A Wind Tunnel Investigation of the Growth of Graupel 
Initiated from Frozen Drops

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ABSTRACT

The formation of graupel by the riming of freely suspended frozen drops has been studied in the UCLA Cloud Tunnel. The following parameters were varied: 1) initial frozen drop size, 2) ambient temperature, 3) liquid water content, 4) growth time and 5) cloud droplet size. The various combinations yielded a total of 16 separate growth environments each consisting of approximately 10 individual growth events. In all of the growth environments, heat dissipation was efficient enough so that all the water acquired by the growing ice particle immediately turned into ice (“dry growth” regime). From the observed growth rates of the ice particles, collection kernels were calculated. These kernels are significantly different from those of drops colliding with drops or those derived from the theoretically determined efficiency with which plate-like and columnar ice crystals collide with drops. In addition, the densities of rimed acquired by the frozen drops during their transition to graupel were measured and found to be somewhat higher than expected from the earlier work of Macklin (1962).

1. Introduction

One of the fundamental precipitation formation mechanisms active in the earth’s atmosphere is the growth of ice particles by the accretion and subsequent freezing of supercooled water drops. Through this process, called riming, ice crystals or frozen drops evolve into distinct entities known as graupel. The growth of graupel is an extremely important stage in the evolution of precipitation in supercooled clouds. Not only have these particles often been found to serve as hailstone embryos (List, 1958; Knight and Knight, 1970; among others) but there is also evidence to indicate they play a dominant role in precipitation development within many convective clouds (Knight et al., 1974; Cannon et al., 1974; Dye et al., 1974). Furthermore, hail suppression and rainfall enhancement efforts which involve the injection of ice nuclei into supercooled clouds undoubtedly create large numbers of riming ice particles.

Unfortunately, little quantitative knowledge is available on the riming growth of ice particles. Despite the absence of such knowledge, cloud modelers have found it necessary to include ice-phase accretional growth in their models in order to simulate the atmosphere more realistically. Some investigators have specified their ice particles to grow by geometric sweepout (see, e.g., Dennis and Musil, 1973; Orville and Kopp, 1977).

Others such as English (1973) have modified the use of geometric sweepout during the latter stage of growth with the results of Macklin and Bailey (1968), who experimentally studied the collection efficiencies of freely falling hailstones. The use of geometric sweepout during the prehailstone growth stage in both of the above approaches is very questionable since it has long been known that ignoring the hydrodynamic flow field around a falling particle leads to larger collection efficiencies than are actually observed. Ludlam (1958), Braham (1968) and Danielsen et al. (1972), among others, have used a collection kernel for ice particles (assumed spherical) which was similar to that of water drops of the same size. However, wind tunnel studies of Pitter and Pruppacher (1973), Pruppacher and Schlamp (1975) and Pflaum et al. (1978) suggest that the efficiency with which frozen drops collide with supercooled water drops cannot a priori be assumed similar to the efficiency with which two liquid water drops collide, because of their observations that frozen drops exhibit various spinning and helical motions which disturb the collision-determining flow field surrounding the frozen drop compared to that of the idealized sphere. Still other investigators (Takahashi, 1976; Beheng, 1978) have incorporated the numerical solutions of Pitter and Pruppacher (1974) who determined the initial efficiency with which supercooled water drops collide with plate-like crystals. While these results are applicable for the initial stage of riming, during which the particle’s density, shape and Reynolds number remain relatively unchanged, extension to even...
moderately rimed crystals is questionable. Furthermore, as pointed out by PfLaum et al. (1978), growing graupel exhibit considerable surface roughness coupled with a variety of spinning, helical and oscillatory motions. To incorporate such motions in theoretical models, using solutions to the Navier-Stokes equation of motion to determine the hydrodynamic and collision behavior of densely rimed ice crystals and graupel particles, will prove very difficult if not impossible. Therefore, it is necessary to resort to experiments.

In the past, although much work has been done on various aspects of the “hail-stage” of riming growth [for a review see Mason (1971), Gokhale (1975) or Foote and Knight (1977)], experimental investigations of graupel growth rates have been essentially non-existent. In an attempt to fill this void, we performed a series of experiments investigating the growth rates of riming ice particles inside the UCLA Cloud Tunnel.

2. Experimental setup and procedure

The present study was carried out by means of the UCLA Cloud Tunnel described in detail by Pruppacher and Neuburger (1968), Beard and Pruppacher (1969), and in its modification for suspending drops and ice particles in the air stream at temperatures below 0°C by Pitter and Pruppacher (1973) and Pitter (1973). In order to adapt the tunnel to the needs of the present experiment, some minor modifications of the previous tunnel setup were needed (for details, see PfLaum, 1978).

The cloud of supercooled droplets injected into the tunnel air stream was produced by a steam condensation method. The liquid water content of the cloud was inferred from the measured dew point of air containing the moisture of the evaporated cloud. Hereafter we refer to this as the absolute dew point. In this study of the riming growth of frozen drops, the drops were injected into the tunnel with a hypodermic needle carrying an insulated metal screen hood which allowed exposing the detachable water drops to an electric potential in order to counteract drop charging by contact potentials and shearing. With the use of an electrometer amplifier it was insured that drop charges were less than $10^{-6}$ esu. In addition, as was established by Beard (1970) and is consistent with condensational growth theory, the droplets of the steam cloud also had charges less than $10^{-6}$ esu. From the theoretical studies of Schlamp et al. (1976), one can readily infer that the accretional growth of ice particles was not affected by the remaining electric charge.

A typical experiment proceeded as follows. The airstream was stabilized at a desired temperature and steam was allowed to enter the humidifying section of the tunnel well upstream of the observation section. This provided ample time for the cloud to reach thermal and vapor equilibrium with the tunnel air. The initial temperature and absolute dew point of the air, continuously monitored by a chart recorder, were noted at this juncture and a sample of the cloud droplet spectrum was taken. A water drop was then injected into the wind tunnel and quickly stabilized at terminal velocity by adjusting the air speed. From this air speed and the drag-size relationships of Beard (1976), the initial mass of the drop was identified. A few seconds later, as the drop approached thermal equilibrium, the few impurities in the drop initiated freezing and the timer was started. At warmer growth temperatures, freezing was initiated by means of contact nucleation with clay particles injected upstream of the drop. Once frozen, the riming particle was prevented from colliding with the surrounding walls by manipulating the direction of the airstream by means of an adjustable inner tunnel [for details see Pitter (1973) and PfLaum (1978)]. Thus, at all times during its growth, the particle remained freely floating in the airstream. The airstream velocity was continuously monitored by a chart recorder. At the end of the desired growth time, the final temperature and absolute dew point of the air were noted. The particle was then removed from the tunnel by means of a specially constructed suction-operated collection apparatus which allowed the graupel to settle into a cup filled with a chilled preserving fluid. At no time during the capture process did the rimed particle come into contact with the walls of the setup ducting the graupel particle into the collection cup. This eliminated the possibility of mass loss due to collisional fragmentation. Additionally, there were never any fragments of ice found in the collection cup, thus assuring that the settling impact of the graupel particles on the surface of the preserving fluid was low enough to prevent fragmentation. The preserving fluid used in the collection cup consisted of a mixture of xylene and diethylphthalate.

After capture the collection cup was quickly brought into a walk-in cold chamber where the graupel particle was photographed for later volume determination. The particle was then transferred to a warm solution of xylene and diethylphthalate wherein it quickly melted into a liquid sphere. This solution was characterized by a gentle density gradient from top to bottom which allowed the liquid drop to float relatively undistorted in an easily photograpahable position. The final mass was determined from these photographs.

Graupel grown from frozen drops were studied under a variety of growth conditions. The following parameters were varied: 1) initial frozen drop size, 2) ambient temperature, 3) cloud liquid water content, 4) growth time and 5) cloud droplet size. The various combinations yielded a total of 16 separate growth environments, each being utilized for approximately 10 individual growth events. The growth environments are given in Table 1.

As previously mentioned, the supercooled cloud was generated by a steam condensation method, the hardware for which is a semi-permanent part of the UCLA Cloud Tunnel. As presently constructed, this method of
Table 1. Specification of the 16 graupel growth environments used in present experiment.

<table>
<thead>
<tr>
<th>Growth environment</th>
<th>Average initial drop radius (cm×10^4)</th>
<th>Average ambient temperature (°C)</th>
<th>Average liquid water content (kg m^-3×10^3)</th>
<th>Average time of growth (s)</th>
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<tbody>
<tr>
<td>1</td>
<td>312</td>
<td>-15.1</td>
<td>1.7</td>
<td>70</td>
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<td>2</td>
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<td>4</td>
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<td>1.0</td>
<td>65</td>
</tr>
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<td>308</td>
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<td>2.3</td>
<td>70</td>
</tr>
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<td>6</td>
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<tr>
<td>7</td>
<td>318</td>
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<td>120</td>
</tr>
<tr>
<td>8</td>
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<tr>
<td>16</td>
<td>156</td>
<td>-3.0</td>
<td>0.7</td>
<td>189</td>
</tr>
</tbody>
</table>

Fig. 2. As in Fig. 1 except for frozen drops of 150 μm radius.

cloud injection does not allow for detailed control over cloud droplet sizes. However, for similar airstream velocities, the size spectra were quite consistent from one experiment to the next. Figs. 1 and 2 represent the overall average cloud droplet size spectra for the 300 and 150 μm frozen drop growth studies, respectively. These distributions were determined by the rod-impact method described in detail by Beard and Pruppacher (1971a). Liquid water contents calculated from the droplet spectra were in good agreement with measured water contents.

3. Results and discussion

a. Graupel growth rates and collection kernels

The collection kernel is defined by

\[ K = \frac{dM/dt}{W}, \]  

(1)

where \( R \) is the collector radius, \( r \) the cloud droplet radius, \( V \) and \( v \) the fall velocities of the collector and the droplet, respectively, \( E_1 \) the collision efficiency of \( r \)-sized droplets with an \( R \)-sized collector, and \( E_2 \) the efficiency with which two colliding particles remain attached to each other.

For the case of drop-drop collisions, it has been customary to publish values for the collision efficiency of various sized collector drops as a function of the collected droplet radius. Collection kernels were then calculated from the collision efficiencies. In the present experiment, collection kernels were directly determined from

\[ \frac{dM/dt}{W} = \frac{\text{final mass} - \text{initial mass}}{\text{growth time}}, \]

The initial and final masses were determined as discussed in Section 2. The time of growth was directly measured. The liquid water content \( W \) of the air was determined as discussed in Section 2. During any particular experimental run, the liquid water content varied inversely with air speed producing deviations of up to 25% from the initial liquid water content. These variations were well documented as a function of time and in all calculations a weighted average of \( W \) was used.

Since the environmental air in which the graupel particles were grown was saturated with respect to water, there existed the possibility of diffusional growth contributing to the accretional accumulation of mass. Estimates of this contribution were obtained from a consideration of the growing equations as outlined in Appendix B. For eight of the growth environments,
diffusional growth was negligible. For two growth environments, diffusion contributed \( \sim 5\% \) of the total growth. Four environments produced a diffusional contribution of \( \sim 10\% \), while two contributed \( \sim 25\% \). Whenever diffusional growth was significant, experimentally derived values of the growth rates were adjusted to represent only the mass acquired by collision. The accuracy of the measured growth rates is discussed in Appendix A.

Following tradition, the experimentally derived kernels were initially plotted against the collector radius. Doing so, it was quickly realized that the radius is a vague predictor for the growth rates of graupel particles. Unlike water drops which have a constant bulk density, graupel exhibit a wide range of densities which allow particles of equal size to be falling at considerably different terminal velocities. Such variations in terminal velocity invariably reflect themselves in the hydrodynamic flow fields surrounding the growing graupel thereby altering their rate of growth. Hence, it was necessary to find another parameter for exhibiting graupel growth rates.

On examining numerous possibilities, the collection kernel was plotted versus the momentum of the collector. Momentum was the best parameter found and was chosen for several reasons:

1) Considering the complexities of shapes and variabilities of densities of the graupel particles, their masses and terminal velocities appeared to be fundamental quantities characterizing their essence. The simplest combination of these two parameters entering the problem is the particle's momentum.

2) Empirical equations for both masses and terminal velocities of naturally occurring graupel are readily available in the literature (Zikmund and Vali, 1972; Locatelli and Hobbs, 1974; Heymsfield, 1978).

3) When theoretical collection kernels (Beard and Grover, 1974) of liquid drops collecting 6 \( \mu \)m droplets \( K_{w,6} \) and 10 \( \mu \)m droplets \( K_{w,10} \) were plotted for comparison, these yielded convenient relationships between the variables (see Fig. 3) which are given by

\[
K_{w,6} = 7.98 (MV)^{0.768},
\]

\[
K_{w,10} = 9.13 (MV)^{0.738}.
\]

It can be seen from Fig. 3 that our experimental data points are quite well-distributed around the theoretical relationships given by (4) and (5).

That this agreement is not just fortuitous can be inferred from the graupel collection efficiencies which ranged from 0.47–1.05. Such numbers assure that the growth process was sampling from a large fraction of the droplet spectra and was not proceeding through a selective capturing of only the largest cloud droplets. Thus, Fig. 3 implies that, regardless of whether the collector is a graupel particle or a water drop, the growth kernel is largely dependent on the collector's momentum and the cloud droplet size. No dependence on the ambient temperature, liquid water content or growth time was found. Hence, relations of the type given by Eqs. (4) and (5) may be used to represent the collection kernel of graupel particles interacting with cloud droplets. Additional relationships for other cloud droplet sizes can be obtained from Beard and Grover (1974). This scheme appears to be satisfactory for graupel up to several millimeters diameter as long as the droplets are small, i.e., as long as \( r/R \leq 0.1 \), a condition which is usually fulfilled during atmospheric graupel growth. Beyond these limits, the subject is open for further in-

![Fig. 3. Collection kernel versus average momentum of collector. \( K_{w,6} \) and \( K_{w,10} \) represent results of Beard and Grover (1974) for water drops collecting 6 and 10 \( \mu \)m radius water droplets, respectively. Dots represent experimental results for drop size distribution given in Fig. 1. Crosses represent experimental results for drop size distribution given in Fig. 2.](image-url)
vestigations. However, it may well be that equations with similar form as (4) and (5) will be applicable here as well.

The results presented above pertain explicitly to graupel initiated from frozen drops. Naturally occurring graupel often form from vapor grown ice crystals as well as frozen drops. While no quantitative work on the riming growth of ice crystals was done in the present investigation, some general recommendations can be made for plate-like crystals from the qualitative observations of Pfaff et al. (1978). In that study, graupel grown from simulated ice plates were found to develop into similarly shaped particles as those initiated from frozen drops. Consequently, after a certain initial stage of growth, plate-initiated graupel would be expected to obey the same growth equations as frozen-drop-initiated graupel. In this regard, riming plates appear to develop into roughly spherical particles without greatly increasing their original plate diameter (e.g., see Pfaff et al., 1978, Fig. 3, 4th sequence).

From consideration of these general characteristics, the authors suggest using the results of Pitter and Pruppacher (1974) for the early stages of riming growth on plates. When enough mass has accumulated such that the plate-like shape has tended toward sphericity, the present results can be applied. The details of this switchover can be left to the ingenuity of the respective modellers. No speculations, based on experimental evidence, can be made regarding columnar shaped ice crystals.

In concluding this section on growth rates, it is important to point out that the compatibility between collection kernels of graupel and water drops, if plotted as a function of their momenta, does not imply that previous computations of growth rates (see Section 1) have been correctly applied to graupel. For example, the present results predict that a 200 μm radius graupel, of bulk density 0.3 g cm⁻³, would have a collection efficiency of ~0.33 for 6 μm radius cloud droplets. In contrast, a 200 μm radius water drop would have a collection efficiency for 6 μm radius cloud droplets of ~0.67 (Beard and Grover, 1974), while a 200 μm radius ice plate would not collect any 6 μm radius cloud droplets (Pitter and Pruppacher, 1974). Efficiencies have been used here to provide a more meaningful comparison with earlier work. It is apparent there exist significant differences between previously estimated growth rates of graupel and the growth rates measured in the present experiment. Such differences undoubtedly have important consequences for numerical simulations of precipitation development within mixed phase clouds.

b. Graupel and rime densities

It has long been recognized that the density of an ice particle plays an important role in determining its in-cloud lifetime and eventual size. The bulk densities of atmospheric graupel particles have been measured by a number of investigators (Nakaya and Terada, 1935; List, 1958; Brahm, 1963; Bashkirova and Peshchina, 1964; Zikmunda and Vaili, 1972; Locatelli and Hobbs, 1974; among others). Reported values range from 0.05×10³ kg m⁻³ (Locatelli and Hobbs, 1974) to 0.89×10³ kg m⁻³ (Braham, 1963). Such large variations reflect the different cloud environments within which graupel particles grow.

In an attempt to quantify the density variation of accreted ice (also called rime) as a function of environmental growth conditions, Macklin (1962) performed a series of icing tunnel experiments using rotating cylinders as his collectors. He found that the density of accreted ice could be expressed as a function of the cloud droplet radius, the impact velocity of the droplets and the surface temperature of the collector. Through the years, Macklin’s results have been used in numerical growth models to specify the density of accreted ice acquired by growing ice hydrometeors. Accordingly, it was of interest for us to compare the results for ice accretion on freely falling frozen drops. Hence, as part of the present growth rate experiments, the densities of the rime ice acquired by the frozen drop embryos were determined for each graupel that was grown.

The density of the accreted ice around a frozen drop embryo was estimated from

\[
\text{rime density} = \frac{\text{final mass} - \text{initial mass}}{\text{final volume} - \text{initial volume}}. \tag{6}
\]

The initial and final masses were determined according to the methods discussed in Section 2. Assuming a density reduction 1.0×10³ to 0.9×10³ kg m⁻³ on freezing, the initial frozen drop radius was calculated from a knowledge of the initial water drop radius according to

\[
\text{initial frozen drop radius} = (0.9)^{-1} \times \text{initial water drop radius}. \tag{7}
\]

The initial volume was calculated from the initial frozen drop radius. The final volume was determined from photographic analysis of the graupel from various views. Typically, four different photographic orientations were documented and analyzed for each particle. An average final graupel radius was determined from these measurements and from this radius the final volume was calculated.

Rime densities were calculated in this manner for 160 graupel particles grown in a variety of environmental conditions. An analysis of the accuracy of the density measurements appears in Appendix A. Following Macklin (1962), the experimentally measured rime densities were plotted in Fig. 4 as a function of cloud droplet median-volume radius \( r \), impact velocity \( V_0 \) and surface temperature \( T_s \). A least-squares fit to the data yielded

\[
\rho_{\text{rime}} = 0.261 \left( \frac{r V_0^{0.28}}{T_s} \right), \tag{8}
\]
of density $\rho$. Impact velocities for intermediate cloud droplet sizes can be obtained through interpolation.

Both surface temperatures and impact velocities were determined for smooth spheres of the same densities and average sizes as those of the graupel particles under consideration. This approximation was necessitated by the unknown combined effects of varying shape and surface roughness. However, in the low Reynolds number regime of this experiment, major effects on the hydrodynamic and heat transfer processes would only occur for extreme cases of surface roughness and distortion of shape (Pruppacher and Klett, 1978). Such extremes were never attained by the graupel particles of the present study and it is the opinion of the authors that the smooth sphere approximation introduces only minor errors to the above calculations.

From an inspection of Fig. 4 it is apparent that the present results lie somewhat above those of Macklin, particularly at the lower densities. Two explanations may be given for this discrepancy. First, because of the various spinning and oscillating motions exhibited by freely falling graupel (Pflaum et al., 1978), the deposition of rime upon such an ice particle proceeds in a different manner than rime deposition on a spatially fixed rotating cylinder. At low values of the parameter $rV_0/T_s$, rime on cylinders is of the "kernel" or "feathery" type. This means there are regions of higher density rime interspersed with relatively large voids. Such conditions did not develop to any great extent upon the graupel. Consequently, since Macklin measured the mean density over the whole cylinder, it is not surprising that his measurements were somewhat lower than those obtained in the present experiment. Second, different
cloud-producing mechanisms were employed by each investigator. Macklin generated his cloud through the use of sprayers and it is well known (see, e.g., Iribarne, 1972) that such a mechanical disruption of the water substance results in electrical charging. Since it appears from Macklin’s paper that no precautions to prevent charging were taken, it seems probable that Macklin’s cloud was significantly charged. The significant effects of electrical charging on the collision process have been demonstrated by the theoretical model computations of Schlamp et al. (1976), among others. Additional evidence comes from the early stages of the present experiment when ultrasonic nebulizers were used to generate cloud. Measurements indicated that cloud produced by such nebulizers carried a rather high negative charge which significantly lowered both growth rates and rime densities of the growing graupel. Therefore, the nebulizers were subsequently abandoned in favor of the steam condensational method to avoid droplet charging. In view of this and previously discussed precautions, it is expected that the density values of the present experiment more accurately reflect those occurring during accretional growth which is relatively unaffected by electrical forces.

Several additional remarks are worth mentioning in regard to our rime density measurements. First, all growth events in the present experiments were carried out at relatively low impact velocities (<2.5 m s⁻¹). It is not known whether the new empirical relationship given by Eq. (8) would also be valid for higher impact velocities.

Second, a linear relationship was fit to the data in Fig. 4 following Macklin. However, an inspection of the data suggests that the rime density does not continue to decrease with decreasing abscissa values but levels off somewhere between 0.20x10⁸ and 0.15x10⁸ kg m⁻³. Such behavior is supported by the experimental findings of Buser and Aufdermaur (1973) who obtained a value of 0.17x10⁸ kg m⁻³ as the lowest possible density of rime for simulated random attachment of droplets to an ice particle. To obtain lower rime densities, it would appear necessary for electrical forces to intervene.

Finally, Macklin’s empirical relationship of Fig. 4 was limited to situations where the temperature of the accreting surface was colder than −5°C. For direct comparison with Macklin, the data plotted in Fig. 4 were also limited to surface temperatures colder than −5°C. However, the present experiments contained a number of cases with surface temperatures warmer than −5°C and these density measurements appear in Fig. 5 along with the best fit empirical relationship. Although no quantitative results were presented for surface temperatures warmer than −5°C, Macklin found that the density not only depended on droplet radius, impact velocity and surface temperature, but in a complicated fashion on the air temperature as well. His research, which was specifically designed to investigate rime densities, examined a greater variety of growth environments than the present work. Accordingly, while applicable for the specific growth conditions listed in Table 1, the empirical relationship of Fig. 5 cannot

\[
\rho = \text{Exp} \left[ -2.335 \left( -\frac{rV_o}{T_s} \right) + 0.479 \left( -\frac{rV_o}{T_s} \right)^2 - 0.0329 \left( -\frac{rV_o}{T_s} \right)^3 + 4.027 \right]
\]

![Fig. 5. Rime density versus predicting parameter \( rV_o/T_s \), where the solid line represents best fit to present experimental data for surface temperatures \( T_s \) warmer than −5°C.](image-url)
necessarily be applied to all “warm” accretion situations which might appear to fall under its jurisdiction.

c. Application

The results given above may be used to determine the rate of growth of a graupel by carrying out the following steps:

1) Specify or ascertain initial conditions: ambient temperature $T_a$, ambient pressure $P$, liquid water content $W$, median volume droplet radius $\bar{r}$, graupel radius $R$, graupel density $\rho_g$, graupel drag coefficient $C_D$.\(^3\)

2) Choose (i) or (ii):

(i) Determine initial mass $M$ and terminal velocity $V$ of graupel from available empirical relationships and initial conditions. Velocity adjustment procedure for pressure variations can be found in Pruppacher and Klett (1978).

(ii) Calculate initial mass $M = f(\rho_g, R)$ and terminal velocity $V = f(\rho_g, R, C_D, T_a, P)$ from initial conditions.

3) Let graupel grow for short time increment ($\Delta T$) using $K = f(MV)$.

4) From $K$ and $W$ calculate $dM/dt$ and $dM$.

5) Calculate surface temperature $T_s = f(dM/dt, T_a, W, V)$ from the scheme outlined in Appendix B.

6) Calculate impact velocity $V_0$ of the impinging cloud droplets: $V_0 = f(\rho_g, R, \bar{r}, V)$ from Eqs. (9)- (11).

7) Calculate rime density $\rho_r = f(\bar{r}, V_0, T_s)$ from Eq. (8).

8) Calculate new graupel radius $R_{new} = f(R, \rho_g, dM, \rho_r)$.

9) Calculate new bulk graupel density $\rho_g_{new} = f(M, dM, R_{new})$.

10) Calculate new graupel mass $M_{new} = f(M, dM)$.

11) Update environmental conditions if necessary $(T_a, P, W, \bar{r})$.

12) Calculate new terminal velocity of graupel: $V_{new} = f(\rho_g, R_{new}, C_D, T_a, P)$.\(^4\)

13) Return to step 3, and so on.

14) Terminate after desired time.

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APPENDIX A

Discussion of Experimental Errors

A careful analysis of our experimental errors revealed the following: 1) the temperature of the air in the wind tunnel was conditioned with thermocouples to within $\pm 0.25^\circ$C; 2) a similar error was made in measuring the absolute dew point of the wind tunnel air; 3) the air velocity in the wind tunnel was determined to within $\pm 2$ cm s$^{-1}$; 4) the initial diameter of the suspended drop was measured to $\pm 5$ $\mu$m; 5) a similar error applies to the diameter of the melted graupel; 6) the diameter of the experimentally grown graupel was measured to $\pm 50$ $\mu$m; 7) the diameter of a cloud droplet was measured to $\pm 1$ $\mu$m; and 8) time was determined to $\pm 1$ s. After applying standard error analysis methods, these individual errors combined to an error of $\pm 12.5\%$ in the growth rate $dM/dt$, an error of $\pm 22\%$ in the liquid water content $W$ of the cloud, an error of $\pm 29\%$ in the collection kernel $K$, an error of $\pm 16\%$ in the ice particle momentum $MV$, and errors of $\pm 30\%$ in the rime density, and $\pm 26\%$ in the density predictor $rV_0/T_s$.

APPENDIX B

Surface Temperature and Diffusional Growth Estimation Technique

Following Mason (1971), the heat balance of a spherical ice particle growing by accretion, with allowance for diffusional mass exchange, is formulated for the simplified case of $r<<R$.

The mass of water collected per unit time under steady-state conditions by a spherical ice particle of radius $R$, falling at velocity $V$ relative to the cloud droplets, is $E\pi R^2 W$, where $E$ is the average collection efficiency and $W$ the concentration of liquid water. If the entire mass of collected water is to be frozen completely, the rate of release of latent heat is

\[
\frac{dQ}{dt} = E\pi R^2 V W [L_f + C_w(T_a - T_0) + C_i(T_0 - T_s)],
\]

where $T_0$ is the melting temperature of ice, $T_a$ the ambient air temperature, $T_s$ the surface temperature of the ice particle, $L_f$ the latent heat of fusion, and $C_w, C_i$ the specific heats of water and ice, respectively.

\(^3\) Calculated drag coefficients for the experimentally grown graupel ranged from $\sim 2.0$ at Reynolds number 40 to $\sim 1.1$ at Reynolds number 150. These values are only slightly higher (5 to 20%) than drag coefficients of smooth spheres but are reasonable since the graupel tended to maintain fairly spherical shapes during growth. Previous estimates of graupel drag coefficients have been somewhat higher [for a review see Heymsfield (1978)] and are probably reflective of the particular mode of growth and eventual shape of the graupel. Accordingly, the authors suggest using drag coefficient values which have been determined for graupel more similar to the ones being modeled.

\(^4\) Calculated terminal velocity of graupel.
In addition to the accretional growth, there is also a
diffusional exchange of mass. The rate of heat transfer
due to this diffusional mass is given by
\[
\left( \frac{dQ}{dt} \right)_D = 2\pi DRL_s (N_{Sh}) (\rho_{se} - \rho_{sr}), \tag{B2}
\]
where \( L_s \) is the latent heat of sublimation, \( D \) the coeffi-
cient of diffusion of water vapor in air, \( \rho_{se} \) the water
vapor density at the surface of the sphere, \( \rho_{sr} \) the water
vapor density in the environment, and \( N_{Sh} \) is the Sherwood
number, which is approximated by the expres-
sions given by Beard and Pruppacher (1971b).

The net release of latent heat from accretional and
diffusional processes causes the surface temperature of the
ice particle to rise above that of its surroundings.
The rate at which heat is transferred by conduction and
forced convection from such a ventilated sphere is
given by
\[
\left( \frac{dQ}{dt} \right)_C = 2\pi kR (N_{Nu}) (T_s - T_\infty), \tag{B3}
\]
where \( K \) is the thermal conductivity of the air, and
\( N_{Nu} \) the Nusselt number, which also can be approxi-
imated by the expressions of Beard and Pruppacher
(1971b).

From a consideration of heat balance requirements of the
ice particle, an expression for \( T_s \), the surface tem-
perature, can be obtained. Thus
\[
\frac{1}{T_s} \frac{dQ}{dt} = \frac{dQ}{dt}_C + \frac{dQ}{dt}_D - \frac{dQ}{dt}_A \tag{B4}
\]
From Eqs. (B1)–(B4) one finds after suitable manipula-
tions that
\[
T_s = \frac{ERVW[L_f + C_w(T_a - T_0) + C_e(T_0 - T_s)] + 2DL_sN_{Sh} (\rho_{se} - \rho_{sr}) + 2kT_sN_{Nu}}{2kN_{Nu} + ERVWC_i} \tag{B5}
\]
Since \( \rho_{se} \) is a function of the surface temperature, \( T_s \)
is computed using standard iterative techniques. An initial
guess is made for \( T_s \), whereupon calculation of
\( \rho_{se} \) immediately yields a new \( T_s \) from equation (B3).
Using the new \( T_s, \rho_{se} \) is recalculated and Eq. (B5)
yields a newer \( T_s \). This process is continued until the
values obtained for \( T_s \) converge.

In our experiments, \( E \) was unknown so that a solution of
Eq. (B5) required that the collection efficiency be
expressed as a function of the surface temperature.
This was done in the following manner: From the
experimental measurements, total \( dM/dt \) was directly
measured. From the conservation of mass requirement,
\[
(dM/dt)_{total} = (dM/dt)_{rainfall} + (dM/dt)_{diffusional} \tag{B6}
\]
substitution yields
\[
(dM/dt)_{total} = \pi R^2 VW[L_f + C_w(T_a - T_0) + C_e(T_0 - T_s)] + 2\pi RDN_{Sh} (\rho_{se} - \rho_{sr}). \tag{B7}
\]
Solving for \( E \), we have
\[
E = \frac{(dM/dt)_{total} - 2\pi RDN_{Sh} (\rho_{se} - \rho_{sr})}{\pi R^2 VW[L_f + C_w(T_a - T_0) + C_e(T_0 - T_s)]}. \tag{B8}
\]
Substitution of (B8) into (B5) yields one equation with
one unknown which was solved by iteration until
the computed values of \( T_s \) converged.

Once \( T_s \) was known, the diffusional growth con-
tribution to the total growth measured in our experiment
was computed from the relation
\[
(dM/dt)_{diffusional} = 2\pi RD(N_{Sh}) (\rho_{se} - \rho_{sr}). \tag{B9}
\]

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