Identification of Rain-Rate Profiles from Radar Returns at Attenuating Wavelengths Using an Inverse Method: A Feasibility Study

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(Manuscript received 8 June 2002, in final form 27 November 2002)

ABSTRACT

Attenuation in rainfall is recognized as one of the most significant limitations in rain-rate estimation from weather radar returns at X- or C-band wavelengths. This paper introduces a radar measurement correction as an inverse problem that accounts for attenuation effects in rainfall. First, a direct theoretical model, relating radar returns at attenuating wavelengths to the rainfall rates between the radar and the point of measurement, is presented. Second, the inverse algorithm used to identify rain-rate estimates from radar returns is described and its application to the attenuation correction is discussed, with the well-known characteristics of the attenuation model (i.e., instability, underdetermination, and nonlinearity) receiving particular attention. Third, a sensitivity analysis is then performed to test the influence of the raindrop size distribution, radar measurement features, and statistical parameters involved in the inverse method. The sensitivity analysis allows for establishing the application conditions of the method. Last, a preliminary evaluation of the method is provided, through simulated radar rainfall measurements and through a limited case study. Various attenuation correction methods are compared with the inverse algorithm. These methods include the standard radar reflectivity–rainfall rate algorithm and two versions of the Hitschfeld–Bordan algorithm. In the simulation exercise, various examples of rainfall field, with different characteristics, are tested. The case study confirms the utility of the proposed method and its ability to provide a robust and stable solution. The method consistently provides better results than the well-known Hitschfeld–Bordan algorithm.

1. Introduction

Attenuation effects in rainfall were recognized very early on as a major source of error in measuring rainfall by radar using “wavelengths of less than about 7 cm” (Atlas and Banks 1951). Geotis (1975) and Delrieu et al. (1997) provided illustrative examples of attenuation effects for a C- and X-band radar, respectively. Based on the weather radar equation for an attenuated wavelength, Hitschfeld and Bordan (1954) derived the analytical solution for the rain-rate profile as a function of the measured reflectivity profile. Their formulation has served as the basis for various discretization schemes aimed at a forward correction of the reflectivities from one range gate to the next along a radar tilt, starting from the radar site (see e.g., Meneghini 1978; Hilderbrand 1978). However, the practical application of such a forward approach gives rise to two major difficulties. The first difficulty lies in the numerical instability of the solution at short wavelengths (e.g., X band). The second difficulty pertains to the underdetermination of the studied problem. Haddad et al. (1995) proved that, depending on the values of the attenuation model’s coefficients, a given attenuated measured reflectivity profile can be generated by many different rain-rate profiles. In other words, the attenuation correction problem is underdetermined because the data (the attenuated reflectivities measured along a radar path) are not informative enough to determine the model parameters uniquely. Delrieu et al. (1999a) confirm this finding when they address the sensitivity of attenuation correction to drop size distribution coefficients and to an eventual radar miscalibration.

Since the early work in the 1950s, which led to abandoning the use of the X band in ground-based weather radar applications, the development of spaceborne and airborne radar projects has renewed interest in the use of attenuating wavelengths (X and K band). The choice of these bands is motivated by technical constraints with respect to both the size of the system being implemented and the measurement resolution. A number of original observation techniques [see Meneghini and Kozu (1990)
for a complete review] have accompanied these developments. Specific mention should be made of the surface reference technique (Meneghini et al. 1983) in which path-integrated attenuation (hereinafter denoted PIA) measurements are derived from surface returns. The availability of a PIA measurement at a given reference range provides two advantages:

1) it allows reformulating the attenuation equation in a way that is numerically stable for various backward-attenuation correction schemes, starting from the reference range (Marzoug and Amayenc 1994); and
2) it contributes to reducing, but not suppressing altogether, the underdetermined status of the attenuation correction (Haddad et al. 1995).

Delrieu et al. (1997) demonstrated the feasibility of applying the surface reference technique to a ground-based configuration with an X-band radar, using mountains as the reference targets. The accuracy of mountain-derived PIA was also assessed by means of a comparison with direct PIA measurements carried out with a receiving antenna located on a mountain ridge (Delrieu et al. 1999b). This approach makes it possible to determine an appropriate correction factor, which represents either a radar miscalibration or a bias in the reflectivity-attenuation coefficient relationship (Serrao et al. 2000). The use of polarimetric techniques is certainly the most promising approach to constrain attenuation corrections. Sauvageot (1996), Testud et al. (2000), or Matrosov et al. (1999) proposed various algorithms based on additional radar observables, such as the differential reflectivity and the differential phase shift.

Unfortunately, the availability of such PIA estimates or additional radar parameters does not yet represent a usual situation in an operational environment. In Europe, a complete networks of C-band radars (United Kingdom, France, Germany, Italy, Switzerland, Spain, Holland, etc.) is being operated and their radars deserve to be processed regarding this problem of attenuation. Therefore, the development of a robust correction method essentially based on the analysis of measured reflectivity profiles remains a need of users. Such a robust correction method for attenuation effects applicable in usual situation in an operational environment. In

2. Methodology description

This section presents both the problem posed by the retrieval of rain rate from attenuated radar reflectivity and the inverse method used to solve it. The attenuation model and the underdetermination of the rain-rate profile retrieval problem are described in section 2a. In section 2b, we review the main features of the inverse algorithm used to solve this retrieval problem, on the basis of the works of Menke (1989) and of Tarantola and Valette (1982a).

2a. The attenuation correction: Formulation of the problem

1) THE ATTENUATION MODEL

The formulation used hereafter was discussed by Delrieu et al. (1999a), who derived the attenuation equation as follows [see p. 58 of their paper, their (3)]:

\[ Z_m(r) = \delta CaR(r)^a \exp \left[ -0.46 \int_0^r cR(s)^d \, ds \right] , \tag{1} \]

where \( Z_m(r) \) is the reflectivity measured by the radar at range \( r \), \( R(r) \) is the corresponding rainfall rate, and \( \delta C \) represents a possible radar calibration error. The coefficients of the relations between the radar measurables and the rainfall rate values are \( a, b, c, \) and \( d \), as represented by power-law models with the following notations:

\[ Z = aR^b \quad \text{and} \quad k = cR^d, \tag{2} \]

where \( Z \) is the actual reflectivity (mm\(^6\) m\(^{-1}\)) and \( k \) is the attenuation coefficient (dB km\(^{-1}\)); \( Z \) and \( k \) depend on the wavelength, drop size distribution (DSD), and temperature of the raindrops.

The attenuation factor at distance \( r \), denoted \( A(r) \), is expressed as

\[ A(r) = \exp \left[ -0.46 \int_0^r k(s) \, ds \right] . \tag{3} \]

The term “attenuation” or “path-integrated attenuation” used herein refers to the quantity PIA\((r) \) (dB), defined as PIA\((r) = -10 \log[A(r)] \). The formulation by Delrieu et al. (1999a) guarantees the consistency of the relationships between reflectivity, attenuation coefficient, rainfall rate, and the underlying drop size distri-
bution model. The bulk variables are expressed as a function of the drop size distribution, as represented by the negative exponential model,
\[ N(D, R) = N_0(R) \exp[-\Lambda(R)D] \quad \text{and} \quad \Lambda(R) = \Lambda_1 R^{\Lambda_2}, \]
where \( N(D, R) \) is the function describing the drop size distribution, \( D \) is the raindrop diameter, and \( N_0, \Lambda_1, \) and \( \Lambda_2 \) are three coefficients. From this drop size distribution model, the bulk variables (reflectivity, attenuation rate, and rainfall rate) are modeled according to the following conditions:

1) the terminal velocity of raindrops is computed using Beard's model (1976) and
2) Mie's theory is applied for calculating backscattering and total attenuation cross sections.

For each rainfall rate \( R, N_0 \) is calculated so that rain-rate, as a bulk variable, and drop size distribution are consistent. It then becomes possible to compute both \( Z \) and \( k \) as a function of \( R \) and to deduce the coefficients of the \( Z-R \) and \( k-R \) relations in (2). As a result of the rain-rate constraint equation, the coefficients \( a, b, c, \) and \( d \) thus appear as functions of two (out of the three) coefficients of the DSD model. This approach is described in detail and justified in Delrieu et al. (1999a, see 58–59). As further work, it would be possible to introduce a more sophisticated parametrization of the relationships between \( Z, k, \) and \( R \) like the one proposed by Testud et al. (2001).

In this attenuation model, the parameters \( \Lambda_1 \) and \( \Lambda_2 \) summarize the rain microphysics and \( \delta C \) summarizes the radar working conditions. We assume that these parameters remain unchanged for the set of radar images forming a rain event. In other words, the radar calibration is assumed to be stable and the \( Z-k-R \) relations are assumed to be constant in time and space for a given rainfall event. The latter assumption is certainly coarse, however it is a standard assumption performed in many operational radar systems relying on reflectivity data alone. In addition, we believe that this simplification is second rate compared to the influence of the initial choice of the coefficients of the DSD and/or of the possible radar calibration error. However, an important issue in the future will be to study the possibility of variation in space and time of the raindrop size distribution and its influence.

In the absence of a simple analytical relationship between rain-rate profiles and attenuated measured reflectivity profiles, the attenuation model defined in (1) must be discretized to achieve a numerical solution. We describe it in the appendix. It should be pointed out that the expression “rain-rate (or reflectivity) profile” used hereinafter denotes a series of discretized rainfall rates (reflectivities) along a radar tilt.

2) The underdetermination of the problem

According to the values adopted for the parameters of the attenuation model (\( \delta C, \Lambda_1, \) and \( \Lambda_2 \)), different rain-rate profiles can generate the same attenuated measured reflectivity profile. The problem of the retrieval of the rain-rate profile from the attenuated measured reflectivity profile is underdetermined, as shown by Haddad et al. (1995).

Based on numerical simulation at X and C bands and on a case study at the X band, Delrieu et al. (1999b) gave some insights about the rain-rate profile retrieval problem. Basically they showed that a number of DSD parametrizations (and the related \( Z, k, R \) relations) may provide acceptable solutions in terms of rain-rate retrieval. In the space of the DSD model parameters (\( \Lambda_1, \Lambda_2 \)), also considered in the present paper, these acceptable parametrizations are organized on patterns whose shapes depend mostly on the wavelength and the corrected PIA. The radar calibration errors were also considered with evidence of the shifting effect on the patterns of the (\( \Lambda_1, \Lambda_2 \)) acceptable parametrizations. From this work, it was concluded that a possible strategy for the parametrization of rain-rate retrieval schemes at attenuating wavelengths is

1) to choose a DSD parametrization (and the resulting \( Z, k, R \) relations), and
2) to optimize a \( \delta C \) correction term representing both a radar calibration error and an eventual misrepresentativity of the DSD parameterization.

We will refer hereinafter to the optimized \( \delta C \) as “an equivalent calibration coefficient.” Note that a procedure for optimizing \( \delta C \) with mountain returns was further proposed by Serrar et al. (2000).

b. The inverse method

The inverse method is applied in two different but interdependent stages. The next paragraph deals with the retrieval of the rain-rate profile \( \mathbf{R} \). It is supposed that the optimal value of \( \delta C \) has yet been determined. The optimization procedure of \( \delta C \) within the inverse approach framework will be addressed in the next section.

1) Retrieval of the rain-rate profiles

On the basis of a given observed reflectivity profile, the problem then consists of retrieving the rain-rate profile, which best reconstitutes the measured reflectivities according to the theoretical attenuation model. The method used to approach such a problem is rooted in the inverse theory (Menke 1989; Rodgers 2001), with the algorithm proposed by Tarantola and Valette (1982a) being applicable herein. The solution process involves maximizing a likelihood function and then minimizing the following expression:
\[
\Phi(Z, R) = (Z - Z_m)C_z^{-1}(Z - Z_m)^T
+ (R - R_{\text{prior}})C_R^{-1}(R - R_{\text{prior}})^T
\]
(5)

where the function \( \Phi \) is a least squares criterion, \( m \) is the attenuation model defined in the appendix (A4), \( Z_m \) corresponds to the observed reflectivities, \( R_{\text{prior}} \) represents the a priori information on the rain-rate profile, \( C_z \) and \( C_R \) are the covariance matrices associated with the residuals \( (Z - Z_m) \) and \( (R - R_{\text{prior}}) \), respectively, and \( T \) stands for transpose. The statistical distributions \( (Z - Z_m) \) and \( (R - R_{\text{prior}}) \) are assumed to be normal. Menke (1989) demonstrates that the solution vector \( R' \) satisfies the following relation:

\[
R' = R_{\text{prior}} + C_p M^{-1}(M^T C_p M + C_z)^{-1}
\times [Z_m - m(R')] + M(R' - R_{\text{prior}})],
\]
(6)

where \( M \) is the matrix of partial derivatives of \( m \). If \( m \) is nonlinear, (6) can be solved using an iterative method, then

\[
R_{k+1} = R_{\text{prior}} + C_p M^{-1}(M^T C_p M + C_z)^{-1}
\times [Z_m - m(R_k) + M_s(R_k - R_{\text{prior}})],
\]
(7)
in which \( R_k \) is the result of the \( k \)th iteration and \( M_s \) is the matrix of partial derivatives at this point and \( R_{\text{prior}} \) has been chosen as the starting point of the iterative procedure. The method converges after three to four iterations. Numerically, we stopped the iterative process when the least squares criterion decreases by less than 5% from an iteration to another.

Let us consider various aspects of the studied problem, which impact both the application conditions of the inverse algorithm and the potential for reaching a reliable solution.

1) The a priori information (introduced in the terms \( R_{\text{prior}} \) and \( C_z \)) contains the initial knowledge of the parameters to be identified (i.e., the rain-rate profile) and also the level of confidence, which can be ascribed to this knowledge. The solution derived by the inverse algorithm is the result of a compromise between two extreme states: 1) a perfect fit of the observed data with the theoretical model, and 2) a solution very close to the a priori information on the parameters. The assistance provided by the a priori information actually depends on the nature of the problem itself. If the observed data are insufficient or if the level of confidence in the data is low, the problem becomes underdetermined and the a priori information assumes a dominant role. If the problem is overdetermined (availability of very high-quality data in sufficient quantity), the solution then no longer depends on the a priori information. As highlighted before, the correction of a rain-rate profile for attenuation is intrinsically an underdetermined problem. Consequently, special attention must be paid in the present context to the a priori information, which heavily influences the solution. The importance of the a priori information in initialing the identification procedure is enhanced by the nonlinearity of the attenuation model, as discussed below.

2) The nonlinearity of the model adds to the underdetermination and makes the problem of attenuation correction a difficult one to solve. The stability, convergence, and uniqueness of the solution to such nonlinear problems have been addressed in detail in Chapter 9 of Menke (1989). One particular assumption in the inverse method is for the function \( \Phi \) of (5) to be monotonously decreasing in the parametric space, between the initialization and the solution (Menke 1989). The nonlinearity of the model might invalidate this assumption, in which case local minima-related problems would interfere with the solution, as the inverse algorithm would converge on the relative minimum closest to the initial position. Sensitivity tests have confirmed that such a situation could indeed arise, indicating the uppermost importance of the a priori rain-rate profile used to initialize this algorithm. The initialization must be as close as possible to the solution. From this perspective, the assumption of a normal and unbiased distribution of residuals \( (R' - R_{\text{prior}}) \) takes on particular importance.

3. Application of the inverse algorithm

This section addresses the application of the algorithm for identifying rain-rate profiles. The first paragraph describes the data and associated matrices, while the second paragraph discusses the a priori parameterization. The inverse application protocol of the method is then presented in the third and final paragraph.

a. The data and associated covariance matrix

The data providing the basis for the attenuation correction are composed of the measured reflectivity profile, \( Z_m \), expressed in decibels. The covariance matrix...
of attenuated reflectivities ($C_z$) summarizes the statistical features of the measurement errors. As proposed by Haddad et al. (1996), these errors are assumed to be characterized by a noise term added to the data from a zero mean Gaussian distribution. The error on a measured reflectivity is presumed to be correlated in terms of distance. An exponential correlation model, consistent with (i) the Gaussian hypothesis and (ii) the definition of the covariance matrix of the a priori rain-rate profile, has been adopted in the following form:

$$\text{cov}[Z_m(i), Z_m(j)] = \sigma_z^2 \exp\left\{ -\frac{(r_i - r_j)^2}{D_z^2} \right\}. \tag{9}$$

where $Z_m(i)$ and $Z_m(j)$ are the measured reflectivities at distances $r_i = (i - 1)\Delta P$ and $r_j = (j - 1)\Delta P$, respectively, $\sigma_z$ is the standard deviation of the measurement error, and $D_z$ is the decorrelation distance. The parameter $D_z$ can be set to zero; the measurement errors are then supposed to be spatially uncorrelated.

b. The a priori rain-rate profile and associated covariance matrix

The a priori information can stem from different sources, such as physical observations or results of another inverse problem (Tarantola and Valette 1982b). Regarding the a priori rain-rate profile, a possible choice would be to adopt the apparent rain-rate profile, which is defined as the rain-rate profile deduced directly from attenuated reflectivities by applying a Z–R relationship (Meneghini 1978). In that case, rainfall rates are increasingly underestimated as the distance from the radar increases. The assumption of a zero mean Gaussian distribution for the residuals ($\mathbf{R} - \mathbf{R}_{\text{true}}$) is, therefore, clearly violated. Simulation tests have confirmed that choosing the apparent rain-rate profile because the a priori profile does not ensure finding the solution in a satisfactory manner. Within the context of this underdetermined problem, it would appear highly advisable to utilize unbiased a priori information, so that the mean value of the a priori rain-rate profile does not differ too greatly from the mean value of the actual rain-rate profile. Additionally, this strategy could help decrease the risk of facing local minima related to the nonlinearity of the model.

The above assessments incite seeking other solutions for defining the a priori rain-rate profile. A radar image is composed of a sequence of juxtaposed radar tilts, forming as many contiguous reflectivity profiles one after the other. The problem then consists of identifying a sequence of juxtaposed rain-rate profiles. When the identification of a given rain-rate profile is conducted, the previous (contiguous in space) profiles have, hence, already been identified and can be used as the a priori profile for the current identification. This initialization technique is valid provided that the distribution of the residuals between the a priori profile and the true profile is Gaussian and potentially unbiased. Testing this assumption is equivalent to studying the statistical distribution of the residuals between contiguous rain-rate profiles from radar data not influenced by attenuation. The residuals are defined by $e(i, j) = Z_m^0(i) - Z_m^{i+1}(i)$, with $Z_m^j(i)$ being the $i$th component of rain-rate profile $j$. Data from the “Cévennes 1986–88” experiment, discussed in section 4, have been used for this test. The applicable profiles are part of the image sector shown in Fig. 1. For the 60 rainfall images under consideration, the mean of the residuals is 0.02 mm h$^{-1}$ and the standard deviation is 4.7 mm h$^{-1}$. Figure 2 illustrates the statistical distribution of rain-rate residuals and confirms that this distribution is indeed unimodal, symmetric, and close to being Gaussian.

The assumption of a normal and unbiased distribution of residuals between the a priori and true parameters is, thus, plausible. In addition, this approach ensures the spatial consistency of successive rain-rate profiles. Its associated covariance matrix complements the a priori information. Practically speaking, this matrix has an important influence on the solution, insofar as it tends to constrain the components of the rain-rate profiles about their a priori value. As in the previous paragraph, data from the Cévennes 1986–88 program have served to characterize the covariance of residuals, thereby leading to the results illustrated in Fig. 3, which demonstrate that (i) the standard deviation of the residuals between two contiguous profiles is related to the mean rainfall rate of the profiles (Fig. 3a), and (ii) the spatial correlation of residuals along a profile can be represented by an exponential function (Fig. 3b). According to these results, the covariance of the a priori rain-rate profile components can be expressed as follows:

$$\sigma_R^2 = AR + B,$$

$$\text{cov}[R(i), R(j)] = \sigma_R^2 \exp\left\{ -\frac{(r_i - r_j)^2}{D_R^2} \right\}. \tag{10}$$

where $\sigma_R$ is the standard deviation of the rain-rate profile component, $\bar{R}$ is the mean value of the a priori rain-rate profile, $A$ and $B$ are two parameters, $R(i)$ and $R(j)$ are two components of the rain-rate profile located at ranges $r_i$ and $r_j$, $D_R$ is the decorrelation distance, $A$ is a proportionality coefficient, and $B$ is introduced in order to maintain a strictly positive standard deviation. In the considered case study, the values obtained are $A = 0.4$, $B = 0.05$, and $D_R = 2$ km.

c. Application of the inverse method

The inverse algorithm will be applied to the attenuated radar images, according to the following steps:

1) An initial set of coefficients ($\delta C$, $\Lambda_1$, $\Lambda_2$) is chosen. Here, $\delta C$ can be initialized according to the results of a previous rain event or according to information available on the radar calibration (for instance resulting from the comparison of radar and rain gauge...
data). The coefficients of the drop size distribution can be affected according to the precipitation, for example, $\lambda_1 = 41.0$ and $\lambda_2 = -0.21$ (Marshall and Palmer 1948) for a stratiform rainfall, and $\lambda_1 = 30.0$ and $\lambda_2 = -0.21$ for a more convective event (Joss and Waldvogel 1969).

2) A reflectivity profile in a direction of low rain-rates is selected. The apparent rain-rate profile serves as the a priori profile. Starting the identification in the sector that is the less affected by attenuation ensures the proper initialization of the procedure. The identification of the rain-rate profile can then be performed by considering the contiguous azimuth. The identification procedure begins with the result from the previous identification as the a priori profile. The subsequent profiles are then identified by successively considering every radar tilt since the beginning. The result of the $(k-1)$th rain-rate identification becomes the a priori profile for the $k$th rain-rate profile identification. Once all of the profiles of the radar image are identified, the global likelihood criterion is calculated according to (8).

3) The value of $\delta C$ is modified and stage 2 is performed again with this modified value. The minimization of the global likelihood criterion can be performed with simple procedures: gradient or dichotomy. The overall identification protocol is summarized in Table 1.

4. Sensitivity analysis

The sensitivity analysis helps define the application conditions for the proposed attenuation correction method. This section starts with a brief presentation of the set of observed data used in this work. The principle behind the sensitivity analysis is discussed in the second paragraph. The third paragraph addresses the influence of the statistical parameters introduced in the retrieval of the rain profiles, assuming that the three coefficients $(\delta C, \lambda_1, \lambda_2)$ are perfectly known. The last paragraph
focuses on the optimization of the coefficient ($\delta C$) and addresses the influence of the drop size distribution in order to validate the proposed procedure.

### a. Presentation of radar data

The radar images used in this work have been taken from the Cévennes 1986–88 experimental project (Andrieu et al. 1997). Four intense rain events were recorded by the S-band Anatol weather radar system (Pointin et al. 1988), which totaled 120 recording hours representing different points of ground rain amounts over 400 mm. The Anatol radar has the following characteristics: a 3-dB beamwidth of 1.8° and a peak power of 250 kW. During the experiment, the radar operated at two alternating elevation angles: 1.0° and 3.0°. The angular resolution of a radar tilt is about 2° in azimuth. This dataset has already been carefully validated (Creutin et al. 1997). We assume that the radar images can be used to produce the actual rainfall fields. Seventy rain-rate profiles within azimuth interval 30°–170°N have been used for both the sensitivity tests and evaluation procedure (see Fig. 1b). This dataset provides immediate availability of contiguous rain-rate profiles for a series of radar images.

### b. Presentation of the sensitivity analysis

A sequence of 70 contiguous rain-rate profiles, extracted from a radar image recorded at 0812 UTC 5 October 1987 by the Anatol weather radar, has been examined. It has been verified that the number of profiles is large enough to correctly assess the accuracy of the identification procedure. The radar image itself is presented in Fig. 1b. The mean value of the rainfall rates along these 70 profiles is 11 mm h$^{-1}$ with a standard deviation of 3 mm h$^{-1}$. This rate is intense enough to test the method for a large attenuation range (ranging from 6–21 dB for a wavelength of 3.2 cm at a distance of 60 km).

The sensitivity analysis is performed as follows:

1) Attenuated reflectivity profiles are simulated from the observed rain-rate profiles as described by Delrieu et al. (1999a). Each profile encompasses a total of 60 radar reflectivity values, with a spatial resolution of 1 km. Observation errors are then simulated by adding a Gaussian random noise to the computed reflectivities. The measured reflectivity can then be written as $Z^*_m(i) = Z_m(i) + \varepsilon_i$, where $\varepsilon_i$, the observation error, has been taken from a Gaussian distribution $N(0, \sigma_i)$. The standard deviation of the observation error ($\sigma_i$) maintains the same value over the entire set of rain-rate profiles.

2) The inverse method is then applied to the 70 attenuated reflectivity profiles according to the protocol described in section 3c.

3) The results are evaluated by comparing the identified profile with the initial profile, whereby the latter represents the true rain-rate profile. This step is carried out using the mean absolute deviation (hereinafter denoted MAD) as the likeness criterion, such that

$$MAD = \frac{1}{n_p n_z} \sum_{j=1}^{n_p} \sum_{i=1}^{n_z} |R^*(i) - R(i)|$$

where $R^*(i)$ and $R(i)$ are the $i$th component of the $j$th identified and true rain-rate profiles, respectively, $n_p$ is the number of rain-rate profiles considered, and $n_z$ is the number of values per profile.

Table 2 lists the default values adopted for this sensitivity analysis. The simulated attenuation represents an X-band weather radar (wavelength: 3.2 cm). The drop size distribution of the measured rainfall corresponds to a stratiform precipitation ($\Lambda_1 = 41.0$ and $\Lambda_2 = -0.21$) and provides the coefficients for both the $Z-R$ ($a = 184$, $b = 1.64$) and $k-R$ relationships ($c = 0.0060$, $d = 1.30$). The radar is assumed to be perfectly calibrated ($\delta C = 0$).
The standard deviation of the measurement noise is $\sigma_z = 0.5$ dBZ, which corresponds to 70 independent pulses for each radar measurement.

The identification of rain-rate profiles is conducted with the following parameters. It is assumed that the standard deviation of the reflectivity measurement errors is $\sigma_z = 1$ dB, with a decorrelation distance of $D_z = 1$ km. The coefficients of both the $Z$–$R$ and $k$–$R$ relationships are perfectly chosen and the radar calibration is also supposed to be perfect. The standard deviation of the rain-rate residuals is chosen to be proportional to the mean a priori rainfall rate, with $A = 0.5$ and $B = 0.1$. Last, the corresponding decorrelation distance is 2 km. Figure 4 shows the identification results obtained under these standard conditions for the first and fifteenth rain-rate profiles. In Fig. 4a, the a priori rain-rate profile is generated by the apparent profile and proves to be biased. It seems that the inverse algorithm is not able to fully compensate for the initial bias between the a priori and the true rain-rate profile. Figure 4b illustrates a more general case. The a priori rain-rate profile is obtained from the adjacent azimuth. In Fig. 4b, the initial bias (i.e., difference between a priori rain-rate profile and true rain-rate profile) appears more randomly distributed. It is clear that the algorithm performs better here than in the previous case and does yield satisfactory results.
c. Sensitivity to the a priori information on data and parameters

1) SENSITIVITY TO THE COVARIANCE OF DATA

The covariance of attenuated reflectivities depends on two parameters: the standard deviation $\sigma_z$ and the decorrelation distance $D_z$. Figure 5 shows the MAD criterion for values of the couple $(\sigma_z, D_z)$, which vary within realistic limits. The identification method appears only moderately sensitive to the decorrelation distance and more sensitive to the standard deviation. The choice of $D_z = 1$ km and $\sigma_z = 1$ dB seems to be a good compromise for attenuated reflectivities affected by a low level of noise ($\sigma_b = 0.5$ dB). Similar tests have been performed for attenuated reflectivities with higher noise (up to $\sigma_b = 4$ dB). Results obtained are numerically stable. Nevertheless, it does appear important for the a priori standard deviation of the measurement error to be consistent with the standard deviation of the actual error.

2) SENSITIVITY TO THE COVARIANCE OF THE A PRIORI RAIN-RATE PROFILE

The standard deviation of the residuals between the a priori rain-rate profile and the true profile introduces parameters $A$ and $B$. The main purpose of parameter $B$ is to maintain the standard deviation as being strictly positive; a value of $B = 0.1$ is used. Parameter $A$ serves to adapt the standard deviation to the current a priori rain-rate profile. The definition of covariance requires one additional parameter, the decorrelation distance $D_R$. The influence of $A$ and $D_R$ on the accuracy of the attenuation correction is illustrated in Fig. 6, which shows that the algorithm’s effectiveness is sensitive to both of these parameters. For the decorrelation distance, $D_R = 2$ km appears to be an appropriate choice. The value of $A$ must be greater than 0.3.
d. Influence of the drop size distribution coefficients and radar miscalibration

This section is dedicated to the procedure proposed to determine the equivalent calibration coefficient \( \delta C \), introduced in section 2a(2)) and to compensate for an eventual erroneous choice of the drop size distribution coefficients. Let us recall that an improper choice of drop size distribution coefficients or a radar miscalibration can lead to rain-rate profiles very different from the true profiles. We want to verify if the optimization of the equivalent calibration coefficient makes it possible to compensate for this influence and contributes to improve the robustness of the proposed attenuation correction method. It is important to emphasize that the approach is made possible thanks to the underdetermination of the attenuation correction problem, and that the resulting equivalent calibration coefficient is representative of both the radar miscalibration and the adaptation of the initial DSD coefficients.

1) Optimization of the equivalent radar calibration coefficient

The objective of this test is to verify that the overall identification procedure summarized in section 3e allows retrieving the true radar calibration coefficient in case of a perfect choice of the drop size distribution coefficients. Attenuated reflectivity profiles are simulated using a “true” radar calibration coefficient. We considered three different values: 0.8, 1.0, and 1.2, respectively. An initial value of \( \delta C \) is chosen and the rain profiles are identified from these attenuated reflectivity profiles leading to the global likelihood criterion \( \Psi(\delta C) \). Figure 7a illustrates the sensitivity of this criterion to the coefficient \( \delta C \). It appears very clearly that the maximum likelihood is reached for the true value of the calibration coefficient. We observed that an inaccurate choice of \( \delta C \) statistically increases the relative (with respect to the average rain-rate of the profile) difference between the contiguous identified rain-rate profiles. As a consequence, \( \Psi(\delta C) \) increases because the difference between the identified and a priori parameters increases [see (5) and (8)]. This test confirms the ability of the identification procedure to retrieve the radar calibration coefficient if the DSD coefficients are perfectly chosen. Figure 7b displays the variations of the MAD between true and identified rain-rate profiles versus the evolution of the calibration coefficient. It shows that the best retrieval (minimum of MAD) is obtained for the true (and correctly identified) value of \( \delta C \).

2) Influence of the drop size distribution coefficients

Let us recall that the DSD coefficients \( (\Lambda_1, \Lambda_2) \) serve to calculate the coefficients of both the \( Z-R \) and \( k-R \) relationships. Unfortunately, the exact values of these coefficients are not known and the adopted values are liable to be erroneous. This negative influence could be corrected thanks to the optimization of the equivalent calibration coefficient. The purpose of the paragraph is to verify if the proposed algorithm for attenuation correction can deal with erroneous values of the DSD coefficients. The evaluation test has been conducted as follows. The attenuated reflectivity profiles are simulated using different values of the coefficient couple; \( \Lambda_1 \) varies from 36 to 46, and \( \Lambda_2 \) varies from \(-0.23 \) to \(-0.19 \). These variation intervals include very distinct rainfall situations: \( \Lambda_1 = 35, \Lambda_2 = -0.195 \) for the Cévennes precipitations (Delrieu et al. 1997); \( \Lambda_1 = 50, \Lambda_2 = -0.21 \) for drizzle (Joss and Waldvogel 1969); and \( \Lambda_1 = 36, \Lambda_2 = -0.14 \) for oceanic precipitations (Savaveot and Lacaux 1995). Rain-rate profiles are identified using the default coefficients listed in Table 2, that is in assuming that the DSD coefficients are \( \Lambda_1 = 41 \) and \( \Lambda_2 = -0.21 \). Two cases are considered:

1) First, \( \delta C \) remains constant and equal to 1. Figure 8,
which displays the variations of the MAD between true rain-rate profiles and identified rain-rate profile, summarizes the effectiveness of the identification of rain-rate profiles as a function of the DSD coefficients. It confirms that these coefficients have a significant influence and that the identification effectiveness degrades as much as the initial values of the DSD coefficients differ from the true ones.

2) The equivalent calibration coefficient is optimized according to (8). Figure 9 illustrates the obtained results. Figure 9a shows the equivalent calibration coefficients obtained. Figure 9b shows that the MAD criterion remains in any case very close to the value reached when the identification is performed with the true DSD parameters. The optimization of the equivalent calibration coefficient allows for compensation of the influence of an improper choice of the raindrop size distribution coefficients. It contributes to improve the robustness of the proposed attenuation correction method.

5. Evaluation of the proposed method: A simulation exercise

The objective of this section is to test the proposed attenuation correction algorithm, considering various examples of rainfall field. Such an evaluation is performed by comparing the proposed method to classical attenuation correction methods and, more specifically, to the Hitschfeld–Bordan algorithm. It takes on the form of a simulation exercise applied to actual rain-rate profiles. The first paragraph describes the principle behind this evaluation procedure, and the second paragraph presents the results.

a. Evaluation principle

Forty rainfall fields recorded during the Cévennes 1986–88 experiment have been selected on the basis of both intensity and rainfall variability. These fields are illustrated in Fig. 1b, which shows an intense and spatially homogeneous field, whereas the field in Fig. 1a displays stronger spatial variability and contains high-intensity rain cells. As described previously (see section 4a), we extracted 70 rain-rate profiles from each image. These observed rain-rate profiles are then used to simulate attenuated reflectivity profiles for a wavelength of 3.2 cm. This simulation is performed with the following radar calibration and DSD coefficients: \( \delta C = 1.05 \) (equivalent to a 0.2-dBZ overestimation of the measured reflectivity), and \( \Lambda_1 = 40, \Lambda_2 = -0.22 \), respectively. A maximum range of 60 km has been considered. Table 3 indicates the distribution of the PIAs associated with the 2800 simulated profiles of attenuated reflectivity. It reveals that 47% of these radar profiles have a simulated
PIA below 10 dB, whereas about 17% of the simulated PIA’s are higher than 20 dB.

The following attenuation correction procedures have been applied to retrieve the initial rain-rate profiles:

1) “ZR” method: Neglecting attenuation, the rain-rate profiles are directly deduced from radar echoes at attenuated wavelengths with a simple Z–R relationship.

2) “HB1” method: The rain-rate profiles are estimated using the algorithm proposed by Hitschfeld and Bordan (1954). In this case, the rainfall rates are obtained from the following formula:

\[ R(r) = \left( \frac{Z_w(r)}{a} \right)^\frac{1}{b} \times \left\{ C_{db} - 0.46 \frac{c}{b} \int_{0}^{L} \left[ \frac{Z_w(s)}{a} \right]^d ds \right\}^{-1/d}. \]  

3) “HB2” method: The Hitschfeld–Bordan algorithm is adapted in order to control eventual numerical instabilities. As suggested in Delrieu et al. (1999a), the PIA undergoing the HB algorithm’s correction is limited to a maximum value of 10 dB.

4) “INV” method: The rain-rate profiles are estimated using the proposed inverse method applied in accordance with the conditions defined in section 3c (see Tables 1 and 2).

All of the tested methods have been applied under the standard conditions laid out in Table 2. In particular the values of the DSD coefficients are \( \Lambda_1 = 41 \), \( \Lambda_2 = -0.21 \). Without attenuation effect, the radar calibration as well as the choice of DSD coefficients would lead to an overestimation of rainfall rates. The resultant accuracy is then evaluated using two criteria:

- The percentage of rainfall rate profiles affected by numerical instabilities, denoted “PC_{unstab}” is used. A correction is considered unstable if the mean rainfall rate of the profile exceeds 30 mm h\(^{-1}\). This threshold has been established with the knowledge that the mean value of every true rain-rate profile is below 23 mm h\(^{-1}\).

- The mean absolute deviation ([11]) between the retrieved rainfall rates and the true values is calculated for retrieved rain-rate profiles whose mean value does not exceed 30 mm h\(^{-1}\).

### Table 3. Importance of the attenuation quantified by the PIA’s values for the 3.2-cm wavelength.

<table>
<thead>
<tr>
<th>PIA values</th>
<th>10 &lt; PIA</th>
<th>&lt; 20 dB</th>
<th>&lt; 30 dB</th>
<th>&gt;30 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of profiles</td>
<td>47</td>
<td>36</td>
<td>14</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 4. Influence of the PIA on the effectiveness of the different attenuation correction procedures in term of the MAD criterion.

<table>
<thead>
<tr>
<th>MAD (mm h(^{-1}))</th>
<th>All values</th>
<th>0 &lt; PIA &lt; 10 dB</th>
<th>10 &lt; PIA &lt; 20 dB</th>
<th>20 &lt; PIA &lt; 30 dB</th>
<th>PIA &gt; 30 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZR</td>
<td>3.60</td>
<td>1.01</td>
<td>4.50</td>
<td>8.6</td>
<td>10.4</td>
</tr>
<tr>
<td>HB1</td>
<td>1.54*</td>
<td>0.81*</td>
<td>3.74*</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>PC_{unstab} = 36%</td>
<td>PC_{unstab} = 4%</td>
<td>PC_{unstab} = 44%</td>
<td>PC_{unstab} = 100%</td>
<td>PC_{unstab} = 100%</td>
</tr>
<tr>
<td>HB2</td>
<td>2.1</td>
<td>0.77</td>
<td>2.23</td>
<td>5.16</td>
<td>7.46</td>
</tr>
<tr>
<td>INV</td>
<td>1.41</td>
<td>0.56</td>
<td>2.10</td>
<td>2.27</td>
<td>2.11</td>
</tr>
</tbody>
</table>

* The criterion MAD is calculated without accounting for the divergent profiles.
seen that the performance of the INV algorithm is not very sensitive to the PIA of the profiles (see Table 4), whereas the performance of the Hitschfeld–Bordan algorithm (both HB1 and HB2) is severely degraded as the PIA of the radar profiles increases. This finding indicates that the correction of the attenuation using an inverse algorithm is more suited to cases of strong attenuation than the Hitschfeld–Bordan algorithm.

6. Evaluation of the proposed method: A case study

In this section, the evaluation of the proposed approach is completed using actual attenuated radar data. The first paragraph presents the data, the second one the results.

a. Presentation of the radar data

The correction attenuation method is applied to the rain event that occurred on 23 September 1993 in the vicinity of the city of Marseille, located in the south of France, during the Marseille hydrometeorological experiment. A detailed presentation can be found in Delrieu et al. (1997). During an 18-month period, an X-band radar system covered the city at ranges from 5 to 15 km. The main radar characteristics are: peak power of 25 kW, 3-dB beamwidth of 1.8°, radial resolution of 500 m, and incremental azimuth of 1.4°. The radar was operated according to a two-elevation scan procedure: 1.0° and 5.0°. The first elevation angle allows detecting the relief surrounding the city (Fig. 11). The last elevation angle is free of ground clutters. The radar operated three sweeps during a 6-min time step, corre-
sponding to the rain gauge data resolution. Concerning rainfall measurement, the city of Marseille is equipped with a network of 25 rain gauges that are distributed over a surface area of 230 km² (see Fig. 11).

The studied event is dominated by convective rain cells moving north. It lasted 2 h with an intense period of 1 h. The maximum 2-h rain accumulation reached 30 mm, and rain rates as great as 60 mm h⁻¹ over a 6-min period have been observed at different gage stations. Figure 12 provides two reflectivity maps illustrating this rain event. Using the mountain reference technique (MRT, Delrieu et al. 1997), the effective calibration coefficient of the radar was estimated to δC = 1.48, that is, a 1.7-dBZ overestimation of the measured reflectivities when the following DSD parametrization was considered: \( \Lambda_1 = 41.0, \Lambda_2 = -0.21 \). The mountain-measured PIAs reached maximum values of about 15 dB over a distance of 15 km or so.

### b. Results

The evaluation of the proposed method has been performed according to the protocol defined in section 5, by comparing the different correction techniques. The attenuation correction is applied to the upper-elevation radar profiles included in the azimuth sector (70°, 330°), which regroups most of the rain gauges as illustrated by Fig. 11. The HB1 and HB2 algorithms are applied with the following coefficients: \( \delta C = 1.0, \Lambda_1 = 41.0, \) and \( \Lambda_2 = -0.21 \). Using the same DSD parametrisation, the inverse method leads to an equivalent radar calibration coefficient \( \delta C = 1.30 \), that is, a 1.14-dBZ overestimation of the measured reflectivities, a value consistent (though significantly lower) with the MRT estimate. In this case study, rain gauge data (available at the 6-min time step) serve as reference rain-rate measurements.

Figure 13 displays the scatterplot of radar versus rain gauge rain rates and the MAD criterion for each of the compared correction method is displayed in Table 5. The classical HB1 algorithm induces unacceptable prob-
lems of numerical instability because of the overestimation of the measured reflectivities. The limitation of the correctable PIA (HB2) offers a rather satisfactory solution in the present case owing to the range of observed PIAs. Last, the INV algorithm provides the best estimates in terms of the MAD criterion. Note that in Fig. 13d a small trend to underestimate the highest rain rates certainly related to the light underestimation of the effective radar calibration coefficient. These results are encouraging insofar as they indicate that an attenuation correction performed according to the proposed method provides better estimates than classical methods.

### Table 5. Comparison of different correction methods for the attenuation correction; 240 pairs of rain gauge–radar data at a 6-min time step are used to calculate the MAD.

<table>
<thead>
<tr>
<th>Method</th>
<th>ZR</th>
<th>HB1</th>
<th>HB2</th>
<th>INV</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD (mm h⁻¹)</td>
<td>5.21</td>
<td>102.37</td>
<td>4.61</td>
<td>4.28</td>
</tr>
</tbody>
</table>

### 7. Conclusions

In this paper, the correction of radar reflectivities for the rain-attenuation effect has been formulated as an inverse problem. The inverse identification of rainfall rates from attenuated reflectivity measurements needs a specific application protocol. In effect, the attenuation correction encounters two major difficulties: (i) the nonlinearity of the theoretical model; and (ii) the underdetermination of the problem, a condition that renders the solution heavily dependent upon the a priori information. Application conditions, which are consistent with the hypotheses imposed by the inverse method, have been defined. A sensitivity analysis has been performed in order to parameterize the inverse method and obtain information on its potential utility. This sensitivity analysis has been complemented by an evaluation based on both a simulation exercise and a real-world case study. This evaluation is performed by comparing
The following expression for the attenuation model is ultimately obtained:

\[ Z_m(i) = 10 \log \left( \delta CaR(i)^6 \frac{\exp \left[ -0.46 \sum_{j=1}^{i-1} cR(j)^6 \Delta P \right] - \exp \left[ -0.46 \sum_{j=1}^{i} cR(j)^6 \Delta P \right]}{0.46cR(i)^6 \Delta P} \right). \]  

\[ A(i) = \frac{1}{\Delta P} \int_{(i-1)\Delta P}^{i\Delta P} \exp \left[ -0.46 \int_0^r cR(s)^4 ds \right] dr. \]  

APPENDIX

Discretization of the Attenuation Model

The measured reflectivity profile in a given direction is discretized according to a spatial resolution \( \Delta P \) and is composed of \( n \) different values grouped into the vector \( Z_n \), with \( Z_n(i), i = 1, n \). The value \( Z_n(i) \) represents the attenuated reflectivity associated with the distance increment \( (i - 1)\Delta P, i\Delta P \). The rain-rate profile is then defined in a similar way and can be represented by the vector \( R \), \( [R(i), i = 1, n] \), where \( R(i) \) is the rain-rate corresponding to the attenuated reflectivity \( Z_n(i) \).

The discretization of the attenuation model is based on an evaluation of the attenuation factor over a given element of the discretized rain-rate profile. The attenuation factor \( A(i) \) associated to \( Z_n(i) \) is the averaged path-integrated attenuation over the \( i \)th element. The path-integrated attenuation over the \( i \)th element lies between the path-integrated attenuation at distance \( (i - 1)\Delta P \) and the path-integrated attenuation at distance \( i\Delta P \). The attenuation factor \( A(i) \) is then expressed as

\[ A(i) = \frac{1}{\Delta P} \int_{(i-1)\Delta P}^{i\Delta P} \exp \left[ -0.46 \int_0^r cR(s)^4 ds \right] dr. \]  

because we have

\[ \exp \left[ -0.46 \int_0^{(i-1)\Delta P} cR(s)^4 ds \right] = \exp \left( -0.46 \sum_{j=1}^{i-1} cR_j^4 \Delta P \right) \]

\[ \exp \left[ -0.46 \int_0^{i\Delta P} cR(s)^4 ds \right] = \exp \left( -0.46 \sum_{j=1}^{i} cR_j^4 \Delta P \right). \]  

A good approximation of \( A(i) \) is then

\[ A(i) = \frac{\exp \left[ -0.46 \sum_{j=1}^{i-1} cR(j)^6 \Delta P \right] - \exp \left[ -0.46 \sum_{j=1}^{i} cR(j)^6 \Delta P \right]}{0.46cR(i)^6 \Delta P}. \]  

Acknowledgments. This work was supported by the Commission of the European Communities, DG XII, Environment and Climate Program, Project HYDROMET, Contract ENV 4CT 960290. The authors thank three anonymous reviewers who provided very helpful comments and suggestions to improve the paper.
where, for the sake of convenience, the measured reflectivity \( Z_m(i) \) is expressed in dBZ.

**REFERENCES**


