A Numerical Simulation of Wintertime, Orographic Precipitation: Part I.
Description of Model Microphysics and Numerical Techniques

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ABSTRACT

A numerical model which extends treatment of microphysical cloud processes to more than one level through use of the continuous bin technique is described. A general solution to the diffusion growth equation including latent heat release due to accretion is presented. Collection processes of coalescence, accretion and aggregation, activation of CCN, and ice phase nucleation through sorption and contact freezing nucleation are combined with a diffusion treatment which allows for calculation of the supersaturation in a multi-level framework and with sedimentation of particles between levels.

1. Introduction

The use of a numerical, microphysical cloud model to predict the microphysical structure of real clouds and the development of precipitation depends on being able to treat the interactions between microphysical processes occurring at several levels within the cloud. Current microphysical models are generally limited in assuming a uniform cloud or that the remainder of the cloud is fixed and unmodified by the particles of interest which fall through it. The models of Cotton (1970), Jiusto and Holroyd (1970) and Hobbs et al. (1973) are examples of this type. Such models are physically unrealistic since clouds in general are not uniform and interaction through sedimentation does alter the rates at which microphysical processes occur.

Dynamical models such as that described by Takeda (1971) may treat microphysical processes occurring at many levels within the cloud and simulate the development of precipitation. The limitations of such existing models lie in their extensive parameterization and simplification of the microphysical processes. This model demonstrates some of the feedback interactions which may occur between the various microphysical processes and which may be eliminated from such highly parameterized models.

The development of the multi-level model described herein, with a minimum of parameterization of the microphysical processes, allows a quantitative assessment of precipitation. The continuous bin technique described can be adapted to either a Lagrangian or Eulerian dynamical framework, although in this paper the former is employed. Subject to field verification, this model may be applied to situations in which the dynamics are well known in order to assess the effects of modifications designed to alter precipitation output.

The formulation and verification of such a model not only bridges the gap between theory and observation in a relatively complex cloud but also provides a rational basis for weather modification activities. Herein, demonstration of a “seeding effect” has relied on a properly designed statistical experiment. A case in point is the Climax, wintertime, orographic seeding experiment (e.g., Mielke et al., 1971). The formulation of this model in an orographic framework and founded on basic physical principles rather than utilizing extensive parameterization or drastic simplifications is a major step toward producing a predictive model which can be used to demonstrate the physical relationship between “cause” and “effect” in the Climax experiment, for example. In addition, such a model can be used to optimize the seeding conditions to maximize the desired effect (or to minimize the cost/benefit ratio). In such a situation, a cost/benefit analysis involves consideration of factors such as the spatial distribution and type of the precipitation produced in order to determine snow removal costs, etc. (Grant, 1969). An application to a wintertime orographic snowfall modification using AgI is described in the following paper (Young, 1974a).

2. Particle-tracing versus bin models

Two basic methods of numerically solving the complex of equations which constitute a microphysical model are evident, those in which individual particles are followed and those in which particles are transferred between fixed bins. Particle-tracing models regard the
particulate population as consisting of distinct groups whose histories are known (e.g., Hobbs et al., 1973). Bin models continually mesh particles of different histories so that individual particle histories are not known (e.g., Berry, 1966).

The particle-tracing model encounters difficulties in tracing layers of particles as they fall through other layers. Within any given time increment, only some of the particles within a layer enter the next lowest cloud layer. Since the portion transferred is now encountering a new environment, the original group of like particles must be subdivided into two subgroups. In the next time increment, another subgroup of particles enters the lower cloud layer, now modified by interaction with the first such subgroup. This results in three slightly different subgroups, each modifying their environment slightly differently. Thus, in a multi-level, particle-tracing model, the required number of groupings increases with the number of time steps used and rapidly becomes too large for a computer to handle efficiently.

In the bin model, the number of particle groupings, i.e., bins, remains constant as similar particles are meshed into the same bin. However, complications arise due to the requirement of maintaining continuity in both number and mass. This leads to two methods of solution, the commonly used discrete bin method and the continuous bin method advanced in this paper.

Under the bin method, growth is a consequence of the addition of mass to the particles in a given bin. In the discrete bin method, particles generally are not permitted to exist at their "new" radius and need to be partitioned between permissible radii so that number and mass are conserved. Hence, in each time step, some particles are always transferred to a larger size, no matter how small the added mass may be. These "larger" particles then accrue more mass during the next time step and again are partitioned between permissible radii, transferring some particles to still larger sizes. Clearly, the growth of the largest particles is restricted only by the spacing of permissible radii and the time step utilized.

Growth under the continuous bin method results in transfer of the larger particles within one bin to the smaller sizes of the next larger bin. In this case, the entire range of the bin need not contain particles. With successive time steps, the larger bin gradually fills until the range is complete and only then do particles advance to the next larger bin.

3. Bin structure

a. Particle classification

Five classes of particles are distinguished in this model: liquid water (LW), freezing water (FW), ice crystals (IW), spherical ice water or graupel (SIW) and snowflakes (SFW). The spherical particles (LW, FW, SIW, SFW) are allowed to occupy any of 50 bins in each layer, ranging in particle radius from 2.6 µm to 4.73 mm. The bins are spaced logarithmically and are contiguous, i.e., all radii within this range are permissible.

There are 900 ice crystal bins ranging from 2.6 to 470 µm along both a- and c-axes, forming a two-dimensional array (30X30). Thus, ice crystals may take on a columnar habit (c>a), a plate-like habit (a>c), or one that is roughly spherical (a=c).

Within each bin, the total number (N), the total mass (M), and mean particle temperature are known. For ice (or mixed phase) particles, the total volume is known which gives the particle density for that bin. In addition, the NH parameter, defined as the product of the number concentration and mean particle height within the layer is known.

b. Distribution within a bin

Particles within a bin are taken to be distributed over radius (or a- and c-axes for ice crystals) when their growth is being considered and over height when their sedimentation is being considered. Since these processes are treated separately in the model, the distribution need only be determined over the parameter (or parameters) pertinent to the process being considered.

Three sets of equations are developed reflecting the radial growth of spherical particles [Eqs. (1), (4), (7)], the growth of ice crystals along their a- and c-axes [Eqs. (2), (5), (8)], and the vertical transfer of particles through sedimentation [Eqs. (3), (6), (9)]. Each set of equations is separate and distinct and are related only in that the number concentration given by Eqs. (4) or (5) for growth has the same value as the number concentration given by Eq. (6) for sedimentation. The number density functions [Eqs. (1), (2), (3)] are meaningful only in the context of the process being considered.

The number density function is assumed to be linear over radius for spherical particle growth,

\[ n_r = n_{r_0} + k_r (r - r_0), \]  

(1)

linear over a- and c-axes for ice crystal growth,

\[ n_{ac} = n_{ac_0} + k_{ac} (a - a_0) + k_c (c - c_0), \]  

(2)

and linear over height for sedimentation,

\[ n_h = n_{h_0} + k_h (h - h_0). \]  

(3)

The terms for \( r_0, a_0 \) and \( c_0 \) refer to size interval midpoints, \( h_0 \) refers to the midpoint of the layer, and the

\[ \text{Version B considers small ice crystals to be spheres (for } r < 18 \mu m \text{ and the range of } a- \text{ and } c-\text{axes is } 18 \mu m \text{ to } 1.5 \text{ mm. Note that } a- \text{ and } c-\text{axes are given as semi-dimensions, consistent with the use of particle radius.}\]
For radial growth, the mass concentration within a bin is
\[ M = \int \frac{(4\pi/3)}{2r^2} n \rho \, dr \]
\[ = \left( \frac{4\pi \rho_0}{3} \right) \left[ R_2(n_{r,0} - k_0 r_0) + k_0 R_3 \right]. \]  

Similarly, for ice crystals
\[ M = \int \int 2\pi^2 cp n_{a} dr dc \]
\[ = 2\pi^2\rho_a C_a [n_{a,0} - k_a a_0 - k_a c_0] + k_a A_1 C_1 + k_a A_2 C_2. \]

For sedimentation, the \( NH \) parameter is
\[ NH = \int n_h dh = (n_{h,0} - k_h h_0) H_2 + k_h H_3. \]

These concepts are presented in Fig. 1.

For each set of equations, these expressions result in pairs of equations in the two unknowns (the solution for \( k_a \) requires setting \( k_a = 0 \) and conversely) which are then solved simultaneously. The mean number densities are
\[
\begin{aligned}
\bar{n}_{r,0} &= \frac{N}{R_1} \\
\bar{n}_{a,0} &= \frac{N}{A_1 C_1} \\
\bar{n}_{h,0} &= \frac{N}{H_1}
\end{aligned}
\]
and the respective $k$'s

$$k = \frac{3R_1M/(4\pi\rho_post)}{R_1R_4 - R_3R_4}$$

$$k_a = \frac{A_1C_1M/(2\pi\rho_post)}{C_1(C_1A_1 - A_2A_1)}$$

$$k_c = \frac{A_1C_1M/(2\pi\rho_post)}{A_1A_3(C_1C_3 - C_2^2)}$$

$$k_h = \frac{H_c(NH) - H_2N}{H_1H_3 - H_2^2}$$

Values of $N$ and $M$ (or $NH$) may exist such that the number density function based on $n_0$ and $k$ so calculated, includes negative values of $n$ within the defined limits of the bin. In such cases, the bin is iteratively redefined until the largest interval (within the original limits of the bin) not containing negative values is found.

4. Transferring particles

Virtually all processes result in transferring particles between bins, frequently between bins of different particle classes or different layers. In some cases, all particles within a given bin are transferred, e.g., through melting. In other cases, only a fraction of the particles are transferred, e.g., through diffusion, accretion or sedimentation. New particles may be created, e.g., by activation of condensation nuclei; or particles may be eliminated, e.g., through evaporation, accretion or precipitation. Such transfers, in general, require meshing particles which are not quite identical in all their properties.

a. Meshing particles

When particles from different sources are meshed into the same bin, parameters such as number, mass, volume and $NH$ may be added directly. Meshing particles with slightly different temperatures, however, requires determining the total particle heat per unit volume (heat concentration) before determining the final temperature after meshing.

For single phase (liquid or ice) particles, the final temperature is

$$T = \sum_j M_jT_j/\sum_j M_j$$

summing over all particle sets ($j$) being meshed.

The treatment of mixed-phase particles is more complex since it is necessary to specify the fraction of the particle mass which is liquid. Rather than create another variable for this purpose, it is more efficient to define a "pseudo-temperature" $T > 273.16K$ which reflects the mass fraction remaining unfrozen. Since ice or mixed-phase particles cannot exhibit particle temperatures $> 273.16K$, any such value is recognized as a pseudo-temperature and the actual particle temperature is known to be $0C$. The pseudo-temperature is defined as that temperature the particle would have if all the liquid were converted to ice and the latent heat so released were used to warm the particle with a specific heat of unity, i.e., the pseudo-temperature elevation above $0C$ is the heat loss (cal gm$^{-1}$) necessary to completely freeze the particle.

The contribution to the total heat concentration by a mixed-phase particle set ($j$) is

$$H_j = 273.16c_iM_j + [1 \text{ cal gm}^{-1}(°K)^{-1}](T_j - 273.16)M_j.$$  

The contribution by an ice particle set ($k$) is

$$H_k = T_kc_iM_k.$$  

If the total heat concentration $(\Sigma H_j + \Sigma H_k) > 273.16c_i \times (\Sigma M_j + \Sigma M_k)$, representing the maximum heat that ice particles of mass $\Sigma M_j + \Sigma M_k$ may possess without melting partially, then the meshed particles are recognized to be mixed phase with a pseudo-temperature $^6$

$$T = 273.16(1 - \epsilon_i) + (\Sigma H_j + \Sigma H_k)/(\Sigma M_j + \Sigma M_k).$$

If the total heat concentration is less than or equal to this value, the temperature of the meshed particles is

$$T = (\Sigma H_j + \Sigma H_k)/[\epsilon_i(\Sigma M_j + \Sigma M_k)].$$

b. Transfer through freezing

The activation of freezing nuclei contained within or collected by the drops of a given bin results in the transfer of a number of drops to the FW class equal to the number of nuclei activated.  

The particles transferred conserve their number, mass and $NH$ but acquire a volume of 10/9 their mass ($\rho = 0.9 \text{ gm cm}^{-3}$) and a pseudo-temperature 79.9K above their liquid-phase temperature. FW particles are transferred to the equivalent SIW bin when their pseudo-temperature $T \leq 273.16K$.

c. Transfer through melting

Any particle (bin) whose pseudo-temperature $T > 353.06K$ is transferred to the liquid phase. Number, mass and $NH$ parameters are conserved but the particle is considered to "collapse" to a density of 1.0 gm cm$^{-3}$ and a "new" radius is calculated from the average particle mass ($M/N$). The particles are transferred to the appropriate LW bin based on this "new" radius with a temperature reduced by 79.9K (to compensate for the pseudo-temperature elevation).

$^6$ It is understood that the right-hand side is divided by 1 cal gm$^{-1}$ $(°K)^{-1}$.

$^7$ If the fraction of drops frozen $> 10\%$, then the probability that two or more nuclei are activated in conjunction with the same drop is considered.
Partial melting is considered in the case of ice crystals whose shape is generally dependent on a rigid ice structure. In this model, a critical value of 25% water is assumed to result in “collapse” of the crystal structure and crystals whose pseudo-temperature $T > 293.2K$ are “collapsed” to a density of 0.9 gm cm$^{-3}$. These are transferred to the appropriate FW bin with conservation of number, mass, temperature and $NH$ parameter.

d. Transfer through growth

Mass gained by the particles within a bin as a result of diffusion or collection processes is assumed to be uniformly partitioned among all the particles within the bin, contributing to the extension of their dimensions. The dimensions resulting from the addition of mass at a density of $\rho_s$ are

$$r = [r^4 + 3AM/(4\pi\rho_s N)]^{1/3}$$  

(for radial growth)

and

$$a = [a^4 + 3AM_{ax}/(2\pi\rho_s a^2 N)]^{1/3}$$  

$$c = c' + 3AM_{cz}/(2\pi\rho_s a^2 N)$$

(for ice crystal growth)

For particles undergoing diffusion growth, $\rho_s = 1.0$ gm cm$^{-3}$ for liquid particles and $\rho_s = 0.9$ gm cm$^{-3}$ for mixed phase particles. For ice particles, $\rho_s$ is dependent on the particle temperature and the effective vapor density excess, $(\Delta D/D) (\rho_m - \rho_{s,2})$, according to Table 1. Values of $\rho_s$ and the relative axial growth rate ($\Gamma = dc/da$) are based on the laboratory data of Kobayashi (1957) and the observations of natural ice crystals by Ono (1970). Supplementary data were derived from Cotton’s (1970) thesis and Heymsfield (1972). Small ice crystals are observed to grow nearly isometrically under all growth conditions. Crystals whose maximum semi-dimensional is less than 25 $\mu$m are grown as though the vapor density excess were less than 0.05 gm m$^{-3}$.

Mass is added to accreting particles with a density dependent on the particle temperature. For ice particles ($T_s < 273.16K$), the accretion density is taken as 0.5 gm cm$^{-3}$. For mixed-phase particles, the density is assumed to be 0.9 gm cm$^{-3}$. Densities given by List (1958) are 0.5–0.7 gm cm$^{-3}$ for graupel (dry growth) and 0.8–0.9 gm cm$^{-3}$ for small hail (wet growth).

Based on the observations of Magome and Nakamura (1965), Jiusto and Holroyd (1970) found the density of snowflakes to be described by 0.0085/$r_f$ where $r_f$ is the flake radius. This value was used for $\rho_s$, for snowflake aggregation although, more properly, $\rho_s$ can be shown to be two-thirds of this value. For small snowflakes, $\rho_s$ is restricted to values $\leq 0.6$ gm cm$^{-3}$. The density of newly formed snowflakes is based on the maximum crystal dimensions.

For some (or all) of the particles within a bin, the new dimensions resulting from growth may exceed the bin limits and particles are transferred as appropriate. When particles of the original bin are divided (by transfer) into two or more bins, there exist sizes ($r_1, r_2, \ldots$) within the original bin which represent particles which “grow” to sizes equal to bin boundaries. These particular sizes divide the original bin into subgroups within which all particles are transferred to the same bin. The number of particles in such a subgroup is

$$N_{\alpha} = \int_{r_s}^{r_{\alpha}} n_s dr,$$

and their mass is

$$M_{\alpha} = \int_{r_s}^{r_{\alpha}} (4\pi r^3/3) n_s dr.$$

The equivalent expressions for $a$- and $c$-axis transfer of ice crystals are similar in form. The transfer of volume is in proportion to mass transfer; the transfer of $NH$ is in proportion to number transfer.

The growth of ice crystals requires that the newly acquired mass be partitioned between $a$- and $c$-axis

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Table 1. Growth aspect ratio and sublimation density.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Crystal habit</th>
<th>0 to 0.05</th>
<th>0.05 to 0.10</th>
<th>0.10 to 0.15</th>
<th>0.15 to 0.20</th>
<th>&gt; 0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to -3</td>
<td>plate</td>
<td>0.50*</td>
<td>0.90*</td>
<td>0.12</td>
<td>0.90</td>
<td>0</td>
</tr>
<tr>
<td>-3 to -6</td>
<td>needle</td>
<td>1.40</td>
<td>0.90</td>
<td>3.00</td>
<td>0.50</td>
<td>3.50</td>
</tr>
<tr>
<td>-6 to -10</td>
<td>column</td>
<td>1.40</td>
<td>0.90</td>
<td>3.00</td>
<td>0.50</td>
<td>3.50</td>
</tr>
<tr>
<td>-10 to -12</td>
<td>plate</td>
<td>0.80</td>
<td>0.90</td>
<td>0.50</td>
<td>0.90</td>
<td>0</td>
</tr>
<tr>
<td>-12 to -17</td>
<td>dendrite</td>
<td>0.80</td>
<td>0.90</td>
<td>0.50</td>
<td>0.90</td>
<td>0</td>
</tr>
<tr>
<td>-17 to -22</td>
<td>plate</td>
<td>0.80</td>
<td>0.90</td>
<td>0.50</td>
<td>0.90</td>
<td>0</td>
</tr>
<tr>
<td>-22 to -40</td>
<td>column</td>
<td>1.40</td>
<td>0.90</td>
<td>3.00</td>
<td>0.50</td>
<td>3.50</td>
</tr>
</tbody>
</table>

* Growth aspect ratio.

** Sublimation density.
growth. Partitioning is based on $\Gamma$ (Table 1) and may be expressed as partial differentials with terms contributing to the extension of the a-axis,

\[ dm_a = 4\pi p_a dca, \]

the c-axis,

\[ dm_c = 2\pi p_a a^2 dc, \]

and the sector formed by their extensions,

\[ dm_{a,c} = 4\pi p_a dada. \]

Solution for $dm_a$ and $dm_c$ follows from substitution of $\Gamma$ with

\[ dm_a (or dm_c) = \frac{-B^* + (B^{**} - 4A^* C^*)^{1/2}}{2A^*}, \quad (10) \]

using the quadratic coefficients:

\[ A^* = \Gamma/(4\pi p_a a^2) \quad \text{and} \quad B^* = 1 + a\Gamma/(2c), \quad \text{for} \quad dm_a, \]

\[ A^* = 1/(\pi p_a a^2\Gamma) \quad \text{and} \quad B^* = 1 + 2c/(a\Gamma), \quad \text{for} \quad dm_c, \]

\[ C^* = \Delta M/N. \]

The term $dm_{a,c}$ is simply

\[ dm_{a,c} = \Delta M - dm_a - dm_c. \]

Axial ratios for small ice crystals are restricted to $\Gamma \leq 1.4$ for $c > a$ and $a \leq 17 \mu m$, and to $\Gamma \geq 0.6$ for $a > c$ and $c \leq 13 \mu m$ in accordance with the observed growth of small ice crystals. If the actual axial ratio of an ice crystal differs from that prescribed by the environmental conditions by more than 30% of their mean, all the mass added extends that axis which tends to grow the crystal into a habit conforming with its environment.

Ice crystal growth requires alternating between a- and c-axis growth since the slopes $k_a$ and $k_c$ must be solved individually. The order $(a, c$ or $c, a)$ is alternated each time step to avoid bias.

e. Transfer through evaporation

Transfer of particles through evaporation is similar to growth except when some or all of the particles within a bin evaporate entirely. If the mass evaporated from a single bin is equal to the total mass in the bin, all the particles are removed. Otherwise, if the largest particle remains greater than the minimum allowable size ($2.6 \mu m$), all particles which evaporate to a size $< 2.6 \mu m$ are removed. If the largest particle evaporates to a size $< 2.6 \mu m$ but some of the mass remains, the mass is divided equally among drops of the smallest allowable size which then defines the concentration of drops remaining within the bin.

Evaporation of ice particles is assumed not to alter the particle's density. For ice crystals, their axial ratio is also assumed to remain unchanged.\textsuperscript{13}

f. Transfer through sedimentation

All particles within the same bin (and layer) are assumed to fall with the same velocity with a net change in $NH$ of $n\Delta t$. Particles which lie below a height of $v\Delta t$ above the layer base ($h_i = h_i + v\Delta t$), i.e.,

\[ \Delta N = \int_{h_i}^{h_f} n dh, \quad (11) \]

are transferred to the next lower layer. Mass and volume are transferred in proportion to the number transferred. The $NH$ transferred is

\[ \Delta(NH) = \int_{h_i}^{h_f} n dh - \Delta N x h\Delta t. \]

In this model of orographic clouds, uniform distance steps are used to maintain vertical alignment of the layers in wind shear. Time steps are defined for each layer as the standard distance step (100 m) divided by the horizontal wind velocity ($u_h$) at that position. Thus, conservation of mass flux requires that the extensive properties of particles\textsuperscript{14} transferred between layers be multiplied by a factor of $v_{x,a,c}/v_{x,a,c}$.\textsuperscript{11}

\[ g. \ \text{Transfer through redefinition} \]

Ice particles are defined according to the mode in which they are initiated, i.e., isometric ice crystals ($a = c$) of low density are distinguished from snowflakes of the same density although they are generally treated similarly otherwise. When the particle in question is sufficiently altered in size or density to be unrecognizable with respect to its classification, it is transferred to a more appropriate category.

Graupel (SIW) are considered to be isometric ice crystals if their radii $< 470 \mu m$.\textsuperscript{15} Snowflakes $> 250 \mu m$ radius are considered to be graupel if, through deposition or accretion, for example, their density exceeds 0.6 gm cm$^{-3}$.

5. Nucleation processes

a. Condensation nuclei

Cloud condensation nuclei (CCN) are activated when the water supersaturation exceeds its previous maximum value [$>0$]. A concentration of liquid particles

\textsuperscript{11} Again, version B obviates this restriction.

\textsuperscript{13} In version B, the minimum allowable size is 1.6 $\mu m$.

\textsuperscript{14} Extensive properties in this instance are in terms of per unit original volume rather than cm$^{-3}$.

\textsuperscript{15} In version B, these limits are 18 $\mu m$ $< r < 19 \mu m$.\textsuperscript{11}
equal to the number of new nuclei activated is added to the smallest LW bin and an equivalent mass is added to the "extra" category for mass conservation.

The number of CCN activated is defined by the activity spectrum

\[ N_{ccn} = N_{ccn,0} S_w^p, \]

where \( S_w \) is the water supersaturation (\%) and \( N_{ccn,0} \) the concentration of CCN active at \( S_w = 1\% \).

The modified version considers the temperature dependence of the CCN activity spectrum. This treatment is based on the formulation of critical supersaturation required to activate a condensation nucleus of a given mass as given by Fletcher (1962). The supersaturation required to nucleate a given number of CCN is 30\% greater at \(-10\) \(^\circ\)C than at \(+20\) \(^\circ\)C where most CCN activity spectra are determined.

b. Contact freezing nuclei

The activation of ice nuclei as contact freezing nuclei requires that they be collected by the liquid phase, it being assumed that the collection is due to (Young, 1974b):

Brownian motion

\[ \frac{1}{N_a} \frac{dN_a}{dt} = 4\pi\nu D_a (1 + 0.3 \text{Re}^1 S_c a^1) \]

Thermophoresis

\[ \frac{1}{N_a} \frac{dN_a}{dt} = 4\pi\nu f \frac{K_a^*}{P} (T_w - T_a) (4.185 \times 10^7 \text{ ergs cal}^{-1}) \]

Diffusiophoresis

\[ \frac{1}{N_a} \frac{dN_a}{dt} = 4\pi\nu g D_a^* (\rho_w - \rho_a) \]

\[ \times (3.34 \times 10^{22} \text{ molecules gm}^{-1}). \]

Only nuclei active as contact nuclei at the drop temperature at the moment of contact are considered to nucleate the ice phase. If contact is made at a temperature too warm for nucleation, the nucleus is considered to be taken up by the drop and has no effect since immersion freezing nucleation thresholds are generally much colder than for contact nucleation. Although version A does not treat the removal of nuclei through these processes, version B allows for scavenging of nuclei by water and ice particles.

Version A considers the radius of natural contact nuclei to be fixed at 0.3 \(\mu\)m based on deductions made by Young (1974b). Similarly, the shape of the activity spectrum is formulated as

\[ N_{acen} = N_{acen,0} (270.16 - T_w)^{1.3}, \]

with a contact nucleation threshold temperature of \(-3\) \(^\circ\)C. The concentration of natural contact nuclei active at \(-4\) \(^\circ\)C \((N_{acen,0})\) is estimated to be \(\sim 200\) liter\(^{-1}\) at sea level. It is felt that natural contact nuclei would be produced at the surface and would be expected to decrease with height in a manner similar to CCN. In lieu of observational data, the concentration active at \(-4\) \(^\circ\)C was taken as 50 liter\(^{-1}\) at 2500 m MSL, decreasing linearly to 10 liter\(^{-1}\) at 5000 m MSL.

Version B allows for a variable nucleus size spectrum from 0.015 to 2.0 \(\mu\)m in 15 size classes. The model treats a log-normal distribution of natural nucleus sizes with a modal radius of 0.3 \(\mu\)m with virtually all particles between 0.1-1.0 \(\mu\)m. The fraction of nuclei active at a given temperature in a given size class is determined from a given surface density of active sites, randomly distributed on the nuclei surface. In lieu of reliable experimental data, the temperature dependence of the surface density of active sites \((d)\) is estimated from Young (1974b) and formulated as

\[ d_{acen} = \begin{cases} 2.6 \times 10^3 (271.16 - T_w)^{1.73}, & \text{for } 271.16K > T_w > 263.16K \\ 1.396 \times 10^4 (271.16 - T_w)^{1.3}, & \text{for } T_w < 263.16K \end{cases} \]

This formulation was undertaken to provide for an increased likelihood of nucleus activity with increasing particle size and decreasing temperature.

In version A, the shape of the AgI contact nucleus spectrum is assumed to be of the same shape as for natural contact nuclei with a nucleation threshold of \(-3\) \(^\circ\)C based on Gokhale and Spengler (1972), formulated as

\[ N_{acen} = N_{acen,0} (270.16 - T_w)^{1.3}. \]

The radii of AgI aerosol active as contact nuclei is taken to be 0.05 \(\mu\)m after Young (1974b) and the concentration active at \(-4\) \(^\circ\)C \((N_{acen,0})\) is a variable specified by the AgI input and turbulent diffusion.18

The improved version allows for a spectrum of AgI nuclei sizes from 0.015 to 2.0 \(\mu\)m radius. The model treats a narrow distribution with a modal radius of 0.045 \(\mu\)m and restricted to the range 0.03-0.07 \(\mu\)m. As done for natural contact nuclei, a surface density of active sites was estimated on the assumption that 10\% of the AgI nuclei in the assumed distribution would be active as contact nuclei at \(-4\) \(^\circ\)C and that 90\% would be active at \(-12\) \(^\circ\)C. This led to the formulation

\[ d_{acen} = 1 \times 10^3 (271.16 - T_w)^{3}. \]

c. Sorption nuclei17

In version A, the activation of ice nuclei as sorption nuclei is considered to require an ambient temperature colder than the activation threshold and an ambient vapor density greater than or equal to water saturation

18 Based on an eddy diffusion coefficient of 500 cm\(^2\) sec\(^{-1}\).
17 The term deposition nuclei is also in common use.
except for AgI nuclei which may be activated provided the ice supersaturation \( \geq 12\% \) (Bryant et al., 1959). Nuclei are assumed not to undergo deactivation or to have been pre-activated. A concentration of ice crystals equal to that of sorption nuclei activated is added to the smallest IW bin \( (a = c = 3.1\, \text{mm}) \) and an equivalent mass is added to the "extra" category for mass conservation.

The shape of the activity spectrum for natural sorption nuclei is based on Fletcher (1962) but using a nucleation threshold of \(-12\, \text{C}\). A rough fit to typical activity spectra gives

\[
N_{\text{asn}} = N_{\text{asn,0}}\left(261.16 - T_a\right)/8\right)^{1.7}
\]

While the shape of the sorption nucleation activity spectra for AgI is based on the theoretical curve presented by Fletcher, the nucleation threshold temperature is taken to be \(-8\, \text{C}\) based on Mason and Van den Heuvel (1959). The activity spectrum is formulated as

\[
N_{\text{asn}} = N_{\text{asn,0}}\left(265.16 - T_a\right)^{1.5}
\]

The concentration active at \(-9\, \text{C}\) \( (V_{\text{asn,0}}) \) is a variable parameter specified by the AgI input and turbulent diffusion within the modeled cloud.

The activation of sorption nuclei in version B is based on Vali’s (1973) observations. These findings indicate that the activation of ice nuclei via sorption nucleation is solely dependent on the ice supersaturation at temperatures colder than the nucleation threshold. Based on Vali’s data, the surface concentration of active sites is estimated to be

\[
d_{\text{asn}} = 10^4(S_{\text{c}}/16\%)^4
\]

for natural nuclei and

\[
d_{\text{asn}} = 10^4(S_{\text{c}}/16\%)^4
\]

for AgI nuclei. The activation of nuclei through sorption nucleation is considered to remove that fraction of the most active nuclei, making these unavailable for activation via contact nucleation.

6. Collection processes

Collection processes may be categorized as liquid-liquid (coalescence), ice-liquid (accretion) or ice-ice (aggregation). In these, a faster falling particle collects a number of smaller particles defined by the size of the larger particle, the difference in their fall velocities, their mutual collection efficiency (CE), and the concentration of smaller particles. In version A, the larger particle is considered to encounter an average number of smaller particles per unit time which are then added to the larger particle (continuous collection). In version B, the smaller particles are assumed to be uniformly distributed and only that fraction of large particles corresponding to the number of collection events is "grown" (discrete collection).

a. Coalescence

The number of smaller drops \( (j) \) transferred to larger drops in a given bin \( (k) \) is

\[
\Delta N_j = \pi r_j^2 N_j N_k |v_{i,k} - v_{i,j}| E_{jk} \Delta t
\]

where \( E_{jk} \) is the mutual collection efficiency (CE) between drops of radii \( r_j \) and \( r_k \) and is based on Young (1973). The total mass transferred to a given bin \( (k) \) is

\[
\Delta M_k = \sum_{j=1}^{k-1} \left( \frac{\Delta N_j}{N_j} \right) M_j
\]

The particle height distribution is not considered for collection processes, i.e., particles are assumed to be uniformly distributed throughout the layer.

b. Accretion

Accretion processes include the collection of liquid particles by freezing water (FW), graupel (SIW), snowflakes (SFW) and ice crystals (IW). The spherical particles are assumed to grow radially, collecting particles in accordance with Eqs. (14) and (15). For these, sphere-drop CE are from Young (1973), where the CE are based on the difference in particle fall velocities.

Ice particles are assumed to fall with their longer axis horizontal, thereby collecting and growing along their shorter axis. For example, columnar crystals grow along their axis until \( a = c \), after which they grow as graupel.

The number of drops \( (j) \) collected by an ice crystal \( (k,m) \) is

\[
\Delta N_1 = \pi r_1^2 N_1 |v_{l,k,m} - v_{l,j}| E_{k,m} \Delta t
\]

with an area cross section of \( \pi a^2 \) for \( a > c \) and of \( 4ac \) for \( c > a \). Collection efficiencies \( (E_{k,m}) \) are based on Ranz and Wong (1952) for discs \( (a > c) \) and cylinders \( (c > a) \). A polynomial least-squares fit to their data gives

\[
E_{k,m} = \begin{cases}
6.50\psi^4 - 7.00\psi^3 + 2.56\psi^2 - 1.20\psi^1, & \text{for } a > c \\
1.29\psi^4 - 0.478\psi^3 + 0.0621\psi^2 - 0.326\psi, & \text{for } c > a
\end{cases}
\]

with

\[
\psi = \frac{r_j^2 |v_{l,k,m} - v_{l,j}|}{97a^2}
\]

Values of CE are restricted to the range 0-1. These values of CE were derived and experimentally verified.

\footnote{In version B, this is the mass transferred to \( \Delta N_j \) drops of the bin, or, if \( \Delta N_j/N_j > 1 \), multiple integral transfers of small particles to the larger are treated.}

\footnote{Version B considers sphere-sphere CE to be a function of the Reynolds number for collector and collected particles and is based on values of CE presented by Young (1973).}
for large collectors and small collected particles but are used in lieu of more suitable values, recognizing that wake effects between particles with similar fall velocities and incomplete structures such as dendrites may significantly alter these values in real clouds.

c. Aggregation

Three types of interaction are described, snowflakes collecting snowflakes (SFW/SFW), snowflakes collecting ice crystals (SFW/IW) and ice crystals collecting ice crystals (IW/IW). The latter case describes the formation of new snowflakes on two ice crystals. Two processes of snowflake formation and growth are considered. For relatively large differences in fall velocity between the particles, collection based on the difference in fall velocity is assumed and the number of particles of radius $r_j$ collected is:

$$\Delta N_j = \pi N j N_k (R_k + r_j)^3 |v_{l,k} - v_{l,j}| \Delta t.$$  

(17)

For relatively small differences in fall velocities, a collision rate based on a Gaussian distribution of fall velocities (Sasyo, 1971) is used and the number of collected particles is:

$$\Delta N_j = \frac{1}{2} \pi N j N_k (R_k + r_j)^3 (v_{l,k} + v_{l,j}) \Delta t.$$  

(18)

The Sasyo collision rate is used when the difference in fall velocities between the particles is less than

$$\frac{1}{2} \pi^{-1} (v_{l,k} + v_{l,j}).$$

Eqs. (17) and (18) assume a geometric collection efficiency.

7. Diffusion processes

The exchange of water vapor and heat between the particulate phases and the ambient air occurs through molecular diffusion. The ambient vapor density is determined by the particulate phases; the ambient temperature is determined by the particles and the adiabatic temperature change as the layer is lifted or descends.

The model extrapolates values of the ambient temperature and vapor density from previous time steps to obtain initial trial values. These values are employed to check mass and heat balance in accordance with (19) and (20). The ambient temperature and vapor density are adjusted until these equations balance within the limits given. In this manner, the supersaturation is calculated directly. In the procedure outlined, the particle temperature is calculated so that effects of latent heat release (through accretion on ice particles) and radiational cooling may be included properly.

$$\text{Eqs. (19) and (20) distinguish between mean values of temperature during the time step and end-point values (}$T_e$). All diffusion equations are solved using mean values of $T_m$ and $p_m$ which are iteratively modified until Eqs. (19) and (20) are balanced. The net heat change is expressed as the difference between successive end-point temperatures ($T_{e,n} - T_{e,n-1}$).31

b. Solution for particle temperatures

Once trial values for $T_m$ and $p_m$ have been selected, the particle temperatures may be solved for through the mass flux equation

$$\frac{dm}{dt} = 4\pi CD_s (p_m - p_{s,i}),$$  

(21)

$^a$ The net change in water vapor density is the difference between successive mean values.
and the heat flux equation:\[\frac{dH}{dt} = \frac{dm}{dt} \cdot dT_x + \frac{dH}{dt} = \frac{L_x}{\eta} \cdot (T_x - T_x) - 0.8 \sigma \varepsilon A T_x^4. \quad (22)\]

Solution requires defining the relationship between particle temperature \(T_x\) and the saturation vapor density over its surface \(\rho_{s,s}\). Mason (1953) used the Clausius-Clapeyron equation for this purpose in his derivation of the growth equation. Here, a linear approximation, based on tabular values for \(\rho_{s,s}\), is utilized in a two-step approximation to the particle temperature, formulated as

\[\rho_{s,s} = sT_x + b. \quad (23)\]

The explicit solution for particle temperature follows through substitution of (21) and (23) in Eq. (22):

\[T_x = \frac{4\pi CD_s^2 L_x (\rho_{m} - b) + c_s m_s (T_x - T_x')/\Delta t) + 4\pi CK_s T_x + L_x (\Delta m/\Delta t)_{suc}}{4\pi CK_s^2 + 4\pi CD_s^2 sL_x + (c_s m_s/\Delta t) + 0.8 \sigma \varepsilon A T_x^3}. \quad (24)\]

This method of solution allows for inclusion of other sources and/or sinks of heat, such as accretion or radiation transfer, without the assumptions used by Cotton (1970), for example, in his rederivation of Mason's growth equation to include the latent heat released with accretion.

The term in Eq. (24) for heat released due to accretion actually considers all liquid/ice phase changes including melting and freezing resulting from exchange of heat with the surrounding air. In practice, all the mass is assumed frozen and the resulting latent heat is carried along (as a pseudo-temperature) until the particle is able to dissipate it to freeze completely or acquires sufficient heat to melt completely. If the calculated temperature of an ice particle exceeds 273.16K, it is recognized to be mixed phase and the heat exchange with its environment is calculated for \(T_x = 0\). This allows determination of the resultant mass fraction of water and thereby the pseudo-temperature.

c. Solution for ambient temperature and vapor density

Once the particle temperatures are known for the trial values of ambient temperature and vapor density, the mass balance equation (19) is solved using values for \(\rho_{s,s}\) based on tabular interpolation. The trial value for ambient vapor density is adjusted in proportion to the mass imbalance until the imbalance is < 10\(^{-4}\) parts of the total interphase mass flux. The remaining mass is then defined as "extra" and carried along to the next time step in order to conserve mass.

Once mass balance has been achieved, the change in sensible heat of the particles and the total mass flux between phases may be determined. The heat balance equation (20) is used in terms of temperature changes, considering the adiabatic change

\[\Delta T|_{ad} = -\frac{g\varepsilon \Delta T}{c_p}, \quad (25)\]

and the contribution due to diffusion processes involving the particles

\[\Delta T|_{diff} = \frac{2R_d T_w}{c_p \varepsilon} [L_x \Delta LWC + L_x (\Delta FWC + \Delta SIWC + \Delta SFWC)] + \sum_{all \text{ particles}} c_s m_s (T_x' - T_x). \quad (26)\]

A new trial value of ambient temperature is defined as the mean between successive endpoints, i.e.,

\[T_w = T_{w,s} + \frac{1}{2} [\Delta T|_{ad} + \Delta T|_{diff}]. \quad (27)\]

Eq. (20) is considered balanced if successive trial values of \(T_w\) differ by < 0.001C. Otherwise, both sets of balance equations are reiterated. A third-order extrapolation to predict \(T_w\) and \(\rho_{s,s}\) was used to reduce the number of iterations required.

8. Sedimentation
a. Particle fall velocities

Particle fall velocities were based on Gunn and Kinzer (1949) for water drops and Heymsfield (1972) for ice crystals.\^2 Using these as a basis, a relationship describing the fall velocity for any particle within the anticipated limits of the model was derived.

\^2 When radiational cooling is considered, an iterative solution for \(T_x\) is employed with an error < 0.002C.

\^3 Version A does not include the radiational heat transfer term. Young (1974b) shows that this term may be highly significant in promoting contact nucleation at the cloud top.

\^4 In version B, this treatment was improved using the treatment presented by McDonald (1960).
Heymsfield gives a polynomial fit to Reynolds numbers for ice crystals as

\[ \text{Re}^* = \exp[A^* + B^* \ln Y + C^*(\ln Y)^2 + D^*(\ln Y)^3] \]  

where \[ Y = \frac{16\rho_a^2 \nu_a}{\rho_d T_0} \] and \( \text{Re}_{\text{plate}} = \text{Re}^* \).

The coefficients \( A^* \), \( B^* \), \( C^* \), and \( D^* \) are dependent on the crystal habit as set forth in Table 2. For columnar crystals, this equation becomes

\[ \text{Re}_{\text{col}} = \text{Re}^* + [1.81 - 2.56(\alpha/c)](\text{Re}^* - \text{Re}^v), \]

where \( \text{Re}^v \) and \( \text{Re}^v \) are defined through coefficients \( x \) and \( y \), respectively. These \( \text{Re} \) were modified to agree with Gun and Kinzer for spherical particles.

In order to reduce computational time, the variation of \( \text{Re} \) with pressure, viscosity, temperature and particle density was investigated. A dimensionless parameter was defined as

\[ X = \frac{P_{\text{atm}}}{T_0} \left( \frac{\eta_{\text{air}}}{\eta_{\text{water}}} \right) \left[ 10^{-5} \text{ cm}^4 \text{ sec}^{-2} \text{ gm}^{-2} \right]. \]

For a particle of given dimensions, the \( \text{Re} \) is dependent only on \( X \) through Eq. (28). Plots of \( \text{Re} \) vs \( X \) for various crystal habits were found to be quite linear for \( X < 8 \) with a slope of \( 0.15 \text{Re}_d \) with \( \text{Re}_d \) being the \( \text{Re} \) for \( X = 5 \) and calculated from Eq. (28). Thus, to a reasonable approximation, the \( \text{Re} \) may be calculated from

\[ \text{Re} = \text{Re}_d + 0.15 \text{Re}_0 (X - 5), \]

and values for \( \text{Re}_0 \) are calculated once and serve as input to the model.

b. Conservation of mass flux

Since horizontal velocities may differ between layers within the cloud, particles being transferred between layers must either become more concentrated (falling into a slower moving layer) or more diffuse (falling into a faster moving layer) so as to satisfy the conservation of mass flux.\(^{27}\) The necessary transfer factor is simply

\[ \frac{v_{z,0,i}(\Delta h)_{0,i}}{v_{z,0,i-1}(\Delta h)_{0,i-1}}, \quad \text{for } i \geq 2; \]

or, since in this case, the initial thickness of each layer is the same (300 m), this factor reduces to

\[ \frac{v_{z,0,i}}{v_{z,0,i-1}}. \]

9. Structure of the model

Three levels are stored in the computer at one time, designated, from top to bottom, as \( G \), \( X \) and \( H \). Two auxiliary storage units are utilized, one to receive information on levels for which computations have been completed, the other to supply the information necessary to begin computations for a level. Information from the proper auxiliary storage unit is buffered into the \( G \) array and buffered out from the \( H \) array except at the model bottom and top when the \( X \) array also takes part in these transfers.

The sequence of operations on a single level begins with transfer of information from an auxiliary storage unit into the \( G \) array. The array \( G \) then contains the number, mass, volume, temperature and \( NH \) parameter for all five particle classifications and for all sizes for one level.\(^{28}\) Nucleation and collection processes are then modeled, using the \( G \) array. Following these, the levels are transferred downward, i.e., the \( X \) array is moved into the \( H \) array and the \( G \) array is moved into the \( X \) array. This allows the next higher level to be buffered into the \( G \) array. Diffusion and sedimentation processes are then modeled on the \( X \) array. Following sedimentation of particles into the \( H \) array, it is transferred to auxiliary storage.

10. Summary and conclusions

This model represents a significant advance over the single-level, microphysical models advanced by Cotton (1970) and Jiusto and Holroyd (1970), for example. The development of the continuous bin technique allows treatment of the microphysical processes on any number of levels and permits interaction between levels through sedimentation. Thus, the model is capable of realistically predicting the precipitation output for clouds whose dynamical structure is relatively simple and fairly well understood. The model is inherently capable of realistically simulating microphysical processes in more complex dynamical systems but is limited by uncertainties in the dynamical treatment, in particular turbulent mixing.

Treatment of the microphysical processes is improved by introducing a general solution to diffusion transfer

\( ^{28} \) If no cloud was present for a level from the previous step, no transfer from auxiliary storage takes place and the \( G \) array is zeroed internally.
of mass and heat, including latent heat release due to accretion, heat transfer through radiational processes, and changes in the sensible temperature of the particle. This treatment includes a determination of the ambient vapor density (and supersaturation) based on the calculated interphase mass transfer. Treatment of ice phase nucleation is markedly improved using the model of contact nucleation advanced by Young (1974b). In this, nucleation is suppressed for growing drops and enhanced for evaporating drops due to the effects of thermo- and diffusiophoresis. Determination of contact nucleation rates depends on calculating the particle temperature which is a direct result of the general solution to diffusion transfer described herein.

A number of deficiencies may be noted in this model. Data are lacking on the collision efficiency for snowflakes and ice crystals undergoing aggregation; it is assumed to be geometric here. The observed side-wise motion of snowflakes (e.g., see Sasyo, 1971) would suggest collision efficiencies much greater than geometric based on mean fall velocities. In this model, the sticking probability has been assumed to be unity whereas Hobbs et al. (1973) use a temperature-dependent probability. The sticking probability may also be dependent on the crystal habit. Uncertainties exist regarding collection efficiencies for ice crystals such as dendrites undergoing accretion. Activity spectra for natural and artificial (AgI) contact nuclei are not well known. The dependence of the activation of natural and artificial sorption nuclei on supersaturation similarly is not well-documented.

The application of this model to an orographic situation is demonstrated by Young (1973, 1974a).

**Acknowledgments.** The patience and support of my thesis advisor, Prof. R. R. Braham, Jr., is acknowledged. The model was developed in part and run at the NCAR computer facility funded by the National Science Foundation. Other parts of this research were supported under NSF Grants GA28190X and GI33373.

### APPENDIX

#### List of Abbreviations and Symbols

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CACN</td>
<td>concentration of artificial contact nuclei (AgI) active at −4°C</td>
</tr>
<tr>
<td>CCN</td>
<td>cloud condensation nuclei</td>
</tr>
<tr>
<td>FW(C)</td>
<td>freezing water (content)</td>
</tr>
<tr>
<td>IW(C)</td>
<td>ice water (content), in reference to ice crystals</td>
</tr>
<tr>
<td>LW(C)</td>
<td>liquid water (content)</td>
</tr>
<tr>
<td>SFW(C)</td>
<td>snowflake water (content)</td>
</tr>
<tr>
<td>SIW(C)</td>
<td>spherical ice water (content), in reference to graupel or hail</td>
</tr>
</tbody>
</table>

**Dimensionless numbers**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kn</td>
<td>Knudsen number (\equiv \lambda/r_s)</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (\equiv \eta_c c_p/K_v)</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number (\equiv 2\pi r_s \rho_v \eta_c)</td>
</tr>
<tr>
<td>Re_5</td>
<td>Reynolds number at (X = 5) (\text{[see Eq. (30)]})</td>
</tr>
<tr>
<td>Sc_(x)</td>
<td>Schmidt number (for phase (x) (\equiv \eta_x/D_x \rho_v)</td>
</tr>
</tbody>
</table>

**Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(a)</td>
<td>(a)-axis semi-dimension area</td>
</tr>
<tr>
<td>(A, A^<em>, B^</em>, C^*)</td>
<td>coefficients for solution of quadratic Eq. (10)</td>
</tr>
<tr>
<td>(b)</td>
<td>intercept for calculation of (\rho_s , x) (\text{[see Eq. (23)]})</td>
</tr>
<tr>
<td>(c)</td>
<td>(c)-axis semi-dimension</td>
</tr>
<tr>
<td>(\epsilon, \epsilon_x)</td>
<td>specific heat—a of air at constant pressure, of particle (x)</td>
</tr>
<tr>
<td>(C)</td>
<td>shape factor for diffusional processes for ice crystals (\text{[based on McDonald (1963)]})</td>
</tr>
<tr>
<td>(d)</td>
<td>surface density of sites active for nucleating the ice phase</td>
</tr>
<tr>
<td>(D_\alpha)</td>
<td>aerosol diffusivity</td>
</tr>
<tr>
<td>(D_v)</td>
<td>diffusivity of water vapor in air</td>
</tr>
<tr>
<td>(D^*_\alpha)</td>
<td>modified (D_v) (\equiv f_1 D_v)</td>
</tr>
<tr>
<td>(e)</td>
<td>extinction cross section</td>
</tr>
<tr>
<td>(E_{km})</td>
<td>collection efficiency between drops (or spheres) of radii (r_k) and (r_m)</td>
</tr>
<tr>
<td>(f_{i}, f_{i}^*)</td>
<td>factors describing the effect of convection on mass and heat diffusion (after Koenig, 1968):</td>
</tr>
<tr>
<td>(f_{i} = 1 +\left(0.74 \text{Re}^1 \text{Sc}^1\right)\left(4\pi C\right)^{-1})</td>
<td></td>
</tr>
<tr>
<td>(f_{i}^* = 1 +\left(0.56 \text{Re}^1 \text{Pr}^1\right)\left(4\pi C\right)^{-1})</td>
<td></td>
</tr>
<tr>
<td>(f_i)</td>
<td>thermophoretic factor:</td>
</tr>
<tr>
<td>(f_i = 0.4[1 + 1.45 \text{Kn} + 0.4 \text{Kn} \times \exp(-1/\text{Kn})] (K_v + 2.5 \text{Kn} \sigma) + [(1 + 3 \text{Kn}) (2K_v + \sigma + 5 \text{Kn} \sigma)])</td>
<td></td>
</tr>
</tbody>
</table>
acceleration due to gravity \([=980 \text{ cm sec}^{-1}]\)

diffusiophoretic factor:

\[ g_d = \left[ \frac{m_s^1}{N_s m_s^1 + N_o m_o^1} \right] \]

with \( g' = 0.8-1.0 \)

\( h \)

height

\( H \)

heat (cal cm\(^{-3}\))

\( k \)

Boltzmann's constant \([=1.38047 \times 10^{16} \text{ erg (K)}^{-1}]\)

\( k_a, k_c \)

slope of number density function over \( a \)- and \( c \)-axis [see Eq. (2)]

\( k_h, k_r \)

slope of number density function over height and radius [see Eqs. (1) and (3)]

\( K_{sa}, K_{sc} \)

thermal conductivity—aerosol particles, air

\( K_0^* \)

modified \( K_0 [= f_{s} f_{a} K_0] \)

\( L_f, L_s, L_v \)

latent heat of—fusion, sublimation, vaporization

\( m \)

mass of individual particles

\( M \)

total mass cm\(^{-3}\) (within a bin), i.e., mass concentration

\( n_{ac}, n_{acn}, n_{asn} \)

number density function for particles over \( a \)- and \( c \)-axes, height, radius [see Eqs. (1), (2) and (3)]

\( N \)

number concentration of particles (within a bin) or molecules

\( NH \)

number-height parameter (see Section 3)

\( p \)

power in CCN activation spectra

\( P \)

total pressure (dyn cm\(^{-2}\))

\( P^* \)

semi-perimeter normal to fall axis

\( r \)

radius

\( R_a \)

gas constant for dry air \([=2.8704 \times 10^4 \text{ ergs gm}^{-1} (\text{K})^{-1}]\)

\( R_w \)

gas constant for water vapor \([=4.6150 \times 10^4 \text{ ergs gm}^{-1} (\text{K})^{-1}]\)

\( s \)

slope for calculation of \( \rho_{s,a} \) [see Eq. (23)]

\( S \)

supersaturation (%)

\( t \)

time

\( T_a, T_m \)

temperature—particle \( x \), ambient

\( T_e \)

temperature at end of time step

\( v_f, v_h, v_v \)

velocity—particle fall, horizontal, vertical

\( X \)

variable for determining \( Re \) for ice crystals [see Eq. (29)]

\( \beta \)

condensation or deposition coefficient \([=0.04]\)

\( \Gamma \)

axial growth ratio \([=dc/da]\)

\( \eta_{ac}, \eta_{acn}, \eta_{asn} \)

viscosity of air, viscosity at \(-10^\circ\text{C}\)

\( \lambda \)

mean free path of air molecules \([=0.0270 \text{ cm}]\)

\( \rho_a \)

ambient vapor density

\( \rho_{sa}, \rho_{sbn} \)

saturation vapor density at particle temperature—over ice, over water

\( \rho_a \)

density of mass added to particles

\( \rho_s, \rho_d \)

particle density, density of air

Stephan-Boltzman constant

\([=1.355 \times 10^{-12} \text{ cal cm}^{-2} (\text{K})^{-4} \text{ sec}^{-1}]\)

parameter for calculating the collection efficiency for ice crystals [see Eq. (16)]

Subscripts

\( a \)

aerosol over \( a \)- and \( c \)-axes

\( ac \)

artificial contact nuclei (reference \(-4^\circ\text{C}\))

\( acn \)

artificial sorption nuclei (reference \(-9^\circ\text{C}\))

\( asn \)

\( ax \)

\( c \)

\( ccn \)

cloud condensation nuclei (CCN) (reference \(S_w=1\%\))

\( cx \)

\( g \)

air (or air molecules)

\( h \)

over height

\( i \)

ice or ice particle

\( j, k, m \)

general subscripts

\( l \)

level

\( lb \)

layer base

\( ncn \)

natural contact nuclei (reference \(-4^\circ\text{C}\))

\( nsn \)

natural sorption nuclei (reference \(-20^\circ\text{C}\))

\( 0 \)

reference (interval mid-point or other specified value)

\( r \)

over radius

\( sf \)

snowflake

\( v \)

water vapor (molecules)

\( w \)

liquid water or water drops

\( x \)

dummy subscript

\( \infty \)

ambient

Superscripts

\( r \)

refers to value on previous step or iteration

\( x, y \)

associated with \( Re \) calculations [see Eq. (26) and Table 2]

Condensed algebraic notation

\[ A_a \int_{a_1}^{a_2} a^{n-1} da \]

\[ C_a \int_{c_1}^{c_2} c^{m-1} dc \]

\[ H_a \int_{h_1}^{h_2} h^{n-1} dh \]

\[ R_a \int_{r_1}^{r_2} r^{n-1} dr \]

REFERENCES


