

## Propagation of Sound Through Atmospheric Fog

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### ABSTRACT

The propagation of sound in an atmospheric fog is described by considering a system of liquid droplets suspended in a mixture of their vapor and a noncondensing gas. The perturbation of this system by a sound field causes the droplets to exchange mass, momentum and heat with the surrounding gaseous mixture, and results in acoustic attenuation and dispersion. For low mass concentrations of liquid phase, the Napier frequency ( $\bar{\tau} = 1$ , when attenuation per unit wavelength is a maximum) is expressed in terms of  $\bar{\tau} = \omega\tau_i/C_m$ , where  $C_m$  is the liquid mass fraction and  $\tau_i$  is, for example, the thermal relaxation time. For an infinite Lewis number (ratio of thermal diffusivity to mass diffusivity) the results reduce to published expressions for nonvolatile particles. Strong attenuation per unit wavelength due to mass transfer effects is predicted in the sub-audible (infrasonic) region in which signals from weather fronts and atmospheric explosions are monitored.

Dimensional attenuation is found to be nearly constant both for infrasonic and low audible frequencies where  $\omega \ll 1/\tau_i$  (e.g., for a warm air fog of droplets with an  $8 \mu\text{m}$  radius,  $\omega = 2\pi f \ll 1200 \text{ sec}^{-1}$ ) and for high frequencies where  $\omega \gg 1/\tau_i$ . The level of dimensional attenuation for the high frequencies is approximately six times that at the low-frequency level. Transition from the low level to the high level of attenuation depends on fog particle size through the relation,  $\omega\tau_i = 1$ . Computed results for typical fog data are given, and optimal signaling frequencies are discussed.

### 1. Introduction

The propagation of sound through a gas containing small suspended particles has been considered by many investigators. A paper published by Derham (1708) suggested that fogs as well as rain and snow strongly attenuate sound. In evaluating sound signaling devices for coastal use in fogs, however, John Tyndall (see Tyndall, 1964) made some observations in 1873 which contradicted Derham's hypothesis. He found, in fact, that sound was frequently transmitted easily in fog and explained this by observing that fog often was accompanied by a homogeneous atmospheric condition devoid of wind or thermal gradients.

Sewell (1910) published the first theoretical treatment of the problem by considering the suspended fog droplets as solid spheres fixed in a viscous medium. Ostwatitsch (1941) considered the problem of volatile particles which were allowed to transfer heat with the surrounding gas. This analysis allowed for evaporation and condensation of the liquid droplets, which we will find to be a dominant effect in atmospheric fogs, but it did not allow for any viscous interaction between the droplets and the surrounding gas. Sewell's basic approach to the non-volatile particle problem was extended by Epstein and Carhart (1953) who determined the attenuation due to both small particle oscillations in a viscous medium and heat transfer with the medium. Chow (1964) verified Epstein and Carhart's theory for larger oscillations. This method was then used by Chu and

Chow<sup>1</sup> to determine the dispersion of the sound wave. Both the attenuation and the dispersion of a sound wave in a non-volatile particle system were obtained by Temkin and Dobbins (1966) from the conservation equations of a gas and high-density particles. Their results, when specialized to small particle mass fractions, agree with the form of the Epstein-Carhart theory applicable to particles of high density. Marble (1969) and Wooten (1966) have discussed the attenuation and dispersion of sound as influenced by mass transfer but have not dealt in detail with the case of atmospheric fog. The present work uses the approach of Temkin and Dobbins to study the propagation in an atmospheric fog by considering mass, momentum and energy transfer mechanisms between the particles and the surrounding gas.

The system under consideration consists of a liquid phase of small spherical droplets suspended in, and initially in equilibrium with, a gaseous mixture of their vapor and a noncondensing species. The perturbation of this system by a planar acoustic wave results in fluctuations of the velocity, pressure, temperature and density of the gas-phase mixture. At suitably low frequencies the droplet velocity, temperature and mass respond to environmental changes and remain in mechanical and thermodynamic equilibrium with the gas phase. How-

<sup>1</sup>Chu, B. T., and J. C. F. Chow, 1965: On a macroscopic theory of a two-phase fluid. Paper No. 65-8, AIAA 2nd Aerospace Sci. Meeting, New York.

ever, as frequency increases, the finite rates of transfer cause the droplet properties to lag behind their equilibrium values. These lags result in the attenuation and dispersion of sound in a manner similar to that caused by vibrational relaxation in a diatomic gas.

Temkin and Dobbins (1966) have previously studied the influence on sound propagation of particulate relaxation resulting from momentum and heat transfer. In the case of small particle mass fraction these two effects are entirely separable and linearly additive in their contributions to acoustic attenuation and dispersion. When mass transfer is included as an additional particulate relaxation process, we will see that the results are considerably more complicated and of radically different character.

The state of the system is described by the conservation equations of fluid mechanics, considering the gaseous mixture (both vapor and noncondensing species) as one component, the vapor as a second, and the liquid phase as a third. The mass, momentum and energy interactions are described by appropriate source terms in the equations.

The following assumptions are necessary to the analysis:

- 1) Fluctuations of pressure, density and temperature produced by the wave are assumed to be small compared with their equilibrium values so that their squares and cross-products may be neglected. Also, the mixture and droplet velocities are small compared with the speed of sound.
- 2) The noncondensing gas and vapor are thermally and calorically perfect.
- 3) The liquid droplets are spherical and of uniform temperature and size.
- 4) The density of the liquid is much greater than that of the surrounding gaseous mixture, so that the volume fraction of the liquid may be neglected compared with that of the gaseous species.
- 5) The volumetric heat transfer rate, mass transfer rate, and body force due to viscous drag in a volume element containing a large number of droplets is the sum of the effects due to each droplet.
- 6) The temperatures of the noncondensing and vapor species in the gaseous mixture are the same. Similarly, the velocities of the two constituents are the same.
- 7) The gas constant of the mixture ( $R_m$ ) is a mass-weighted quantity. For the water vapor-air system the effects due to the perturbation of  $R_m$  by the sound field are small and will be neglected.

2. Basic equations<sup>2</sup>

The one-dimensional continuity equations for the mixture, vapor and droplets, respectively, are as follows:

$$\partial \rho_m / \partial t + \partial (\rho_m u_m) / \partial x = n \dot{M}, \tag{1}$$

<sup>2</sup> A list of symbols is given in the Appendix.

$$\partial \rho_v / \partial t + \partial (\rho_v u_m) / \partial x = n \dot{M}, \tag{2}$$

$$\partial \rho_p / \partial t + \partial (\rho_p u_p) / \partial x = -n \dot{M}, \tag{3}$$

where  $n$  is the number of droplets per unit volume of the droplet-mixture system,  $\dot{M}$  the evaporation rate per droplet,  $\rho_m$  the sum of the partial densities of the vapor and the noncondensing gas, and  $\rho_p$  the mass of the droplets per unit volume of the droplet-mixture system.

The momentum and energy equations for the mixture and droplets are

$$\rho_m (D u_m / D t) + \partial P_m / \partial x = n F - n \dot{M} (u_m - u_p), \tag{4}$$

$$\rho_p D u_p / D t = -n F, \tag{5}$$

$$\begin{aligned} \rho_m (D h_m / D t) - \partial P_m / \partial t - u_m (\partial P_m / \partial x) \\ = n F (u_p - u_m) + n \dot{Q} \\ + n \dot{M} [1/2 (u_m - u_p)^2 + (h_s - h_m)], \end{aligned} \tag{6}$$

$$\rho_p (L h_p / L t) = -n \dot{M} L - n \dot{Q}, \tag{7}$$

where  $F$  is the viscous drag force of a single droplet on the surrounding gas,  $\dot{Q}$  the rate of heat transfer from a single droplet to the gas,  $L$  the latent heat of vaporization for the liquid, and  $P_m$  the sum of the partial pressures of the vapor and the noncondensing gas.

The equations of state for the gaseous mixture and for the saturated vapor are, respectively,

$$P_m = \rho_m R_m T_m, \tag{8}$$

where

$$R_m = (\rho_g R_g + \rho_v R_v) / \rho_m,$$

and

$$P_s = \rho_s R_v T_p. \tag{9}$$

Finally, the Clausius-Clapeyron equation,

$$d P_s / d T_p = L P_s / (R_v T_p^2), \tag{10}$$

is required to relate the changes in saturation pressure to changes in the temperature of the droplets.

a. Droplet-gas interaction terms

We use the following expressions for the drag force and heat transfer rate per droplet:

$$F = 6 \pi R \mu (u_p - u_m), \tag{11}$$

$$\dot{Q} = 4 \pi R k (T_p - T_m), \tag{12}$$

where  $\mu$  is the dynamic viscosity and  $k$  the thermal conductivity of the gaseous mixture. These expressions are, respectively, Stokes' drag force and a corresponding heat transfer rate which is based on pure heat conduction. Both expressions apply in the limit of low Reynolds number for steady flow. These expressions may be used in an oscillating flow if (Temkin and Dobbins, 1966)

$$\rho_m / \rho_l \ll 1 \text{ and } (\omega R^2 / 2\nu)^{1/2} \ll 1.$$

The mass transfer rate is given by the corresponding low Reynolds number expression, i.e., Maxwell's diffusion

equation,

$$\dot{M} = 4\pi RD(\rho_s - \rho_v), \quad (13)$$

where  $D$  is the mass diffusivity.

The above mass, momentum and energy interaction terms are continuum results and are therefore only strictly valid for droplet diameters much greater than the mean-free-path of the surrounding gas. For normal atmospheric conditions this restricts the use of the interaction terms to droplets whose radius  $> 0.5 \mu\text{m}$ . Corrections for droplets smaller than this may be made by modifying the three transport coefficients; such modifications to the viscous, thermal and mass diffusion coefficients for use in the near continuum region are given by Fuchs (1964), Howell (1948) and Fuchs (1959), respectively.

### b. Linearization and nondimensionalization

Substitution of the above interaction terms into the original conservation equations yields a system of equations in ten dependent variables,  $P_m, P_s, T_m, T_p, u_m, u_p, \rho_m, \rho_p, \rho_s, \rho_v$ .

The equations are linearized by considering small perturbations of the above variables from their equilibrium values. The resulting perturbed variables are typically nondimensionalized as follows:

$$\begin{aligned} P_m &= P_{m0} + P_m' = P_{m0} + \rho_{m0} a_o^2 \bar{P}_m, \\ P_s &= P_{s0} + P_s' = P_{s0} (1 + \bar{P}_s), \\ T_m &= T_{m0} + T_m' = T_{m0} (1 + \bar{T}_m), \\ u_m &= u_m' = a_o \bar{u}_m, \\ \rho_m &= \rho_{m0} + \rho_m' = \rho_{m0} (1 + \bar{\rho}_m), \\ x &= a_o \bar{x} / \omega, \\ t &= \bar{t} / \omega. \end{aligned}$$

Substitution of the above form of the variables into the original system of equations and elimination of squares and cross-products of the nondimensional variables yields a set of linearized nondimensional, homogeneous differential equations. Nondimensional groups of parameters are defined as follows:

$$\begin{aligned} \bar{\tau}_c &= \omega R_o^2 \rho_l / (3D\rho_{m0}), \\ \bar{\tau}_d &= 2\omega R_o^2 \rho_l / (9\nu\rho_{m0}), \\ \bar{\tau}_t &= \omega R_o^2 \rho_l / (3\alpha_l \rho_{m0}), \\ C_m &= \rho_{p0} / \rho_{m0} = 4\pi R_o^3 n_o \rho_l / (3\rho_{m0}), \\ \bar{r} &= \rho_{v0} / \rho_{m0}, \\ \bar{A} &= C_{pv}(\gamma_v - 1) / (C_r \gamma_v), \\ \bar{B} &= C_{pm} / C_l, \text{ where } C_{pm} = (\rho_{g0} C_{pg} + \rho_{v0} C_{pv}) / \rho_{m0}, \\ \bar{L} &= L / (R_v T_{m0}), \\ \gamma_m &= (\rho_{g0} \gamma_g + \rho_{v0} \gamma_v) / \rho_{m0}. \end{aligned}$$

The resulting system of equations is then:

$$\partial \bar{\rho}_m / \partial \bar{t} + \partial \bar{u}_m / \partial \bar{x} = C_m \bar{r} (\bar{\rho}_s - \bar{\rho}_v) / \bar{\tau}_c, \quad (14)$$

$$\partial \bar{\rho}_v / \partial \bar{t} + \partial \bar{u}_m / \partial \bar{x} = C_m (\bar{\rho}_s - \bar{\rho}_v) / \bar{\tau}_c, \quad (15)$$

$$\partial \bar{\rho}_p / \partial \bar{t} + \partial \bar{u}_p / \partial \bar{x} = -\bar{r} (\bar{\rho}_s - \bar{\rho}_v) / \bar{\tau}_c, \quad (16)$$

$$\partial \bar{u}_m / \partial \bar{t} + \partial \bar{P}_m / \partial \bar{x} = C_m (\bar{u}_p - \bar{u}_m) / \bar{\tau}_d, \quad (17)$$

$$\partial \bar{u}_p / \partial \bar{t} = -(\bar{u}_p - \bar{u}_m) / \bar{\tau}_d, \quad (18)$$

$$\partial \bar{T}_m / \partial \bar{t} - (\gamma_m - 1) \partial \bar{P}_m / \partial \bar{x} = C_m (\bar{T}_p - \bar{T}_m) / \bar{\tau}_t, \quad (19)$$

$$\partial \bar{T}_p / \partial \bar{t} = -\bar{B} (\bar{T}_p - \bar{T}_m) / \bar{\tau}_t - \bar{A} \bar{L} \bar{r} (\bar{\rho}_s - \bar{\rho}_v) / \bar{\tau}_c, \quad (20)$$

$$\gamma_m \bar{P}_m - \bar{\rho}_m - \bar{T}_m = 0, \quad (21)$$

$$\bar{P}_s - \bar{\rho}_s - \bar{T}_p = 0, \quad (22)$$

$$d\bar{P}_s / d\bar{T}_p = \bar{L}. \quad (23)$$

In the non-volatile particle problem, the particle-energy equation (20) does not contain the second term on the right-hand side. In that case, the particle-energy equation is a first-order, linear differential equation identical to the relaxation equation used to describe the transfer of energy from an translational to an internal degree of freedom [see, for instance, Herzfeld and Litovitz (1959)]. The form of the resulting attenuation and dispersion for the non-volatile particle problem is therefore closely related to the form of the attenuation and dispersion resulting from processes such as the vibrational relaxation of a diatomic molecule.

When mass transfer is included, the energy equation for the droplets is no longer identical to the classical relaxation equation; rather, it becomes a more complicated equation containing the effects of simultaneous heat and mass transfer. The momentum equation (18) for the droplet phase, however, remains a classical relaxation equation characterized by  $\bar{\tau}_d$  for both the non-volatile and the volatile particle problem.

In order to characterize the momentum, energy and mass interaction processes for the volatile particle problem, both  $\bar{\tau}_d$  and  $\bar{\tau}_t$  are retained from the non-volatile particle problem and an analogous mass transfer time  $\bar{\tau}_c$  is defined. It is noted that for most gases the Prandtl number and the Lewis number are of order unity; therefore, the three characteristic times ( $\bar{\tau}_c, \bar{\tau}_d, \bar{\tau}_t$ ) are of the same order of magnitude.

### 3. Method of solution

The system of linear differential equations is reduced to a system of linear algebraic equations by assuming a steady solution for all the dependent variables proportional to  $\exp[i(a_o K \bar{x} / \omega - \bar{t})]$ , where  $K = K_1 + iK_2$  is the complex propagation number, and  $a_o$  the sound speed in the gaseous mixture ( $a_o^2 = \gamma_m R_m T_{m0}$ ).

This results in a system of ten homogeneous, linear algebraic equations. The system of ten equations is easily reduced to a system of six equations by eliminating  $\bar{P}_m, \bar{P}_s, \bar{T}_p, \bar{u}_p$ . A non-trivial solution of the resulting six homogeneous equations exists if and only if the determinant of the coefficients vanishes. Expanding the determinant and simplifying by row and column operations yields the following  $5 \times 5$  determinant which is identically equal to zero and is solved for  $(a_o K / \omega)^2$ :

$$\begin{vmatrix} 1 & 1 & -i & 0 & \bar{r} \\ 1 & \frac{1}{\bar{r}}\left(1-i\frac{\bar{r}_c}{C_m}\right) & 0 & 0 & 1 \\ i\left(1+\frac{C_m}{1-i\bar{r}_d}\right)\left(\frac{\omega}{a_0K}\right)^2\gamma_m & 0 & 1 & 1 & 0 \\ 0 & 0 & i\frac{(\gamma_m-1)}{\gamma_m} & \left(\frac{C_m-i}{\bar{r}_t\gamma_m}\right) & \frac{N_{Le}}{(\bar{L}-1)} \\ 0 & \bar{A}\bar{L} & 0 & \frac{C_m}{\bar{r}_t} & \left[\frac{(\bar{B}-i\bar{r}_t)}{(\bar{L}-1)}N_{Le}+\bar{A}\bar{L}\bar{r}\right] \end{vmatrix} = 0.$$

We observe that the viscous effect appears in the sole term in which the wave propagation number appears. The determinant may therefore be solved for  $(a_0K/\omega)^2$ , excluding the viscous effect, and the non-viscous result then multiplied by  $[1+(C_m/(1-i\bar{r}_d))]$  to obtain the full result.

We also note that other than in the aforementioned viscous term which involves  $\bar{r}_d$ , either  $\bar{r}_c/C_m$  or  $\bar{r}_t/C_m$  appears in three of the remaining four terms of the determinant which contain a characteristic time. Anticipating a future result, we will choose  $\bar{r}=\bar{r}_t/C_m$  as the characteristic time for much of the remaining discussion.

4. Results

In order to simply display the resulting expression for the complex propagation number, the following combinations of dimensionless thermodynamic parameters are defined:

$$\begin{aligned} C_1 &= 1+\gamma_m N_{Le}, \\ C_2 &= \bar{A}\bar{L}(\bar{L}-1)\bar{r}+\bar{B}N_{Le}, \\ C_3 &= \gamma_m, \\ C_4 &= \gamma_m\bar{A}\bar{L}(\bar{L}-1)\bar{r}+\bar{B}-\bar{B}(\gamma_m-1)(\bar{L}-1)\bar{r}, \\ C_5 &= N_{Le}, \\ C_6 &= N_{Le}+1-\bar{r}, \\ C_7 &= 1-\bar{r}, \\ C_8 &= \bar{A}\bar{L}^2\bar{r}+\bar{B}(1-\bar{r}). \end{aligned}$$

The full result obtained by solving the above determinant is

$$\left(\frac{a_0K}{\omega}\right)^2 = \left[1+\frac{C_m}{\left(1-i\frac{2C_m\bar{r}}{3N_{Pr}}\right)}\right] \times \left[\frac{(C_2+C_mC_1)\bar{r}+i(C_4+C_mC_3-C_mC_5\bar{r}^2)}{(C_2+C_mC_6)\bar{r}+i(C_8+C_mC_7-C_mC_5\bar{r}^2)}\right]. \quad (24)$$

This equation can be solved exactly by complex number digital computation; however, the examination of cases

for which parameters of interest in (24) are small affords greater insight and utility. The specific limits are delineated below in cases A through D. Some results of these limiting cases for attenuation and dispersion have been plotted in subsequent figures using dry air values of pressure and density taken from the data of the U. S. Standard Atmosphere (Dubin *et al.*, 1966). The data corresponding to a temperature of 23.1C have been selected to represent a ground-level warm fog. The properties of a saturated vapor corresponding to the above temperatures have been taken from water vapor tables of Goff and Gratch (1946). Values of thermal conductivity and viscosity have been taken to be dry air values while the value of mass diffusivity has been obtained from Fuller *et al.* (1966). Specific heats and specific heat ratios for water vapor and air have been obtained from Washburn (1929) and Hilsenrath *et al.* (1956), respectively. The complete table of parameters used for plotting the results is shown in Table 1.

Case A. Small droplet mass fraction and  $C_m\bar{r}\ll 1$

Under these limitations, (24) reduces to

$$(a_0K/\omega)^2 = (C_2\bar{r}+iC_4)/(C_2\bar{r}+iC_8). \quad (25)$$

The conditions for case A are fulfilled when the effects of mass transfer are most pronounced. In atmospheric fogs this case includes the low audible and infrasonic frequency regions. For natural fogs  $C_m$  is typically  $10^{-4}$ , corresponding to a liquid water content of  $0.11 \text{ gm m}^{-3}$  [see, for instance, Mason (1957) and Fletcher (1962)];

TABLE 1. Values of thermodynamic properties, transport properties, and groupings of properties for water and air at 23.1C and 1.0 atm.

$\rho_{v0} = 1.192 \text{ kg m}^{-3}$	$\gamma_0 = 1.402$
$\rho_{v0} = 2.044 \times 10^{-2} \text{ kg m}^{-3}$	$\gamma_v = 1.32$
$\rho_l = 1.000 \times 10^3 \text{ kg m}^{-3}$	$C_1 = 2.165$
$L = 5.839 \times 10^2 \text{ kcal kg}^{-1}$	$C_2 = 0.762$
$C_{pv} = 0.239 \text{ kcal kg}^{-1} (\text{°K})^{-1}$	$C_3 = 1.399$
$C_{pl} = 0.453 \text{ kcal kg}^{-1} (\text{°K})^{-1}$	$C_4 = 0.999$
$C_l = 1.000 \text{ kcal kg}^{-1} (\text{°K})^{-1}$	$C_5 = 0.832$
$k = 6.204 \times 10^{-6} \text{ kcal m}^{-1} \text{ sec}^{-1} (\text{°K})^{-1}$	$C_6 = 1.816$
$\mu = 1.828 \times 10^{-6} \text{ kg m}^{-1} \text{ sec}^{-1}$	$C_7 = 0.983$
$D = 2.58 \times 10^{-6} \text{ m}^2 \text{ sec}^{-1}$	$C_8 = 0.832$

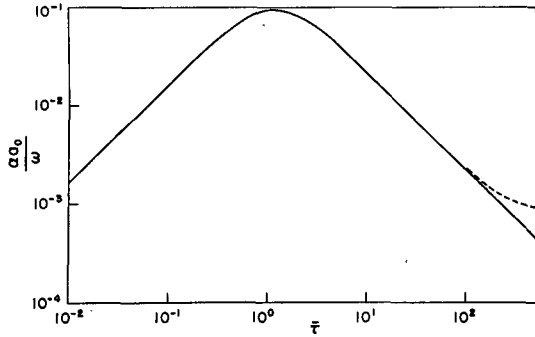


FIG. 1. Attenuation per unit wavelength for small values of  $\bar{\tau}$ : solid line, exact solution (24) for  $C_m \leq 10^{-4}$  and small  $C_m \bar{\tau}^2$  (Case A) solution (27); dashed line, exact solution (24) for  $C_m = 10^{-3}$ .

the value of  $\bar{\tau}$  is therefore restricted to be much less than 100. For example, in a warm fog, the frequency range must, in this case, be small compared with 120 Hz for a droplet radius of 1.0  $\mu\text{m}$ , or 1.2 Hz for a droplet radius of 10  $\mu\text{m}$ .

Attenuation and dispersion are presented in Figs. 1 and 2 using the exact solution (24) and the simplified solution (25) of case A. The frequency range of validity of (25) depends on the magnitude of the mass fraction. Fig. 1 indicates that the Napier frequency (i.e., the frequency for which attenuation per unit wavelength is a maximum) is found near  $\bar{\tau}$  equal to unity. This result can be recovered from Eq. (25) by separating its real and imaginary parts to yield

$$\left(K_1 \frac{a_o}{\omega}\right) \left(2K_2 \frac{a_o}{\omega}\right) = \bar{\tau} \left(\frac{C_4 - C_8}{C_2}\right) / \left[\bar{\tau}^2 + \left(\frac{C_8}{C_2}\right)^2\right]. \quad (26)$$

Since  $K_1 a_o / \omega = a_o / a \approx 1$ , we find the dimensionless absorption given by

$$\frac{\alpha a_o}{\omega} \approx \bar{\tau} \left(\frac{C_4 - C_8}{C_2}\right) / \left[\bar{\tau}^2 + \left(\frac{C_8}{C_2}\right)^2\right], \quad (27)$$

where  $\alpha$ , the coefficient of acoustic energy or intensity attenuation, is equal to  $2K_2$ . From this we find that  $\alpha a_o / \omega$  has a maximum when  $\bar{\tau} = \bar{\tau}_{nap}$  given by

$$\bar{\tau}_{nap} = C_8 / C_2 = \frac{[\bar{A} \bar{L}^2 \bar{\tau} + \bar{B}(1 - \bar{\tau})]}{[\bar{A} \bar{L}(\bar{L} - 1) \bar{\tau} + \bar{B} N_{Le}]}, \quad (28)$$

which is numerically close to unity. Eq. (27) can be used as a simple algebraic expression to calculate attenuation to an accuracy of  $\sim 10\%$  or better for infrasonic and low audible frequencies.

The contrast between the results obtained from the non-volatile and the evaporating particle theories is now apparent. For the non-volatile particle theory, the important characteristic times are the dynamic and thermal relaxation times which are of the same magnitude. When mass transfer effects are included, the important relaxation time is the time  $\bar{\tau}$  (defined by  $\bar{\tau} = \bar{\tau}_t / C_m$  and equal in magnitude to  $\bar{\tau}_d$  or  $\bar{\tau}_e$  divided by

the mass fraction). Here we chose the thermal relaxation time because we expect a thermal accommodation coefficient of unity. A departure from unity of the accommodation coefficient for mass transfer can then be conveniently considered by adjusting the Lewis number. We offer the following explanation for the difference between the characteristic times of non-volatile and volatile particle theories. When only the viscous effects are considered, the particle relaxation time for an incremental velocity change is determined by the particle mass and the drag force and, if the mass fraction is small, the drag force is not related to the number of particles present. If an incremental change of vapor pressure is imposed upon a small mass fraction of volatile particles, then the time for vapor equilibration depends not only on diffusion coefficient and surface area of the droplet but also on the volume of space with which each droplet must exchange vapor. Thus, the time characteristic of mass transfer is proportional to  $R^2/D$  divided by the volume fraction which the droplets represent. Since the volume fraction is  $C_m \rho_m / \rho_i$ , we obtain the important characteristic time with  $C_m$  in the denominator.

Case B. Small droplet mass fraction and  $C_m \bar{\tau}^2 \gg 1$

These conditions are fulfilled when viscous effects play their most important part. The results for this case will normally describe the attenuation of sound for fog at all audible frequencies.

The frequency range for which  $C_m \bar{\tau}^2 \gg 1$  (i.e.,  $\bar{\tau}^2 \gg 1/C_m$ ) gives algebraically tractable results for the separation of attenuation and dispersion. In order to simplify the full expression (24) for this region, the result without the viscous contribution is first considered. Dividing numerator and denominator in (24) by  $C_2 \bar{\tau} + i(C_8 - C_m C_5 \bar{\tau}^2)$ , and setting  $N_{Pr} = 0$ , we obtain

$$\left(\frac{a_o K}{\omega}\right)^2 = \left\{ 1 + \left[ \frac{C_m C_1 \bar{\tau} + i(C_4 - C_8 + C_m C_3)}{C_2 \bar{\tau} + i(C_8 - C_m C_5 \bar{\tau}^2)} \right] \right\} / \left\{ 1 + \left[ \frac{C_m(C_6 \bar{\tau} + iC_7)}{C_2 \bar{\tau} + i(C_8 - C_m C_5 \bar{\tau}^2)} \right] \right\}.$$

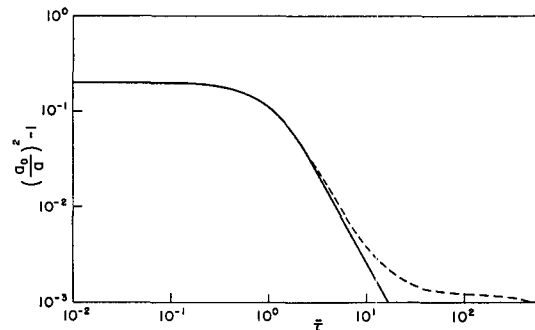


FIG. 2. Dispersion for small values of  $\bar{\tau}$ : solid line, exact solution (24) for  $C_m \leq 10^{-4}$  and small  $C_m \bar{\tau}^2$  (Case A) solution (27); dashed line, exact solution for  $C_m = 10^{-3}$ .

For  $C_m \ll 1$  and all of the  $C_i$  of order unity (see Table 1), the denominator can be expanded as a power series, giving

$$\left(\frac{a_o K}{\omega}\right)^2 = 1 + \frac{C_m \bar{\tau}(C_1 - C_6) + i[C_4 - C_8 + C_m(C_3 - C_7)]}{C_2 \bar{\tau} + i(C_3 - C_m C_6 \bar{\tau}^2)} + \dots$$

This expression is then multiplied by  $\{1 + [C_m / (1 - i\bar{\tau}_d)]\}$  to include the viscous contribution. Retaining terms of order  $C_m$  and using the binomial expansion, the real and imaginary parts of  $K$  are obtained. The explicit dependence of the results on  $C_m$  may be determined by reverting to the original relaxation times,  $\bar{\tau}_d$  and  $\bar{\tau}_t$ ; then,

$$\left(\frac{a\alpha_o}{\omega}\right) = C_m \left\{ \frac{(C_4 - C_8) + (C_1 - C_6) \frac{C_5}{C_2} \bar{\tau}_t^2}{C_2 \bar{\tau}_t \left[ 1 + \left(\frac{C_5}{C_2} \bar{\tau}_t\right)^2 \right]} + \frac{\bar{\tau}_d}{1 + \bar{\tau}_d^2} \right\}, \quad (29)$$

$$\left(\frac{a_o}{a}\right)^2 - 1 = C_m \left\{ \frac{(C_1 - C_6) - (C_4 - C_8) \frac{C_5}{C_2}}{C_2 \left[ 1 + \left(\frac{C_5}{C_2} \bar{\tau}_t\right)^2 \right]} + \frac{1}{1 + \bar{\tau}_d^2} \right\}. \quad (30)$$

The above relations for attenuation and dispersion are valid for the case in which  $C_m \ll 1$  and  $C_m \bar{\tau}^2 \gg 1$ , and they are linearly dependent on  $C_m$ . The viscous contribution to attenuation and dispersion, the second terms of (29) and (30), is identical with that of the non-volatile particle theory. The first terms of (29) and (30) represents the contribution due to simultaneous heat and mass transfer; a corresponding term in the results of the non-volatile particle theory gives the contribution due to the heat transfer process only.

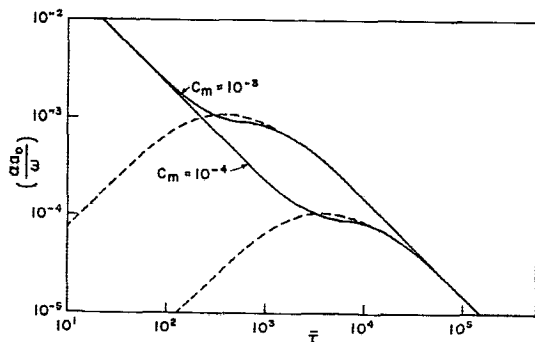


FIG. 3. Attenuation per unit wave length for large values of  $\bar{\tau}$ : solid line, exact solution (24) and large  $C_m \bar{\tau}^2$  (Case B) solution (29); dashed line, no mass transfer (Case C) solution (32).

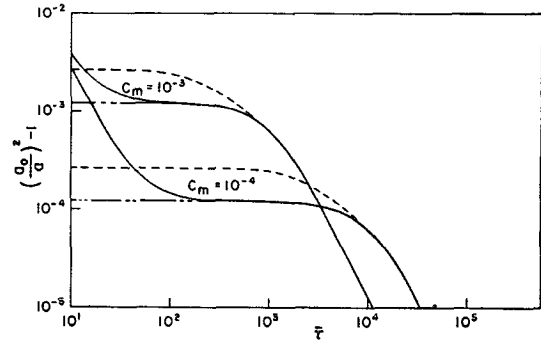


FIG. 4. Dispersion for large values of  $\bar{\tau}$ : solid line, exact solution (24); intermittent dashed line, large  $C_m \bar{\tau}^2$  (Case B) solution (30); uniform dashed line, no mass transfer (Case C) solution (33).

The results of (29) and (30) are compared with the results of the exact solution Eq. (24) in Figs. 3 and 4. The attenuation per unit wavelength from (29) and the exact solution agree with each other over the range of  $\bar{\tau}$  from 10 to  $10^5$  for the two mass fractions indicated in Fig. 3. On Fig. 4, however, the results of (30) for dispersion deviate from the exact solution for low values of  $\bar{\tau}$ .

Summarizing the results for small  $C_m$ , in the region for which  $C_m \bar{\tau}^2 \ll 1$ , we note that both attenuation and dispersion are characterized by a time  $\bar{\tau} = \bar{\tau}_t / C_m$  and are otherwise independent of  $C_m$ . For the region in which  $C_m \bar{\tau}^2 \gg 1$ , however, Eqs. (29) and (30) show that both attenuation and dispersion depend directly on  $C_m$  and may be characterized by times  $\bar{\tau}_d$  and  $\bar{\tau}_t$ .

*Case C. Non-volatile particle limit for attenuation and dispersion*

The full expression may be reduced to the results of the non-volatile particle theory by considering the limit of infinite Lewis number when the Prandtl number remains of order unity. In this case  $\bar{\tau}_e \rightarrow 0$ , and we obtain

$$\lim_{N_{Le} \rightarrow \infty} \left(\frac{a_o K}{\omega}\right)^2 = \left[ 1 + \frac{C_m}{1 - i \frac{2C_m \bar{\tau}}{3N_{Pr}}} \right] \left( \frac{\bar{B} + C_m \gamma_m - i C_m \bar{\tau}}{\bar{B} + C_m - i C_m \bar{\tau}} \right). \quad (31)$$

When the definitions of  $\bar{\tau}$  and  $\bar{B}$  are substituted, for  $C_m \ll 1$ , we have

$$\frac{\alpha\alpha_o}{\omega C_m} = \frac{\omega \tau_d}{1 + (\omega \tau_d)^2} + (\gamma_m - 1) \left[ \frac{\omega \tau_t}{1 + \left(\frac{C_{Pm}}{C_t}\right)^2} \right], \quad (32)$$

$$\left[ \left( \frac{a_o}{a} \right)^2 - 1 \right] / C_m = \frac{1}{1 + (\omega\tau_d)^2} + (\gamma_m - 1) \left[ \frac{C_l / C_{Pm}}{1 + \left( \frac{C_{Pm}}{\omega\tau_t} \right)^2} \right]. \quad (33)$$

This is exactly the result obtained for the non-volatile particle problem (Temkin and Dobbins, 1966).<sup>3</sup>

The predictions of (32) and (33) are shown in Figs. 3 and 4 as the short dashed lines. From these figures it is apparent that the curves for the non-volatile and volatile particles are coincident at higher frequencies but deviate markedly from one another at the lower frequencies.

*Case D. The zero and infinite frequency limits of sound speed*

The zero frequency limit of the exact expression is

$$\lim_{\omega \rightarrow 0} \left( \frac{a_o K}{\omega} \right)^2 = (1 + C_m) \left( \frac{C_4 + C_m C_3}{C_8 + C_m C_7} \right), \quad (34)$$

and for small mass fraction, the zero frequency limit is

$$\lim_{C_m \rightarrow 0} \left[ \lim_{\omega \rightarrow 0} \left( \frac{a_o K}{\omega} \right)^2 \right] = \frac{C_4}{C_8}. \quad (35)$$

The result states that the zero frequency limit of the ratio of the speed of sound in the gaseous mixture to that in the mixture-droplet system is not unity in the case of vanishing mass fraction. The corresponding limiting case for the non-volatile particle predicts a sound speed ratio of unity.

Evaluating the expression for infinite frequency gives the expected result

$$\lim_{\omega \rightarrow \infty} \left( \frac{a_o K}{\omega} \right)^2 = 1.$$

As shown in Fig. 2, the zero frequency limit of the complete solution (24) is the value of the dispersion curve at zero frequency. For  $C_m \leq 10^{-3}$ , the zero frequency sound speed ratio is seen to be independent of  $C_m$  and not equal to unity.

**5. Attenuation of audible frequencies in fog and acoustic signaling**

Fig. 5 shows attenuation  $\alpha$  per meter as a function of dimensional frequency (Hz) for a mass fraction  $C_m$  of  $10^{-4}$  and varying droplet radii. On each curve the distance in meters,  $l_v$ , represents a path for which the optical transmission is equal to  $e^{-2}$ , which approximates

<sup>3</sup> It should be noted that the thermal relaxation time  $\tau_t$  for this analysis is not the identical time as that used by Temkin and Dobbins.

visual range. The dashed curve on Fig. 5 indicates the departure of the non-volatile particle attenuation from that of volatile droplets. The effect of the mass transfer process is to provide a nearly constant level of attenuation for low audible frequencies.

For small mass fraction ( $C_m \ll 1$ ) the solution [Eqs. (24) or (29)] predicts two levels of nearly constant dimensional attenuation in the audible range of frequencies. As shown in Fig. 5, the lower level of attenuation extends from the infrasonic region up to the region in which viscous effects begin to become important. This level of attenuation per unit distance ( $\alpha_o$ ) may be obtained from (27) specialized for  $\omega$  becoming large, i.e.,

$$\alpha_o = \lim_{\omega \rightarrow \infty} \alpha = (\omega/a_o)(C_4 - C_8)/(C_2 \bar{\tau}) = C_m(C_4 - C_8)/(a_o C_2 \tau_t).$$

For the upper level of dimensional attenuation ( $\alpha_\infty$ ), (29) may also be investigated for  $\omega$  becoming large, i.e.,

$$\alpha_\infty = \lim_{\omega \rightarrow \infty} \alpha = (\omega/a_o) \left[ (C_1 - C_6)/(C_5 \bar{\tau}) + \left( \frac{3}{2} \right) N_{Pr} / \bar{\tau} \right] = (C_m/a_o \tau_t) \left[ (C_1 - C_6)/C_5 + \left( \frac{3}{2} \right) N_{Pr} \right].$$

Thus, using the approximate results (27) and (29), both  $\alpha_o$  and  $\alpha_\infty$  are independent of  $\omega$  and directly dependent on mass fraction. The ratio of the two levels of attenuation is given by

$$\alpha_\infty/\alpha_o = \left[ (C_1 - C_6)/C_5 + \left( \frac{3}{2} \right) N_{Pr} \right] \left[ C_2 / (C_4 - C_8) \right]. \quad (36)$$

For typical atmospheric conditions, (36) reduces to  $\alpha_\infty/\alpha_o \approx 6.0$ .

The transition from  $\alpha_o$  to  $\alpha_\infty$  occurs for  $\omega\tau_t \approx 1.0$ . For a warm fog of 8.0  $\mu\text{m}$  droplet radius, this condition implies that the transition frequency is approximately 130 Hz. Thus, for small mass fraction, attenuation per unit distance through a fog is given by  $\alpha_o$  when  $\omega \ll 1/\tau_t$ , while for  $\omega \gg 1/\tau_t$ , dimensional attenuation is found to be  $\alpha_\infty$ .

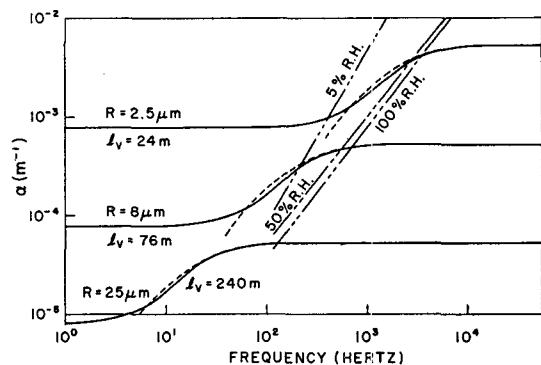


Fig. 5. Attenuation of acoustic energy by fog and by humid air as functions of dimensional frequency: solid line, exact solution (24) for  $C_m = 10^{-4}$ ; uniform dashed line, no mass transfer (Case C) solution (32) for  $C_m = 10^{-4}$ ; intermittent dashed line, humid air data from Harris (1967).

The attenuation in humid air from data by Harris (1967) is also plotted on Fig. 5. The total attenuation of sound through a fog is then given by the sum of the contributions of the droplets and of the surrounding humid air. The data of Harris indicates that the attenuation in saturated air (with no droplets present) is smaller than that in less humid conditions; therefore, in a certain frequency range the total attenuation of sound by a fine fog (i.e., small  $C_m$ ) is less than the attenuation by a clear air of low humidity. At high frequencies, attenuation by the molecular relaxation processes in air always dominates the droplet attenuation.

Since the major effects of mass transfer only appear at very low frequencies, it is difficult to experimentally verify the results of this theory. The data of Wei (1950) taken in fogs at low frequencies indicate the correct trend in the attenuation due to mass transfer but only order-of-magnitude agreement. The data of Knudsen *et al.* (1948) show order-of-magnitude agreement at frequencies where the contribution due to the mass transfer is small. Agreement of a qualitative nature is obtained from the observations of Wiener (1961) who found no appreciable difference in attenuation with or without the presence of a natural fog. He concludes that effects due to velocity and thermal gradients predominate over the effects discussed above in causing attenuation and dispersion of sound in natural fogs and clouds. Our own measurements of acoustic attenuation in fogs will be reported in the future.

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## APPENDIX

### List of Symbols

$a$	speed of sound in the fog
$a_0$	speed of sound in the gaseous mixture (vapor and noncondensing species)
$c_l$	specific heat of the liquid
$C_m$	ratio of the mass of the liquid droplets to the mass of the gaseous mixture for a unit volume of the undisturbed mixture-droplet system
$c_p$	constant pressure specific heat
$c_v$	constant volume specific heat
$D$	coefficient of mass diffusivity of the gaseous mixture
$F$	drag force on one droplet
$h$	specific enthalpy
$K$	complex wave propagation number
$k$	thermal conductivity of the gaseous mixture
$L$	latent heat of vaporization
$l_v$	optical path length

$\dot{M}$	rate of mass evaporated from one droplet
$n$	number of droplets per unit volume of the mixture
$N_{Le}$	Lewis number-ratio of the diffusivity of heat to the diffusivity of mass ( $=\alpha_t/D$ )
$N_{Pr}$	Prandtl number-ratio of the diffusivity of momentum to the diffusivity of heat ( $=\nu/\alpha_t$ )
$P$	pressure
$\dot{Q}$	rate of heat transferred from one droplet
$R$	droplet radius
$R_i$	gas constant for gaseous species $i$
$T$	temperature
$t$	time
$u$	velocity
$x$	distance
$\alpha$	coefficient of energy attenuation, i.e., twice the attenuation coefficient for the fluctuating component of pressure, density, etc.
$\alpha_t$	thermal diffusivity of the mixture
$\gamma$	specific heat ratio
$\mu$	dynamic viscosity of the gaseous mixture
$\nu$	kinematic viscosity of the gaseous mixture
$\rho$	density (mass per unit volume of the gaseous mixture)
$\bar{\tau}$	dimensionless characteristic time for the system
$\omega$	circular acoustic frequency or $2\pi$ times the frequency of oscillation in hertz

### Subscripts

$g$	noncondensing gas
$v$	vapor
$s$	saturated vapor
$p$	droplet phase
$l$	liquid
$m$	mixture of noncondensing gas and vapor
$o$	equilibrium conditions

### Superscripts

'	perturbed quantities due to the sound field
-	nondimensionalized perturbed quantities

### REFERENCES

- Chow, J. C. F., 1964: Attenuation of acoustic waves in dilute emulsions and suspensions. *J. Acoust. Soc. Amer.*, **36**, 2395-2401.
- Derham, R., 1708: Experiments on the motion of sound. *Phil. Trans. Roy. Soc. London*, **5**, 380-395.
- Dubin, M., N. Sissenwine and S. Teweles (Cochairmen, U. S. Committee on Extension to the Standard Atmosphere), 1966: *U. S. Standard Atmosphere Supplement, 1966*. Washington, D. C., Govt. Printing Office, p. 115.
- Epstein, P. S., and R. R. Carhart, 1953: The absorption of sound in suspensions and emulsions. I. Water fog in air. *J. Acoust. Soc. Amer.*, **25**, 553-565.
- Fletcher, N. H., 1962: *The Physics of Rainclouds*. Cambridge University Press, 110-121.
- Fuchs, N. A., 1959: *Evaporation and Droplet Growth in Gaseous Media*. New York, Pergamon, 7-11.
- , 1964: *The Mechanics of Aerosols*. New York, MacMillan, 25-29.



- Fuller, E. H., P. D. Schettler and J. C. Giddings, 1966: A new method for prediction of binary gas-phase diffusion coefficients. *Ind. Eng. Chem.*, **58**, 5, 19-27.
- Goff, J. A., and S. Gratch, 1946: Low-pressure properties of water from  $-160$  to  $212^{\circ}\text{F}$ . *Trans. Amer. Soc. Heating Ventilating Eng.*, **52**, 95-121.
- Harris, C. M., 1967: Absorption of sound in air versus humidity and temperature. NASA Rept. CR-647 (prepared by Columbia University for George C. Marshall Space Flight Center).
- Herzfeld, K. F., and T. A. Litovitz, 1959: *Absorption and Dispersion of Ultrasonic Waves*. New York, Academic Press.
- Hilsenrath, J., C. W. Beckett, W. S. Benedict, L. Fano, H. J. Hodge, J. F. Masi, R. L. Nuttall, Y. S. Touloukian and H. W. Woolley, 1956: Tables of thermal properties of gases. Natl. Bur. Stds., Circ. No. 564, 24-74.
- Howell, W. E., 1948: The growth of cloud drops in uniformly cooled air. Ph.D. dissertation, Massachusetts Institute of Technology.
- Knudsen, V. C., J. V. Wilson and N. S. Anderson, 1948: The attenuation of audible sound in fog and smoke. *J. Acoust. Soc. Amer.*, **20**, 849-857.
- Marble, F. E., 1969: Some gas dynamic problems in the flow of condensing vapours. *Astronautica Acta*, **14**, 585-614.
- Mason, B. J., 1957: *The Physics of Clouds*. Oxford University Press, 84-99.
- Oswatitsch, Kl. von, 1941: Die Dispersion und Absorption des Schalles in Wolken. *Phys. Z.*, **42**, 365-378. Translated as NSAS-TT-F-10939.
- Sewell, C. J. T., 1910: On the extinction of sound in a viscous atmosphere by small obstacles of cylindrical and spherical form. *Phil. Trans. Roy. Soc. London*, **A210**, 239-270.
- Temkin, S., and R. A. Dobbins, 1966: Attenuation and dispersion of sound by particulate-relaxation processes. *J. Acoust. Soc. Amer.*, **40**, 317-324.
- Tyndall, J., 1964: *The Science of Sound*. New York, Philosophical Library, 305-376.
- Washburn, E. W., 1929: *International Critical Tables of Numerical Data, Physics, Chemistry, and Technology*, Vol. 5. New York, McGraw-Hill, 62-82.
- Wei, Y. T., 1950: The absorption of sound in foggy air at low audible frequencies. Ph.D. dissertation, University of California, Los Angeles.
- Wiener, F. M., 1961: Sound propagation over ocean waters in fog. *J. Acoust. Soc. Amer.*, **33**, 1200-1205.
- Wooten, D. C., 1966: The attenuation and dispersion of sound in condensing medium. Ph.D. dissertation (Part I), California Institute of Technology.