

The Adiabatic Lapse Rate in the Venus Atmosphere

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ABSTRACT

The adiabatic lapse rate is derived for a real gas, and the result applied to the Venus atmosphere, which is assumed to consist of pure CO₂. The ratio of the real to the ideal gas adiabatic lapse rate is found to exceed unity at pressures and temperatures currently believed to occur in the lower Venus atmosphere. For a pressure of 100 atm and temperatures of 750, 650 and 550K, the ratios are 1.03, 1.06 and 1.13, respectively. At 70 atm the ratios are 1.02, 1.05 and 1.10, respectively. The expression g/c_p , where g is gravity and c_p the specific heat at constant pressure of a real gas, is inferior to the adiabatic lapse rate of an ideal gas as an approximation to the real gas adiabatic lapse rate. Estimates of Venus surface temperature based on downward adiabatic extrapolation for an ideal gas, or even the quantity g/c_p , do not lead to results seriously different from extrapolations based on the real adiabatic lapse rate. However, the real gas adiabatic lapse rate will be necessary for accurate analysis of hydrostatic stability.

1. Introduction

McElroy (1969) has adiabatically extrapolated the Soviet Venera 4 data, which terminated at 6078 ± 5 km radial distance from the center of Venus, down to the radar radius of about 6053 ± 5 km, to obtain a surface pressure of ~ 100 atm and a surface temperature of ~ 730 K. Gale *et al.* (1969) have used microwave data to find a surface temperature of 670 ± 20 K at the surface. Composition by volume determined from Venera 4 measurements by Vinogradov *et al.* (1968) are the following:

CO ₂	$90 \pm 10\%$ (probably $> 90\%$)
O ₂	0.4% , $< 1.6\%$ (probably $\sim 1\%$)
N ₂	7% (probably $< 2.5\%$)
H ₂ O	from 1–8 mg liter ⁻¹

The Venera 5 and 6 probes (Pravda, 4 June 1969) placed CO₂ in the range from 93–97%, with N₂ and inert gases amounting to 2–5% of the total.

The critical point pressure, temperature and specific volume of CO₂ are at 72.9 atm, 304 K and $2.14 \text{ cm}^3 \text{ gm}^{-1}$, respectively. Therefore, the implied value of surface CO₂ pressure on Venus is of the order of the critical pressure, whereas the surface temperatures are substantially in excess of the critical temperature.

In view of the comparable magnitudes of the apparent surface CO₂ pressure and the critical pressure of CO₂, it is not obvious that the adiabatic lapse rate of an ideal gas,

$$\Gamma_i = \frac{g}{c_{pi}}, \quad (1)$$

where g is gravity and c_{pi} the specific heat at constant

pressure for an ideal gas, can be applied without incurring significant error. In the past, expressions of this form have been somewhat uncritically applied to the Venus atmosphere. Hunten (1968), for example, calculated g/c_p , using for c_p the specific heat at constant pressure tabulated by Hilsenrath (1960) as a function of temperature and pressure. This procedure is erroneous, since the specific heats tabulated by Hilsenrath are the *real* gas values for CO₂, and g/c_p is not the correct form for the adiabatic lapse rate of a real gas.

Avduevsky *et al.* (1968) give without derivation the following, less familiar expression for the adiabatic lapse rate:

$$\Gamma = \frac{g}{c_p} \left[1 - \rho \left(\frac{\partial i}{\partial p} \right)_T \right], \quad (2)$$

where c_p is the specific heat at constant pressure, ρ density, p pressure, T temperature, and i specific enthalpy. This expression is readily shown to be correct, regardless of whether the gas is real or ideal, but it goes little beyond the definition of adiabatic lapse rate, and Avduevsky *et al.* do not give the source of values for c_p , or indicate how $(\partial i / \partial p)_T$ was evaluated.

The purpose of this paper is to derive the adiabatic lapse rate for an arbitrary equation of state and then to calculate numerical values over a wide range of pressures and temperatures in a Venus atmosphere assumed to consist of pure CO₂.

2. Theory

The first law of thermodynamics is

$$dh = du + p d\alpha, \quad (3)$$

where dh is the differential heat added per unit mass, u the internal energy per unit mass, p pressure, and α specific volume. The internal energy may be expressed as a function of temperature and specific volume. Thus, Eq. (3) becomes

$$dh = c_v dT + \left[\left(\frac{\partial u}{\partial \alpha} \right)_T + p \right] d\alpha, \quad (4)$$

where $c_v = \partial u(\alpha, T) / \partial T$. The requirement that the entropy differential be exact yields

$$\left(\frac{\partial u}{\partial \alpha} \right)_T = T^2 \left[\frac{\partial}{\partial T} \left(\frac{p}{T} \right) \right]_\alpha = T \left(\frac{\partial p}{\partial T} \right)_\alpha - p. \quad (5)$$

Now the first law becomes

$$dh = c_v dT + T \left(\frac{\partial p}{\partial T} \right)_\alpha d\alpha. \quad (6)$$

We next introduce an equation of state, $p = p(\alpha, T)$, in order to eliminate the differential of specific volume in Eq. (6), which now becomes

$$dh = \left[c_v - \frac{T(\partial p / \partial T)_\alpha^2}{(\partial p / \partial \alpha)_T} \right] dT + \frac{T(\partial p / \partial T)_\alpha}{(\partial p / \partial \alpha)_T} dp. \quad (7)$$

The relationship of specific heats follows as

$$\left(\frac{dh}{dT} \right)_{p \text{ const}} \equiv c_p = c_v - \frac{T(\partial p / \partial T)_\alpha^2}{(\partial p / \partial \alpha)_T}. \quad (8)$$

Substitution of Eq. (8) in Eq. (7) yields the first law in the form

$$dh = c_p dT + \frac{T(\partial p / \partial T)_\alpha}{(\partial p / \partial \alpha)_T} dp. \quad (9)$$

Introduction of the adiabatic assumption, $dh=0$, and the hydrostatic equation,

$$dp = -g\rho dz, \quad (10)$$

where z is height, into Eq. (9) yields the adiabatic lapse rate

$$\begin{aligned} \Gamma \equiv -\frac{dT}{dz} &= -\frac{T(\partial p / \partial T)_\alpha}{\alpha(\partial p / \partial \alpha)_T} \left(\frac{g}{c_p} \right) \\ &= \frac{T(\partial p / \partial T)_\rho}{\rho(\partial p / \partial \rho)_T} \left(\frac{g}{c_p} \right), \end{aligned} \quad (11)$$

where $\rho = \alpha^{-1}$.

It may be readily verified, in the case of an ideal gas, that (8) reduces to $c_{pi} = c_{vi} + R$, where c_{pi} and c_{vi} are the specific heats of an ideal gas, and R the gas constant, while (11) reduces to (1), the adiabatic lapse rate for an ideal gas.

We are here concerned with the more general problem of evaluating Eq. (11) for a real gas. The derivatives as well as c_p depend on the form of the equation of state; c_p depends additionally on c_v . The equation of state in virial form is

$$\frac{p\alpha}{RT} = 1 + \frac{B(T)}{\alpha} + \frac{C(T)}{\alpha^2} + \dots = Z, \quad (12)$$

where B, C, \dots are the virial coefficients, and Z is called the compressibility factor. Callen (1960) gives the relationship of real and ideal specific heats at constant volume as

$$c_v = c_{vi} - \frac{R}{\alpha} \frac{d}{dT} \left(T^2 \frac{dB}{dT} \right) - \frac{R}{2\alpha^2} \frac{d}{dT} \left(T^2 \frac{dC}{dT} \right). \quad (13)$$

For linear molecules such as carbon dioxide,

$$c_{vi} = \frac{5}{2}R + \sum_{T_v} \left[R \left(\frac{T_v}{2T} \right)^2 \sinh^{-2} \left(\frac{T_v}{2T} \right) \right], \quad (14)$$

where T_v are the vibrational temperatures, which for CO_2 take on the values $T_v(\text{K}) = 960, 960, 2000, 3380$. Now c_p can be expressed more explicitly by substituting (14) in (13) and then substituting the result in (8).

The evaluation of Eq. (11) may be carried out by using the virial expansion in Eq. (12) explicitly. However, the evaluation is much simpler if tabulations of the compressibility factor and c_p are available in the forms $Z = Z(p, T)$ and $c_p = c_p(p, T)$. Hilsenrath (1960) provides these tabulations. It may easily be shown that

$$T \left(\frac{\partial p}{\partial T} \right)_\rho = \frac{p \left[\frac{\partial}{\partial T} (TZ) \right]_\rho}{Z - p \left(\frac{\partial Z}{\partial p} \right)_T}, \quad (15)$$

$$\rho \left(\frac{\partial p}{\partial \rho} \right)_T = \frac{pZ}{Z - p \left(\frac{\partial Z}{\partial p} \right)_T}. \quad (16)$$

Substitution of (15) and (16) in (11) yields

$$\Gamma = Z^{-1} \left[\frac{\partial}{\partial T} (TZ) \right]_\rho \left(\frac{g}{c_p} \right). \quad (17)$$

Division by Eq. (1) yields the ratio of the adiabatic lapse rate to the adiabatic lapse rate of an ideal gas, i.e.,

$$\frac{\Gamma}{\Gamma_i} = Z^{-1} \left[\frac{\partial}{\partial T} (TZ) \right]_p \frac{c_{pi}}{c_p}. \quad (18)$$

Eq. (11) may be written alternatively in the form

$$\Gamma = - \frac{T}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \left(\frac{g}{c_p} \right), \quad (19)$$

and the ratio of lapse rates as

$$\frac{\Gamma}{\Gamma_i} = - \frac{T}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \left(\frac{c_{pi}}{c_p} \right). \quad (20)$$

The compressibility factor Z and its derivative with respect to temperature may be obtained from Hilsenrath's tables of the compressibility factor at 10K intervals for 10, 40, 70 and 100 atm. The density and its derivative with respect to temperature may be obtained from Hilsenrath's tables of density at the same temperatures and pressures. Specific heats are also provided at these temperatures and pressures. Evaluation from the density table yielded the same Γ as that obtained from the tables of the compressibility factor.

Some qualitative properties of the real gas adiabatic lapse rate may be anticipated by evaluation of (18) [or (20)] at the critical point c of a Van der Waals gas. It may easily be shown that

$$\left(\frac{\Gamma}{\Gamma_i} \right)_c = \frac{2(c_{pi})_c}{3R}, \quad (21)$$

where $(c_{pi})_c$ would be determined by the critical point temperature. Reference to Eq. (1) shows that the adiabatic lapse rate at the critical point is

$$\Gamma_c = \frac{2g}{3R}, \quad (22)$$

or two-thirds of the autoconvective lapse rate of an ideal gas. For an ideal CO_2 gas, the adiabatic lapse rate would be Rg/Rc_{pi} , or $0.22g/R$ at 300K and $0.16g/R$ at 800K. Hence, we anticipate that Γ will exceed Γ_i in a large region extending out (to larger p and α) from the critical point. The adiabatic and autoconvective lapse rates must be equal, of course, at the critical point of all real gases, since, if $T = T(p, \alpha)$, we have $dT/dp = (\partial T/\partial p)_\alpha + (\partial T/\partial \alpha)_p (d\alpha/dp)$, in which the first term on the right corresponds to the autoconvective lapse rate, and the second term vanishes at the critical point. Elsewhere the second term subtracts from the first.

3. Numerical results and conclusions

Table 1 and Fig. 1 show the ratio Γ/Γ_i based on computations for pressures of 1, 10, 40, 70 and 100 atm and temperatures at 50K intervals from 250 to 800K. At 70 atm Γ/Γ_i is 1.02, 1.05 and 1.10 for $T = 750, 650$ and 550K , respectively. At 100 atm Γ/Γ_i is 1.03, 1.06 and 1.13 at the same temperatures. It follows that if the Venus atmosphere is adiabatically stratified, the surface temperature will be underestimated. However, the underestimate would amount to only about 10K for a 25-km extrapolation using an average ratio of 1.04 for Γ/Γ_i . This error is tolerably small in view of the crude state of our present knowledge of the Venus atmosphere.

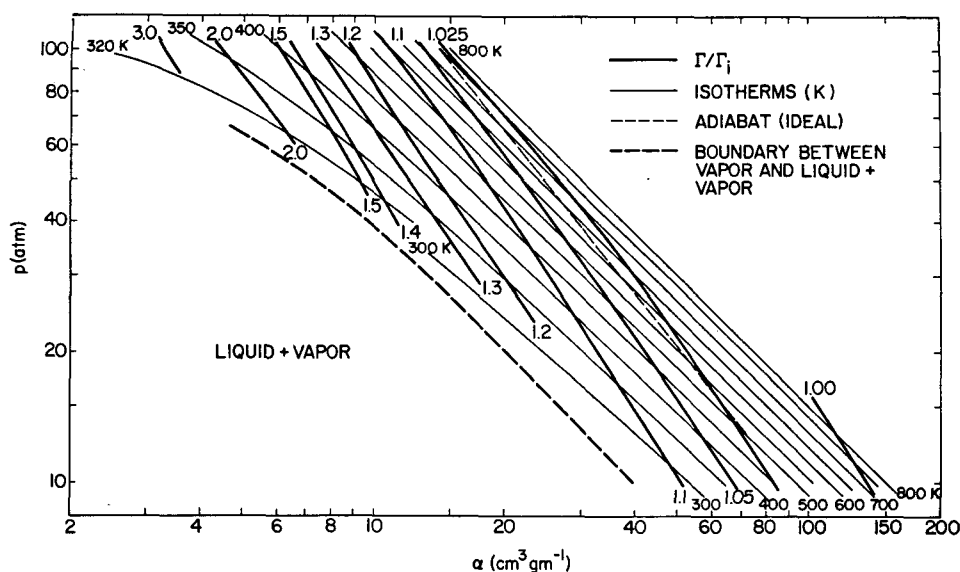
The ratio decreases slightly with height, if the temperature distribution is adiabatic. This is shown by the variation of the ratio along the approximate adiabat shown in Fig. 1. This adiabat is actually based on the ideal gas law, since the adiabat may then be obtained analytically and is sufficiently accurate to illustrate the point.

If the lower Venus atmosphere is not adiabatically stratified, surface temperatures may be less than current estimates. If the lapse rate in the lower atmosphere is of the order of 60–70% of adiabatic, as in the earth's atmosphere, the surface temperature may be as much as 100K less than the value of about 750K obtained by adiabatic extrapolation. Such temperatures would be comparable to those found by Gale *et al.* (1969). In this case, Γ/Γ_i near the surface would be of the order of 1.05–1.10. If the percentage variation globally of the Venus surface temperature is as large as that of the surface temperature on earth, temperatures of the order of 600K would be implied, and Γ/Γ_i would again be of the order of 1.05–1.10. Such departures as these would be important in any analysis of hydrostatic stability.

It is important to note from Table 1 that the quantity $\Gamma^* = g/c_p$ is a worse approximation to the actual lapse rate than g/c_{pi} , the adiabatic lapse rate of an ideal CO_2 gas.

The theory and numerical computations of this paper have been based on the assumption of 100% CO_2 composition. The influence of trace constituents is probably small, but their influence is unfortunately neither in direct proportion to their percentage of the composition nor susceptible to easy evaluation. The difficulty (Callen, 1960) is that the virial coefficients of a multi-component gas are not entirely determined by the virial coefficients of each of the components. For a binary mixture, consisting of n_1 moles of component 1 and n_2 moles of component 2, the first virial coefficient in Eq. (12) is

$$B(T) = \left(\frac{n_1}{n} \right)^2 B_1(T) + \frac{2n_1 n_2}{n^2} B_{12}(T) + \left(\frac{n_2}{n} \right)^2 B_2(T), \quad (23)$$

FIG. 1. The ratio of real-to-ideal gas adiabatic lapse rates for CO₂.

where $n=n_1+n_2$, $B_1(T)$ and $B_2(T)$ are the virial coefficients of the individual components, and $B_{12}(T)$ a new coefficient depending on interactions of the two gases. The third virial coefficient is cubic in the mole fractions, and the n th virial coefficient is n th order in the mole

fractions. In the case of Venus, the final term in Eq. (23) would be entirely negligible by reason of the trace amount of the second most abundant component, N₂, and perhaps also by reason of $B_2(T)$ being small at temperatures greatly in excess of the critical tempera-

TABLE 1. Numerical results of adiabatic lapse rate computations for Venus atmosphere (100% CO₂) at various temperatures and pressures: c_{pi} [10⁶ ergs gm⁻¹ (°K)⁻¹], Γ_i (°K km⁻¹ for $g=870$ cm sec⁻²).

T (K)	c_{pi}	Γ_i	$(\Gamma/\Gamma_i \text{ and } \Gamma/\Gamma^*)^a$ p (atm)					$(\alpha \text{ and } c_p)^b$ p (atm)				
			1	10	40	70	100	1	10	40	70	100
250	7.91	11.00	1.013 1.031	1.073 1.478				461.8 8.05	42.1 10.90			
300	8.42	10.33	1.004 1.017	1.089 1.188				556.6 8.53	53.1 9.19	10.64		
350	8.91	9.76	1.002 1.010	1.056 1.110	1.276 1.559	1.584 2.327	2.099 3.859	650.7 8.99	63.3 9.37	14.26 10.88	7.18 13.09	4.25 16.38
400	9.34	9.31	0.999 1.007	1.036 1.071	1.174 1.333	1.322 1.665	1.498 2.109	744.5 9.42	73.2 9.66	17.25 10.61	9.26 11.76	6.08 13.15
450	9.72	8.95	0.996 1.005	1.024 1.050	1.110 1.210	1.202 1.390	1.305 1.592	838.2 9.80	82.9 9.98	20.02 10.60	11.05 11.25	7.50 11.86
500	10.07	8.64	0.995 1.004	1.014 1.035	1.078 1.147	1.145 1.262	1.215 1.371	931.7 10.15	92.6 10.28	22.66 10.71	12.69 11.10	8.73 11.36
550	10.40	8.37	0.996 1.002	1.010 1.026	1.055 1.106	1.097 1.189	1.134 1.271	1025 10.47	102.1 10.57	25.22 10.91	14.27 11.27	9.90 11.65
600	10.71	8.12	0.997 1.002	1.008 1.020	1.038 1.076	1.070 1.137	1.095 1.192	1118 10.76	111.16 10.83	27.73 11.10	15.78 11.37	11.02 11.66
650	10.98	7.92	0.997 1.002	1.006 1.015	1.030 1.059	1.047 1.098	1.063 1.137	1212 11.03	121.1 11.08	30.21 11.29	17.26 11.51	12.10 11.75
700	11.22	7.75	0.997 1.001	1.003 1.012	1.022 1.045	1.033 1.073	1.047 1.104	1305 11.27	130.5 11.31	32.66 11.48	18.70 11.66	13.14 11.84
750	11.42	7.62	0.995 1.001	1.000 1.009	1.012 1.034	1.021 1.056	1.030 1.078	1398 11.49	139.9 11.53	35.08 11.66	20.13 11.81	14.17 11.95
800	11.60	7.50	0.993 1.000	0.996 1.007	1.006 1.026	1.013 1.044	1.016 1.057	1492 11.69	149.3 11.72	37.49 11.84	21.54 11.95	15.17 12.07

^a Γ/Γ_i , upper numbers; Γ/Γ^* , lower.^b α (cm³ gm⁻¹), upper numbers; c_p [10⁶ ergs gm⁻¹ (°K)⁻¹], lower.

ture of N_2 . The terms involving interactions of the components are more difficult to estimate, but their importance appears severely restricted by the presence of the factor representing the mole fraction of the trace component.

The direct measurements of composition have been limited to pressures ≥ 10 atm. It has been assumed that composition does not vary significantly between the 10-atm level and the surface. This assumption seems perfectly safe in view of the great heights to which mixing occurs on earth, but so little is presently known about the Venus atmosphere that complete mixing cannot be regarded as a certainty.

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