On the Height of the Tropopause and the Static Stability of the Troposphere

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ABSTRACT

Speculative arguments are presented that describe how radiative and dynamical constraints conspire to determine the height of the tropopause and the tropospheric static stability in midlatitudes and in the tropics. The arguments suggest an explanation for the observation that climatological isentropic slopes in midlatitudes are close to the critical slope required for baroclinic instability in a two-layer model.

1. Introduction

Radiative equilibrium temperature profiles are not very useful as first approximations to the vertical temperature structure of the lower atmosphere, as they are generally statically unstable in a layer adjacent to the ground. "Radiative-convective" models have been developed to correct this deficiency in as simple a way as possible. In a radiative-convective model, the atmosphere is forced to transport sufficient heat vertically to prevent the lapse rate, $-\partial T/\partial z$, from exceeding some prescribed critical value, γ. The result is a temperature profile with a stratosphere in radiative equilibrium above a troposphere in which the lapse rate is maintained at the value γ by unspecified dynamical fluxes. Models of this sort have played an important role in providing preliminary estimates of temperature sensitivity to perturbations in atmospheric composition [Manabe and Wetherald (1967), Ramanathan and Coakley (1978)]. In these sensitivity studies the tropospheric lapse rate is simply set equal to the observed lapse rate. A theory for the vertical distribution of the dynamic heating, or a numerical model in which the responsible circulations are computed explicitly, is required to improve upon this semi-empirical theory.

In the absence of a complete dynamical theory, the following approach may be of at least pedagogic value. Assume that $-\partial T/\partial z \equiv \gamma$ is independent of z for $z < H_T$, and that temperatures are in radiative equilibrium for $z > H_T$. The standard radiative calculation can then be thought of as determining the tropopause height H_T, given the tropospheric lapse rate γ. If one can find another relation between H_T and γ from dynamical considerations, one can then determine the tropospheric lapse rate and the height of the tropopause simultaneously. This note consists of speculations on the form such a relation might take, both in midlatitudes and in the tropics.

2. The radiative constraint

Radiative-convective models are usually thought of as models of the global-mean vertical temperature profile. However, the character of the dynamic heating changes dramatically from the tropics, where moist convection is dominant, to midlatitudes, where large-scale eddies play a significant role, so there is little point in attempting to construct a theory for the global mean tropospheric static stability. Fortunately, there is no need to restrict the radiative-convective formalism to the global mean profile.

Consider the time-averaged, zonally-averaged atmospheric heat balance at a particular latitude. The temperature profile at this latitude, $T(z)$, is assumed to have the form described in the previous section—constant lapse rate below the tropopause, in the region within which all of the dynamical fluxes are confined, and radiative equilibrium above the tropopause. The surface temperature T_* is set equal to the atmospheric temperature at the ground $T(0)$, a simplifying assumption common to most radiative-convective calculations. Let F be the upward heat flux across the ocean surface at this latitude, and C the convergence of the horizontal flux of energy (moist static energy, to an excellent approximation) integrated through the depth of the troposphere. If $F + C$ is given from other considerations, a standard radiative-convective calculation can easily be modified by depositing this total non-radiative flux within the troposphere. The vertical distribution of the non-radiative heating need not be specified since it can be determined as a residual from the radiative fluxes in the equilibrium state.

As a simple illustration, consider an atmosphere transparent to the incident solar flux and gray at those wavelengths at which radiation is re-emitted by the surface. If Q is the absorbed solar flux, then for an atmosphere in steady state the outgoing long-wave flux must be $I = Q + F + C$. In the Eddington approximation, the equations of radiative transfer reduce to

$$\left.\begin{aligned} \frac{dU}{d\tau} &= \frac{3}{2}(U - B) \\ \frac{dD}{d\tau} &= \frac{3}{2}(B - D) \end{aligned}\right\}, \qquad (1)$$

with the boundary conditions $U(\tau(0)) = \sigma T_*^4$ and $D(0) = 0$. U and D are the upward and downward infrared fluxes, $B = \sigma T^4$, and $\tau(z)$ is the optical depth. In the stratosphere, $U - D = I$ from which it follows that

$$(U, D, B) = \left(1 + \frac{3\tau}{4}, \frac{3\tau}{4}, \frac{1}{2} + \frac{3\tau}{4}\right)I.$$

Therefore, the stratospheric temperatures are

$$T(z) = \left\{\frac{I}{\sigma}\left[\frac{1}{2} + \frac{3}{4}\tau(z)\right]\right\}^{1/4}, \quad z \geq H_T. \qquad (2)$$

In radiative-convective equilibrium, $T(z)$ is assumed to be continuous at $z = H_T$, so the tropospheric temperatures must be given by

$$T(z) = \left\{\frac{I}{\sigma}\left[\frac{1}{2} + \frac{3}{4}\tau(H_T)\right]\right\}^{1/4} + \gamma(H_T - z), \quad z \leq H_T. \qquad (3)$$

T_* is set equal to $T(0)$, which in turn is obtained from (3). Given γ, $\tau(z)$, and H_T one can integrate the first of the equations in (1) from the surface, where $U = \sigma T_*^4$, to the top of the atmosphere. H_T can then be adjusted until the resulting flux at the top of the atmosphere equals I.

Thus, in this simple example $I = Q + C + \bar{F}$ contains all of the information on the non-radiative heating needed to determine the radiative-convective equilibrium state. The resulting relation between H_T and γ for $I = \sigma(255\ \mathrm{K})^4$ and $\tau(z) = \tau_* e^{-z/z_0}$, with $\tau_* = 4.0$ and $z_0 = 2$ km as in Goody (1964, p. 333), is plotted as a solid line referred to as $R(\gamma)$ in Fig. 1. The same calculation can be performed with more realistic radiative models, more elaborate techniques being required to find the equilibrium states. We expect the basic structure of Fig. 1, the rather modest increase of the tropopause height with increasing static stability over the range of stabilities of interest, to be reproduced by these more realistic models.

3. The dynamical constraint in midlatitudes

In Charney's model of baroclinic instability on a beta-plane, the vertical structure of the most unstable wave is controlled by the parameter h/H, where

$$h \equiv -\frac{f}{\beta}\frac{\partial\bar{\Theta}/\partial y}{\partial\bar{\Theta}/\partial z} = \frac{f^2 \partial\bar{u}/\partial z}{\beta N^2},$$

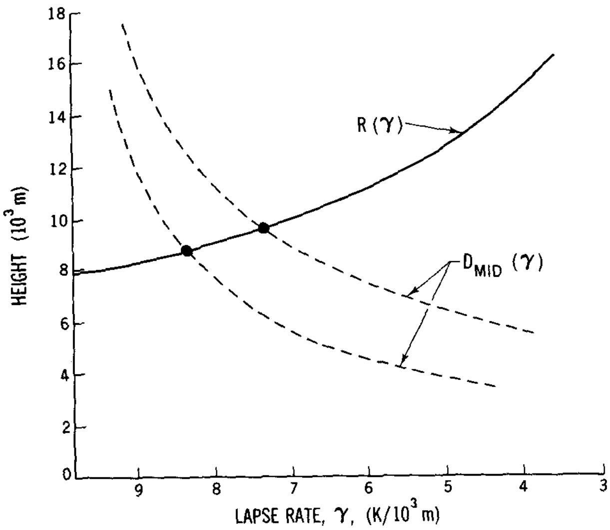

FIG. 1. The radiative constraint between tropopause height and static stability $R(\gamma)$, and the dynamical constraint in midlatitudes, $D_{mid}(\gamma)$. The two D_{mid} curves are drawn for $\partial T/\partial y$ fixed at those values which yield $h/H = \xi$ when $\gamma = 6.5\ K\ \mathrm{km}^{-1}$, where $\xi = 2$ for the upper curve and $\xi = 1$ for the lower curve. The scale height H is set equal to 7.5 km.

H is the scale height and $\bar{\Theta}$ is the potential temperature of the mean state. (The notation is otherwise standard.) When $h/H \ll 1$, the most unstable wave decays above the ground roughly exponentially, with an e-folding height proportional to h. As discussed in Held (1978a), this fact suggests that in a statistically steady state of a forced, dissipative quasi-geostrophic flow on a beta-plane, characterized by mean temperature gradients such that $h \ll H$, the bulk of the dynamic heating may also be confined to a layer of depth $\propto h$ above the ground.

Significant dynamic heating must extend to the tropopause, by definition; $R(\gamma)$ in Fig. 1 then implies that dynamic heating must extend to heights comparable to the scale height for any positive static stability. If one accepts the hypothesis that heat redistribution due to quasi-geostrophic dynamics is confined to a small fraction of a scale height when h/H is sufficiently small (and also assumes that heating due to small-scale motions such as moist convection is negligible), then one must conclude that an atmosphere in which $h/H \ll 1$ is, in fact, unrealizeable. h must be at least comparable to a scale height, implying that the mid-latitude temperature gradient must correspond to a temperature difference across a distance of one Earth radius a that is at least as large as the difference in potential temperature in the vertical over one scale height: $h/H \gtrsim 1$ and $\tan\bar{\Theta} \approx 1$ imply that

$$\left|\frac{\partial\bar{\Theta}}{\partial y}\right| a \gtrsim \frac{\partial\bar{\Theta}}{\partial z} H.$$

The validity of this hypothesis on the vertical distribution of the dynamic heating can be questioned in a number of ways. For example, one can argue that the eddy fluxes averaged over the life cycle of a disturbance extend further into the atmosphere than the fluxes in the linear, exponentially growing stage, as in Fig. 3 of Edmon *et al.* (1980). However, any linear disturbance in the vertical shear flow $\bar{u} = z\partial\bar{u}/\partial z$ with total horizontal wavenumber k and phase speed c propagates vertically only between its steering level, where $\bar{u} = c$, and its turning point in the increasing westerlies, where $\bar{u} = c + \beta k^{-2}$ (in the case $h \ll H$). The distance between these two points for a wavenumber comparable to that of the most unstable wave [$k \approx f/(Nh)$] is

$$\beta \bigg/ \left(k^2 \frac{\partial \bar{u}}{\partial z}\right) \approx \beta \left(\frac{Nh}{f}\right)^2 \bigg/ \frac{\partial \bar{u}}{\partial z} = h.$$

Thus, as long as horizontal scales and phase speeds comparable to those of the most unstable wave predominate, one might still expect the vertical extent of the time-averaged heating to *scale* with h, although the penetration of the unstable mode's dynamic heating may indeed underestimate this vertical extent.

Of more concern is the fact that waves with horizontal scales larger than that of the most unstable mode are able to penetrate much more deeply into the atmosphere. These larger waves can be generated either by relatively weak instabilities of the zonal flow or by transfer of energy from smaller scales. However, it is important to keep in mind that stable or weakly unstable waves can dominate the energy spectrum and yet contribute little to the dynamic heating. One can picture more or less turbulent mixing confined to a depth $\approx h$ above the ground, with more nearly linear long waves penetrating into the stratosphere. These longer waves may affect the flow at upper levels, but in a manner fundamentally different from the "tropospheric" mixing. Without denying that the vertical distribution of the heating is dependent to some degree on other parameters, particularly those dissipative parameters helping to control the energy level of the flow and the strength of the energy cascade to longer, more deeply penetrating waves, the contention is that the scale h remains of paramount importance.

The same qualitative conclusion follows from an argument in Lindzen and Farrell (1980). What is the minimum height above the ground up to which one need modify the mean flow in order to stabilize it, using the generalized Charney-Stern sufficient condition for stability as the criterion? The meridional temperature gradient or vertical wind shear at the ground must first be destroyed. This can be achieved by modifying the flow in a very small layer at the ground, but this will in turn create destabilizing curvature, that is, the potential vorticity gradient,

$$\partial\bar{q}/\partial y \equiv \beta - f^2 \frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial \bar{u}}{\partial z}\right),$$

will change sign in the vertical (working in the Boussinesq limit, $h \ll H$, and ignoring horizontal curvature of $\bar{u}$). Modifying the shear to have the value $\partial\bar{u}_n/\partial z \equiv \beta N^2 z/f^2$ in a layer adjacent to the ground results in $\partial\bar{q}/\partial y = 0$ in this layer. To avoid destabilizing curvature at the top of this layer, it must extend at least up to the height at which $\partial\bar{u}_n/\partial z$ first matches the environmental shear. The minimal adjustment required is, therefore, the creation of a layer of thickness $f^2(\partial\bar{u}/\partial z)/(\beta N^2)$ within which $\partial\bar{q}/\partial y = 0$. The claim is that this construction describes how a typical eddy modifies the mean flow.

It should be emphasized that the time-averaged flow need bear no resemblance to an adjusted state with $\partial\bar{q}/\partial y \approx 0$ in a layer adjacent to the ground. An eddy can stabilize its immediate environment by destroying low-level temperature gradients, but in so doing it will likely destabilize surrounding regions. Other eddies will grow in these regions and in the process undo much of the work of the original eddy. If the restoring forces are of sufficient strength, whether they be due to other eddies in this fashion or to diabatic effects, an unstable time-averaged state will be maintained.

Let $D_{\text{mid}}(\gamma)$ be the depth of significant dynamical heating in mid-latitudes as a function of lapse rate. The preceding arguments suggest that $D_{\text{mid}} \ll H$ if γ and the horizontal temperature gradient are such that $h \ll H$. A more precise specification of $D_{\text{mid}}(\gamma)$ must necessarily be very tentative. Scaling arguments alone do not provide any information on the depth of penetration of the dynamic heating when h/H is not small. We have chosen to use as a guide the neutralizing construction described in Lindzen and Farrell (1980) for a compressible atmosphere with arbitrary vertical structure. In pressure coordinates,

$$\partial\bar{q}/\partial y = \beta - f^2 \frac{\partial}{\partial p}\left(\frac{1}{\sigma}\frac{\partial \bar{u}}{\partial p}\right),$$

where

$$\sigma = -\frac{\alpha}{\bar{\Theta}}\frac{\partial\bar{\Theta}}{\partial p}.$$

and α is the specific volume. If

$$\frac{\partial \bar{u}_n}{\partial p} \equiv \frac{\beta\sigma}{f^2}(p - p_*), \tag{4}$$

in a layer adjacent to the ground, then $\partial\bar{q}/\partial y = 0$ within this layer. As before, to avoid destabilizing curvature the layer must extend at least to the pressure p_T at which $\partial\bar{u}_n/\partial p$ first matches the environmental shear. Thus

$$p_* - p_T = \frac{f}{\beta}\left(\frac{\partial\bar{\Theta}/\partial y}{\partial\bar{\Theta}/\partial p}\right)_{p=p_T}, \tag{5}$$

or, transforming from p to $z \equiv H \ln(p_*/p)$,

$$H_T = H \ln(p_*/p_T) = H \ln[1 + h(H_T)/H], \quad (6)$$

where

$$h(H_T) \equiv -\frac{f}{\beta}\left(\frac{\partial\bar{\Theta}/\partial y}{\partial\bar{\Theta}/\partial z}\right)_{z=H_T}.$$

If the lapse rate γ is independent of height, then

$$H_T = D_{\text{mid}}(\gamma) \equiv H \ln\left[1 - \frac{f\partial T/\partial y}{H\beta(\gamma_d - \gamma)}\right], \quad (7)$$

γ_d being the dry adiabatic lapse rate. Given the horizontal temperature gradient, the hope is that (7) provides a qualitative guide to the dynamical constraint between tropopause height and lapse rate when eddies produced by baroclinic instability are responsible for the bulk of the dynamic heating in the free atmosphere.

According to (7), the depth of penetration of the dynamic heating increases indefinitely as $\gamma \rightarrow \gamma_d$. This conclusion should be contrasted with that based on the vertical structure of the most unstable mode in Charney's model, which asymptotes to a form independent of the static stability as the static stability tends to zero. In choosing (7) as an estimate of the vertical extent of the dynamic heating, one is assuming that disturbances penetrating more deeply than the most unstable mode come into play when h/H is large. In this regard, it may be noteworthy that the wavenumber k_n of the "neutral" point in Charney's model dividing the Charney modes ($k > k_n$) from the ultra-long wave instabilities ($k < k_n$), equals $(0.5\,Nh/f)^{-1}$ when $h/H \ll 1$, but tends to zero [rather than asymptoting to $(NH/f)^{-1}$] as $h/H \rightarrow \infty$. Calculations with suitably idealized dynamical models are needed to decide if (7) is indeed a useful guide to the depth of penetration.

Using the zonally averaged Northern Hemisphere temperature fields for January and July in Oort and Rasmussen (1971), (5) yields the pressure p_T shown by the dotted lines in Fig. 2. These values are obtained by starting at the surface and decreasing p_T until (5) is satisfied, linearly interpolating the data in latitude and log(p). It is clear that (5) will be satisfied at some point below that at which $\partial T/\partial y = 0$ in the observations. The "predicted" tropopause is located at $\approx$400 mb north of 30° in the winter and north of 45° in the summer, and drops rapidly to the surface equatorward of these latitudes. The actual extratropical tropopause is located at 200–300 mb. Our view is that the comparison with the observed heights is encouraging, given the qualitative character of the arguments presented. (If a construction providing more realistic tropopause heights is desired, the arbitrary correction $H_T \approx 1.5D_{\text{mid}}(\gamma)$ would serve this purpose and yet perhaps retain something of the correct dependence of height on γ.)

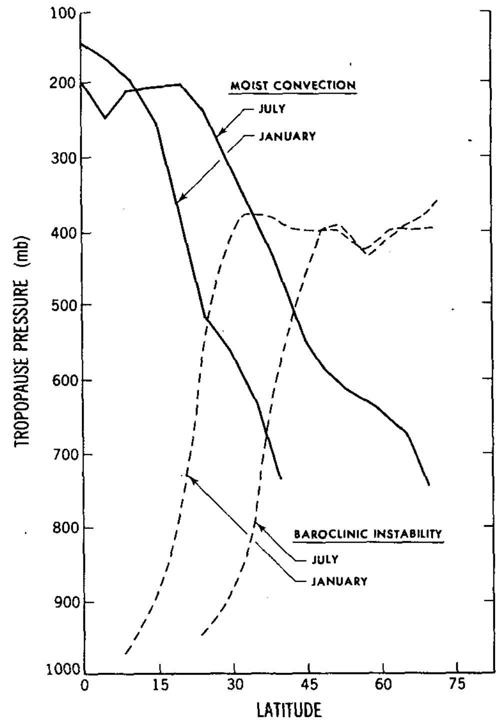

FIG. 2. The pressure at the tropopause predicted by a moist convective model and by a model of mixing by baroclinic instabilities, as discussed in the text, using the January and July zonal mean temperatures in Oort and Rasmusson (1971).

The function $D_{\text{mid}}(\gamma)$ is plotted for two values of $\partial T/\partial y$ (corresponding to $h/H = 1$ and $h/H = 2$ at $\gamma = 6.5$ K km^{-1}) in Fig. 1. The intersection of the curves $R(\gamma)$ and $D_{\text{mid}}(\gamma)$ yields the predicted tropopause height and lapse rate. In response to an increase in horizontal temperature gradient, the change in tropopause height is relatively small, since R is a slowly varying function of γ. The lapse rate adjusts so that h/H, or the isentropic slope, remains nearly constant. Recall that

$$h/H = (H\partial\bar{u}/\partial z)/(\beta\lambda^2) = \text{constant}, \quad (8)$$

(where $\lambda \equiv NH/f$) is also the condition for marginal stability in a two-layer atmospheric model with layer depths proportional to H. Therefore, this construction provides a possible alternative explanation to that offered in Held (1978a) for the fact that the observed extratropical isentropic slopes can be matched rather closely to those corresponding to marginal stability in a two-layer model. In Held (1978a) the static stability was taken as given and it was argued that the shallow disturbances that would dominate the flow if $h \ll H$ would be too inefficient at transporting heat poleward to maintain such small isentropic slopes. It is argued here that these shallow disturbances could not, in any case, consistently maintain a radiative-convective thermal structure.

4. The dynamic constraint in low latitudes

Upper tropospheric heating in the tropics is assumed to be due to protected core clouds in which the loss of buoyancy due to entrainment of environmental air is small. Thus, the deepest convective elements are assumed to consist of parcels that possess moist static energy equal to that of the time-mean state near the ground, conserve their moist static energy as they rise, become saturated at cloud base, and penetrate to their level of zero buoyancy where the moist static energy once again equals that of the environment. [The difference between the depth of penetration computed using equivalent potential temperature and that computed using moist static energy is of the order of 100 m (Betts, 1974) and is negligible for our purposes.] Similar assumptions are common to most simple models of the tropical temperature profile (cf. Schneider, 1977). To emphasize that this picture may be a significant oversimplification, note that in the GCM study of Manabe and Mahlman (1976, Figs. 3.2 and 8.3), a tropical tropopause is produced at 100 mb, but moist convective heating is confined below 200 mb, with the dynamic heating in the intervening layer due to large-scale eddies.

The moist static energy is $c_pT + gz + Lr \equiv s$ where r is the water-vapor mixing ratio. One has the choice of specifying cloud base or the relative humidity near the ground. Choosing the latter alternative, we set $s = s_0 = c_pT(0) + h_*Lr_s[T(0), p_*]$ at the surface, h_* being the relative humidity and $r_s(T, p)$ the saturation mixing ratio. At the tropopause, where water vapor concentration is negligible, $s = s_T = c_pT(H_T) + gH_T$. Using the observed January and July temperature fields and choosing a relative humidity of 0.80, the height at which one finds $s_T = s_0$ is plotted as a solid line in Fig. 2.

If one assumes that $\gamma \equiv -\partial T/\partial z$ is constant with height, then $s_T = s_0$ implies that

$$H_T = D_{\mathrm{trop}}(\gamma) \equiv h_*Lr_s[T(0), p_*]/[c_p(\gamma_d - \gamma)]. \quad (9)$$

For fixed T_0, H_T as defined by (9) is inversely proportional to $\gamma_d - \gamma$, and therefore the radiative and dynamical constraints bear the same qualitative relationship to each other as in the mid-latitude case pictured in Fig. 1. T_0 can be determined self-consistently from the radiative-convective calculation. A radiative-convective atmosphere so defined, with constant lapse rate in the troposphere, has the advantage over a simple adjustment to the local moist adiabat in that it is conditionally unstable in the lower troposphere.

Although one can formulate such a radiative-convective calculation at a particular latitude, one does not expect the vertical extent of the dynamical fluxes at one latitude to be controlled solely by the mean flow parameters and forcing at that latitude. The elliptic character of the quasi-geostrophic equations forces any theory of midlatitude eddy dynamic heating to be non-local in the horizontal, at least over a distance equal to N/f times the vertical scale of the eddies. The Hadley circulation gives an even stronger non-local character to the tropical dynamic heating, perhaps with convection in the intertropical convergence zone controlling the tropical tropopause height and the large-scale circulation communicating this information to the rest of the tropics.

With this consideration in mind, one is still tempted to apply the midlatitude and tropical constraints latitude by latitude and then choose that one which produces the largest static stability or, equivalently, the deepest troposphere, naively assuming that moist convection and large-scale baroclinic eddies are unaffected by each others' presence. The point at which $D_{\mathrm{mid}}(\gamma) = D_{\mathrm{trop}}(\gamma) \approx 25°$ latitude in the winter and 40° in the summer in Fig. 2, then provides a rough estimate of the boundary poleward of which large-scale eddies dominate the static stability balance (cf. Stone and Carlson, 1979). From the discussion in Section 3, it is clear that this construction will place the boundary near the point predicted by the two-level model's stability criterion.

5. A final caveat

An attempt has been made to provide a qualitative theory for the height of the tropopause and the tropospheric static stability that does not require detailed information on the vertical structure of the dynamic heating. The lapse rate is assumed to be independent of height within the troposphere, so that the dynamical theory need only provide the tropopause height as a function of this lapse rate. Unfortunately, even if the arguments presented for the vertical extent of the dynamic heating have some value, the utility of the resulting theory is limited by this assumed form of the temperature profile. High northern latitudes in winter are one region where this approach is clearly inadequate, the deep lower tropospheric inversion resulting in a profile unlike that postulated.

A peculiar, and perhaps implausible, feature of these constructions is their prediction of a static stability that is not explicitly dependent on the *magnitude* of the dynamic heat fluxes, being dependent only on the vertical extent of these fluxes. The magnitude is determined implicitly by the radiative-convective model. This should be contrasted with the more common approach to the problem of the static stability of the troposphere in midlatitudes, in which the height of the dynamic heating is fixed [as in Stone (1972) or in the analysis of a two-level model in Held (1978b)] and the static stability is determined by the magnitude of the vertical flux of heat from lower to upper troposphere. Our concern here is to point out that variations in the vertical extent of the heating should not be ignored, as they may, in fact, play a key role in determining the static stability.

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