A Measurement of Mountain Drag

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ABSTRACT

The pressure drag on the Blue Ridge Mountain in the central Appalachians has been determined by measuring the pressure on each side of the ridge. During the first two weeks of January 1974 several periods with significant drag were observed with pressure differences typically 0.5 mb (50 Pa) across the ridge. This corresponds to a mountain drag which is at least equal to the skin friction drag in the Appalachian region if the other ridges in the area experience a similar drag. As expected, the drag seems to occur when the environmental wind has a component perpendicular to the ridge. Such a cross flow appears necessary but not sufficient for the production of drag. The measured mountain drag seems also to be associated with blocking of the surface flow upstream, a condition which occurs most frequently in stable nighttime conditions. While none of the possible drag mechanisms can be absolutely excluded or verified, drag due to flow separation appears unlikely and the mechanism of wave drag (of some sort) appears a likely candidate.

The sources of error in mountain drag measurements and ways to improve the measurement techniques are also discussed.

1. Introduction

The general problem of the transfer of momentum between the atmosphere and the surface of the earth is one of the central concerns of dynamical meteorology. This drag force acting between the earth and the atmosphere is complicated by the wide range of size of the surface irregularities and the fact that the drag mechanism will be different for each size range, examples of which are as follows: 1) on smooth water, sand grains, pine needles, etc. \( L \approx 1 \text{ mm} \) viscous forces dominate due to the small Reynolds number 2); on trees, buildings, small hills \( L \approx 10 \text{ cm} \) to 100 m) pressure forces associated with fluid inertia dominate 3); on large mountains, small mountain ranges \( L \approx 10-100 \text{ km} \) pressure forces associated with fluid buoyancy forces dominate 4); on large mountain ranges, orogenic basins and plateaus \( L \approx 1000 \text{ km} \) pressure forces associated with the Coriolis force dominate. At intermediate scales, say \( L \approx 2 \text{ km} \), the drag forces are associated with combinations of the above mechanisms.

There has been a great deal of progress made in understanding the flow around small roughness elements such as houses, crops and trees and the composite effect when these irregularities are distributed in a statistically uniform manner on the surface (see, e.g., Sutton, 1953). Much of this work was a direct extension of the earlier engineering work on aerodynamic drag and turbulent boundary layers started by von Kármán, Prandtl and others. The major results of this work have been the use of a roughness length parameter to describe the pressure drag on the surface irregularities and the concept of a mixing length to describe how the momentum is transported through the boundary layer.

There has been some progress also on the determination of pressure drag on the largest scale of irregularities \( L \approx 1000 \text{ to } 10000 \text{ km} \). White (1949), Newton (1971) and Oort and Bowman (1974) have used the synoptic-scale pressure patterns at the earth's surface to compute the pressure difference across major mountain ranges. The use of a severely smoothed representation of the topography allowed the mountain drag or mountain torque (the moment of the drag about the earth's axis) to be determined. In general, it was found to be comparable to, but in most latitudes smaller than, the frictional drag. As this synoptic scale mountain drag is affected strongly by geographic distribution of surface heating, cyclone tracks, etc., it was not surprising that no simple relationship was found between mountain drag and the local winds passing over the mountains.

In between these two extremes of scale there exists a wide range of surface irregularity \( L \approx 100 \text{ m} \to 100 \text{ km} \) about which little is known. Lorenz (1951) speculated that the drag on these scales might be important but there have been, to my knowledge, no direct measurements reported in the literature. Lilly and Kennedy (1973), however, have calculated the vertical flux of momentum above the Front Range \( L \approx 100 \text{ km} \) of the Rocky Mountains from aircraft measurements of
mountain waves and this represents at least a partial measure of the total drag on that feature. They found the momentum flux to vary widely but on at least one occasion to be comparable with the total drag around the world in that latitude belt.

There have been several attempts to theoretically predict the drag on mountains within this size range. The results of Sawyer (1959), Blumen (1965), Bretherton (1969) and Miles (1969) and others have helped to understand how mountain drag can be produced in an inviscid stably stratified fluid by the generation of internal gravity waves. Unfortunately, it is not known to what extent these idealized theories are applicable to the atmosphere or even if the wave drag mechanism is the most important one.

It is the goal of this paper to improve somewhat our understanding of the importance of mountain drag and the mechanisms which cause it. We will examine the drag on the Blue Ridge Mountain in the central Appalachians—a ridge of simple shape near the small end of the "mountain" size range. We consider in turn 1) the technique of mountain drag measurement, 2) the relationship of the measured drag to prevailing wind conditions and 3) possible drag mechanisms. It must be recognized at the outset that the data reported on here represent a first attempt at the direct measurement of mountain drag and the results do not attain the level of accuracy that I believe is possible in subsequent studies. The data are of special interest because the large number of microbarographs used (7) allowed measurement of the pressure difference between independent pairs of instruments—thus making possible a check on the data—and a continuous record of surface winds at two stations on the mountain support the pressure data and provides some information about the mechanism of mountain drag.

2. Techniques of pressure drag measurement

A straight section of Blue Ridge, running southwest from Harpers Ferry, WV, was chosen for the study. This section of the Blue Ridge is nearly two-dimensional (see Fig. 1) over a length of 35 km with a height of about 300 m and a width of about 2 km. To the west is the Great Valley of the central Appalachians—34 km of flat lying farmland and beyond, the Valley and Ridge Province—100 km of closely spaced ridges. To the east are the Piedmont and Coastal plains—featureless terrain leading to the Chesapeake Bay. Long straight ridges similar to the Blue Ridge are common throughout the world and are generally of common origin. The erosion which sculptured the topography was controlled by the nearly two-dimensional folding and faulting of initially flat sedimentary rocks, associated with tectonic movements (in many cases the closing of ancient ocean basins). The origin of the mountains is of course irrelevant here except as an explanation for the frequent occurrence of this type of topography. Examples include the Jura in France, the Canadian Rockies, the western Urals, etc. The Blue Ridge was chosen for this study because of its 1) relatively small size to avoid the effects of the earth's rotation and to a certain extent, the uncertainties associated with synoptic and mesoscale disturbances, 2) simple geometry, and 3) isolation. In this latter trait the Blue Ridge is somewhat atypical.

On 1 January 1974, several meteorological instruments were placed across the ridge as shown in Fig. 1 and Table 1. The object was to measure the pressure difference across the mountain and the associated pressure drag per unit length

\[ D/l = \int_{z_0}^{z_h} \Delta \rho dz, \]  \hspace{1cm} (1)

where \( \Delta \rho \) is the pressure difference at each height \( z \). No attempt was made to locate the instruments on a single cross section. Because of the availability of existing roads and trails, the microbarograph positions along the ridge vary up to 2 km, but if the flow is truly two-dimensional the evaluation of (1) will not be affected.

3. Difficulties in the measurement of \( \Delta \rho \)

There are three independent difficulties in the measurement of \( \Delta \rho \) for use in (1).

a. Absolute pressure accuracy

To measure a pressure difference of 0.5 mb to an accuracy of ±20% requires an absolute accuracy of at least ±0.1 mb (neglecting covariant errors). If the two pressures could be compared directly, this level of accuracy would be easily obtained, but here each instrument must record the absolute pressure (~1000 mb) to an accuracy of one part in ten thousand. The microbarographs used here (Belfort Instrument Company 5-800, Weather Bureau model), are rated at this accuracy (~0.1 mb) by the manufacturer and a simple side-by-side comparison test indicates that the different instruments usually read within ±0.1 mb of each other over most of their range. No further calibration of these instruments was attempted.
b. Simultaneity

A typical pressure tendency \( (\partial P/\partial t) \) in the atmosphere is 1 mb h\(^{-1}\) for rapid synoptic systems and as large as 5 mb h\(^{-1}\) for mesoscale waves and convective systems. Thus if we require an accuracy of the \( \Delta \phi \) measurement of \( \pm 0.1 \) mb the pressure must be recorded on the two sides of the mountain within a few minutes of each other. While there is no fundamental difficulty in obtaining this level of temporal accuracy, the simple drum recording type of microbarograph used here did not have this kind of accuracy or resolution in time when weekly recording charts were used. This problem was clearly the principal cause of the errors in the present data. In future experiments it is likely that better clock calibration and shorter recording periods could largely overcome this problem. In a more sophisticated (and expensive) project, digital data recording with quartz crystal timekeeping or telemetry to a single recording station, the simultaneity problem could be completely overcome.

c. Reduction to a standard level

In the evaluation of (1) it is assumed that the pressures on each side of the mountain are measured at the same elevation. Even in a quiescent homogeneous atmosphere, microbarographs at different elevations would measure different pressures due to the hydrostatic pressure gradient but this would not suggest the existence of drag. The height of each instrument must be known to within 1 m if a reduction correction is to be made within \( \pm 0.1 \) mb = 10 Pa \( (\delta \phi = -\rho g \delta z) \). Geodetic leveling, using surveying techniques or satellite ranging, is possible to this accuracy (and much better) but is extremely expensive and time consuming. Within the scope of this study it is necessary to consider an \textit{in situ} calibration to determine the zero point for the determination of \( \Delta \phi \).

4. The \textit{in situ} calibration

In order to interpret the pressure difference between two instruments at different heights it is necessary to assume that under calm conditions there will be no
pressure difference and no mountain drag. Calm conditions will be defined so that the component of the wind perpendicular to the ridge, measured at mountain top level (300 m above the surrounding country), is less than 3 m s$^{-1}$. The following factors could make this assumption invalid:

1) DYNAMIC EFFECTS

Pressure differences ($\Delta \rho$) associated with inertial (Bernoulli) effects should be the order of $q=\frac{1}{2} \rho U^3$, the dynamic pressure. At $U=3$ m s$^{-1}$, $q=5$ Pa which is within the allowable inaccuracy of $\pm 0.1$ mb.

2) CORIOLIS EFFECTS

A strong geostrophically balanced wind blowing along the mountain ridge will have a pressure gradient associated with it of

$$\rho \frac{\partial}{\partial x} U = \nabla \rho.$$

Microbarographs placed within 4 km will not record a pressure difference exceeding 10 Pa unless the wind-speed exceeds 25 m s$^{-1}$. Winds exceeding this value are infrequent in the low troposphere.

3) DENSITY EFFECTS

When there is no horizontal pressure gradient above the ridge top and no air motion there still may be a $\Delta \rho$ and mountain drag if the air temperature below ridge top is different on the two sides of the ridge. Thus from the hydrostatic law we have $\Delta \rho \approx \frac{\partial}{\partial y} (\Delta T/T)H/2$ for the pressure difference halfway up the ridge; here $\rho$, $\Delta T$, $T$ and $H$ are the average density, temperature difference, average temperature and mountain height, respectively. Taking $T=300$ K, $H=300$ m, $\Delta \rho=0.1$ mb gives $\Delta T \approx 2^\circ C$. Thus, if $\Delta T$ exceeds $2^\circ C$, a measurable $\Delta \rho$ may result even with no wind. Such a $\Delta T$ could be produced in the morning or evening by differential heating on the slopes associated with a low sun angle. Still, on the average, during many calm periods there might be no systematic $\Delta T$ and by looking at several calm periods, a zero point for $\Delta \rho$ may be determinable.

The above simple calculation points out how sensitive $\Delta \rho$ is to temperature differences. This effect could, as shown above produce a force on the ridge without requiring flow over the ridge. In other situations the $\Delta T$ could be directly associated with the flow over the ridge. This is, in fact, closely related to the mechanism by which wave drag in a stratified fluid acts. We shall see later that these buoyancy forces must play an important role in producing drag on the Blue Ridge during periods of time when the flow aloft is passing over the ridge.

5. Results of the pressure drag field experiment

The pressure differences computed between three independent pairs of microbarographs (5 and 2, 4 and 3, 6 and 1) for the period 13–18 January 1974 are shown in Fig. 2. The reader should refer back to Fig. 1 and Table 1 to locate microbarographs 1–6 on the mountain. The absolute measurement errors associated with the points on Fig. 2 are difficult to estimate but are probably

![Fig. 2. The pressure difference (mb) across the ridge for three independent pairs of microbarographs, plotted against time (EST) for the period 13–18 January 1974: (a) microbarograph numbers 5 and 2, (b) 4 and 3 (c) 6 and 1. See Fig. 1 for the locations of these microbarographs.](http://journals.ametsoc.org/jas/article-pdf/35/9/1644/3420438/1520-0469(1978)035_1644_amomd_2_0_co_2.pdf)
quite large (e.g., ±0.5 mb). An elaborate system of
clock corrections has been used to try to minimize the
deviations from simultaneity discussed earlier. These
corrections are themselves uncertain, however, and in
the end, confidence in the data can be gained only by
comparing independent data sets as in Fig. 2. No zero
point calibration has been applied to this data so the
absolute values on the ordinate have no meaning.

The primary feature of the data is that the pressure
differences do fluctuate by ±0.5 mb or so and that the
variations are noticeably (but not perfectly) coherent
between the three pairs of sensors. The Δρ vs t time
series in Fig. 2 are not really long enough to allow any
definite statistical conclusions but the apparent simi-
larity between the three independent data sets suggests
that the data may be a real measure of a mean pressure
difference across the mountain, not simply instrumental
error or local pressure variations caused, for example,
by boundary layer turbulence. Note also, that the
Δρ variations often persist in time for several hours,
and the magnitude of the Δρ variations are largest near
the bottom of the mountain—smallest for the pairs of
microbarographs located near the mountain top.

The obvious question is, are the variations in Δρ
and the associated pressure drag correlated with the
wind blowing over the mountain? Figs. 3 and 4 show
the comparison between Δρ and wind for the periods
5–10 January and 13–18 January, respectively. The
data in Fig. 4a are the same as those in Fig. 2a. Corrobo-
rating Δρ data for the 5–10 January period similar to
those in Figs. 2b and 2c, are not available due to
instrument malfunction. The winds at mountain top
level (300 m above ground) were determined from the

regularly scheduled National Weather Service rain-
sonde releases at Sterling, VA, 35 km east southeast
from the instrumented part of the Blue Ridge. This
station is far enough away to be reasonably unaffected
by the mountain but close enough to give a fair represen-
tation of the relevant synoptic flow (except during
frontal, squall line or severe convection conditions).
The component of the mountain top wind which is
perpendicular to the ridge (Uv) is shown in Figs. 3b
and 4b. The 12 h sampling rate (0000 and 1200 GMT)
is too infrequent to fully resolve the temporal variations
in the flow, especially during the passage of a front.
The smooth curve through the points in Figs. 3b and 4b
is therefore drawn only as an aid to the eye, not as a
quantitative interpolation. Additional information on
the weather patterns in the Appalachian region during
this period can be found in the Daily Weather Maps
(available from the Department of Commerce, Wash-
ington, DC) and in the rawinsonde data stored at the
National Climatic Center in Asheville, NC.

Comparison of Δρ and U (Figs. 3a, 3b and 4a, 4b)
shows at once a strong correlation and in the expected
sense. For example on 13 January with the wind from
the west the pressure ρw on the east side (lee side) is
relatively low compared to ρw on the west (windward)
side. This pattern reverses on the 14th. It is now
possible to qualitatively apply the in situ calibration
discussed earlier. Comparing Figs. 3a and 3b, a choice
of Δρw = 14.9 mb ± 0.2 mb would be reasonable, while
for Figs. 4a and 4b perhaps Δρw = 15.0 mb ± 0.2 mb is
better. We may now compute Δρc = Δρ − Δρw the

Fig. 3. A comparison of the differential pressure (mb) with the
component of the wind (m s−1) perpendicular to the mountain for
the period 5–10 January 1974: (a) pressure difference between
micorbarograph numbers 6 and 1, (b) wind at 300 m above the
ground at Sterling, VA, (c) surface wind at site 1 west of the
ridge, and (d) surface wind at site 6 on the eastern slope of the ridge.

Fig. 4. As in Fig. 3 except for 13–18 January 1974: (a) pressure
difference between microbarograph numbers 5 and 2, (b) wind
at 300 m above the ground at Sterling, VA, (c) surface winds at
site 1 west of the ridge, and (d) surface wind at site 6 on the
eastern slope of the ridge.
pressure difference corrected for differing heights of the instruments. This quantity can be used in (1) to compute the mountain pressure drag. Qualitatively \( D/l = H \Delta \rho_a \), where \( H \) is the mountain height and \( \Delta \rho_a \) is the average pressure difference across the mountain. Choosing a value of \( \Delta \rho_a = 0.5 \) mb = 50 Pa from Figs. 3a and 4a, and with \( H = 300 \) m, gives

\[ D/l = 15 \times 10^3 \text{ N m}^{-1} \]

of ridge (about equal to the weight of 2 tons). The relative importance of this drag can be illustrated by noting that the average skin friction around the globe in these latitudes (\( \sim 40^\circ \text{N} \)) and in this winter season is approximately \( \tau = 0.2 \) N m\(^{-2} \). The ridge pressure drag is then equivalent to the skin friction drag on 75 km of the earth's surface. Since the ridgetop in the Appalachian region are on the average more closely spaced than this, we conclude that the mountain drag is at least of equal importance to the skin friction drag in this area some of the time. There is of course no information about whether these two types of drag are correlated, perhaps inversely, leading to a partial compensation.

Probably the most useful evidence available in the present study regarding the mechanism of mountain drag are the records of surface wind at sites 1 and 6. These records are shown in Figs. 3c, 3d, 4c and 4d. Each point in these figures represents the component of the hourly averaged wind perpendicular to the ridge. In the design of the experiment it was expected that the wind at site 1, located some 5.7 km west of the mountain, would to a large degree reflect the undisturbed flow aloft both in direction and in relative magnitude. Note, however, that the wind at site 6 on the eastern slope of the ridge (Figs 3d and 4d) follows much more closely the wind aloft (Figs. 3b and 4b). On several occasions when the wind is from the west the wind at site 1 seems to stop for a period of several hours. These events are marked with a capital B on Figs. 3c and 4c. Probably the most marked of these was B6 during the morning of the 16th but events B2, B4 and B5 are also particularly noticeable. These blocking events (as we will call them) seem to occur preferentially during the early morning hours. The correspondence between the blocking events and the drag events during the periods of westerly flow aloft is especially striking. Unfortunately, the uncertainty in the \( \Delta \rho \) measurement and the shortness of the record prevent us from seeing how good the correlation really is. We can go so far as to say that the data are consistent with the idea that each period of drag occurs during one of the blocking events (B1-B6).

During the two weeks of observations, shorter periods of blocking were also present. These were too short (30–90 min) to show up clearly in Figs. 3 and 4 and too short for the mountain drag to be determined at all. However, a description of the changes in surface winds will further illustrate the blocking phenomena.

**Fig. 5. Schematic illustration of the surface winds at sites 1 and 6 during the evening of 15 January 1974: (a) unblocked conditions, (b) blocked conditions.**

The most striking example occurred during the evening of 15 January. Over a period of 10 h the flow alternated between blocked and unblocked, undergoing three complete cycles. The winds during each type of flow is shown in Fig. 5; and a quantitative summary of the pattern at sites 1 and 6 is shown below:

<table>
<thead>
<tr>
<th>Site</th>
<th>Block</th>
<th>Wind Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Blocked</td>
<td>SSW 0.5 m s(^{-1})</td>
</tr>
<tr>
<td>6</td>
<td>Unblocked</td>
<td>WNW 3 m s(^{-1})</td>
</tr>
</tbody>
</table>

Each condition remained for an hour or two while the change between blocked and unblocked flow took less than 10 min. The alternation between the blocked and unblocked condition did not appear spontaneously but seemed to be associated with the passage of a quasi-periodic mesoscale system (perhaps internal gravity waves or an organized convective system) producing variations of surface pressure of \( \pm 1 \) mb.

### 6. The mechanism of drag

In the previous sections we have described the pressure drag and the surface wind fields around the mountain, insofar as the available instrumentation could resolve it. In this section a more deductive approach will be used to try to determine the flow mechanism responsible for the mountain drag. Five possibilities will be considered (see Fig. 6):

- (a) Flow separation—buoyancy forces are negligible.
- (b) Wave drag from vertically propagating internal gravity waves.
- (c) Wave drag from trapped (resonant) internal gravity waves.
- (d) Upstream blocking of cold surface air.
- (e) Regional trapping of cold surface air between mountains.

These mechanisms are physically distinct but of course combinations could also occur. The fact that such a wide range of possibilities must be considered is an indication of how little is known about aerodynamics on this scale.
where $h$, $L$ and $U$ are the obstacle height, the total width and the wind speed away from the obstacle at the height $z=h$. Fitting a cosine to the Blue Ridge would give $h=300$ m, $L=3$ km, so that $h/L=0.1$ and $C_D=0.15$.

To determine an approximate upper limit on the drag produced by separation (i.e., form drag), a wind tunnel experiment was performed on an obstacle with a slightly exaggerated $h/L=0.13$ and of triangular shape to increase separation. The obstacle was placed on the floor of the tunnel several meters from the entrance region thus allowing a thick turbulent boundary layer to form upstream of the "mountain." The structure of the incoming boundary layer and the pressure distribution on the obstacle were determined by probing the flow with a small pitot-static tube. The results are shown in Fig. 7. The ratio of the boundary layer thickness to obstacle height in the experiment is roughly what one would find in the atmosphere for a 300 m high mountain; thus the laboratory model is at least crudely similar to an atmospheric flow without buoyancy forces. The drag coefficient computed from the pressure distribution using Eq. (1) is $C_D=0.25\pm0.05$ based on the free-stream velocity but is $0.34\pm0.01$ based on the velocity at $z=h$ upstream.

The approximate drag coefficient for the real Blue Ridge can be computed from the data in Figs. 3 and 4. Choosing $\Delta p=50$ Pa and $U=10$ m s$^{-1}$ gives $C_D=0.8$.

To summarize:

- Empirical formula of Hood (1939) $C_D=0.15$
- Wind tunnel measurements on exaggerated model $C_D=0.37\pm0.07$
- Direct measurement of Blue Ridge drag $C_D=0.8$.

It seems then that the measured drag on the Blue Ridge is significantly larger than could be produced by the separation mechanism alone acting on such a gentle obstacle. In addition, the field evidence during periods of drag that 1) stable nighttime conditions are most common, 2) the flow upstream is blocked, and 3) the lee side flow is strong and smooth is hardly in accord with the flow shown in Fig. 6a.

b. Wave drag from vertically propagating internal gravity wave

The generation of vertically propagating internal gravity waves can be an important drag mechanism. The fluid inertia is no longer the only cause of pressure differences as buoyancy forces become important. For small mountains (so that linear theory is applicable) it can be shown that because the vertical transport of momentum by wave motion is proportional to the square of the wave amplitude, the wave drag is proportional to the square of the mountain height. Furthermore, it can be shown that mountains with small horizontal dimensions (e.g., 1 km) produce disturbances
with an intrinsic frequency (i.e., a frequency based on the time it takes for a fluid particle to pass through the disturbance) greater than the buoyancy frequency

\[ N = \left[ \frac{g}{\theta} \frac{d\theta}{dz} \right] . \]

Since these disturbances cannot propagate, they produce no drag. Broad mountains (i.e., \( L \approx 50 \) km) are very efficient wave generators as most of the disturbances they produce have intrinsic frequencies substantially smaller than \( N \). These two effects together mean that the drag from this mechanism will depend strongly on the size of the mountain.

Using the results from Miles (1969), a smooth Witch-of-Anges shaped mountain (see Fig. 1) which is made very broad will have a drag coefficient

\[ C_D = \frac{D/l}{\frac{1}{2} \rho U^2 h} = 0.5 \pi (Nh/U). \]

The Blue Ridge however is rather narrow. The halfwidth factor \( b \) in the Witch-of-Anges formula is best chosen to be about 900 m, which results in a non-dimensional width of \( bN/U = 0.9 \) (using \( N = 0.01 \) s\(^{-1}\) and \( U = 10 \) m s\(^{-1}\)). A mountain this narrow has only half the drag coefficient of a broad mountain of the same height. Then we can estimate that

\[ C_D \approx 0.24. \]

Like that for the separation mechanism, this computed drag coefficient is too small to account for the measured drag.

The surface wind observations are not inconsistent with this type of wave drag. The simple models which have been developed for stratified flow over a bump show weak winds upstream of the bump, with strong winds on the lee side. The complete blocking of the upstream surface flow would not, however, be predicted by the linear models for such a small mountain as the Blue Ridge. One could reason that the stong upstream blocking was caused by the inability of the cold dense air near the ground to climb over the mountain. The slowing of the upstream air caused by the wave generation could accentuate this effect. The stalled air upstream of the mountain could, in turn, increase the effective width of the mountain, improving its efficiency as a wave generator and increasing its drag.

The computation of the drag coefficient associated with the generation of waves is quite strongly dependent on the value of \( N \). If the static stability in the lower troposphere is larger than \( N = 0.01 \) s\(^{-1}\), the estimated drag coefficient could be significantly larger due to the addition of some higher frequency waves to the propagation spectrum. In many cases these higher frequency waves cannot propagate up into the middle and upper troposphere and are reflected, giving rise to trapped (or resonant) lee waves. This is the topic of the next section.

c. Wave drag from trapped (resonant) internal gravity waves

The mechanism of drag associated with the generation of trapped waves is similar to that in the preceding discussion except that these shorter waves propagate horizontally along layers of large static stability and low wind speed in the lower troposphere. Trapped waves are responsible for the long trains of lee wave clouds seen so frequently on satellite photographs in
many parts of the world. This region of the central Appalachians is particularly well known for the frequent occurrence of lee wave clouds. In a companion study (Smith, 1976) the waves produced by this same section of the Blue Ridge were studied. According to linear theory (see Smith, 1976) the lee wave drag for a Witch-of-Agnesi shaped ridge is

$$D/l = \frac{2\pi^2 \rho U_0^2 (\frac{3}{4} H b e^{-kb})^2}{\bar{z}^4},$$

where $\rho$, $U_0$, $H$ and $b$ are the mean air density, wind speed near the ground, mountain height and half-width, respectively. The values of $k$ and $\bar{z}$ describe horizontal and vertical scale of the trapped lee wave. The wave number $k = (2\pi/\text{wavelength})$, while $\bar{z}$ is a crude measure of the vertical distance between the ground and the height where the wave energy is concentrated, typically 0.5–5 km. Both of these values can be determined theoretically from the solution to the trapped wave eigenvalue problem. Trying the values of $\rho = 1.2$, $U_0 = 10$, $H = 300$, $b = 900$, $k = 2\pi/10^4$, $\bar{z} = 1000$ (all in SI units), we have

$$D/l = 14,000 \text{ N m}^{-1}$$

or

$$C_D = \frac{D/l}{\frac{1}{2} \rho U_0^2} = 0.78,$$

close to the measured value. According to this theory there is a strong inverse dependence of drag on $\bar{z}$. If the stable layer along which these waves propagate is high above the ground the waves excited by the mountain will be of small amplitude and wave drag negligible. These theoretical estimates should not be relied on however. Aircraft measurements (Smith, 1976) under conditions leading to a predicted $\bar{z} = 3000$ showed wave amplitudes of vertical motion of 2.8 m s$^{-1}$ instead of the predicted 0.7 m s$^{-1}$. Because of the large value of $\bar{z}$ the predicted wave drag was only about 700 N m$^{-1}$, while the momentum carried away by the measured waves (proportional to the amplitude squared) is $(2.8/0.7)^2 = 16$ times larger or about 11,000 N m$^{-1}$. A similar discrepancy has been found in laboratory experiments for certain ranges of the parameters and has been shown to be caused by nonlinear effects.

The frequent occurrence of lee wave clouds in the area and the aircraft measurements of wave amplitudes large enough to explain the drag measured in this study make the lee wave mechanism a viable possibility. Like the generation of vertically propagating waves, this mechanism would be associated with 1) stratified conditions, 2) slow flow on the windward side, and 3) strong flow on the lee side, and this is consistent with the observations but there is still no direct evidence pointing to this mechanism.

d. Upstream blocking of cold surface air

The blocking of cold surface air is likely if the air has less kinetic energy than the potential energy barrier presented by the mountain. A simple way to estimate the drag which could be produced by such a situation is to assume the air upstream is blocked up to mountain top level and that this air has a temperature $\Delta T$ lower than the surrounding air. If the flow in the blocked region is hydrostatic and if the Bernoulli function is constant along the streamline which passes over the blocked region, the pressure difference halfway up the mountain is approximately

$$\Delta P = \frac{(gH/2)\bar{z}(\Delta T/T)}{\frac{1}{2} \rho (U_1^2 - U_H^2)},$$

where $U_H$ and $U_1$ are the wind speeds at mountain top level upstream and on the lee side of the mountain. The first term alone, if $\Delta T$ is taken as 6°C, contributes a $\Delta P$ of about 30 Pa or 0.3 mb. If the air above is unstratified, so that no waves are being produced, the speed on the lee side would probably be less than upstream due to the increase in stream tube area and the second term could partially offset the first. Our choice for the depth of the blocked region (300 m) and the temperature difference ($\Delta T = 6^\circ$C) are probably over-estimates and if indeed the velocity slows on the lee side then the blocking phenomena is not adequate to explain the observed pressure difference. If at the same time waves are being generated, either vertically propagating or trapped, the velocity on the lee side could be larger than upstream and the two mechanisms could cooperate.

The surface observations are certainly in agreement with the idea of blocking. The drag seems to increase during periods of blocking and those occur primarily at night during periods of strong static stability near the surface. The measured temperatures at sites 1 and 6 near the Blue Ridge also support this idea for in spite of the scatter in the data the upstream temperature is commonly 2–4°C lower than the lee side temperature at site 6. Considering that site 6 is 100 m higher than site 1 the difference in potential temperature becomes 3–5°C. Still, nothing is known of the depth of the blocked air and in light of the above calculation it seems unlikely that the drag can be accounted for on this basis alone. It seems more likely that the stagnation of the surface flow upstream is more or less a passive symptom of the real drag mechanism. Either mechanism of wave generation would produce a slowing of the flow upstream and an adverse (i.e., positive) pressure gradient there. Even in a homogeneous fluid an adverse pressure gradient can cause stagnation of the slow flow near the bottom of the boundary layer. With a layer of cold dense air present near the surface, the adverse pressure gradient would make it even more difficult for that air to climb over the mountain.
e. Regional trapping at cold surface air between mountains

In the previous discussion of blocking it has been tacitly assumed that the stagnation measured at site 1 is a local feature of the flow over the Blue Ridge even though that site is located almost 6 km away. This is almost 20 times the mountain height. If the blocking is present this far from the mountain, might it not be regional in character? It is known that over uniform terrain the presence of strong static stability tends to decouple the surface flow from the flow aloft. The surface flow (in a crude sense) reaches a local force balance between friction at the ground and the synoptic-scale horizontal pressure gradient. The surface air then moves rather slowly at a large angle (\(\sim 90^\circ\)) to the geostrophic flow aloft. In rougher terrain, where the skin friction is greater, the surface winds are slower still and in regions of closely spaced hills the wind in the valleys blows parallel to the valleys or not at all. The component of the pressure gradient perpendicular to the ridges does nothing but bank the cold surface air up against the ridges. If the ridges are close together the banking is slight and the increased hydrostatic pressure associated with the pool of cold air acts equally on both sides of the valley.

In regard to the flow near the ridge, this mechanism is indistinguishable from the blocking discussed in the previous section and the conclusion that the mechanism is probably inadequate to totally explain the measured drag, applies here as well. The primary differences between the two descriptions are 1) that the blocked region extends farther from the mountain, 2) the strong lee side flow is interpreted as air which has come down from aloft rather than surface air which has temporarily lifted to pass over the mountain, and 3) the flow problem is looked at in the context of a stably stratified boundary layer over rough terrain.

This completes the discussion of the five drag mechanisms shown in Fig. 6. It has not been possible to categorically exclude any of the five possibilities but the generation of internal gravity waves seems at this stage to be the most likely candidate. Clearly, more work is needed both to gain a better climatological view of the magnitude of mountain drag and to understand the drag mechanism and the associated effects of the mountain on the local winds.

7. Conclusions

1) The measurement of the pressure difference across a mountain is possible even on a relatively small mountain. Further development of instruments and techniques could substantially reduce the measurement errors.

2) Pressure differences of from 0.2 to 0.5 mb (20-50 Pa) are not uncommon on the Blue Ridge in the central Appalachians.

3) The pressure differences across the mountain correlate reasonably well with the component of the environmental wind, measured at mountain top level, which is perpendicular to the ridge. Pressures on the windward side are higher than on the leeward side leading to drag in the expected sense.

4) During the periods of large measured pressure difference, the corresponding drag is the order of 10,000 N m\(^{-1}\) of ridge. This is roughly the frictional force that would act on 50-100 km of open country. Thus, if a similar drag acts on the other Appalachian ridges, the total mountain drag must be as important as the skin friction in this region.

5) The phenomenon of flow separation, which is associated with most of the drag on smaller objects, cannot account for the relatively large drag found on the rather gentle Blue Ridge. Furthermore, the observed surface winds are inconsistent with this type of flow. Thus, even though the Blue Ridge is small as mountains go (height 300 m, width \(\sim 2\) km) the density stratification of the flow seems to be largely responsible for the drag.

6) There seems to be some correspondence between periods of drag and periods when the flow upstream on the surface is blocked, at least for westerly winds. The blocking, which usually occurs in the early morning, is probably not itself responsible for the drag; it may cooperate with some other drag mechanisms or simply be a passive symptom of the active drag mechanism. The generation of internal gravity waves seems to be a possible explanation for the observed drag but there is little evidence to either prove or disprove it.

7) Subsequent attempts at mountain drag measurement should ensure simultaneity of measurement within 10 min or so, the use of microbarographs with an absolute accuracy of about 0.1 mb (10 Pa) and concurrent wind measurements to allow an \textit{in situ} calibration. Further understanding of the drag mechanism will probably require surface measurements of wind and temperature at (say) four points on the surface and some type of boundary layer profiling (e.g., pibal, tethered balloon, etc.) along with the pressure measurements. Eventually cooperative aircraft measurements could prove valuable but the difficulty in properly timing such flights is considerable.

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