Parametric Study of Large-Scale Eddy Properties.

Part II: The Zonal Scale

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ABSTRACT

Laboratory experiments reveal an approximate inverse dependence of the azimuthal wavenumber upon the radial temperature gradient, in the steady Rossby regime. To verify the occurrence of a similar relationship in the atmosphere, it being an unsteady system, we define an average zonal wavenumber \( \bar{n} \) and correlate it with the meridional temperature gradient \( \Delta T \). The values of the correlation coefficient \( r(\bar{n},\Delta T) \) vary considerably from month to month and therefore are unreliable. The values of \( r(\bar{n},\Delta T) \), where the bars denote averages over a month, are very small except in the vicinity of 35°N.

Linear baroclinic theory shows that there is a functional dependence between the zonal wavelength of the most rapidly amplifying waves and static stability \( S \). In the case of the atmosphere \( r(\bar{n},S) \) is as unreliable as \( r(\bar{n},\Delta T) \), but \( r(\bar{n},S) \) is considerably superior to \( r(\bar{n},\Delta T) \). Therefore, the meridional and vertical temperature gradients in the atmosphere must be decoupled, at least at 45 and 55°N where \( r(\bar{n},S) \) is very large.

1. Introduction

Extratropical atmospheric circulation is dominated by axially asymmetric perturbations which have zonal wavelengths of several hundred to several thousand kilometers. But since these perturbations do not show any simple periodic variations around latitude circles, no single integral zonal wavenumber associated with a Fourier or spherical harmonic series can completely, or even nearly completely, explain all of the zonal variances. Although relatively small sets of zonal harmonics can together produce patterns of eddy distribution similar to those observed, these "small sets" are still not small enough to be called a single scale of longitudinal distribution. As an example, Saltzman and Fleisher (1960) divide the first 15 zonal harmonics into groups of five wavenumbers each, called long waves, cyclone waves and short waves. The cumbersome method of spatial Fourier analysis, while yielding valuable insights into eddy characteristics, does not provide directly a mean azimuthal scale that can be used to compare the behavior of the atmosphere with the results of laboratory experiments and analytic theories. Thus there is an essential need to define statistical (or objective) spatial scales for atmospheric eddies. Failing to find such scales, one would resort to the counting of ridges and troughs and high and low pressure cells. Although such a subjective method may not be difficult to apply while studying limited numbers of weather charts, as shown by Breistin and Parry (1954), the currently available 5- and 10-year compilations, not to speak of the more extensive ones which would result with the sheer passage of time, would strain even large groups of technicians employed in data analysis.

These considerations had led Lorenz (1951) to define objective parameterizations of eddy properties as expressed in terms of zonal harmonics. Lorenz defined mean horizontal and vertical tilts of atmospheric eddy axes which are very useful in the study of the transports of heat and momentum. Using the geostrophic approximation as Lorenz did, Srivatsangam (1976a) extended the parameterization to longitudinal wavenumbers and other quantities. The average zonal wavenumber \( \bar{n} \) thus defined was computed for several months. It was found then, in Srivatsangam (1976b), that the meridional distribution of \( n \) agrees very well with the meridional distribution of the zonal wavenumber of the most unstable baroclinic waves on a sphere, as determined by Moura and Stone (1977). The agreement was shown to hold good for transient atmospheric states as well as when \( n \) is averaged over a month.

In this paper \( \bar{n} \) will be studied in relation to theoretical and experimental considerations not taken into account in Srivatsangam (1976b).

A result of the laboratory experiments with rotating cylindrical annuli is the inverse relationship between the horizontal temperature gradient and the azimuthal wavenumber. We will look for such a relationship in the atmosphere, to delineate in which zonal belt, if

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anywhere, there is a semblance in this regard between the atmosphere and the laboratory experiments. We will also relate $\mathcal{H}$ to baroclinic wave theory through the local, temporal variations in $\mathcal{H}$, in a manner different from Srivatsangam (1976b).

To bring out our results clearly, we have arranged the subject matter in this fashion: In Section 2 we present the results of experimental studies which are of use here. Similarly, Section 3 contains a summary of baroclinic theory results. In Section 4 we have the results on the dependence of $\mathcal{H}$ upon the meridional temperature gradient, and, in Section 5, a set of results on the correlation between $\mathcal{H}$ and static stability. The discussion in Section 6 concludes the paper.

2. Results of laboratory experiments

The laboratory model that is generally agreed as reproducing the characteristics of planetary circulations is a rotating circular cylinder or cylindrical annulus in which a "working fluid" is contained (see, e.g., Fultz et al., 1959). The effect of baroclinicity, introducing vertical shears of the horizontal motion, is induced usually by an imposed radial temperature difference. The motions at the top surface of the fluid are usually observed with visible particles such as aluminum powder. The motions at the lower levels may then be determined by geostrophy, when it is applicable. Experimenters have more recently used neutrally buoyant particles to directly photograph lower level motions (see Douglas et al., 1972). Thus, much of the early data relate to the transitions in flow regimes as determined from the top surface.

The baroclinicity of the model, represented by a vertical shear or a radial thermal contrast $\Delta T_r$, and the rotation rate of the cylinder $\Omega$ are obviously the independent variables which can be controlled and with respect to which the results can be presented. The baroclinicity is often given as a vertically averaged value of $\Delta T_r$ (Lorenz, 1967), and is included in the thermal Rossby number

$$ R_{OR} = \frac{g \rho \beta}{a \Omega} \frac{\Delta T_r}{\Delta T}, $$

where $g$ is the acceleration due to gravity, $d$ the depth of the fluid, $\rho$ the coefficient of volume expansion of the fluid and $a$ the width of the annulus. The effects of rotation and of fluid viscosity are contained in the Taylor number

$$ Ta = \frac{4 \Omega d^2}{\nu}, $$

where $\nu$ is the kinematic coefficient of viscosity. The different modes of circulation are plotted in the plane of $R_{OR}$ and $Ta$ (see, e.g., Lorenz, 1967). Although such a presentation of data is intuitively obvious, and works well in practice, according to Fowlis and Hilde (1963) there are 16 dimensionless parameters governing the equations of motion and boundary conditions. Thus, although the $R_{OR}$ vs $Ta$ plots might be useful for presenting experimental results, these two nondimensional parameters may not be of particular relevance in the resolution of atmospheric motions. We will return to this question later in this paper.

The $R_{OR}$, $Ta$ plane diagrams for the experiments reveal an axially symmetric regime and several wave regimes. The symmetric regime is separated from the wave regimes by a knee- or anvil-shaped curve, and the symmetric modes above and below the knee are called the upper and lower symmetric regimes, respectively. The wave regime is divided into the steady Rossby regime, the unsteady Rossby regime, amplitude vacillations and tilted-trouch vacillations. In the steady Rossby regime and the vacillation regimes the (integral) azimuthal wavenumber $n$ is conserved in time. These regimes are the best studied quantitatively. The unsteady Rossby regime does not have a temporal conservation of wavenumber and the nearly random distribution and development of eddies suggest that this regime resembles the atmosphere most closely; this regime is the least understood of all the circulation types in the experiments, since the azimuthal wavenumber in this regime does not have a single integral value.

The steady Rossby regime is such that, roughly speaking, the wavenumber $n$ increases as $\Delta T_r$ is decreased. That is, for a constant value of $Ta$,

$$ n = \gamma R_{OR}^{-\alpha}, $$

where $\gamma$ and $\alpha$ are undetermined. The validity of (3) depends on the quality of the observations and the smoothing of the data. In most of the recent publications (e.g., Kaiser, 1970; Fein, 1973; Matsuo et al., 1976), the data in the $R_{OR}$, $Ta$ plane are presented exactly, and in none of these publications can (3) be considered as being exact.

3. Results of baroclinic theory

Kuo (1956, 1957) has studied baroclinic instability especially with the aim of developing a theory of the regime transitions in the laboratory experiments. He finds that the azimuthal wavenumber is determined by the parameter

$$ \Pi = \frac{g \beta S}{4 \Omega d^2}, $$

where $d$ is the vertical scale of motion (or the depth of the fluid), $a$ the radial scale of motion (or the width of the annulus), and $S$ the static stability is given by

$$ S = -\alpha \partial \theta_0 / \partial \sigma = \alpha (\Delta \theta_0 / d). $$

In (5), $\alpha$ is the specific volume of the fluid and $\theta_0$ the basic state (or, equivalently, azimuthal mean) temperature. In the experiments, especially when the depth of the fluid is small compared to the horizontal scale of motion, $\Delta \theta_0$ is proportional to $\Delta \theta_0$, the radial temperature difference. Kuo (1957) has given the proportionality constants for $\Delta \theta_0 \propto \Delta \theta_0$ for certain regime transitions. Under the conditions stated here, the
parameter $Pi$ becomes proportional to the thermal Rossby number $Ro_T$. As a consequence, the azimuthal wavenumber is determined completely by the radial temperature difference and the rotation rate.

The conclusion reached by Kuo on the dependence of azimuthal wavenumber on static stability has also been made by Eliassen (1952) and others. For a quasi-geostrophic, baroclinic zonal current Eliassen finds that the wavelength of maximum instability increases in proportion to the square root of static stability and decreases in proportion to the Coriolis parameter.

Stone (1969) examined the addition of the meridional shear to a quasi-geostrophic, two-layer, $\beta$-plane model. He concludes that the baroclinic waves have zonal and meridional scales equal to the radius of deformation $L$, where

$$L = (Sg d^2/f_0)^{1/3},$$

and $f_0$ is a constant Coriolis parameter. Stone's result on the meridional scale has been contradicted by Simmons (1974; see also Stone, 1975). Warn (1976) states that for her two-layer model on a sphere both the zonal and meridional scales are larger than the radius of deformation $L$. But all these studies do show that the zonal and the meridional scales, whether or not they are exactly equal to $L$, are indeed functions of $L$.

We would like to note, moreover, that Moura and Stone (1976), considering baroclinic instability on a sphere, find also that the horizontal scales are approximately proportional to $L$. We mentioned above the agreement between the meridional distribution of $\tilde{n}$ in the atmosphere and the zonal wavenumber in the Moura and Stone model. Since the latter is scaled by the radius of deformation, $\tilde{n}$ may also be scaled by $L$. However, are the variations in $\tilde{n}$ related to static stability changes? We would seek an answer to this question in this paper.

4. Relationship between the average wavenumber and meridional temperature gradient

The correlation between the average zonal wavenumber $\tilde{n}$ and the meridional temperature gradient $\Delta T$ was studied with many sets of data. All the data, however, were derived from the distributions of heights of isobaric surfaces in the National Meteorological Center.
(NMC) tapes. The values of $\mathbf{H}$ were computed by using

$$\mathbf{H} = \left[ \frac{\left[ \mathbf{v}^2 \right]}{\left[ \mathbf{z}^2 \right]} \right] g^{-1} \bar{a} \cos \phi,$$

(7)

where square brackets represent zonal averages and asterisks departures therefrom. Therefore, $\left[ \mathbf{v}^2 \right]$ and $\left[ \mathbf{z}^2 \right]$ are the zonal variances of the geostrophic wind at, and the height of, an isobaric surface, respectively. Also, $f$ is the Coriolis parameter, $a$ the mean radius of the earth, $\phi$ latitude and $g$ the acceleration of gravity. The derivation of (7) is presented in Srinivasanam (1976a).

We would like to note that $\mathbf{v}^*$ was obtained through the geostrophic equation and the temperatures were obtained from the height fields by the hydrostatic equation and the equation of state for air.

Initially, scatter diagrams between $\mathbf{H}$ and $\Delta T$ were prepared for daily data for periods equal to a month. Also, we considered the data for the 300 mb level since this level approximately represents the top of the troposphere and therefore corresponds to the top surface of the laboratory experimental fluid. We repeat that it is to the latter surface that many of the regime transition diagrams pertain.

The first few plots had $\Delta T$ measured at the 300 mb level or the vertically averaged values of $\Delta T$ as abscissas. The latter were used in the belief that being closer to the procedure for obtaining ROF in the experiments, these vertical means would be more closely related to $n$ than the unaveraged values.

We present a considerable portion of our results in the form of scatter diagrams. This we hope would give the reader a better understanding of the actual distribution of data and of the presence of nonlinearities than the tabulated values of correlation coefficients.

Fig. 1 represents the average zonal wavenumber at 35°N plotted against the meridional temperature difference, $\left[ T \right](20°N) - \left[ T \right](50°N)$, for the 300 mb level (square brackets denote zonal averages). These data are for October 1968. Fig. 1 does show an inverse dependence of $\mathbf{H}$ on $\Delta T$. But if the two points at the top left-hand corner of this diagram were absent, no inverse relationship between $\mathbf{H}$ and $\Delta T$ would exist. Thus the validity of the $\mathbf{H}$, $\Delta T$ relationship depends on just two points in Fig. 1.

Fig. 2 has the daily values of $\mathbf{H}$ at 65°N plotted against the temperature difference $\left[ T \right](50°N) - \left[ T \right](80°N)$. It is also for the data at the 300 mb level in October 1968. This diagram also reveals an inverse dependence between $\Delta T$ and $\mathbf{H}$, and the relationship is more dependable than the one in Fig. 1.

Fig. 3 is for the same data as in Fig. 2, but for November 1968. There is no relationship between $\mathbf{H}$ and $\Delta T$ in this diagram.

In addition to Figs. 1–3, we made several other plots, averaging the temperature gradient in the vertical and also plotting the data for other months. The result of these efforts is that there is no reliable

![Fig. 2](image-url)
Fig. 3. As in Fig. 2 except for November 1968.

Fig. 4. Monthly average values of $\bar{v}$ at 35°N plotted against $\Delta T$ between 20 and 50°N, for 700 mb. Symbols: solid circles are for October–December 1968 and February–April 1969, solid triangles for October 1969–April 1970, and solid squares for November and December 1970 and February–April 1971. Units as in Fig. 1.
relationship between $\mathbf{\mathbb{H}}$ and the meridional temperature gradient on a daily basis. This may be because of the inability of hemispheric scale waves to make short-term adjustments in their spatial scales. However, over longer periods of time the eddies may be able to adjust their wavenumber to correspond to $\Delta T$. Such a possibility has led us to consider the correlation between the monthly mean values of $\mathbf{\mathbb{H}}$, denoted by $\overline{\mathbf{\mathbb{H}}}$, and of $\Delta T$ denoted by $\overline{\Delta T}$. The reader may note that $\mathbf{\mathbb{H}}$ is calculated using daily weather data and hence includes both standing and transient eddies. Since $\overline{\mathbf{\mathbb{H}}}$ is the monthly mean of the daily values of $\mathbf{\mathbb{H}}$, $\overline{\mathbf{\mathbb{H}}}$ also includes both standing and transient eddies.

Fig. 4 constitutes the scatter of $\overline{\mathbf{\mathbb{H}}}$ at 35°N against $\overline{\Delta T}$ between 20° and 50°N. Fig. 4 is for the 700 mb level, from a three-winter set of NMC data. It reveals a dependable negative correlation between $\overline{\mathbf{\mathbb{H}}}$ and $\overline{\Delta T}$. There is a slight nonlinearity in the dependence of $\overline{\mathbf{\mathbb{H}}}$ on $\overline{\Delta T}$.

Fig. 5 is a $\overline{\mathbf{\mathbb{H}}}$, $\overline{\Delta T}$ scatter diagram for a 120-month set of NMC data, with $\overline{\mathbf{\mathbb{H}}}$ at 35°N, 300 mb and the temperature gradient evaluated between 20° and 50°N at 300 mb. Despite a rather wide scatter of points, this diagram also shows an inverse relation between the time-mean average wavenumber and meridional temperature gradient. The linear correlation coefficient for these data is $-0.35$.

Fig. 6 is for the same set of data as Fig. 5, but for the 500 mb level. These data have a correlation coefficient of $-0.57$. There is also a noticeable nonlinearity in this diagram.

From Figs. 4–6 we can conclude that there is a weak inverse relationship between $\overline{\mathbf{\mathbb{H}}}$ and $\overline{\Delta T}$ in the latitude belt 20°–50°N. The relationship at the 500 and 700 mb levels is superior to that at the 300 mb level.

The similarity between the experiments and the atmosphere, as obtained by the inverse dependence of the azimuthal wavenumber and the meridional temperature gradient, is closest in the vicinity of 35°N. This is proved by Table 1 as well as Fig. 7, the latter revealing the lack of a relationship between $\overline{\mathbf{\mathbb{H}}}$ at 65°N and $\overline{\Delta T}$ in the belt 50–80°N, all data pertaining to the 300 mb level.

Column A of Table 1 contains the values of the coefficient of determination, which is the square of the linear correlation coefficient $r^2$ is equal to the ratio of the reduction in the sum of squares of deviations obtained by using the linear regression model to the total sum of squares of deviations about the sample mean of $\mathbf{\mathbb{H}}$, which would be the predictor of $\overline{\mathbf{\mathbb{H}}}$ if $\overline{\Delta T}$ were ignored (see, e.g., Mendenhall, 1975). In other words, $r^2$ is a measure of the variance of $\overline{\mathbf{\mathbb{H}}}$ which $\overline{\Delta T}$ (taken at 10° latitude intervals) "explains." We see from these data that $\overline{\Delta T}$ explains <2% of the variance of $\overline{\mathbf{\mathbb{H}}}$ everywhere except 35°N. (The percentage value is obtained by multiplying Column A by 100.)

5. Relationship between $\mathbf{n}$ and static stability

In linear baroclinic theory the zonal wavenumber of the most rapidly amplifying waves is inversely proportional to the radius of deformation, as discussed in Section 3. While considering the local, temporal varia-
Fig. 6. As in Fig. 5 except for 500 mb.

Fig. 7. As in Fig. 4 except for 300 mb, $\bar{T}$ at 65°N and $\Delta T$ between 50 and 80°N.
tions of the zonal wavenumber, however, since the Coriolis parameter is then a constant, one would expect from (6) an inverse relationship between $\mathbb{H}$ and the zonal-mean static stability $S$.

The linear correlation between wavelength and $S$ has been investigated by Breis et al. (1954) who had, however, considered the data for only a 10-day period. Also, their definition of wavelength is arbitrary. For instance, they state “Semipermanent long waves are to be excluded since their amplitudes are usually large and their frequency such that they unduly weight the statistics.” Since they have considered only a 10-day period of data, it appears that Breis et al. and Parry have taken the “semipermanent long waves” from climatological charts. As noted before, this kind of approach is good only for the analysis of a limited amount of data.

In the following results, as before, $\mathbb{H}$ represents the average wavenumber of the total perturbation field, including the transient and the standing eddies. We are considering finite-amplitude waves while the theory considers linear waves. The finite-amplitude atmospheric eddies are such that the large-scale, quasipermanent component of these eddies is maintained against frictional dissipation by a transfer of kinetic energy from the smaller, cyclone-scale component through nonlinear interaction (see Saltzman, 1959; Saltzman and Fleisher, 1960). For these reasons, it appears that we might study the entire atmospheric eddy field for comparison with theoretical results.

The coefficients of determination $r^2$ between $\mathbb{H}$ and the static stability $S$ defined as

$$S = \frac{\langle \theta \rangle(200 \text{ mb}) - \langle \theta \rangle(500 \text{ mb})}{\langle \theta \rangle(300 \text{ mb})}$$

are presented in column B of Table 1. A comparison of the values of $r^2$ in columns A and B of Table 1 shows that $r^2$ is greater between $\mathbb{H}$ and $S$ than between $\mathbb{H}$ and $\Delta T$ at all latitudes except $35^\circ$N. Also, $S$ explains a minimum of 5% of the variance of $\mathbb{H}$, whereas $\Delta T$ explains, as a minimum, only 0.01% of the variance of $\mathbb{H}$. Therefore, $S$ is a much superior predictor of $\mathbb{H}$ than $\Delta T$.

That the correlation between $\mathbb{H}$ and $S$ is also negative is seen from Fig. 8, which has $\mathbb{H}$ at 300 mb, 45$^\circ$N plotted against $S$ at 45$^\circ$N.

We also evaluated the coefficient of correlation $r$ between static stability defined as

$$r = \frac{\langle \theta \rangle(250 \text{ mb}) - \langle \theta \rangle(850 \text{ mb})}{\langle \theta \rangle(600 \text{ mb})}$$

and $\mathbb{H}$ at 500 mb for a few months. In Table 2 we present $r(S, \mathbb{H})$ for October and November 1968. The very considerable inter-monthly variations in $r(S, \mathbb{H})$ are obvious from Table 2. For instance, at 25 and 30$^\circ$N the value of $r(S, \mathbb{H})$ changes from -0.43 and -0.51, respectively, in October 1968 to 0.6 and 0.44, respectively, in November 1968. Furthermore, the values of $r(S, \mathbb{H})$ are usually very small. Thus the estimate of $r(=0.82)$ obtained by Breis et al. and Parry (1954)

<table>
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<tr>
<th>Month</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
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<th>80</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.06</td>
<td>-0.43</td>
<td>-0.51</td>
<td>-0.21</td>
<td>0.26</td>
<td>-0.12</td>
<td>0.15</td>
<td>0.12</td>
<td>-0.04</td>
<td>-0.32</td>
<td>-0.14</td>
<td>-0.14</td>
<td>-0.30</td>
<td>-0.05</td>
</tr>
<tr>
<td>November 1968</td>
<td>0.05</td>
<td>0.60</td>
<td>0.44</td>
<td>0.24</td>
<td>0.38</td>
<td>0.08</td>
<td>-0.05</td>
<td>-0.41</td>
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<td>-0.18</td>
<td>-0.07</td>
<td>0.00</td>
<td>-0.17</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

Fig. 8. Dependence of $\mathbb{H}$ on $S = \frac{\langle \theta \rangle(200 \text{ mb}) - \langle \theta \rangle(500 \text{ mb})}{\langle \theta \rangle(300 \text{ mb})}$ for 300 mb at 45$^\circ$N. Data as in Fig. 5. Units: $\mathbb{H}$ and $S$ are nondimensional, $\theta$ (K).
for their hemispheric data, comparing wavelength and static stability, appears to be fortuitous; it may also be due in part to the subjectiveness of their analysis and the inadequacy of the network of hemispheric stations available in 1952–53, the period of their analysis. Brestein and Parry’s estimate of \( r = 0.37 \) for the period 15 February 1952–1 June 1953, obtained with the United States data only, is, however, comparable to the values of \( r \) for 40°N in Table 2.

We can conclude from the above that the correlation between static stability and \( \vec{F} \) is both small and unreliable, on a daily basis. Since this is also the case with the meridional temperature gradient versus \( \vec{F} \) correlation, we deduce that small periods of about a day are insufficient for the atmosphere to reach an equilibrium between the zonal scale and the meridional or vertical temperature gradient. However, the correlation between the static stability and \( \vec{F} \) is sufficiently large, especially in the zone 35–65°N, so that an equilibrium between \( S \) and \( \vec{F} \) is apparently reached in periods longer than a day but shorter than or equal to a month.

6. Discussion

In view of the poor correlation between the horizontal temperature gradient and the average zonal wavenumber in the atmosphere we would like to reconsider the inverse relation between the radial thermal contrast and the azimuthal wavenumber in the experiments. This is due to the fact that a survey of more recent laboratory modeling suggests that earlier results might have overemphasized the dependence of wavenumber on \( \Delta T_r \).

It has been recognized from the earliest stages of experimentation that the wavenumber observed in the steady Rossby regime is subject to hysteresis, as shown by Hide (1958). That is, the manner in which a particular steady state is arrived at in the Ro_T plane affects the resulting wavenumber. A given combination of Ro_T and Ta can be reached by increasing or decreasing \( \Omega \) or \( \Delta T_r \),—four possibilities in all. Fein (1973) has considered all four possibilities. His conclusions are stated as follows: “Different wavenumbers were observed at different times at the same point in parameter space. Never, however, were more than three different wavenumbers observed at any one point in parameter space and never were those numbers anything but consecutive.” Lambert and Synder (1966) have conducted a different kind of experiment, in which the two cylinders (between which the working fluid is confined) are rotated independently of each other so that there is an imposed horizontal shear on the fluid. With this model, Lambert and Synder have varied \( \Delta \Omega / \vec{F} \) where \( \Delta \Omega = \Omega_2 - \Omega_1 \), and \( \vec{F} = (\Omega_1 + \Omega_2) / 2 \). Here \( \Omega_1 \) and \( \Omega_2 \) are the angular velocities of the inner and outer cylinders, respectively. The results of Lambert and Synder for increasing and decreasing values of \( \Delta \Omega / \vec{F} \), plotted in Ro_T, \( \Delta \Omega / \vec{F} \) plane are considerably different. In other words, although the radial temperature difference might be constant, increasing or decreasing the imposed horizontal shear to reach a particular value of \( \Delta \Omega / \vec{F} \) leads to different azimuthal wavenumbers. We would like to note that these experiments are closer to atmospheric conditions than the experiments in which \( \Omega_1 \) and \( \Omega_2 \) are the same. In the atmosphere, the prevailing meridional shear of the zonal wind may be reached from higher or lower values of the shear.

We see from the statements above that even for the same value of \( \Delta T_r \), the steady-Rossby-regime azimuthal wavenumber may vary by one or two units. The extent of variation of \( \vec{F} \) (though not of \( \vec{F} \)) in the diagrams presented here are, therefore, in accordance with the experimental results.

Two factors which should not be overlooked in our study of the \( \vec{F}, \Delta T_r \) (and \( \vec{F} \) and \( \Delta T_r \)) relationships are that the atmosphere is an unsteady Rossby regime which occurs in a system which has no walls. Spectral analysis of the unsteady Rossby regime is still in a primitive stage of development (see, e.g., Rao and Ketchum, 1975, 1976). Not much is known concerning the azimuthal harmonics of the unsteady wave regime in the experiments and nothing is known concerning their average scales. Most of the currently available experimental results are for cylindrical annuli. Only one published report (Snyder and Youutz, 1969) deals with periodically varying imposed temperature gradients. Since the different zones in the atmosphere interact freely with adjoining zones, locally observed values of \( \vec{F} \) and \( \vec{F} \) are not entirely attributable to local conditions. A spherical model, which can be placed in a low-gravity or no-gravity environment, as has been described by Srivatsangam (1976c), might lead to a better understanding of inter-latitudinal exchanges.

An important result of our study is that the static stability is a better predictor of \( \vec{F} \) than the mean meridional temperature gradient. It was noted above that the horizontal and vertical temperature gradients are well correlated in many dishpan experiments (see also Pfefer and Chiang, 1967). Kaiser (1977) has questioned the universality of this correlation, stating that it arises through the balance between vertical heat conduction and radial heat advection when the depth of the fluid is much smaller than the width of the annulus. He suggests that the correlation between the radial and vertical temperature gradients would be small for a deep layer of fluid.

Ketchum (1972) has made a comprehensive study of the thermal structure of the axisymmetric and wave regimes. He has defined dimensionless measures of the vertical and radial temperature differences away from the conduction layers given by

\[
\sigma_z = (d/\Delta T) \frac{\partial T}{\partial z} \quad \text{and} \quad \sigma_r = (a/\Delta T) \frac{\partial T}{\partial r}.
\]
Here $\Delta T$ is the imposed radial temperature difference and the tilde indicates an average over the interior volume of the fluid. It is seen from the data presented by Ketchum that the mean vertical stability of the fluid, proportional to $\sigma_v$, remains essentially the same irrespective of the regime type, i.e., $\sigma_v$ is independent of $\Delta T$ and $\Omega$. But $\sigma_v$ varies considerably with respect to both $\Delta T$ and $\Omega$. Our results, in which $\tilde{H}$ is better correlated with $S$ than with $\Delta T$ indicate that in the atmosphere, as in the experiments of Ketchum, the vertical and horizontal temperature differences are poorly related to each other, at least at 45 and 55°N, where $r^2(\tilde{H}, S)$ is very large and $r^2(\tilde{H}, \Delta T)$ is very small. Thus the midlatitude lower atmosphere is to be considered a deep layer of fluid, in the sense of Kaiser (1977). We would like to note that in Ketchum’s experiments $a = 15.28$ cm and $15.1 \text{ cm} < d < 15.3 \text{ cm}$.

A final statement we would like to make is that we have considered the finite-amplitude eddies in the atmosphere in the light of linear (i.e., negligible amplitude) baroclinic wave theory. A survey of the literature has not produced any relationship between the zonal scale of finite-amplitude baroclinic waves and other parameters which can be estimated. However, if, following Moura and Stone (1976), we assume that baroclinic waves which occur near neutral stability may correspond to finite-amplitude waves, then our results indicate that, like waves near neutral stability, atmospheric eddies have a zonal scale that varies inversely with static stability, at least in the middle latitudes (see Table 1).

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