A Dynamic Model of Thundercloud Electric Fields

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ABSTRACT

A dynamic interactive computer model of the electrical behavior of a thundercloud surrounded by the distributed atmosphere, earth, ionosphere circuit is described. The electrification mechanisms in the model are represented by current or voltage generators. The electrical breakdown is taken care of by switches that close when the voltages between shortest paths exceed breakdown electric fields and open again when the voltages fall below those necessary to sustain the arc. The model is thus able to simulate the very complicated interactive processes that occur in and around a thundercloud due to the linear conduction and displacement currents, as well as the nonlinear charge redistribution processes that occur during arc breakdown within and outside the cloud.

The new models are intended to serve as an extension of the convective dynamic and microphysical models. They allow predictions to be made of the interrelation of the observable electrical parameters of the storm to the geometry and parameters of the electrical generating mechanisms. It is shown that there are major differences between the behavior of a fully interactive dynamic model and the simple monopole or dipole models on the one hand and models which neglect atmospheric conduction and displacement currents and electrical breakdown on the other.

To demonstrate the possibilities of this type of model, several exploratory studies were carried out. The effects of variations in the height of the cloud above ground and of the vertical separation between the top and bottom of the cloud were investigated. Changing the cloud conductivity from one tenth to ten times the conductivity of the surrounding atmosphere was shown to have only a small effect on the external electric fields and on the current to the ionosphere. Several generator configurations were examined to study the effect of partial neutralization of the precipitation at the bottom of the cloud. The effect of neutralization between the cloud and ground was also examined. It was shown that only in cases where the current was almost completely neutralized in the bottom of the cloud or when almost all the current flowed to the ground could cloud-to-ground strokes occur.

1. Introduction

The electrical behavior of thunderclouds and its relation to the environment is extremely important to the understanding of severe weather. The electrical activity, the static and dynamic electric fields, the currents and the lightning frequency all provide the most easily measurable parameters of the storm. It is thus necessary that the understanding of the electrical mechanisms within the storm and their relationships to these measurable parameters be as advanced as the models of the storm itself.

It is of interest to examine the role thunderstorm electric fields play in maintaining the potential of the ionosphere, in transporting ions, and in heating ions and electrons in the D and E regions. It is also desirable to have a model that can be used to examine the effects of ionospheric or magnetospheric electric fields on thunderstorms because of possible sun–weather coupling mechanisms that have been suggested.

The main factors controlling the storm are modeled in convective dynamic storm models such as those of Klemp and Wilhelmson (1978), Miller (1978), Schlesinger (1978), Chen and Orville (1980), Clark (1979) and Tripoli and Cotton (1980, 1983). These models start with dynamic boundary conditions and solve the energy and momentum equations. Of particular importance are the inputs associated with the phase changes of water. The observables predictable by the models are the velocity fields, the temperatures and pressures, the sizes of water drops, and the types and sizes of ice particles. The state of the art is well developed and typical models solve the equations as a function of time for three-dimensional grids having tens of thousands of grid points.

Microphysical cloud electrification models follow the charged precipitation particles in the dynamic environment provided by the convective dynamic storm models, keeping track of the charging and charge neutralization processes and the divergences in the currents produced by such things as dramatic size changes on collision, melting or vaporization. Examples of such models are those of Ziv and Levin (1974), Illingworth and Latham (1977), Chiu (1978), Spangler and Rosenkilde (1979), Wagner and Telford (1981) and Tzur and Levin (1981). Models such as HELSDON'S (1980) combine a microphysical cloud electrification model with a convective dynamic model.
They consider the cloud in isolation and, usually, little attention is paid to the earth-ionosphere circuit or to the displacement currents in the surrounding medium. While the electrical forces are much smaller than the gravitational and kinetic forces for the large precipitation particles, this is not true for the small ions, and large-scale conduction and displacement currents flow in response to the dynamic electric fields that are set up in and around the cloud. Lightning occurs when the local breakdown electric field is exceeded; and this dramatically alters the charge distributions, but again, this is not included in these models. The microphysical cloud electrification models thus relate the convective dynamic storm models to the driving generator currents in the thunderstorm, those carried by the charged ice and water drops. Some of the models are very highly developed and, for example, that of Tzur and Levin (1981) handles the continuity equations of 38 categories of water drops, 30 categories of ice particles, positive, negative and large ions, and water vapor. The spatial resolution of this model is of the order of 200 m.

To relate the driving generator currents to the dynamic charge distributions and electric fields, dynamic electrical storm models, which are the subject of this present paper, are required. They should allow the three-dimensional structure of the static and dynamic currents and electric fields around the storm to be calculated as a function of time and they should be able to include realistic electrical models of the atmosphere and ionosphere. The charge distributions should be determined, as they are in a real cloud, by driving currents, conduction and displacement currents, and rapid charge transfer after electrical breakdown. The models should be able to handle the complicated generator geometries predicted by the microphysical models, and they should ultimately be designed to have comparable temporal and spatial resolution to the convective dynamical models and be able to simulate real storms.

Models such as those of Holzer and Saxon (1952), Tamura (1955) and Park and Dejnakantrintra (1973) assume discrete charges at the top and bottom of the cloud and calculate the resulting electric field patterns in realistic atmospheric and ionospheric circuits. All of the above models are essentially static. Kasemir (1959) gives a model in which the cloud is represented by the parallel connection of a current generator, a capacitor, a resistor and a spark gap. The circuit to the earth is replaced by a resistor and that to the ionosphere by the parallel connection of a resistor and a capacitor. Anderson and Freier (1969) and Park and Dejnakantrintra (1977) have studied the effect of dynamic changes in dipole charge configurations in realistic models of atmospheric conductivity.

It is apparent, however, that the monopole or dipole models fall very far short of the requirements of the truly dynamic electrical model specified above, and it was decided to attempt to develop a first generation model of this type.

Requirements of a dynamic electrical model. It is first necessary to model the generator processes in the cloud. A large number of mechanisms have been shown to be capable of separating positive and negative charges in clouds ranging from cosmic ray ionization, ion diffusion and conduction to ice particles and water drops (Wilson, 1929; Whipple and Chalmers, 1944; Gunn, 1954). Charging in collisions between water drops and ice particles has been discussed by Workman and Reynolds (1950) and Außerdermaur and Johnson (1972). Ice–ice collisions have been extensively studied in terms of temperature, collision velocity, particle size and liquid water content by Reynolds et al. (1957), Latham and Mason (1961), Latham and Stow (1965), Buser and Außerdermaur (1977) and Gaskell and Illingworth (1980). Growth and decay of different types of particles occur and particles change state. These changes, while they may conserve charge, may also drastically alter the transport properties. Ions and charged precipitation particles, such as ice or raindrops, can be neutralized by various processes, and the location where this happens and the paths of the particles involved critically affect the current system. In addition, ions can be generated in coronal discharges and externally generated ions can be entrained into clouds by the large velocity fields.

The electrification of thunderclouds requires that there be relative motion between the net positive and negative charges and divergence of the resulting current system. Two main mechanisms have been proposed to provide this relative motion. In the first, the heavy particle of one sign is moved vertically by gravity relative to the lighter charge of opposite polarity. Motion of the lighter and more mobile charge in the resulting electric field must also be considered. In the second mechanism, the relative velocities are produced by differences in the entrainment of different polarity ions across one of the large shear gradients in a thundercloud. In one mechanism, for example, a pre-existing field causes ions to drift across the shear layer in opposite directions. In another, proposed by Grenet (1947) and Vonnegut (1953), charges from outside the cloud are entrained into the cloud's convection system. In addition, coronal and convection effects at the ground introduce charged particles and there are regions of stored charge at the border of clouds, for example, where the type of ion carrying the conduction current changes abruptly. These also can be represented by generators. These mechanisms are all part of the microphysical models. All that is important for the dynamic electrical models is the field of the divergences of the current system produced by regions of charge separation and charge neutralization. The models should be sufficiently flexible to accommodate arbitrary
generator mechanisms as required to simulate these fields.

The conductivity varies greatly with height, and in the ionosphere it is anisotropic. The exchange layer close to the ground contains convection currents and large ions and is variable in conductivity and height (Sagalyn and Fauchier, 1954). Conductivities in the D region and the lower mesosphere are highly variable (Mitchell et al., 1982). It is thus desirable for the model to be able to use arbitrary conductivity profiles and to specify different conductivities in regions such as clouds that contain mainly large ions.

A major feature of the dynamic electrification of thunderclouds is the dynamic rearrangement of charge distributions following lightning. Several models have been developed of the propagation of the lightning stroke itself, such as those of Szpor (1971), Price and Pierce (1972), Griffiths and Phelps (1976), Veverka (1978) and Lin et al. (1980). It would be desirable in the dynamic model to have the lightning stroke models initiated at any arbitrary location in the model where the electric field exceeds the breakdown value. Unfortunately, the time scales of lightning discharges are of the order of nanoseconds and the distance scales are of the order of meters, and these are much smaller than those of the remaining parameters of the cloud. It seems that, for the present, the breakdown must be parameterized in some way. Equivalent amounts of charge should be transferred whenever and wherever the electric field exceeds the breakdown value and arc charge transport should be terminated when the electric field is reduced below some maintenance value. This transfer should take place in times short compared to the smallest time scales of interest. Sufficient flexibility should be included so that different breakdown fields can be assumed at different altitudes or, for example, in locations where particles are present that are likely to initiate coronal discharges.

2. Description of the model

It was decided to build a discrete rather than an analytic model because it was believed that in this way the complex interactions that occur with a wide range of breakdown paths could be simulated and the flexibility discussed in the preceding section obtained. The simplest method for testing this idea was to adapt an already developed piece of software, a general purpose computer program called ECAP (Electrical Circuit Analysis Program). This program has several modes of operation, one of which, that used for the transient analysis of networks, proved suitable for our purposes after modification. It can include impedance elements, as well as dependent and independent voltage and current generators, and switches that may be set to change the impedances of branches under conditions determined by the voltage across any desired branch.

In the thunderstorm model, electric breakdown was simulated by switches that were incorporated to cause the resistances across each branch to change to a predetermined lower value when the potential across the branch exceeded that corresponding to the assumed breakdown electric field. They would remain in that condition until the electric field dropped below the maintenance value, at which time the arc was presumed to be extinguished and the original resistance value restored.

Like any general purpose piece of software developed for another application, the program used for this initial study has its limitations. One of these is the maximum size of the network it can handle and, in the form used for the present study, this is 50 nodes and 200 branches. A 200 branch, 50 node network is quite large from a computational point of view, but is only marginally large enough to model the complete electrical environment of a thundercloud, so some care had to be taken in the development of the model. It was decided, after some experimentation and calculations, to use a cylindrical geometry with nodes at seven radii and seven altitudes in addition to the ground. The radii used were 100 m, 300 m, 900 m, 2.7 km, 8.1 km, 24.3 km and 72.9 km. Typical altitudes used were 1, 2, 4, 8, 16, 32 and 64 km; however, these were varied for some of the studies.

There are two ways of modeling sources in electrical models. The Thevenin equivalent consists of an ideal voltage generator in series with an impedance. The Norton equivalent consists of an ideal current generator in parallel with an impedance. In the present work, Norton generator configurations were employed for modeling the thunderstorm electrical generation mechanism mainly because it was easier to separate the specification of the generator magnitude and the breakdown characteristics of the cloud. Thevenin generators have been used to model the fair weather field potential as well as layers of charge because of the ease with which the generator value could be related to physically observable variables. The two configurations are electrically identical and the choice is quite independent of the actual cloud electrification mechanism.

The time scale of our current interest only extends down to the order of milliseconds, and so the actual arc breakdown may be assumed to be instantaneous. This allowed energy storage in the magnetic field to be neglected, and only resistive and capacitive elements were used.

Determination of the element values. Because of the discrete nature of the circuit, it is necessary to model the conduction and displacement current densities in terms of currents flowing in resistors and capacitors. The equations used to determine these elements are given below. Fig. 1 gives a diagram of the electrical network and of an individual branch.
1) VERTICAL ELEMENTS

For the values of the elements connecting nodes at altitude $z_i$ to altitude $z_j$ at radius $r_m$

$$R_{ijm} = (z_j - z_i)(\sigma_j - \sigma_i)/A_m\sigma_j\sigma_i \ln(\sigma_j/\sigma_i),$$  \hspace{1cm} (1)

$$C_{ijm} = A_m\epsilon/(z_j - z_i),$$ \hspace{1cm} (2)

the voltage between nodes corresponding to an electric field $E_B$ is

$$V_{Bi} = E_B(z_j - z_i)(1 - \sigma_j/\sigma_i)/\ln(\sigma_j/\sigma_i),$$ \hspace{1cm} (3)

where $\sigma_i$ is the atmospheric conductivity at $z_i$, $\epsilon$ is the permittivity of free space, and

$$A_m = \pi[(r_m + r_n)^2 - (r_m + r_j)^2]/4.$$ \hspace{1cm} (4)

2) HORIZONTAL ELEMENTS

For the values of the elements connecting nodes at radius $r_m$ to radius $r_n$ at altitude $z_j$

$$R_{mn} = \frac{\ln(\sigma_j/\sigma_h)\ln(r_n/r_m)}{2\pi(z_j - z_h)\sigma_j^{0.5}(\sigma_j^{0.5} - \sigma_h^{0.5})},$$ \hspace{1cm} (5)

$$C_{mn} = \pi\epsilon(z_j - z_h)/\ln(r_n/r_m),$$ \hspace{1cm} (6)

$$V_{Bmn} = E_Br_m \ln(r_n/r_m).$$ \hspace{1cm} (7)

3) THE FAIR WEATHER FIELD

To reproduce the effect of the fair weather field and the global ionosphere to ground impedance, a Thévenin generator of 200 kV and 145Ω internal resistance in parallel with a 0.046 F capacitor was connected between the node at the top altitude and outer radius and ground.

4) THE CONDUCTIVITY MODEL

For most of these initial studies, unless otherwise stated, the conductivity model of Gish (1944) was used, which gives

$$\sigma(z) = 10^{-13}/[2.94 \exp(-4.52 \times 10^{-3}z) + 1.39(-3.75 \times 10^{-4}z) + 0.369 \exp(-1.21 \times 10^{-4}z)] [\text{S m}^{-1}].$$ \hspace{1cm} (8)

3. Simplified analytic model

The behavior of the dynamic electrical model is very complicated because of the interaction of the large number of breakdown paths and the resulting continually changing charge distributions even for the most simple generator geometries. Before examining the results of simulations produced by the dynamic computer model, it is advisable to attempt to understand the way parameters are interrelated. Unless this is done, the range of combinations of variables becomes completely unmanageable, and it becomes difficult to interpret causes and effects.

The calculations and predictions of the dynamic computer model are completely independent of the approximations in the analysis in this section; however, the simple analysis that follows is strongly dependent on approximations and values derived from the computer model. While most of this analysis has been done before, some of the generalizations applicable to point charge models are not valid. It is thus advisable to develop those relationships that are applicable from first principles.

From the point of view of the electrical circuit, what is of importance is the total concentration of positive
and negative charges in a given volume and the total currents carried by these charges. They range in size from electrons to large raindrops or ice particles which may be multiply charged.

The equation for the conservation of the charge density may be written

\[ \frac{d\rho}{dt} = -\nabla \cdot J, \tag{9} \]

where \( J \) is the total current density. To change the charge density distribution, therefore, requires a divergence of the total current and the introduction of generators in the model.

It is convenient to integrate (9) over the volume surrounding the charge to give

\[ -\frac{dQ}{dt} = I + \oint J_c \cdot ds, \tag{10} \]

where \( I \) is the non-ohmic component of the current out of the volume produced, for example, by gravitational separation, \( J_c \) is the component of the current density due to the motion of ions induced by the electric field, and \( ds \) is the incremental surface area vector. Now from Maxwell’s equations

\[ Q = \oint D \cdot ds = \varepsilon \oint E \cdot ds, \tag{11} \]

\[ J_c = \sigma E, \tag{12} \]

where \( \sigma \) is the conductivity

\[ Q = \varepsilon \sigma^{-1} \oint J_c \cdot ds. \tag{13} \]

Now \( \varepsilon \sigma^{-1} = T \) is the local relaxation time constant of the atmosphere at the height where the charge \( Q \) is located, so that from Eqs. (10) and (13) we have

\[ -I = \frac{dQ}{dt} + \frac{Q}{T}. \tag{14} \]

If \( I \) is constant, the solution to Eq. (14) is

\[ Q = -IT + C \exp(-t/T), \tag{15} \]

where \( C \) is a constant determined by the initial conditions at \( t = 0 \); and \( I_0 \), the minimum current out of the region of the charge that will just cause breakdown for an infinitely large \( t \), is given by

\[ I_0 = -\frac{Q_B}{T}, \tag{16} \]

where \( Q_B \) is the charge stored at breakdown. If we assume that charge buildup begins at \( t = 0 \), where \( Q = Q_E = kQ_B \) and \( Q_E \) is the charge at which the arc is extinguished, then

\[ C = IT + kQ_B = T(I - kI_0). \tag{17} \]

Substituting (16) and (17) in (15) gives, for the time the charge builds up to \( Q_B \), i.e., the time \( t_B \) between breakdowns,

\[ t_B = T \ln[(1 - kI_0/I)/(1 - I_0/I)]. \tag{18} \]

Figure 2 shows the ratio of the time between lightning strokes to the local relaxation time as a function of the ratio of the generator current to that which will just produce breakdown for several values of the ratio \( k \) of the charge stored after breakdown to that stored before breakdown.

The generator current in a typical storm is likely to be several times that required just to produce breakdown so that the time between strokes is likely to be of the same order as the local relaxation time at the altitude where the breakdown is initiated. This is usu-
ally the lower charge center in the cloud. Changes in
the currents between the cloud and the ionosphere
thus take place at a rate slower than the local relaxation
time, and changes below the cloud at a rate faster
than the local relaxation time. Equilibrium solutions for
the potential distributions are thus quite different from
dynamic distributions. This analytic expression is exact;
however, it should be pointed out that \( I_0 \) is the current
which would be just sufficient to cause breakdown
with the charge configuration that is actually present
in the dynamic case just before breakdown. Because
the breakdown occurs when the maximum electric
field exceeds some given value, the actual value of \( Q_B \)
and hence \( I_0 \) will depend on the complete charge
configuration everywhere produced by all the local break-
downs, and this will vary from one stroke to the next.
Only an approximate solution can be obtained from
static calculations and exact solutions of the complete
electric field structure at breakdown can be obtained
only from dynamic simulations. The value of \( k = Q_B / Q_B \)
relating the charge when the arc is extinguished to
the charge at breakdown, is not equal to the ratios of
the electric fields at these two times because the charge
distributions will be different in the two cases, and this
relationship also can be determined only from the dy-
namic model.

Although the actual charge distributions are quite
complicated, it is helpful to have an idea of how the
size, shape and height of the charge affect the ratio of
the current \( I_0 \) in Eq. (18) to the breakdown field \( E_B \).
One simple shape that provides a rough approximation
to the charge distributions observed in nature and in
the model is an ellipsoid. For an ellipsoidal monopole
with a circular horizontal cross section of radius \( a \) and
a semi-major axis \( b \), as is shown in the Appendix,

\[
\frac{|I_0|}{E_B} = \frac{4 \pi e a^2}{3 c T},
\]

where \( T \) is the relaxation time constant at the break-
down location, \( E_B \) is the breakdown electric field, and
\( c \) is a constant dependent on \( m = b/a \). When \( m = 1 \),
\( a = b \), the charge distribution is circular and \( c = \frac{1}{2} \).
With \( n = 1 - (a/b)^2 \) we have, from the Appendix,

\[
c = \left(1 - \frac{1}{n}\right) \left[ 1 - \frac{1}{2 n^{1/2}} \log_e \left(1 + 2 n^{1/2} + n\right) \right].
\]

Values of \( |I_0|/E_B \) for various altitudes and values of
\( m = b/a \) are shown in Fig. 3.

In practice, the ratios of \( |I_0|/E_B \) will always be smaller
than these values for equivalent charge distributions
because of the effects of the upper charge center for
top-to-bottom breakdown in the cloud and because of
image charges induced in the ground below for cloud-
to-ground strokes. These effects also are automatically
included in the dynamic model because of the way
the equations are solved. They are not as easy to include
in the theoretical analysis as might be thought because
of the time delays associated with charge transfer and
the height variation of the conductivity.

4. Studies with the model

This type of dynamic electrical model is extremely
flexible and eventually, with suitable development,
should be able to simulate most of the electrical me-
chanisms in real storms. The spatial resolution of
the present model is quite limited, however, and for this
initial study we will use only very simple idealized
generator configurations. The aim will be to examine,
using a simple atmospheric conductivity model, the
interrelationships of the locations and sizes of the gen-
erators to the observable electrical parameters.

a. General behavior

Figure 4 shows the buildup of the electric field at
the ground and in the cloud, and of the resistive com-

\[ \text{Fig. 3. Variation of the ratio of the equilibrium breakdown current to the breakdown electric field as a function of height for an oblate spheroid monopole compared with values calculated for clouds of different dimensions from the computer model.} \]
ponent of the current density $J_i$ to the ionosphere as a function of time, all normalized to the magnitude of the breakdown electric field in the cloud. For this simulation a 200 kV m$^{-1}$ breakdown field was assumed, and current generators totaling 10 A and simulating $8.7 \times 10^{-6}$ A m$^{-2}$ were included from 4 to 8 km altitude at $r = 100$ m, $r = 300$ m and $r = 900$ m. The ratio $k$ of the restoration electric field to that at breakdown was assumed to be 0.1. Only breakdown within the cloud occurs. A very large number of small-scale horizontal breakdowns occur both before and after the vertical strokes at $t = 5.4$, 8.9, 13.5 and 16.8 s. For this model, the generator current was chosen to be ten times the minimum breakdown current $I_0$ for the equilibrium charge configuration calculated by limiting the horizontal electric fields in the cloud and solving the steady-state equations. As discussed in Section 3, this gives only a very rough approximation to the actual value of $I_0$ in (18) because the static and dynamic charge configurations are quite different due to displacement currents and charge redistribution by lightning. Steady-state equations are very cheap and easy to solve, however, and it is convenient to have even an approximate value of $I_0$ before making dynamic calculations so that a suitable range of generator currents can be chosen. If this is not done, large amounts of computer time can be wasted on runs for which the generator currents are not large enough to produce lightning or for which breakdown occurs at unrealistic rates. As can be seen, the time between strokes, most easily seen as the times for which $E_C/E_B = 1$, is quite variable and lower than the 4.85 s predicted for the approximate value of $I_0$ and $t_B$ as given by (18).

To examine the effect of generator current on the time between strokes, a similar model was taken with only one generator from 4 to 8 km at $r = 100$ m and a series of runs made for different generator currents with the ratio of the restoration to breakdown electric fields again taken to be 0.1. The results of these calculations are shown in Fig. 5. As can be seen there is a very considerable scatter in the times between strokes, especially at the smaller generator currents. The values of $t_B$ calculated from (18) are also plotted for three values of $I_0$ with $k = 0.1$. It can be seen that the values

**Fig. 4.** Buildup of the electric field at the ground, below the cloud, and of the resistive component of the current density to the ionosphere.

**Fig. 5.** Effect of generator current on the time between strokes.
of \( I_0 \) that give best agreement with the average times increased from 2 to 2.5 A as the generator current was reduced from 20 to 5 A.

It is thus apparent that the dynamic charge distributions at breakdown fluctuate from one time to another and depend on the generator current. They must consequently be different under static and dynamic conditions and it is important to understand these differences. Fig. 6 shows the voltage patterns around a cloud model with upper and lower generator terminals at 8 and 4 km. On the left-hand side of the figure, the voltage contours are plotted for the equilibrium case, with the generator current density held uniform over the 100, 300 and 900 m radii to reduce horizontal fields below the breakdown value. On the right-hand side, the contours have been plotted for a time just before the cloud broke down from top to bottom. As can be seen in Fig. 6, the horizontal extent of the charged region is considerably larger for the dynamic case, even though the axial electric fields within the cloud are the same. In the dynamic case, the voltages on the upper part of the cloud are considerably larger and the voltages on the lower part of the cloud considerably smaller than in the static case. This affects the magnitudes of the currents to the ground and to the ionosphere.

Figure 6 shows the charge distribution at only one time, that just before breakdown. During breakdown, very large changes occur in the electric fields as charge is transferred along the arc. While the modeling of the arc itself is primitive, the magnitudes of the charges transferred are approximately correct, and an idea of the charge redistribution processes that must occur can be obtained from a study of a sequence of electric field contours.

Figure 7 shows the electric field patterns within a 7 km radius and up to 16 km during a vertical breakdown in the cloud. For each of the six frames, the contour intervals are spaced by 50 MV. Breakdown is shown by crosses. The ionospheric potential was taken to be 200 kV. The current generator is located at the center of the circle representing the cloud and extends from...
4 to 8 km at \( r = 0 \). For this series of simulations the breakdown field was assumed to be 200 kV m\(^{-1}\) everywhere.

Times in milliseconds are shown for each frame. These depend on the ratio of the resistance values before and after switches operate on breakdown. For this simulation a value of \( 10^6 \) was chosen for reasons having to do with the stability and accuracy of the matrix solutions so that the times are relevant only to the model and not to the atmosphere.

During this time period eleven different breakdowns and recoveries took place, only five of which are shown here. In Fig. 7a the potential contours are those just before and after the vertical breakdown, shown by the series of crosses from 4 to 8 km. Fourteen horizontal breakdowns which occurred since the last vertical breakdown have expanded the region at which the electric field exceeds 25 kV m\(^{-1}\) to a region of \( \sim 6 \) km diameter about the upper generator terminal and 14 km about the lower. In Fig. 7b 4.9 ms after the stroke, the electric fields have decreased by about 25%. Charge removed from the immediate vicinity of the upper terminal has reduced the voltage there, causing an increased electric field in this region and horizontal breakdown. At 9.2 ms in Fig. 7c the electric fields around the upper charge center have been reduced below the breakdown value, but the radial field around the lower charge center causes breakdown there. Continual charge removal from the upper charge center reduces the potential around the top of the cloud and by 9.8 ms the electric breakdown extends to 0.9 km from the axis as shown in Fig. 7d. In Fig. 7e at 13.5 ms the breakdown again occurs around the lower terminal. By 20 ms the electric fields everywhere have fallen below the assumed restoration value of 20 kV m\(^{-1}\). Toroidal rings of charge 2 km in diameter remain around the upper, and 6 km in diameter about the lower, terminals. Following the recovery, the centers are charged up once more by the generator at a rate dependent on the assumed generator current.

Electrical breakdown in the cloud can thus extend the region of charge buildup considerably beyond the region of charge separation. Charge transfer along an arc can cause electric fields to be built up in the region from which the charge has been removed. These electric fields exceed those necessary for breakdown and considerably expand the region of the cloud from which charge is transferred. Even in the present very simple model, there is a great deal of randomness due to the large number of possible breakdown locations and much greater fluctuations must occur in nature. It is thus apparent that the charge distribution in a thundercloud can be very complex indeed, even for very simple driving current geometries such as might correspond to a single column of charged ice particles, for example. Any dynamic model of the electric fields must in some way include these effects.

**b. Effect of the height of the cloud above the ground**

The heights of the upper and lower charge centers in clouds vary considerably with location and from one storm to another. It is thus interesting to examine how the observable electrical properties are related to these variables. For this series of calculations a cloud model with a 4 km vertical separation between the generator terminals was used. Fig. 8 shows the ratio of some parameters to the cloud breakdown electric...
field as the lower terminal was raised from 2 to 7 km and the upper terminal from 6 to 11 km.

Two sets of data are plotted, one for the equilibrium case with \( I = I_0 \) and the second taken from dynamic runs after conditions had stabilized and at a time just prior to vertical breakdown in the cloud. Because the model exhibits a large number of horizontal breakdowns between vertical breakdowns, the actual conditions vary from time to time; however, the values given are reasonably typical.

Three parameters are plotted. The ratio of the magnitudes of the electric field at the ground directly under the cloud to that in the cloud, \( |E_b|/|E_c| \), gives an indication of surface measurements that would be expected in the absence of corona. The ratio of the magnitude of the electric field just below the cloud to that in the cloud, \( |E_{cb}|/|E_c| \), gives an idea of the relative probabilities of cloud-to-ground and vertical cloud breakdowns. The ratio of the resistive component of the current density to the ionosphere directly above the cloud to the breakdown electric field in the cloud, \( J_d/|E_c| \), gives a measure of the currents to the ionosphere which are important in producing the “fair weather field.”

With a constant cloud geometry the voltage between the top and bottom of the cloud for a given cloud breakdown field is essentially independent of altitude because the scale height of the conductivity variations is not strongly altitude dependent. Raising the cloud thus decreases the currents and electric fields below the cloud and increases those above it. This is quite evident in the behavior of \( |E_b|/|E_c| \) and \( J_d/|E_c| \) and in the equilibrium values of \( E_p \). For the 2 km lower generator terminal height, cloud-to-ground strokes occur frequently in the dynamic model calculations, and these reduce the ground electric field considerably because of the large time constants at these altitudes. Even for clouds with a lower charge center as high as 7 km, the electric field at the ground is about 0.38 of the field in the bottom of cloud at breakdown. As has been shown by Freier (1978) and others, fields actually measured at the ground are much smaller than this, indicating that coronal effects play an important role close to the ground (Standler, 1980).

As was pointed out in Section 3, the time constants below (above) the cloud are longer (shorter) than the times between breakdowns when \( I > 2I_0 \). The general effect of this is to reduce the electric fields below the cloud and to increase them above it.

The ratio of \( |E_b|/|E_c| \) is of interest because of the role it plays in controlling the ratio of the number of cloud-to-ground and vertical cloud strokes. For the equilibrium model the ratio is about unity at 4 km, so that for currents close to \( I_0 \) clouds above this altitude would break down more often in a vertical cloud mode and those below would break down from cloud to ground. In the dynamic case with \( I = 10I_0 \) this altitude drops to \( \sim 2 \) km, so that cloud-to-ground strokes would be expected to be very rare at normal cloud altitudes with this cloud generator geometry. In practice, considerable randomness is introduced in the model behavior by the variability of the times of horizontal breakdown in the cloud. In the real situation, there will be considerable variability in local charge densities, so that even more randomness would be expected to occur.

The ratio of the resistive component of current density above the cloud to the ionosphere, \( J_d/|E_c| \), increases from \( 3.43 \times 10^{-16} \) for \( I = I_0 \) to \( 3.16 \times 10^{-15} \) for \( I = 10I_0 \) in the dynamic case for a cloud top at 8 km. It is thus very important to take into account dynamic effects in all ionospheric current calculations. In the dynamic case, \( J_d/|E_c| \) increases from \( 2.35 \times 10^{-15} \) for a cloud top at 6 km to \( 6.21 \times 10^{-15} \) for a top at 11 km. While this variation is by no means negligible, cloud altitude is not obviously a source of major uncertainty in estimating currents to the ionosphere.

As the cloud altitude increases, the relaxation time constant \( T \) decreases, so that the generator current \( I_0 \) increases. As the cloud is raised from 2–6 km to 7–11 km, \( I_0/|E_B| \) increases from \( 3.8 \times 10^{-6} \) mho m to \( 14.2 \times 10^{-6} \) mho m. The minimum current \( I_0 \) required to produce lightning can be calculated by multiplying these figures by the breakdown electric field, so that for \( E_p = 0.2 \) MV m\(^{-1}\) these would correspond to 0.76 and 2.84 A, respectively. Fig. 9 shows the ranges of the times between breakdowns which occurred in the model simulations, plotted as a function of the heights of the generator terminals. Values calculated from Eq. (18) are also shown, based on \( I_0 \) values derived from equilibrium calculations with the models. As can be seen, the upper limit on the times generally corresponds to the theoretical values; hence, the average times correspond to somewhat larger currents and stored charge than would be predicted from equilibrium models. Because of the large differences between the static and dynamic charge distributions shown in Fig. 6, this is perhaps not surprising. Both cloud-to-ground and bottom-to-top breakdown occurred with the 2 km lower height and the values shown represent times between breakdowns in either mode. The shorter time between strokes for alternate modes of breakdown is presumably because cloud-to-ground strokes will not remove as much charge from the top of the lower charge center, nor bottom-to-top of the cloud strokes remove as much from the bottom.

Figure 3 shows, with the symbol ×, the ratio of \( I_0/|E_B| \) for the six models, derived from equilibrium calculations and superimposed on the theoretical ratios from (19) for a simple ellipsoidal monopole with an assumed value of \( a \) of 1 km. Because the height between the upper and lower charge centers in the cloud models remained constant for this series of calculations, the shape and size of the two charge distributions would be expected to remain approximately constant. It is
seen that the actual values for the equilibrium cases lie between those for ellipsoids with \( m \) values between 2 and 3. The effect of the ground image charges is presumably reflected in the lower values of \( I_0/E_b \) for the lower heights. Dynamic distributions are both variable and different from these static values. A first-order approximation to the value of \( I_0 \) can, however, be estimated in this way that is sufficiently accurate for choosing a generator magnitude for an initial dynamic calculation.

c. Effect of the vertical separation of the charge centers of the cloud

For this series of calculations, the height of the midpoint between the two generator terminals was kept at 6 km. Fig. 10 shows the ratio of some of the parameters to the cloud breakdown electric field as the upper terminal is raised from 2 to 5 km and the vertical separation between the two generator terminals is decreased from 8 to 2 km.

For a given electric field within the cloud, the potential across the cloud increases as the separation between the charges in the cloud increases. For this reason, the greater the lower altitude of the cloud and the smaller the vertical separation of the charge centers, the smaller are the electric fields at the ground, below the cloud, and the current to the ionosphere. The percentage change in the ratio of the current densities above the cloud to \( E_c \) is larger than for the cloud with a constant 4 km vertical separation because larger upper-terminal heights correspond to larger potential differences across the cloud.

As with the previous comparisons, the dynamic simulations show smaller fields below the cloud than those based on equilibrium calculations and much larger fields and currents above it. For a separation of less than about 4 km for the equilibrium case with \( I = I_0 \), the electric field below the cloud is smaller in magnitude than that in the cloud, so that breakdown between the top and bottom of the cloud would be
expected to be the dominant breakdown mode with the field below the cloud. This altitude decreases to \( \sim 2.5 \) km for the dynamic case with \( I = 10I_0 \) so that this simple generator model will not reproduce cloud-to-ground strokes at normal thundercloud altitudes. This factor will be treated more completely in Section 4, however.

The electric fields at the ground would be expected to range from 0.32 times the electric field in the lower portions of the cloud for a 2 km separation to 0.6 times the electric field just below the cloud for 8 km vertical separation, in the dynamic case neglecting the effects of corona.

The resistive component of the current density to the ionosphere ranges from \( 1.12 \times 10^{-15} \) times the cloud breakdown field for a 2 km separation to \( 8.84 \times 10^{-15} \) for 8 km separation in the dynamic case with \( I = 10I_0 \). In the equilibrium case, the corresponding values were only \( 1.07 \times 10^{-16} \) and \( 1.13 \times 10^{-15} \).

As the altitude of the lower current terminal decreases, the relaxation time constant increases and the charge and electric field for a given current increase correspondingly. The current needed to provide a given breakdown electric field decreases with decreasing altitude for the lower charge center and thus, in this study, as the vertical separation increases. Fig. 11 shows the range of times between breakdowns compared with theoretical values based on equilibrium calculations from Eq. (18), as described in the previous section. As before, the upper bound on the times corresponds to the charge distribution of the equilibrium calculations except for the cloud with its lower terminal at 2 km, where, as before, both cloud-to-ground and vertical cloud breakdown occurred. The reason for this appears to be in the differing shapes of the charge distributions after the two types of breakdown. This apparently increases the effective value of \( k \) in Eq. (18) when the two types of breakdown occur alternately.

The square in Fig. 3 shows the ratio of \( I_0/E_B \) for this series of models. As can be seen, the ratio \( m \) of the major to the minor axes of the ellipsoid monopole that had the same \( I_0/E_B \) ratio decreased from \( \sim 3.5 \) for the cloud with a separation of 8 km between its upper and lower terminals to about 1.5 for the 2 km separation. These are of the same order as the 4 and 1 that might be crudely estimated, based on a rough approximation to the geometry.

d. Effect of cloud conductivity

There is good reason to believe that the conductivity within a thundercloud may be different from that of the surrounding air. Kasemir (1959) has suggested that a factor of 3 increase in conductivity is not unreasonable and that this could have important effects on the external fields produced by the cloud. Other considerations involving aerosol particles or low mobility ions lead to the possibility of a reduced conductivity in the cloud. It seemed of interest to examine the importance of this parameter in view of these suggestions and their possible relevance.

To model this effect, the values of the resistances of those branches representing the cloud conductivities were modified. For this purpose, the horizontal and vertical resistive elements between the upper and lower generator terminals out to 0.9 km radius were multiplied by a factor that varied from 10 to 0.1. A cloud model was chosen with upper and lower terminals at 8 and 4 km, respectively, and the generator current was held constant at 10 A.

Figure 12 shows the effect of cloud conductivity on three externally observable electrical parameters of the storm. The ratio of the magnitudes of the electric field below the cloud on the ground to the electric field in the bottom of the cloud just at breakdown, \( |E_g|/|E_c| \), is shown on the left block. This parameter gives an
indication of the magnitude of the effects likely to be observable on the ground. As is apparent, an increase by a factor of 100 in the cloud conductivity only increases this ratio from 0.67 to 0.81 for the configuration used.

A similar lack of sensitivity to cloud conductivity is shown by the ratio of the magnitudes of the electric field just below the cloud to that in the cloud at breakdown, $|E_b|/|E_c|$. For the same factor of 100 increase, the ratio only increases from 0.50 to 0.64. It would thus appear that the ratio of cloud-to-ground to vertical cloud strokes is unlikely to be very sensitive to cloud conductivity.

The ratio of the resistive component of the current density to the ionosphere above the cloud $J_t$ to the magnitude of $E_c$ gives an idea of the effect of cloud conductivity on the currents that are believed to produce the fair weather field. As can be seen from the graph on the right-hand side of Fig. 12, this ratio increases from $4.0 \times 10^{-15}$ to $4.6 \times 10^{-15}$ as the cloud conductivity is increased from one-tenth to ten times the value outside the cloud. When this is compared with the effects on this parameter of the height of the cloud, the result is surprisingly small.

The largest externally observable effect of the variation in cloud conductivity is in the time between strokes. There are two effects which partially compensate one another; a reduction in conductivity reduces the relaxation time constant, thus reducing the time between strokes. At the same time, the reduced cloud conductivity changes the charge distribution, reducing $I_0$. The net result, for the model studied here, was an increase from 2.6 to 5.9 s as the conductivity in the cloud was increased from one-tenth to ten times the ambient conductivity.

While only one cloud configuration has been treated here, it seems to be apparent that cloud conductivity is a most unimportant parameter in the relation of the thundercloud current generation and voltage breakdown mechanisms to the externally observable electrical parameters. In this series of calculations, the current generator in the cloud was held constant and all parameters calculated were normalized to the breakdown electric field in the cloud. This helps to explain the insensitivity because the voltage between the top and bottom of the cloud is dependent only on the breakdown electric field and the generator configuration. How this voltage is divided between the regions above and below the cloud is determined mainly by impedances external to the cloud. The resistive component of the current density to the ionosphere above the cloud is determined mainly by the magnitude of the current generator itself and the impedances external to the cloud, unless the cloud conductivities become very small indeed.

It does not seem, therefore, that the internal cloud conductivity is either an important factor, or an important unknown, in understanding the external fields of thunderclouds.

e. Effect of precipitation current

In models considered previously, the current generator has been included only between the top and bottom of the cloud. In physical terms, this might be considered to represent heavy particles, such as hail, being charged at the top of the cloud, separating gravitationally from lighter particles of opposite charge and falling through the cloud. For the generator in the cloud model to be valid, all the net charge must be removed at the bottom of the cloud. While several studies indicate that the current is indeed smaller below the cloud, there is little evidence that it is zero. Indeed, it is difficult to conceive of mechanisms that would result in complete neutralization. It therefore seemed appropriate to study some alternative generator configurations.
The same model used for the study described in Section 4d was employed with the cloud located from 4 to 8 km. The ratio of the precipitation current $I_p$ below the cloud to the cloud current $I_c$ was varied from 0 to 1. Some care had to be taken in the selection of the generator currents to ensure that the effective $I/I_0$ in Eq. (18) remained reasonably constant; otherwise, either the breakdown field would not be reached or strokes would occur at an unreasonably fast rate. As long as breakdown is initiated at the bottom of the cloud, what is of importance is the net current to this generator terminal. For this reason, for $I_p/I_c$ between 0 and 0.8, the difference $I_c - I_p$ was kept constant at 10 A. For $I_p/I_c$ larger than 0.8, this results in unreasonably large currents because the largest fields then occur at the upper terminal. For these calculations $I_c$ was maintained at 50 A.

The results of the calculations are shown in Fig. 13. As can be seen, the ratio of the electric field $E_g$ at the ground to the field $E_c$ in the bottom of the cloud at breakdown decreases in magnitude from −0.71 for $I_p = 0$ to zero for $I_p$ approximately half $I_c$. For larger precipitation currents, the electric field on the ground and in the cloud are of the same sign. The ratio $E_g/E_c$ approaches 0.8 as the precipitation and cloud currents become equal. At these larger rain currents, there are rather large differences between electric fields at the ground when breakdown takes place, because of differing charge distributions for successive strokes.

The ratio of the electric field below the cloud, at 2 km, to the field in the bottom of the cloud at breakdown also changes sign at a ratio of $I_p/I_c$ of ~0.5. This means, of course, that in cases where approximately half of the current is neutralized at the bottom of the cloud, the fields below the cloud would be small and cloud-to-ground strokes would presumably not occur.

The ratio of the resistive component $J_r$ of the current density to the field $E_c$ in the ionosphere above the cloud increases as the precipitation current becomes a larger fraction of the total cloud current. This is as would be expected because the negative charge which is produced by the difference in the two currents serves to neutralize the effect of the positive upper charge above the cloud. The ratio of the current densities to $E_c$ is approximately proportional to the increase in $I_c$.

The reason for the small electric field below the cloud for the intermediate cases can be easily seen by examining Eq. (8). As the precipitation current increases to become a major fraction of the cloud current, the charge in the bottom of the cloud becomes small compared with that in the upper charge center. This causes the electric fields below the lower charge center to decrease to zero and then become in phase with the electric field between the upper and lower charge centers.

Figure 14 shows the potential contours for a range of $I_p/I_c$ values between 0 and 1. These results are very interesting because of the restrictions placed on thundercloud models by the behavior of the ratio $E_g/E_c$. The presence of cloud-to-ground strokes implies either a configuration in which almost all the charge is neutralized at the bottom of the cloud or one in which almost all of the charge flows from the top of the cloud to the ground. Intermediate cases would result in insufficient field below the cloud for cloud-to-ground breakdown. The two configurations are, however, very different indeed with a monopole field in the second case and a dipole plus a monopole of the reverse sign in the first. The requirements on the generator current are approximately five times as large in the second case and the current needed to supply the fair weather field is larger by a corresponding amount.

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**Fig. 13.** Effect of the ratio of the precipitation current below the cloud to the cloud current, on the electric field on the ground and below the cloud and on the resistive component of the current density to the ionosphere.
f. Effect of neutralization of the precipitation charge below the cloud

It may be assumed that the heavy negative ions, such as rain or hail, falling through the cloud are charged either on formation or in their descent through the cloud. Unless these charges are neutralized instantaneously at the base of the cloud, they will continue to carry a current vertically below the cloud. In Section 4e the consequences of such currents were examined.

There also exists the possibility that the precipitation could be neutralized continuously as it falls to the ground. To examine this effect, the current generators were modified by including horizontal and vertical generators to simulate a neutralization current of $K$ A km$^{-1}$. Thus, $K = 0$ corresponds to no effective recombination with ions being formed at 8 km and descending uniformly to the ground at constant velocity and is identical to the case studied in Section 4e in which $I_p = I_c$. The generator current $I_c$ was maintained constant at 10 A so that $K = 2.5$ A km$^{-1}$ corresponds to complete neutralization by the time the precipitation reaches the ground.

Figure 15 shows the electric field at the ground and under the cloud and the resistive component of the current density above the cloud to the ionosphere, all normalized to the electric field in the cloud, as a function of $K$. For values of $K$ between 1 and 4 A km$^{-1}$, breakdown occurs sometimes between the top and bottom of the cloud and sometimes from the ground up. Corresponding values for examples of times when each of these types of breakdown occurred are shown in Fig. 15 with different symbols. As can be seen, only in two cases do the electric fields at the ground always become small. This happens when there is essentially no recombination or neutralization and when the precipitation is almost completely neutralized at the bottom of the cloud. In the cases where there is substantial neutralization of the rain charge below the cloud, the electric fields at the ground sometimes become much larger than they are in the cloud or elsewhere; thus breakdown would occur at the ground between 0 and 1 km but not ascend to the cloud. The reason for this is that when the rain becomes neutralized, the current terminates since the charges are no longer being transported to the ground by gravity. This leads to a buildup of charge between the cloud and the ground. Because these observations are so different from what is normally seen in the way of electric field distributions or lightning breakdown, it does not appear as though either charging or neutralization between the cloud and ground are important under normal circumstances.

g. Cloud-to-ground strokes

As is apparent from the calculations in the preceding sections, the magnitude of the electric field below the thundercloud is generally less than that in the thundercloud for most of the models studied. For these conditions, cloud-to-ground strokes would not occur. There are, however, several ways in which cloud-to-ground breakdown can be induced.

If the lower cloud terminal is below 2.5 km, cloud-to-ground breakdown occurs for the standard model, even with a generator from the bottom to the top of the cloud of 10$I_0$ uniformly distributed over 1.6 km from the axis. This cloud height, however, is lower than is normal for typical thunderclouds.

The thundercloud model in which no neutralization takes place at the bottom of the cloud also breaks
down in a cloud-to-ground mode. In this case, a single current generator from the ground to the top of the thundercloud is used and breakdown starts at the top of the cloud and continues to the ground. Despite the inherent simplicity and attractiveness of the model, it does seem to be contradicted by a large body of electric field measurements in and around thunderclouds.

Our model is discrete. The maximum electric field is calculated as the current density through the branch divided by the smallest conductivity along the path represented by the branch. This gives a good estimate of breakdown conditions for upward breakdown such as takes place in the cloud, but tends to underestimate the maximum electric field for downward breakdown when close to the charge center. It is probable that a somewhat lower equivalent electric field should be used just below the cloud. Another unknown is the ratio of the actual breakdown electric field within the cloud to that below it. Electric field and charge measurements within thunderclouds consistently show a positive charge center at the base of the cloud. This charge center increases the electric fields below the cloud and considerably extends the range of conditions under which cloud-to-ground breakdown occurs.

When the generator current is increased to $\sim 15I_0$ and is confined to within 200 m radius from the axis, charging takes place very rapidly; the charge distributions and electric fields change often, due to frequent horizontal breakdowns and cloud-to-ground breakdown occurs for 4–8 km altitude clouds. The fluctuations introduced in this way in the model probably occur at much lower current levels in real thunderclouds, where the local variations from place to place must be very large.

Figure 16 shows the potential contours at six times during a cloud-to-ground stroke out to a radius of 7 km and up to 16 km. The standard cloud model was used with a single generator of 15 A at $r = 0$ from the bottom at 4 km, to the top of the cloud at 8 km. In all cases the contours are spaced by 50 MV. The breakdown field is assumed to be 200 kV m$^{-1}$ everywhere and breakdown is shown by crosses. The ionospheric potential was assumed to be 200 kV, as in the other studies.

In the first frame at $t = 0$, the potential contours are those just before the stroke and the region at $r = 0$ from 2 to 4 km has reached the breakdown field. At 4.11 ms the region down to 1 km exceeds the breakdown field and breakdown is extending radially at the bottom of the cloud. At 5.76 ms, the breakdown extends to the ground and the radial breakdown is extinguished. At 8.77 ms radial breakdown again occurs at the base of the cloud. At 20.44 ms, the electric fields are reduced below the maintenance value and the arcs are extinguished everywhere. As was stated before, the time scale during breakdown is controlled by the assumptions about the ratio of the resistances between nodes after breakdown to that before breakdown. While the general rearrangement of charge is believed to be accurate, the time scale is not. The series of voltage profiles does show that though breakdown around the charge centers serves to extend the region from which current flows to the arc, this process cannot be complete. Some charge centers located at a distance from the major charge center are likely to retain considerable remnant charge after the arcs are extinguished.

5. Conclusions

This paper describes the first results obtained with a new type of dynamic electrical model of a thundercloud that allows the charge rearrangement produced in arc breakdown, as well as the conduction and displacement currents, to be calculated with realistic generator configurations. Such models are important because they provide the link between the convective dynamic and microphysical storm models and the observable electrical parameters of the thundercloud.
The model demonstrates quite clearly the great complexity of behavior of thunderclouds due to the interaction of the nonlinear breakdown mechanisms, energy stored in the electric field, and a conductivity that varies with altitude. It is also apparent that dynamic charge distributions and electric fields are quite different from static distributions. These differences affect the initial conditions before and after lightning strokes, and it is apparent that this has not been adequately recognized in many previous analyses of thundercloud behavior.

This paper is meant as an introduction to the type of studies possible with a model of this sort and not as a comprehensive examination of thundercloud electrical behavior. The model in its present form is very limited in spatial resolution; however, much larger numbers of nodes could be included in a program of this type. Since the calculations described were made, a series of checks has been conducted using a modified version of the program in which the spatial resolution was doubled. These tests did not indicate major quantitative differences in any of the numbers presented here.

The switches used to simulate arc breakdown appear to handle the redistribution of charge produced by lightning strokes in an adequate manner, but much remains to be done in either parameterizing the results of lightning stroke models or actually including realistic lightning stroke models in a reconfigured model. Until either of these pieces of work is accomplished, quantitative estimates of charge transfer are subject to some uncertainty.

Only very simple generator geometries have been considered and little attention has been paid to relating them to actual storms. The generator currents have been held constant and when changes occur in times that are short compared with the electrical time constants, more complicated simulations are required. The results have been presented in normalized forms whenever possible and some simple approximate theoretical relations are given which should allow the estimation of observable parameters to be scaled for other conditions.

Some of the conclusions that can be drawn from these preliminary studies are:

1) Simple generator geometries, such as would correspond to a single charged precipitation channel, can produce very complex charge distributions. The reason for this is that lightning produces extremely large divergences in current density with corresponding structure in the charge distribution. This effect can make it very difficult to interpret observed charge distributions in terms of cloud electrification mechanisms. The current carried by the precipitation channel is thus much more closely related to the physics of cloud electrification than the charges and electric fields.

2) As shown in Section 4, the conduction current density to the ionosphere is very much larger in the dynamic cases than in static simulations. It is not independent of the generator current, as it is in the equilibrium models, but is approximately proportional to it. This is because the electrical relaxation time high above the cloud is short compared with the time between strokes, while below the cloud it is much longer and there, displacement and lightning currents are important in completing the electrical circuit. Large negative current pulses to the ionosphere take place on strokes between the top and bottom of the cloud and large positive pulses for cloud-to-ground strokes. Ionospheric heating is dependent on the square of the current and the pulses of either polarity are equally effective. As shown in Sections 4b and 4c, currents to the ionosphere increase greatly as the height of the top of the cloud is raised.

3) Such basic properties of thunderclouds as the production of cloud-to-ground strokes appear to be compatible only with a very limited range of thundercloud models. When models are considered in which the charge neutralization at the base of the cloud
is incomplete, the electric fields become very small and cloud-to-ground breakdown would not take place. When a major portion of the precipitation current is neutralized between the cloud and ground, strokes from the ground up occur which do not extend to the cloud. Apart from the case when the charge was neutralized at the bottom of the cloud, only in the case where all the charged particles reached the ground did cloud-to-ground strokes occur. For this case, the charge and voltage configurations were quite different from measurements by Evans (1969), Winn et al. (1974) and others. It does not seem, therefore, as though much current can be carried by the precipitation below the cloud or that much neutralization of the precipitation below the cloud can take place.

4) Cloud conductivity is not a simple parameter due to the presence of different ions and nonlinear processes such as corona. Nevertheless, it does not seem to be an important unknown in the study of the external electrical behavior of a thundercloud. While it would be interesting to conduct more realistic simulations and to study more cases than were considered here, it seems unlikely that this conclusion can be greatly in error. The reason for the insensitivity appears to be that displacement and conduction currents in the medium surrounding the cloud greatly dominate the currents flowing in the cloud itself. It is the total potential across the cloud that controls the electric fields external to the cloud and this is controlled by the cloud dimensions and the breakdown field.

5) Coronal and convection currents cause the electric fields at the surface to be much smaller than they would be in their absence. As was shown in Sections 4b and 4c, the heights of the upper and lower charge centers of the cloud affect the electric field both below the cloud and on the ground much less in the dynamic simulations than in the static calculations, when all parameters are normalized to the breakdown field.

As a result, the electrification processes are much more directly related to the total current density at the surface under a thundercloud than they are to either the conduction current density or the electric field at the surface.

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APPENDIX

Derivation of the Electric Field at the Surface of an Ellipsoidal Monopole

We consider an ellipsoidal monopole with uniform charge density \( \rho \), circular horizontal cross section, and semi-minor axis \( a \) and semi-major axis \( b \) in a vertical plane. If the vertical axis is in the \( z \) direction, the coordinates of the surface are given by

\[
\frac{z^2}{b^2} + \frac{r_0^2}{a^2} = 1,
\]

where \( r_0 \) is the distance from the \( z \) axis. The electric field at \( z = b \) can be calculated using Coulomb's law,

\[
E_z = \frac{\rho}{4\epsilon} \int_{r = b} \int_{r = 0} \frac{2(b - z) r dz dr}{(r^2 + (b - z)^2)^{3/2}}.
\]

Integrating Eq. (A2) with respect to \( r \) gives

\[
E_z = -\frac{\rho}{2\epsilon} \int_{-b}^{b} \frac{(b - z) dz}{[z^2(1 - (a/b)^2) - 2b z + (a^2 + b^2)^{1/2}]} - dz,
\]

where \( n = 1 - (a/b)^2 \). Eq. (A4) may now be integrated with respect to \( z \) to give

\[
E_z = \frac{\rho b c}{\epsilon} \left( 1 - \frac{1}{n} \right)
\times \left[ 1 - \frac{1}{2n^{1/2}} \log \left| \frac{1 + 2n^{1/2} + n}{1 - n} \right| \right],
\]

where

\[
c = \left( 1 - \frac{1}{n} \right) \left[ 1 - \frac{1}{2n^{1/2}} \log \left| \frac{1 + 2n^{1/2} + n}{1 - n} \right| \right],
\]

and the value of the charge stored is

\[
Q = \int_{z = -b}^{b} \int_{r = 0}^{r_0} 2\rho r dz dr,
\]

\[
Q = \frac{4}{3} \pi a^2 \rho b.
\]

From Eq. (16), the magnitude of the charge stored at breakdown is related to the magnitude of the generator current \( I_0 \) that will just produce breakdown, i.e.,

\[
Q_B = |I_0| T.
\]

Substituting (A9) and (A10) in (A5) gives, for breakdown at \( z = b \),

\[
\frac{|I_0|}{E_B} = \frac{4 \pi \epsilon a^2}{3 c T},
\]

where \( T \) is the relaxation time constant at the breakdown location, \( E_B \) is the breakdown electric field and \( c \) is given in Eq. (A7). Values of \( |I_0|/E_B \) for various altitudes and values of \( m = b/a \) are shown in Fig. 3.
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