SCATTERING OF SOLAR ENERGY BY CLOUDS OF "LARGE DROPS"

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ABSTRACT

For the case when absorption is negligible, the direct and scattered solar beams have been followed down into clouds of "large" spherical waterdrops, to calculate the distribution of the generation of diffuse energy in the cloud. The generated diffuse energy is considered to obey a simple diffusion equation, from which the cloud albedo has been computed as a function of the cloud thickness, the mean free path, and the sun's zenith distance. One of the main results is the marked variation of albedo with zenith distance, especially for "thin" clouds. The results are compared with observations, and it appears that the dissipation of stratus clouds is often accompanied by a reduction in the effective scattering radius of the cloud drops.

1. Introduction

Calculations of the scattering of solar energy by clouds are very complex. Yet, to assess the radiative effects of clouds, one must estimate not only the scattering but also the absorption of solar energy by clouds. Here we shall limit the discussion to scattering only, leaving the absorption of solar energy for a subsequent communication. Scattering without absorption would be closely applicable to visible and ultraviolet light.

2. A physical picture of scattering

A monochromatic beam of light may be thought of either as "energy" flowing along the beam, or as a vast number of photons moving swiftly along the beam. When a parallel beam of photons falls on a "thin" layer of cloud of "large" droplets, a portion will pass through the layer without encountering any drops; these photons are transmitted as a direct beam. The remaining photons will collide with droplets.

Those photons which have collided with a single waterdrop will suffer changes in direction, $\theta$, from less than 1 to 180 deg, but most of the scattered photons will retain a forward velocity component. Further, the distribution of the scattered photons per unit solid angle has a maximum at zero degrees (almost no deviation from the original direction), and the number scattered decreases rapidly as the angle increases from the forward direction ($\theta = 0$). These physical facts, together with some simplifying assumptions, are presented schematically in fig. 1, which shows a scattering pattern around a waterdrop. If we consider the drop as a point, the incident photons are scattered with intensity (photons per unit solid angle per unit time) $I(\theta)$. Outside a circle whose points lie 60 deg from the forward direction, the number of photons emerging at every point of the drop's surface is relatively small, and has nearly a constant value $I_d$; in the region outside the 60-deg circle, we shall assume the flow of scattered photons to be nearly isotropic.

In the region inside the 60-deg circle, the number of photons, $I(\theta)$, emerging through each point of the drop's surface, is relatively large and increases as $\theta$ approaches 0. At each point in this zone, subtract the constant number, $I_d$, from $I(\theta)$. This enables us to suppose that everywhere on the drop's surface a constant number of photons is entering radially from each point. In addition, there will still be the remaining photons proceeding forward through the drop's surface inside the 60-deg circle.

It will be convenient, therefore, to define three terms. The first is the "direct" parallel beam; the scattered photons are...
second is the beam of photons “scattered forward” between $\theta = 0$ and $\theta = 60$ deg; and third is the “diffuse,” nearly isotropic photons which radiate in all directions from the drop.

3. Review of previous work

The scattering and absorption of solar energy by clouds have been studied by many writers, and recently by Hewson (1943), Mecke (1944), Bullrich (1948) and Neiburger (1949). They all assume an upward and a downward flowing current of solar energy, each current undergoing scattering and absorption. When the light is actually diffuse, as in the interior of a deep cloud, and if the absorption is zero, it may be valid to assume a “two-stream” approximation with a suitably selected scattering coefficient (Hulburt, 1943). But near the cloud top, the energy is not diffuse; the direct, scattered and diffuse portions of the incident beam are scattered differently, so that a “suitable” scattering coefficient cannot be selected and, in general, the two-stream assumption with a single scattering coefficient breaks down. Following the work of Dietzius (1922) and of Albrecht (1933), advocates of the two-stream method have assumed that the selected scattering coefficient was valid only when the sun’s zenithal distance, $Z$, was $67$ deg. This assumption, even if tenable, has thus effectively eliminated the question of the variation of the albedo and absorption with $Z$. We shall find that the variation of the albedo of “thin” clouds with $Z$ is indeed very large, in marked contrast to Hewson’s results. Further difficulty with the “two-stream” approximation is encountered when absorption is not negligible, as will be discussed in a subsequent paper.

Faigelson (1951) has studied scattering in stratus by solving the correct “energy transfer” equation, in which he used a three-term expression to approximate the scatter by drops. His scattering pattern gives too much scatter toward the sides and not enough scatter forward. The numerical albedos which he obtained were thus too high, as Faigelson himself points out.

4. Method of attack

In view of these difficulties, another method, adhering closely to the physical processes, seems desirable. It seems appropriate, from the physical point of view, to see where the direct beam “generates” diffuse energy (or photons), and then to study the diffuse energy by means of the well-known diffusion equation. Indeed, Langmuir and Westendorp (1931) used a diffusion equation, and the concept of the mean free path, in a study of diffusion of light in clouds. In the case of sunlight, however, the diffusion equation does not apply to all the solar energy found near the cloud “top.”

The present method of attack, with absorption zero, will be to follow the direct and “forward scattered” beams down through the cloud, extracting from them the diffuse energy as it is generated. To the diffuse energy generated, we apply the diffusion equation.

5. Scattering pattern of a single drop

Houghton and Chalker (1949) show that the scattering area coefficient, $K_s$, for waterdrops varies markedly with the parameter $x = 2\pi r/\lambda$, $r$ being the radius of the drop, and $\lambda$ being the wavelength of the light; but as $x$ increases, $K_s$ approaches 2 as a limit. It is therefore often assumed that, for $x \gg \lambda$, the scattered light may be considered as two independent parts: one part which has been scattered by reflection plus refraction, and a second part which has been scattered by diffraction around the drop. The total energy scattered in each of the two parts is about equal to the energy intercepted by the geometric cross-section of the drop.

For very large drops, the portion of the energy which is diffracted is scattered almost entirely into a very small angular distance about the forward direction. Therefore, to a close approximation, the diffracted light may be omitted from the scattered light, but instead included with the direct beam. We take $K_s = 1$ and assume scattering occurs only from reflection and refraction by the drop.

For some clouds, this “large drop” assumption appears valid. For example, with the drop-size distributions published by Neiburger (1949), the amount of solar energy reflected by a unit volume of drops is only a few per cent smaller than is actually the case if we consider all the drops to be large ($x \geq 40$). In clouds where the fraction of small drops is great, such as in some cumulus clouds, the large-drop approximations may not be valid.

![Fig. 2. Intensity of light scattered by large waterdrop, excluding diffraction, according to Wiener. For $\theta > 60$, use right-hand ordinate.](image-url)
With the energy intercepted by the cross-sectional area of a "large" drop taken as unity, the intensity of light, \( I(\theta) \), is given according to Wiener (1907) in fig. 2. The experimental verification of Wiener's scattering pattern is discussed by Bricard (1943); in general, the agreement is good.\(^3\)

On the theoretical side, a comparison of Wiener's pattern with calculations according to the Mie theory, as given by Gumprecht et al (1952), for \( x = 40 \) shows that the agreement is satisfactory. For \( x = 40 \), the diffracted energy is concentrated almost entirely between \( \theta = 0 \) and 10 deg, so that the deflection from parallel light is small enough to be negligible in some cases. When \( x > 40 \), as will generally be the case in stratus clouds and sunlight, the assumption which disregards diffraction should become even better. If one is to choose a single scattering pattern, at present Wiener's seems most applicable for clouds of large drops.

6. Scattering in a cloud of particles

To apply Wiener's scattering pattern, imagine a cloud of spherical waterdrops with smooth boundaries, infinite in horizontal extent; assume, for the present, that the solar beam is perpendicular to the cloud. Imagine, also, a layer of thickness \( \Delta z \), so thin that those photons from the original beam which intercept drops at all are scattered only once. Ideally, the thickness of such a "thin" layer would be infinitesimal. For computational purposes, we adopt for \( \Delta z \) a layer whose thickness is \( L/4 \), where the mean free path, \( L \), of the light through the cloud is

\[
L = 1/(\sum N_i \pi r_i^2) .
\]

\( N_i \) is the number of drops with radius \( r_i \) per cubic centimeter, and the summation is over the drop sizes. The assumption that only one collision occurs even in a distance \( L/2 \) will not introduce a serious error.

With this cloud model, Wiener's scattering pattern, and the assumption that all energy scattered into the zone 60 to 180 deg with the vertical was isotropic (see fig. 1), the direct and "forward-scattered" energies were traced\(^4\) down through the cloud. From both the direct beam and the forward-scattered energies the diffuse-generated energy was extracted, so that finally the distribution of diffuse energy generated in the cloud was obtained. To treat the diffusion of the generated energy mathematically, an analytic expression was fitted by inspection. It was found that

\[
n_{\text{diff}} = 0.02 (4/L)^2 z e^{-0.565/L} \frac{\text{al}}{\text{vol}}
\]

was fitted fairly well. Here \( n_{\text{diff}} \) is the diffuse energy generated per unit volume per unit time, and \( z \) is the depth in the cloud, measured from \( z = 0 \) at the cloud top.

The diffusion equation.—For steady-state diffuse photons, in the absence of absorption and if \( L \) is constant, the diffusion equation is

\[
- D \frac{dn}{dz^2} = n_{\text{diff}} ,
\]

where \( n \) is the number of photons per unit volume, and \( D \) is a diffusion coefficient. In diffusion theory (Goodman 1949), it is shown that

\[
D = L_v v / 3 ,
\]

where \( v \) is the velocity of the photons, and \( L_v \) is the "transport" mean free path, to be described below.

Upon substitution for \( n_{\text{diff}} \) from (1), (2) becomes

\[
- D \frac{dn}{dz^2} = K_1 z e^{-K_1 z} ,
\]

with \( K_1 = (0.02/D) (4/L)^2 \), and \( K_2 = 0.565/L \).

Boundary conditions.—The boundary conditions required to solve (4) can be obtained from analogy with Weisskopf or Placzek (see Goodman, 1949). Let the number of photons at depth \( z \), which are moving in the direction \( \theta \), be given by

\[
n(z, \theta) = n_0(z) + n_1(z) \cos \theta + \ldots
\]

second-order terms, (5)

where \( n_0(z) \) and \( n_1(z) \) are constants for a given \( z \).

The total number of diffuse photons per unit volume at \( z \) is proportional to

\[
n(z) = \int_0^\pi n(z, \theta) \sin \theta \ d\theta = 2n_0(z) .
\]

Also, the net current is given by

\[
S = \int_0^\pi v n(z, \theta) \sin \theta \cos \theta \ d\theta = 2v n_1(z) .
\]

But for diffuse photons, \( S = -D \frac{dn}{dz} \). Therefore,

\[
n_0(z) = \frac{n(z)}{2} ; \quad n_1(z) = - \frac{3}{2} \frac{D}{v} \frac{dn}{dz} .
\]

If the underlying surface albedo is zero, the net current at the lower boundary, \( S_b = -D (dn/dz)_b \), equals the downward-current part of (7). Consequently, at the lower boundary,

\[
n(h) = - \left( 2 D / v \right) (dn/dz)_b .
\]

Analogously, assuming no diffuse sky radiation, we obtain at \( z = 0 \):

\[
n(0) = (2 D / v) (dn/dz)_0 .
\]
The "transport" mean free path.—Because of the very large forward scatter by the drops (fig. 2), the "transport" mean free path, \( L_t \), will be considerably longer than \( L \); the photons will not diffuse randomly after only one collision. To correct for this, Weiskopf (see Goodman, 1949) shows that

\[
L_t = L/(1 - \cos \theta),
\]

where \( \cos \theta \) is the average cosine of the deflection of the photons in a single scatter. This was taken to be

\[
\frac{\int_0^\infty I(\theta) \sin \theta \cos \theta d\theta}{\int_0^\infty I(\theta) \sin \theta d\theta} = 0.745.
\]

\( I(\theta) \) is Wiener's scattering pattern (fig. 2). From (11),

\[
L_t = 3.92 \, L, \quad \text{and} \quad D/\nu = 1.31 \, L.
\]

Langmuir and Westendorp (1931) estimated the coefficient of \( L \) to be 2 or 3, compared to 3.92 above.

7. The cloud albedo

On substituting solutions from (4) into the boundary conditions, (9) and (10), and upon using (12), we find

\[
Dc_1 = \frac{e^{-3.24h/L}[0.48 (h/L) - 0.92] + 6.16}{5.2 + h/L} = f(h/L),
\]

where \( c_1 \) is a constant of integration. Furthermore, the cloud albedo, \( -S_0 = D(\delta n_\nu/\delta z)_o \), becomes, after one integration of (4) and setting of \( z = 0 \),

\[
- S_0 = 1 - Dc_1.
\]

Values of \( -S_0 \), computed from (13) and (14), versus \( h/L \) are shown in fig. 3 on the curve for zenith distance \( Z = 0 \).

If one compares the albedo with the total cumulative diffuse energy generated, \( \Sigma n_\nu \), one finds that the albedo is \( 1/2 \Sigma n_\nu \) for values of \( h/L \leq 6 \). When \( h/L > 6 \), the albedo becomes a greater fraction of \( \Sigma n_\nu \).

Another important fact is the very low albedo when \( Z = 0 \). For example, sometimes in stratus cloud with \( h = 500 \) ft, \( h/L \approx 1 \). Thus, the albedos of these clouds should be about 0.1. But Neiburger's (1949) observations include much larger albedos for such clouds. Obviously, other factors must be important. We turn now to a discussion of these other factors.

8. Scattering in clouds: The general case

When the solar beam is not perpendicular to the cloud, the major question is: "What will be the distribution of \( n_\nu \), the diffuse energy generated?" When \( Z = 0 \), we can appeal to symmetry about the vertical; when \( Z \neq 0 \), this will no longer be so. To account for the asymmetry exactly would greatly complicate the computations.

To avoid prohibitive computations, we assume that symmetry is maintained, and shall attempt to correct later for this false assumption through physical reasoning. By the assumption that symmetry is maintained, the end computations for albedo will, therefore, be too low and further correction will be necessary.

Expression for diffuse generated energy.—A parallel beam, containing \( I_0 \) photons/cm²/min on an area perpendicular to its direction, will pass through a length \( L/4 \) in the vertical distance \( \Delta z = L/(4m) \), where \( m = \sec Z \). Since we assume that symmetry about the incident beam is maintained, if we put \( I_0/m = 1 \), we arrive at the same diffusion equation as (4), but now \( K_1 = (0.02/D)(4m/L)^2 \) and \( K_2 = 0.5656 \) \( m/L \); in general, the underlying-surface albedo

![Fig. 3. Cloud albedo as function of \( h/L \) for various values of zenith distance; symmetry assumed. Underlying surface albedo and sky radiation zero.](image-url)
\[ n(h) = (-2D/v)(dn/dz)h M_s + R/v, \quad (15) \]

where
\[ M_s = \frac{1 + a_s}{1 - a_s}, \]

\[ -Dce = \frac{e^{-3.06}\text{wh} h/(h/L)(1 - 1.48 mM) + 3.54/m - 2.62 M} {h/L + 2.62(1 + M)} + \frac{5.24g - 1.31 R}{h/L + 2.62(1 + M)}. \quad (17) \]

In the present case, albedo = \((|S_0| + g)/(1 + g)\).

The effect of sky radiation.—When the cloud tops are very low, one can obtain approximate values of \(g\) from Kimball (1919). For example, when \(Z = 50\) deg \((m = 1.55), g = 0.19\). Computations from (17) show that, for \(Z = 50\) deg and \(a_s = 0.10\), the cloud has about the same albedo whether \(g = 0\) or \(g = 0.19\). Apparently the albedo of cloud for diffuse radiation is similar to the cloud albedo of a direct beam when \(Z \approx 50\) deg.

The effect of the underlying surface.—The only significant contribution of the surface albedo to the cloud albedo, at least for \(a_s = 0.1\), is made by the last term in (17), which involves \(R\). Since \(R\) approaches zero as \(h\) increases, the surface albedo has no effect on thick clouds, which agrees with Neiburger’s computations.

The effect of the sun’s zenith distance.—When \(g = 0\), \(|S_0| = \text{albedo}\). Values of \(|S_0|\) for several values of \(Z\) and \(a_s\) were computed; those for \(a_s = 0\) are shown in fig. 3, where the cloud albedo is plotted as a function of \(h/L\) for specific values of \(Z\) (or \(m\)). The noteworthy feature of the curves is the considerable increase of cloud albedo as \(Z\) increases. But even this albedo increase is too small, because of the asymmetry of the scatter; fig. 3 can be taken only as a first approximation to the albedo.

Correction of albedos for asymmetry of scatter.—We can improve the results of fig. 3 by physical reasoning.

In a distance along the beam equal to 0.1 \(L\) and 0.2 \(L\) from the cloud top, we may certainly assume that only one collision occurs. Dividing the incident energy by the reflected energy, we note that the albedo becomes \((1 - e^{-a_s}) P(90\degree, Z)\) when \(h = 0.1 L/m\), and \((1 - e^{-a_s}) P(90\degree, Z)\) when \(h = 0.2 L/m\), where \(P(90\degree, Z)\) is the amount of energy scattered upward by a single drop irradiated by a beam incident at an angle \(Z\) with the vertical (see Dietzis, 1922). A few points, computed for \((h/L) < (1/m)\), appear in fig. 4.

When \(h/L = 1/m\), we can approximate the albedo as follows. The concept of the mean free path assumes

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**Fig. 4.** Same as fig. 3, except asymmetry of scatter is taken into account.
that, if a large number of particles traverses a diffusing medium, on the average each particle will undergo just one collision in a distance equal to one mean free path. If the total direct beam is considered as a vast number of photons which collided just once with drops in the vertical distance \( k/L = 1/m \), the cloud albedo is \( P(90°, Z) \). Such values of albedo are plotted against \( h/L \) as a function of \( Z \) in fig. 4.

When \( (h/L) > (1/m) \) and \( Z \) is large, qualitative reasoning sometimes indicates an upper limit to the albedo. For \( Z = 80 \) deg, such a point has been plotted in fig. 4.

With the curves of fig. 3 as a guide, and with the points already plotted in fig. 4, curves were arbitrarily extended so that respective curves in figs. 3 and 4 coincided in the vicinity of \( h/L = 50 \), as they must eventually do.

Fig. 3 (symmetry assumed) and fig. 4 (corrected for asymmetry) are almost identical when \( Z = 0 \), but naturally the difference between the corresponding curves in the figure becomes larger as \( Z \) increases. At \( Z = 80 \) deg, the curve of fig. 3 is no longer a good approximation to the actual albedo when \( h/L \) lies between 0.1 and 1.

Correction for albedo of underlying surface.—Cloud-albedo observations include the influence of the albedo \( a_0 \) of the underlying surface. From (17), the cloud albedo was computed for several values of \( Z \), with \( g = 0 \) and \( a_0 = 0.1 \). When the albedos of fig. 3 (\( a_0 = 0 \)) are subtracted from these latter computations, we obtain the influence of the surface albedo on the total cloud albedo as a function of \( h/L \) for each value of \( Z \). Thus, we may define a factor, \( F \), which is a measure of the influence of the cloud itself on the energy reflected by the surface:

\[
F = \frac{(S_0)_{a_1} - (S_0)}{E_t}.
\]

(18)

In (18), \((S_0)_{a_1}\) and \((S_0)\) are the cloud albedo when \( a_0 = 0.1 \) and 0, respectively, and \( E_t \) is the total transmitted energy when \( a_0 = 0 \).

As might be expected, computed values of \( F \) varied little with \( Z \). The values of \( F \) are given in table 1 as a function of \( h/L \).

By use of the factors of table 1 (based on assumed symmetry of scatter) on the albedos of fig. 4, the albedos for \( a_0 = 0.1 \) can be computed for the case of asymmetrical scatter. The results of such computations are shown in fig. 5, which is supposed to represent the actual cloud albedo when \( a_0 = 0.1 \), sky radiation is zero, and \( L \) is constant with depth.

9. Comparison of results with theory

Comparison with two-stream theory.—Some advocates of the two-stream theory have maintained (see Hewson, 1943) that the albedo does not vary much with \( Z \). This is diametrically opposite to the results of fig. 4. It should be pointed out, however, that actually Hewson varied only the spectral distribution of the incident light; he did not vary \( Z \) in his computations, and thus could not have found the albedo as a function of \( Z \).

Moreover, even at \( Z = 67 \) deg, at which the two-stream method is supposed to apply, there is disagreement between figs. 3 (and 4) and the results of the two-stream method, because the latter is not applicable near the cloud top. In the case of zero absorption and zero albedo of the underlying surface, the two-stream method gives for the cloud albedo:

\[
S_0 = \frac{s(h/L)}{1 + s(h/L)},
\]

(19)

where \( s \) is a scattering coefficient chosen to be applicable at \( Z = 67 \) deg.

Equation (19) has been plotted on figs. 3 and 4. In fig. 4, (19) is not near the curve for \( Z = 67 \) deg, but rather is near \( Z = 50 \) deg and even coincides with \( Z = 40 \) deg at times. Even in fig. 3, which yields too-low values of albedo, (19) again gives lower values when \( Z = 67 \) deg and \( h/L \) exceeds 0.6.

<table>
<thead>
<tr>
<th>Table 1. Values of ( F ) when ( a_0 = 0.1 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h/L )</td>
</tr>
<tr>
<td>( F )</td>
</tr>
</tbody>
</table>

Fig. 5. Cloud albedo as function of \( h/L \) for various values of zenith distance; \( a_0 = 0.1 \); asymmetry of scatter included.
Table 2. Comparison of observed with computed albedos, flight 46.

<table>
<thead>
<tr>
<th>Z (deg)</th>
<th>64</th>
<th>58</th>
<th>47</th>
<th>43</th>
<th>35</th>
<th>33</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (PST):</td>
<td>0754-0800</td>
<td>0804-0904</td>
<td>0935-0939</td>
<td>0941-1032</td>
<td>1053-1100</td>
<td>1105-1140</td>
<td>1155-1156</td>
</tr>
<tr>
<td>h (ft):</td>
<td>1900</td>
<td>1800</td>
<td>0.34</td>
<td>0.30</td>
<td>0.19</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>ρ (g/m²):</td>
<td>14.3</td>
<td>13.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>h/L:</td>
<td>78</td>
<td>73</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Albedo (observed):</td>
<td>66-73</td>
<td>66-69</td>
<td>64-67</td>
<td>64-67</td>
<td>64-67</td>
<td>64-67</td>
<td>64-67</td>
</tr>
</tbody>
</table>

* This high albedo was not included in the published albedos.

Part of the reason for the discrepancy was pointed out by Whipple and Gold (see Hewson, 1943). The equations which lead to (19) are of the form

$$dS_{up} = s(S_{up} - S_{down}) \frac{dz}{L}.$$  (20)

Even if we accept the assumption of a single s near the cloud top, the correct equation should be of the form

$$dS_{up} = s(S_{up} - S_{down} \sec Z) \frac{dz}{L},$$  (21)

which is obviously different from (20).

Furthermore, diffraction, which has been ignored so far, also increases the cloud albedo with Z. Near Z = 0, diffraction plays a minor role; but when Z is very large, the diffracted energy can escape from the cloud after one collision with cloud drops. Numerical estimates indicate that, if drops for which χ = 40 are present, the effect of diffraction becomes pronounced when Z exceeds 80 deg.

Comparison with Faigelson's theory.—Using approximations to Neiburger's (1949) published cloud parameters, Faigelson (1951) computed the albedo as a function of Z for h = 300 m and h = 600 m. These cloud thicknesses correspond to h/L = 3 and h/L = 12, if we use rₜ = 14 μ. The corresponding values of albedo from fig. 4 are considerably lower. For example, at Z = 0 and h = 300 m, Faigelson gives an albedo of 74 per cent at an albedo of only 26 per cent in fig. 4. For h = 600 m, Faigelson’s albedo is over 90 per cent even when Z = 0. Naturally, as h and Z increase, the discrepancy becomes smaller, since the albedo approaches 100 per cent in both methods.

Faigelson recognized that his albedos were "somewhat" high, and cites the fact that the "real" scattering pattern is more "elongated" than the one which he used, so that at Z = 0, at least, his computed energy is readily reflected from the cloud top instead of being directed downward.

10. Comparison with observations

Comparison between fig. 5 and Neiburger's data.—Neiburger made about 200 measurements of cloud albedo, but "observations of drop size and liquid water content were obtained on only a few flights." The published data show that large drops were relatively more frequent near the cloud base than elsewhere in the cloud; the liquid-water content, w, was low near the cloud base and increased with height to a distance of 100 to 200 ft below the cloud top, above which level w decreased rapidly.

To utilize the measured data, we require h/L. When the drop size is not uniform, we obtain

$$L = \left(\frac{4}{3}\right)\left(\frac{r_{av}}{w}\right),$$  (22)

in which rₜ = (ΣNₜrₜ²)/(ΣNₜrₜ). We may think of rₜ as an effective drop radius in scattering problems. If the drops were uniform in size (but they were not), they would scatter light as though their uniform radius were rₜ. For the distributions which occur in clouds, rₜ will always be considerably larger than rₘ, the radius of most frequent size. The use of rₘ instead of rₜ makes a given cloud appear to reflect more radiation than it actually does. Since in Neiburger's clouds w varied with height, the most adequate definition of h/L is probably

$$h/L = \frac{\Sigma \left(1/L \Delta z\right)}{\Sigma (w/rₜ) h},$$  (23)

where w is the average liquid-water content per cubic centimeter, averaged with respect to height in the

Table 3. Comparison of observed with computed albedos, flight 42.

<table>
<thead>
<tr>
<th>Z (deg)</th>
<th>61</th>
<th>50</th>
<th>45</th>
<th>35</th>
<th>33</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (PST):</td>
<td>0751-0805</td>
<td>0847-0902</td>
<td>0909-0925</td>
<td>1011-1037</td>
<td>1035-1103</td>
<td>1116-1117</td>
</tr>
<tr>
<td>h (ft):</td>
<td>250</td>
<td>300</td>
<td>8.3-6.2</td>
<td>0.34</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>ρ (g/m²):</td>
<td>0.42</td>
<td>0.82</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>h/L:</td>
<td>15</td>
<td>16-25</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>
Table 4. Comparison of observed with computed albedos, flight 44.

<table>
<thead>
<tr>
<th>Z (deg)</th>
<th>64</th>
<th>58</th>
<th>52-44</th>
<th>39</th>
<th>37</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (PST):</td>
<td>0756-0805</td>
<td>0809-0847</td>
<td>0853-0941</td>
<td>1014-1020</td>
<td>1024-1033</td>
</tr>
<tr>
<td>h (ft):</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w (g/m³):</td>
<td></td>
<td>0.99</td>
<td></td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>k/L:</td>
<td>1.01</td>
<td></td>
<td></td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>Albedo (fig. 5):</td>
<td></td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Albedo (observed):</td>
<td>63-87</td>
<td>33-35</td>
<td>18-40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

could. From Neiburger's (1949) curve of flight 46, we can find w. On this flight, rₚ was not measured; rₑ = 10 μ appears reasonable from other flight data, and moreover leads to agreement between observed and computed albedo for portions of the flight (table 2). Using rₑ = 10 μ and the observed w and h data in (23), we find h/L, with which the albedo can be found in fig. 5. The agreement between computed and observed albedos, summarized in table 2, is fairly good.

If the albedo of 88 per cent is correct, it indicates that rₑ decreased appreciably because w decreased with time, and even h had decreased slightly at Z = 31 deg. To find an albedo of 88 per cent at Z = 31 deg in fig. 5, we require h/L = 45, which corresponds to rₑ = 1.7 μ if w = 0.19 as observed. Such a small value of rₑ indicates the presence of numerous small drops. Therefore, fig. 5 does not apply. Nevertheless, the measurements of albedo (88 per cent), w and h, together with fig. 5, indicate that the cloud changed rapidly, either in situ or by advection, from a "large droplet" cloud to one in which many small drops were present.

For another flight (flight 43), an analysis essentially similar to table 2 reveals that again agreement between computed and observed albedos was good in the morning; but the agreement deteriorated as time progressed, until at 1115 PST the computed albedo was 41 per cent and the observed albedos were 62 to 66 per cent. This again indicates that rₑ decreased near noon, although other explanations may be possible.

A decrease of rₑ with time is also indicated in Aldrich's cloud, to be discussed presently. These cases, although few in number, suggest that drops often become smaller as the cloud begins to dissipate, coincident with a decrease in w and a shrinkage of h. This may be due to increased vertical motion, with perhaps admixture of dry air from above and below the cloud; perhaps direct solar heating in the cloud is partly responsible.

In thin clouds, the comparisons with Neiburger's observations should be even more valid because absorption plays a reduced role, and w is more likely to represent the effective w. Measurements in flight 42 permitted comparisons which are summarized in table 3. The agreement between observed and computed albedos in table 3 is not bad; it would take relatively small changes in w and in rₑ to make the agreement perfect. Thus, the assumption that the scattering effect of the small drops was negligible in this cloud seems valid.

But in flight 44 (table 4) the situation is different. Drop size was not measured on this flight. Therefore, rₑ = 8 μ, as suggested by flight 42, was used as an approximation. The agreement between observed and computed albedos is good at times, but not in the early morning. For example, one albedo was 87 per cent, although the measured w had a maximum value of 0.194 g/m³, suggesting w = 0.09. This albedo requires rₑ ≈ 0.6 μ, a very low value indeed, and indicates the presence of many small drops (but probably an actual rₑ > 0.6 μ but less than 8 μ) even though fig. 5 is not applicable under those conditions. During the albedo measurements on this flight, "small breaks below" were frequently noted. Thus, in line with the discussion of flight 46 (table 2) and flight 43, it seems that sometimes, at any rate, when stratus

![Fig. 6. Observed cloud albedo (Neiburger) vs. zenith distance for "thin" clouds.](http://journals.ametsoc.org/jas/article-pdf/11/4/291/3921992/1520-0469(1954)011_0291_sosebc_2_0_co_2.pdf)
clouds are in the dissipating stage or actually are broken, they contain more numerous small drops than non-dissipating stratus.

That there are sometimes many small drops is also indicated by the direct drop measurements. In the few data furnished by Neiburger, the linear average drop radius, \( r \), varied from 3.7 to 8.9 \( \mu \). In the cloud where \( r \) was 3.7 \( \mu \), there must have been a good proportion of drops with \( r < 3 \mu \), even though these become increasingly difficult to catch. For the smaller drops, the drop scattering pattern and scattering cross-section would vary considerably with wavelength, and it would be necessary to modify the "large-drop" theory to account for the albedos of such clouds.

**Variation of albedo with zenith distance.**—When \( h/L \) lies between 0.1 and 10, fig. 5 shows a large variation of albedo with \( Z \). In examination of Neiburger’s data for such a variation, the true relationship is masked by a large range of values of \( L \) in the clouds and by the presence of small drops in some clouds. Nevertheless, to test the albedo variation with \( Z \), Neiburger’s data for “solid” clouds, with \( h \) between 200 and 650 ft, were plotted (fig. 6) and analyzed; plotted points represent average albedos in a 4-deg range of \( Z \). The points are, of course, widely scattered; but, in spite of that, a variation of albedo with \( Z \) is suggested. A statistical analysis reveals that the correlation coefficient between cloud albedo and \( Z \), holding \( h \) constant, is about 0.67; this means that, on an average single day, about 45 per cent of the observed variation in albedo is due to changes in \( Z \) for “thin” clouds. The correlation coefficient between albedo and \( h \), holding \( Z \) constant, was about 0.3; thus, in this thin-cloud sample, variations in \( h \) accounted for only about 10 per cent of the variation of albedo. This suggests that variations in \( L \) were quite important in causing changes in albedo from day to day and within a single day, and that for thin clouds the variation of \( Z \) is more important than the variation in \( h \) in explaining observed changes of albedo. In fig. 6, lines for \( h/L = 1, 3 \) and 5 have been added from fig. 5; these lines indicate that, for the clouds in this sample, \( h/L \) had values mainly between 1 and 5 if we assume the clouds to have contained large drops only. One also notes in these clouds a tendency, on several occasions, for the albedo to increase sharply as the lower zenith distances are approached, perhaps indicative of a decrease in \( r \).

Neiburger measured some albedos near 8 per cent when \( Z \) was 16 deg. Since the albedo of the sea surface may be as low as 0.02 for the direct beam, an albedo of about 8 per cent can be obtained from fig. 4 with \( h/L \approx 0.25 \). Such low cloud-albedos do not seem possible when \( Z \) is large.

**Comparison with Aldrich’s data.**—Aldrich (1919) measured the albedos of a stratus cloud from morning to near noon on one day; \( h \) decreased from 1220 to 600 ft, while \( Z \) varied from 69 to 34 deg. If we assume a regular decrease of \( h \) with time, and use fig. 5 to find \( h/L \) from his albedos, we can compute values of \( L \). Actually, Aldrich’s albedos were so high that they fall in regions of fig. 5 where variation with \( Z \) is relatively small. Nevertheless, we should still have expected a decrease of 0.10 in the albedo as the sun rose. Moreover, the measurements show that the cloud thickness decreased considerably; probably \( \bar{\sigma} \) also decreased, as in Neiburger’s case. All these three effects should have reduced the albedo as \( Z \) decreased. Yet, except for the very last measurement, there was no decrease in albedo. On the contrary, the highest acceptable albedo, 87.4 per cent, occurred when \( h \) had already decreased to 800 ft and \( Z \) was 51 deg, whereas earlier albedos were near 78 per cent. The measurements indicate that \( L \) decreased from more than 100 ft to a low acceptable value of 23 ft, and only increased somewhat at the last measurement. Since we expect the contribution to \( L \) from \( \bar{\sigma} \) to go in just the opposite direction, we must fall back on a reduced value of \( r \) to give a decreased value for \( L \). The reduction of the cloud from below, where larger drops are more prevalent (Neiburger, 1949), and the increased corrugations\(^7\) on the top surface which Aldrich observed, suggesting increased vertical motion and mixing, make the conclusion of greater frequency of small drops reasonable.

**Comparison with Haurwitz’s data.**—Haurwitz (1948) analyzed the solar-radiation measurements at Blue Hill, Massachusetts, in relation to cloudiness. He found that the “transmission,” \( T_H \), varied very little with \( Z \). Despite that, it is not difficult to reconcile Haurwitz’ data with the requirement for variation of albedo with \( Z \). The variation of \( T_H \) with \( Z \) is to a large extent a seasonal variation. Hence, at least three factors operate to reduce the variation of \( T_H \) with \( Z \). These are:

1. The atmosphere is moister in summer; consequently, the air under the cloud can absorb more (diffuse) energy, thus reducing \( T_H \) in summer relative to winter;
2. It may well be that the cloud parameters, and therefore \( h/L \), are different in summer than in winter; if \( h/L \) is greater in summer, \( T_H \) would be decreased in summer compared to winter;
3. It is well known that snow cover will increase \( T_H \) appreciably; thus, the snow effect also decreases \( T_H \) in summer as compared with winter.

A non-seasonal influence is also present.

It should be noted that \( T_H \) is computed relative to clear-sky measurements at the ground. Fig. 5 refers to the energy incident on the cloud top. This factor also

\(^7\) The influence of the corrugations on the cloud surface is too complicated a problem to discuss in detail.
tends to make $T_H$ more nearly alike at the same values of $m$.

In view of all these factors, it is easy to bring the constancy, or even increase, of $T_H$ with $Z$ into agreement with fig. 5, which postulates an increase of albedo with increasing $Z$.

11. Conclusion

The results of figs. 3 to 5 indicate that the albedo of clouds depends strongly on the solar zenith distance. In some studies of the albedo of the whole earth, the mean distribution of cloud amount is estimated for each cloud type, and a “representative” albedo is somehow assigned to each cloud type. One way of assigning such cloud-type albedos, which may serve as an independent check, is to assemble the best available estimates of the cloud parameters, such as liquid-water content, drop-size distribution, and thickness, and compute from those and fig. 5 the average albedo expected for each cloud type. A suitable summation would then yield an estimate of the total energy reflected to space by clouds on the average and would aid in estimating the albedo of the whole earth.

The results of figs 3 to 5 may also have some relation to the visibility in clouds of large droplets. Visibility depends on $L$. Consequently a measure of cloud albedo (or transmission) in light which is negligibly absorbed would yield an estimate of $L$, if $k$ is known. This value of $L$ may be useful in estimating the visibility in clouds. Conversely, a measure of visibility in clouds may be useful in estimating the albedo of, or transmission through, clouds.

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REFERENCES


