NOTES AND CORRESPONDENCE

A Note on the Condition for Existence of More than One Steady-State Solution in Budyko-Sellers Type Models

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Since Budyko (1969) and Sellers (1969) published their climate models considerable effort has been devoted to studying the existence and stability of steady-state solutions for such energy-balance models. Recently Ghil (1976) presented a survey of different parameterizations that had been employed in these models, and their relation to the number of steady-state solutions. Solving the energy balance equation for steady states, Ghil obtained three stationary temperature distributions representing the following:

1) The present climate $\bar{T}_1 = 287.76$ K
2) Ice-age climate $\bar{T}_2 = 267.44$ K
3) Ice-covered earth $\bar{T}_3 = 175.43$ K

Here $\bar{T}_j$ is the global mean temperature, $j=1, 2, 3$.

In studying the stability properties of the different solutions Ghil used a refined technique of eigenvalue analysis. We have found, however, that a globally averaged model already qualitatively gives an interesting insight into the stability problem. In such a model the governing equation is

$$C\frac{d\bar{T}}{dt} = R_i(\bar{T}) - R_o(\bar{T}), \quad (1)$$

with $\bar{T}$ the globally averaged temperature, and R_i and R_o the incoming and outgoing radiation, respectively.

Following Sellers' parameterization, we write

$$R_i(\bar{T}) = Q[1-\alpha(\bar{T})], \quad (2)$$

$$R_o(\bar{T}) = \sigma\bar{T}^4[1-0.5\tanh(1.9\times10^{-15}\bar{T}^6)], \quad (3)$$

where Q is the solar constant, $\alpha(\bar{T})$ the albedo function, σ the Stefan–Boltzmann constant and C the heat capacity of the system. Here $\overline{T^4}$ and $\overline{T^6}$ have been approximated by $\bar{T}^4$ and $\bar{T}^6$, respectively, neglecting the meridional temperature distribution. An estimate of the error thus introduced can be made, using data from Ghil's (1976) calculated temperature distributions. The error is at most 2%.

Our model is similar to the one treated by Held and Suarez (1974) who, however, have included the meridional transport terms in parameterized form. We have neglected this internal redistribution of energy in order to study a more straightforward ordinary differential equation.

The albedo function $\alpha(\bar{T})$ [which gives the shape of the curve $R_i = R_i(\bar{T})$] is assumed linear in a temperature interval $\bar{T}_i < \bar{T} < \bar{T}_n$. $\bar{T}_i$ is the highest temperature that can exist when the whole earth is ice-covered and $\bar{T}_n$ the lowest possible temperature when no ice is present. Above $\bar{T}_n$ and below $\bar{T}_i$, the albedo is held constant.

The steady-state solutions to this equation are then simply given by a radiation balance, where the albedo-temperature coupling $\alpha = \alpha(\bar{T})$ determines the number of solutions. From Fig. 1, we see that there are three temperatures for which there is balance. In their vicinity $R_i - R_o$ can be approximated as $k_j(\bar{T} - \bar{T}_j)$, where $k_j < 0$ for $j=1, 3$ and $k_2 > 0$. The solution to (1) around these points will thus be of the form

$$\bar{T}(t) = Ae^{k_j t}. \quad (4)$$

Consequently, solutions (1) and (3) are internally stable, while (2) is unstable. It can also be seen in Fig. 1 that a decrease of Q will lower the curve R_i, thus moving the solutions (1) and (2) closer to each other until they eventually disappear for a critical value of Q.

Fig. 1 also agrees qualitatively with earlier results that the existence of bounds (upper and lower) on $\alpha(\bar{T})$ may be a necessary condition for multiple steady-state solutions. Whether we get one or three solutions depends on the slopes of R_o and R_i; in particular, we can directly conclude that the linearization of R_o is not a critical approximation in determining the number of steady-state solutions.

In Fig. 2, two different parameterizations for R_i are plotted. These are the parameterizations studied by Schneider and Gal-Chen (1973); they were originally proposed by Faegre (1972) and Sellers (1969). R_o has been plotted according to (3), with the bars indicating the range of variation due to the meridional temperature

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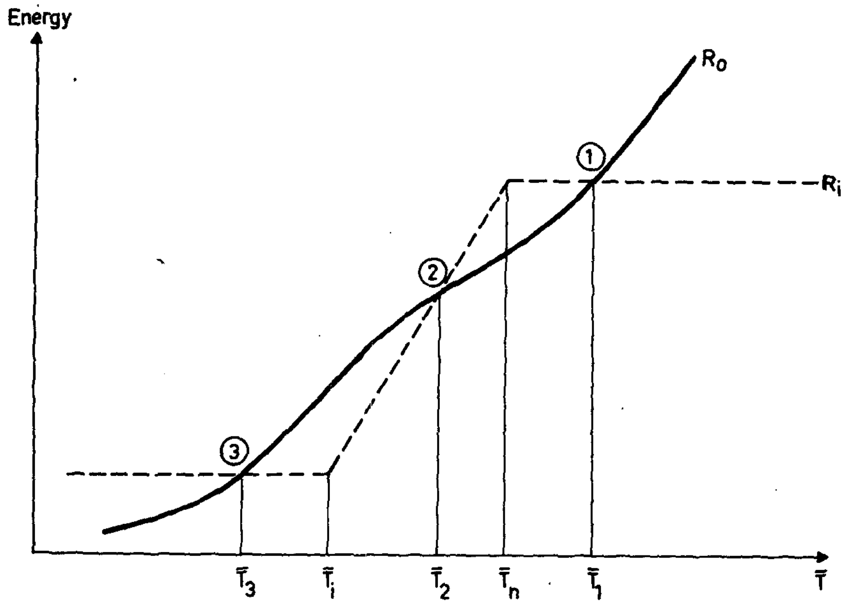

FIG. 1. Incoming (R_i) and outgoing (R_o) radiative energy as functions of global average temperature. Points of intersection give the steady-state solutions of (1).

distribution. The albedo formulations are (Sellers, 1969)

$$\alpha=\begin{cases} b(\phi)-0.009T, & T<283.16 \\ & (0.25\leqslant\alpha\leqslant0.85, \text{ all } T) \\ b(\phi)-0.009\times283.16, & T>283.16 \end{cases}$$

and (Faegre, 1973)

$$\alpha=0.4860-0.0092\ (T-273),\quad (0.25\leqslant\alpha\leqslant0.85, \text{ all } T),$$

respectively, where T is the zonally averaged surface temperature, and $b(\phi)$ is an empirical coefficient determined by Sellers (1969). In Fig. 2 a mean value for $b(\phi)$ has been chosen, such that $\alpha(283.16)=0.25$.

Schneider and Gal-Chen studied the time-dependent behavior of temperature perturbations from the present climate. Their result was that the Faegre formulation is very sensitive to negative perturbations, i.e., the

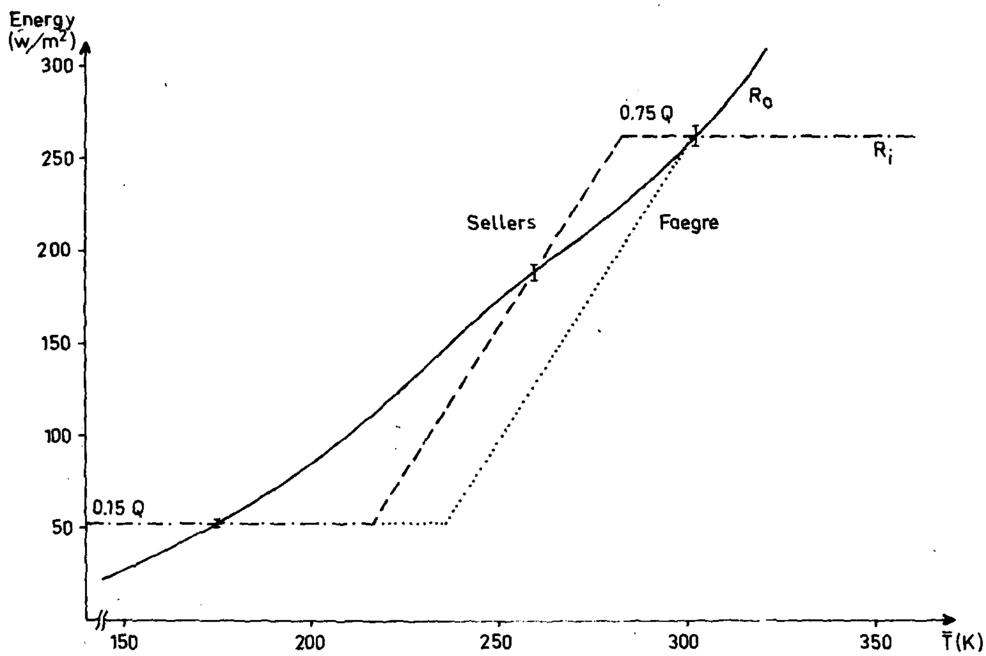

FIG. 2. Solid line: $R_o=\sigma\bar{T}^4[1-0.5\ \tanh(1.9\times10^{-15}\bar{T}^6)]$; dashed line: R_i with Sellers' albedo formulation; dotted line: R_i with Faegre's albedo formulation. I indicates the range of variation in R_o. $Q=349.7$ W m^{-2}.

model would quickly end up in the ice-covered earth climate, even though the initial temperature distribution was very close to a present day climate steady state. The Sellers formulation, however, required considerably larger negative temperature perturbations in order to result in the ice-covered earth climate. This is qualitatively in good agreement with Fig. 2. $\bar{T}_1$ and $\bar{T}_2$ with Faegre's albedo formulation are seen to be very close to each other, and as $\bar{T}_2$ is unstable, only a slight perturbation from $\bar{T}_1$ is needed for a transition to $\bar{T}_3$. With Sellers' formulation T_1 and $\bar{T}_2$ are further apart, thus being less sensitive to perturbations.

The results from the present simple model will of course not agree quantitatively with those from Ghil (1976), due to the effect of the neglected diffusion terms. The qualitative results, however, are in good agreement with the stability analysis presented by Ghil and the model also shows how different parameterizations will affect the number of steady states in a Budyko-Sellers type model

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A Note Concerning the Effect of Varying Extinction on Radiative-Photochemical Relaxation

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ABSTRACT

The effect of varying ozone column density on radiative-photochemical relaxation is examined. It is shown that neglecting variations in the total number of ozone molecules between a particular level and the sun can result in a spurious 50% increase in the damping rate of temperature perturbations. The relaxation rate of ozone perturbations, on the other hand, is decreased by as much as 50% if variations in extinction are neglected. The distribution and magnitude of the effect of varying column density on radiative-photochemical relaxation depends on elevation, latitude and season.

1. Introduction

In the stratosphere radiation and photochemistry jointly determine the equilibrium thermal structure and chemical composition and also interact strongly in determining the rate at which perturbations return to this equilibrium state. The stabilizing effect of ozone and temperature on each other in the stratosphere through radiative photochemical processes was first pointed out by Craig and Ohring (1958). Other studies of these mechanisms have been reported by Lindzen and Goody (1965) for oxygen-only photochemistry and by Blake and Lindzen (1973) for a more complicated photochemical model involving the important nitrogen and hydrogen reactions. In these papers the authors pointed out that the effect of coupling between chemistry and radiation in the stratosphere is to increase the relaxation rate of temperature perturbations over that produced by infrared radiative transfer. The acceleration is reduced somewhat by the inclusion of nitrogen and hydrogen photochemistry since this additional chemistry results in a weaker dependence of O_3 concentration on temperature.

The purpose of this note is to evaluate the importance for radiative-photochemica lrelaxation of the variations in available radiation at a particular level caused by changes in ozone concentration above that level. The discussion will be limited to disturbances with vertical wavelengths greater than a scale height and periods longer than a few days. The conclusions will thus be relevant to the damping of planetary waves and the zonal mean changes associated with them, which are the most important dynamical phenomena in the

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