape $K_{DP} \varphi_{R}$ relation at X band ($R = 12.3 K_{DP}^{0.84}$); the mean Wallops $Z_{DR} \varphi_{R}$ relation ($i.e., Z_{DR} \approx 250 R^{0.68}$); and a case-tuned $Z_{DR} \varphi_{R}$ relation. The case-tuned $Z_{DR} \varphi_{R}$ relations were obtained for each of 15
rainfall events by considering DSDs measured by the disdrometer only during the particular event. The parameters for these case-tuned relations are given in Table 1, where it can be seen that the coefficient and the exponent varied widely from case to case. In addition to the radar estimates, data from the ETL high-resolution rain gauge are shown in Fig. 9. All the data in the figure correspond to the prime validation site mentioned above.

From different radar approaches, the combined polarimetric estimator gives the best agreement with the rain gauge. This agreement is not as good for the period of the very heavy rain (e.g., after 1530 UTC in Fig. 9b). For light and moderate rains combined, polarimetric estimators and from the high-resolution rain gauge for two rain events: (a) 25 Feb 2001 and (b) 21 Mar 2001.

### Table 2. Relative biases and standard deviations of rainfall accumulations obtained using different rainfall estimators (as compared with the tipping-bucket-type rain gauge data): the combined polarimetric estimator (4), the equilibrium drop shape \( \text{K}_\text{DF}-R \) relation (\( R = 12.3 \text{K}_\text{DF}^{0.31} \)), the mean Wallops \( Z_r-R \) relation (i.e., \( Z_r = 250 \text{R}^{0.81} \)), and the case-tuned \( Z_r-R \) relations (reflectivity/differential reflectivity measurements were corrected for the attenuation/differential attenuation effects).

<table>
<thead>
<tr>
<th>Estimator</th>
<th>( Z_{\text{obs}} )</th>
<th>( Z_{\text{DR}} )</th>
<th>( K_{\text{DF}} )</th>
<th>( K_{\text{DF}}-R )</th>
<th>Mean ( Z_r )</th>
<th>( Z_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>(-8%)</td>
<td>(-9%)</td>
<td>(-18%)</td>
<td>(-6%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std dev</td>
<td>(22%)</td>
<td>(30%)</td>
<td>(32%)</td>
<td>(23%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The combined polarimetric estimator (4) was used more than 95% of the time.

The mean relative bias and standard deviation (sd) between radar and gauge estimates of accumulation (A) were calculated for each radar estimator:

\[
\text{bias} = \langle (A_r - A_{\text{g}})A_{\text{g}}^{-1} \rangle, \quad (17)
\]

\[
\text{(sd)}^2 = \langle (A_r - A_{\text{g}})^2A_{\text{g}}^{-2} \rangle, \quad (18)
\]

where the angular brackets denote averaging, and subscripts \( r \) and \( g \) refer to the radar and gauge, respectively. Results of estimates using (17) and (18) are given in Table 2. Note that results obtained with all the estimators considered here were corrected for the altitude according to (5).

Assessments of bias and the standard deviation for the combined polarimetric estimator (\( K_{\text{DF}}, Z_{\text{DR}}, \) and \( Z_{\text{ob}} \)), the equilibrium drop shape \( K_{\text{DF}}-R \) relation at X band (\( R = 12.3 \text{K}_\text{DF}^{0.31} \)), the mean Wallops \( Z_r-R \) relation (i.e., \( Z_r = 250 \text{R}^{0.81} \)), and the case-tuned \( Z_r-R \) relations yield \(-8\%\), \(-9\%\), \(-18\%\), and \(-6\%\) for bias, and \(22\%\), \(30\%\), \(23\%\), and \(32\%\) for the standard deviation, respectively. Reflectivity values used in the \( Z_r-R \) relations were corrected for partial attenuation using differential phase shift measurements.

The combined polarimetric estimator gives the smallest overall standard deviation (22%), though the case-tuned \( Z_r-R \) relations provide almost the same value of the standard deviation (23%) with a slightly smaller bias. It should be noted, however, that the case-tuned \( Z_r-R \) relations generally are not known a priori and the polarimetric measurements at X band are still needed for accounting for attenuation in reflectivity measurements even if \( Z_r-R \) relations are used from the rainfall estimates. The use of the mean \( Z_r-R \) relation results in the most significant negative bias (\(-18\%\)) and the largest standard deviation (32%). These bias and standard deviation data for different estimators are summarized in Table 2.

The equilibrium drop shape \( K_{\text{DF}}-R \) relation and the combined polarimetric estimator (4) provided similar biases for the total rainfall accumulation, although (4) was noticeably better in terms of the standard deviation. The similarity of biases is most likely due to the fact that the mean value of the shape factor \( b \) averaged over
all the recorded events was rather close to its equilibrium value for these rain events. The estimator (4) that intrinsically accounts for changes in the drop shape parameter \( b \) is superior to the equilibrium drop shape \( K_{dp} = R \) relation for estimates of instantaneous rainfall rates, and it is also expected to perform generally better for total accumulations for individual rain events when mean values of the shape parameter \( b \) are more likely to deviate from the equilibrium value compared to the average of many rain events. The case-tuned \( Z_e - R \) relations perform notably better than the mean relation, which provides the most significant bias and standard deviation.

One possible explanation of the small negative biases of both estimators that utilize differential phase shift measurements lies in the spread in drop canting angles, which was ignored when modeling \( K_{dp} - R \) relations in section 2a. This spread \( (\sigma_r) \) tends to reduce the coefficient in the \( K_{dp} - R \) relations by a factor of about \( \exp(-2\sigma_r^2) \) (Oguchi 1983). According to Ryzhkov (2001), a typical value of \( \sigma_r \) in rain is about 10°, which can amount to about 6% reduction of this coefficient. The small negative biases for the combined polarimetric estimator, the equilibrium drop shape \( K_{dp} = R \) relation, and the case-tuned \( Z_e - R \) relations are, however, close to the uncertainty of gauge measurements. An average difference in total accumulations measured by the colocated NASA and ETL rain gauges at one of the validation sites was about 10%.

5. Conclusions

One source of uncertainty in \( K_{dp} = R \) relations is due to variability in the raindrop oblateness–size dependence since the commonly assumed equilibrium drop shape is not unique. This dependence results in change of the coefficient in the \( K_{dp} - R \) relations while leaving the exponent almost intact. Tuning \( K_{dp} - R \) relations requires knowing \( b \), the slope parameter of this dependence. A multiparameter scheme for estimating \( b \), similar to the one suggested by Gorgucci et al. (2000) for S band, was derived here for X band. Modeling that led toward an approach to estimate \( b \) was done using the experimental raindrop size distributions rather than modeled ones.

As a result of modeling \( K_{dp} = R \) relations and the drop shape parameter \( b \) estimation algorithm, an X-band polarimetric estimator for instantaneous rainfall rates based on combined measurements of the specific propagation differential phase shift \( K_{dp} \), differential reflectivity \( Z_{DR} \), and the horizontal polarization reflectivity \( Z_{eh} \) was suggested. This estimator accounts for changes of the slope parameter \( b \) in the drop oblateness–size dependence and also for changes in drop fall velocities due to changes in the ambient air density.

Since the suggested polarimetric estimator uses power measurements (i.e., reflectivity and differential reflectivity), a procedure for correcting effects of the partial attenuation and differential attenuation based on measurements of differential phase shift was suggested. The differential phase capability thus offers a robust and relatively straightforward way of correcting for attenuation effects that traditionally have been a major obstacle to using X-band radar quantitative rainfall estimates based on power measurements.

The suggested X-band multiparameter polarization approach for rain measurements was tested with the transportable NOAA X-band radar that was recently upgraded in a polarimetric sense. The radar was deployed for an eight-week field experiment during February–April 2001 at the NASA Wallops Island base. Fifteen rain events were observed during this period. The ground validation equipment included three high-resolution tipping-bucket-type (0.01 in.) rain gauges deployed at three different locations and additional rain gauges and a Joss–Waldvogel disdrometer at one of these locations.

The observations included a wide range of rain conditions from very light rain with \( K_{dp} \) of about 0.1° km\(^{-1}\) to very heavy rain with \( K_{dp} \) reaching 12–14° km\(^{-1}\). No significant effects of the backscatter phase shift were evident even in heavy rain, though more data are needed to better understand these effects. Comparisons of rainfall rate values estimated from gauges and obtained using the combined X-band polarimetric estimator proposed here were generally in good agreement for rainfall rates greater than about 1.5–2 mm h\(^{-1}\). Lighter rains produce very subtle polarimetric signatures and the accuracy of polarimetric estimates degrades. Estimates of rainfall rates based on \( Z_e - R \) relations work better in these situations.

The quantitative comparisons of rainfall accumulations from different radar estimators and the high-resolution rain gauges were made for all 15 observed rain events. The mean \( Z_e - R \) relation (i.e., \( Z_{eh} = 250R^{0.68} \)) derived from 3450 DSD spectra recorded during the entire period of radar observations provided a mean relative bias of −18% and a mean relative standard deviation of 32% in comparison with the rain gauge measurements. Corresponding values of the bias and standard deviation for the case-tuned \( Z_e - R \) relations (i.e., derived from DSD spectra recorded only for the particular event) were −8% and 23%, respectively. However, case tuning of \( Z_e - R \) relations is not usually possible for real-time estimates.

The simplest polarimetric estimator based on the equilibrium drop shape \( K_{dp} = R \) relation at X band \((R = 12.3K_{dp}^{0.63})\) provided a −9% bias and a 30% standard deviation as compared with gauges. The best results in terms of agreement with the gauge data were obtained with the combined polarimetric estimator that uses measurements of \( K_{dp} \), \( Z_{DR} \), and \( Z_{eh} \) and tunes the value of the shape parameter \( b \) that describes the drop oblateness–size dependence. It yielded mean relative bias and standard deviation values of −8% and 22%, respectively. The similarity of mean biases for both polarimetric estimators is most likely due to the possible prox-
iminity of the mean value of $b$ for all the observed events to its equilibrium value. The combined polarimetric estimator is much superior to the equilibrium shape $K_{\text{def}}$-R relation for estimations of instantaneous rainfall rates and total accumulations in individual events when the factor $b$ deviates from its equilibrium value.

Both polarimetric estimators were used when values of reflectivity corrected for attenuation exceeded about 28 dBZ, which constituted more than 95% of total observation time above the gauges. Note that though the tuned $Z_{\text{R}}$ relation and the combined polarimetric approaches provided similar results (in terms of bias and standard deviation), the latter approach should be considered superior since it does not use a priori information.

A small negative bias in the polarimetric and tuned $Z_{\text{R}}$ relation estimates of rainfall accumulations may be explained by a number of factors, including canting angle spread and sampling issues. The magnitude of this bias is, however, of an order of uncertainty of gauge measurements themselves as determined by comparing data from two collocated tipping-bucket-type gauges.

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