NOTES AND CORRESPONDENCE

A Delayed Action Oscillator for ENSO

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ABSTRACT

A simple nonlinear model is proposed for the El Niño/Southern Oscillation (ENSO) phenomenon. Its key feature is the inclusion of oceanic wave transit effects through a negative, delayed feedback. A linear stability analysis and numerical results are presented to show that the period of the oscillation is typically several times the delay. It is argued such an effect can account for the long time scale of ENSO.

1. Introduction

The purpose of this note is to propose a simple nonlinear model for the ENSO phenomenon. The model relies on the existence of a strong positive feedback in the coupled ocean–atmosphere system and on some unspecified nonlinear mechanism invoked to limit the growth of unstable perturbations. Its key element is the inclusion of the effects of equatorially trapped oceanic waves propagating in a closed basin through a time delayed term. This simple system has multiple stationary states which can all become unstable. When this happens, solutions are self-sustained oscillations whose period is at least twice as long as the assumed delay. We offer this model as an explanation for the results of simple circulation models (Cane and Zebiak 1985; Schopf and Suarez 1988) that produce periodic or nearly periodic behavior, and as a candidate mechanism for ENSO.

2. The model

The model we envision is sketched in Fig. 1. It assumes the strongest coupling takes place in the central portion of the basin and that an important side effect of growing perturbations in this region is the emission of weakly coupled, westward propagating oceanic signals that, after reflecting from the western boundary, return and recouple to the atmosphere. If a coupled mode is growing strongly, the delayed effect of the returning wave will be small. If, on the other hand, the mode has begun to equilibrate, the returning signal (depending on how much has been attenuated) can be an important perturbation. In this section we try to justify this view and discuss in more detail how these mechanisms may act in ENSO.

a. Coupled feedback

The amplitude of observed El Niño variability at low frequency requires that there exist some strong positive feedback mechanism to balance the restoring effects of surface heat exchanges on SST anomalies. Linear models that isolate such mechanisms have been discussed by Lau (1981), Philander et al. (1984), Yamagata (1985) and Hirst (1985). In these models, the feedback arises from the coupling of the tropical ocean and atmoshere: ocean temperature perturbations, produced by advective processes, result in atmospheric heating and wind responses that drive ocean currents so as to enhance the original perturbations. The growth rate of the instabilities in simple models depends on the product of the strength of the two legs of this feedback: how sensitive is the atmospheric heating to SST anomalies and how sensitive are the SSTs to the resulting wind anomalies.

The east–west structure of the ocean in the tropical Pacific is characterized by large gradients in SST and thermocline depth. In the western Pacific, where surface waters are warm, conditions are favorable for SST anomalies to influence the atmosphere, but because the thermocline and the mixed layer are deep and horizontal SST gradients are small, wind anomalies can have only a small effect on the temperature. In the eastern Pacific, where waters are cold along the coast and in a tongue on the equator, the situation is reversed. With a shallow thermocline and large horizontal gradients, advection (both vertical and horizontal) can

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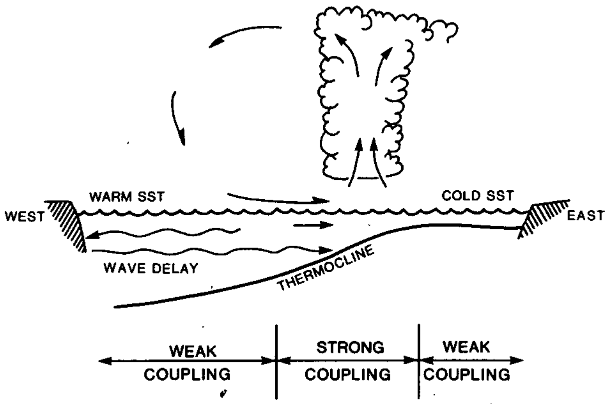

FIG. 1. Model schematic. In the central to eastern part of the basin, a strong positive feedback loop exists between SST anomalies, heating anomalies and surface wind anomalies. In the west, the wind-driven ocean perturbations do not affect SST; but rather perturb the deep thermocline. Uncoupled or weakly coupled waves travel to the western boundary and back to the east, where they affect the SST.

easily produce SST anomalies, but over cold surface waters convective heating in the atmosphere will be less sensitive to SST. We may thus expect the coupled feedback to be most effective for an intermediate region, with largest SST and wind anomalies in the central Pacific.

We assume first that we may think of these coupled processes as acting locally in such a region, and that they can be characterized by a single amplifying mode, with SST anomalies on the equator, and precipitation anomalies producing maximum surface wind anomalies near the center of the basin.

b. Nonlinear effects

As such a mode amplifies from an assumed equilibrium state, nonlinear effects will come into play to limit its growth. The two obvious candidates for this limiting role are advective processes in the ocean and moist processes (availability of water or other nonlinear limitations on the CISK-like amplification of convection) in the atmosphere. The former is the dominant effect in both Cane and Zebiak (1985) and Schopf and Suarez (1988), although the details of the process are not clear in either case. Partly because of this uncertainty, and partly to keep things as simple as possible, we assume the nonlinear form

$$dT/dt = kT - bT^3.$$

Here T represents the amplitude of the growing disturbance, k its growth rate and bT^3 all nonlinear effects acting on it at finite amplitude. Scaling time by k^{-1} and T by $(k/b)^{1/2}$, we have

$$dT/dt = T - T^3. \tag{1}$$

Equation (1) has three stationary states: the assumed unstable state at $T = 0$ and symmetric stable states at $T = 1$. A bifurcation into three stationary states (which occurs here at $k = 0$) is a common feature of these very dynamical models.

c. Wave propagations

As we argued above, coupling effects may be weak in the western Pacific. Whether or not this may be true in nature, it is certainly true in the oscillatory solutions of Schopf and Suarez (1987) and to a lesser degree in those of Cane and Zebiak (1985). Thus, let us for the moment assume that all variations in the western portion of the basin are the uncoupled response emanating from the coupled action in the eastern portion of the basin. We now rely on the picture presented by Schopf and Suarez (1988) for such a situation: The wind anomalies induced by SST perturbations drive Rossby waves on the ocean thermocline. These propagate westward from the coupled region, and on reaching the western boundary, reflect into eastward propagating equatorial Kelvin waves. During this transit to the western boundary, information carried by the signals is "hidden" from the coupled problem. Upon returning to the central/eastern portion of the basin, however, the thermocline displacements have an increasing effect on SST, due to the shallow mean thermocline depth there and the strong horizontal gradients in SST. We may then think of these signals "reentering" the coupled problem after a time delay equal to their transit time.

Thus the equation for coupled perturbations must include, in addition to the coupled feedback and nonlinear terms in (1), a term that represents the effect of these delayed signals. We do this as follows:

$$dT/dt = T - T^3 - \alpha T(t - \delta), \tag{2}$$

where δ is the nondimensional delay (wave transit time), and α measures the influence of the returning signal *relative* to that of the local feedback. To adopt a more precise interpretation of (2), we take the growth term to represent signals indistinguishable, except for the timing of their source, from those represented by the delay term; and that all other "local" effects produce a net damping that is accounted for in the cubic term. Since we expect information to be lost in transit by dissipation and imperfect reflection at the western boundary we will consider only $|\alpha| < 1$. Also, following the arguments presented in Schopf and Suarez (1988), this term represents a negative (delayed) feedback (i.e., $\alpha > 0$). To see this consider, say, a warm SST perturbation in the coupled region. This produces a westerly wind response that enhances the perturbation by tending to deepen the thermocline locally (the coupled positive feedback effect). But these same wind perturbations produce *divergent* westward propagating signals that on returning will tend to produce upwelling and cooling, damping the original perturbation.

In Schopf and Suarez (1988) we pointed out that one might think of these processes as a phase-reversing reflection off the coupled region: a "warming" signal enters from the west and a "cooling" signal is reemitted by wind perturbations produced by the incident signal. We referred to this as "coupled reflection". Because the western reflection (at the rigid boundary) is phase-preserving and the coupled reflection is phase-reversing, it will take two transits to the western boundary and back to return to the original phase. We thus expect the period of oscillation associated with these propagation properties to be no less than *twice* the transit time. We referred to this effect as "period doubling". We will show below that oscillatory solutions of (2) in the range $0 < \alpha < 1$ have periods *longer* than 2δ.

3. Linear stability analysis

Numerical investigation of (2) can be readily undertaken, and will be presented below. As is often the case with such problems, however, an analysis of the linear stability of the equation provides a good indication of the properties of the fully nonlinear system. Here, we consider the linearized equation over the parameter range relevant to our problem: $0 < \alpha < 1$ and $\delta > 0$.

In addition to the stationary solution $T = 0$ (the "inner" stationary state), the model possesses two additional stationary states:

$$T_o = \pm(1 - \alpha)^{1/2} \quad \text{for} \quad \alpha < 1. \tag{3}$$

We will examine only the stability of the outer (T_o) solutions. The inner solution always has at least one unstable nonoscillatory mode in this parameter range, so that small perturbations from it will grow, but do not oscillate. In the absence of another instability, solutions would settle to either the "cold" or "warm" stationary state (3). But depending on α and δ, these "outer" solutions may themselves be unstable and in this case both the growing perturbations and the asymptotic solutions are oscillatory.

Neutral curves for perturbations of the outer solutions. For $T = T_o$, perturbations from the stationary solution obey

$$dT'/dt = (3\alpha - 2)T' - \alpha T'(t - \delta). \tag{4}$$

From (4) we can again emphasize that $\alpha < 1$ does not imply that *net* local effects are more important than delayed effects. In the interval $\frac{1}{2} < \alpha < 1$, the coefficient on the local term is always smaller than that on the delayed term. Seeking solutions of the form $T' = T \times \exp(\sigma t)$, with complex $\sigma = \sigma_r + i\sigma_i$, we obtain

$$\sigma = (3\alpha - 2) - \alpha e^{-\sigma\delta}, \tag{5}$$

or

$$\sigma_r = (3\alpha - 2) - \alpha \cos(\sigma_i\delta)e^{-\sigma_r\delta}, \tag{6}$$

$$\sigma_i = \alpha \sin(\sigma_i\delta)e^{-\sigma_r\delta}. \tag{7}$$

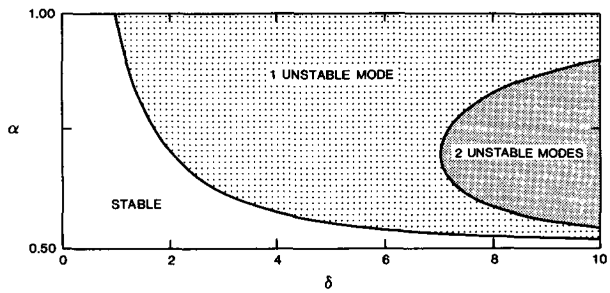

FIG. 2. Neutral stability curves of the outer stationary solution. Parameters lying below the lower line are stable. An infinite number of additional neutral curves exist to the right of the lines shown, but are only found for large δ.

The neutral curves ($\sigma_r = 0$) are those for which

$$\delta = a \cos\{(3\alpha - 2)/\alpha\}/\sigma_i(\alpha), \tag{8}$$

$$\sigma_i(\alpha) = \{\alpha^2 - (2 - 3\alpha)^2\}^{1/2}. \tag{9}$$

For a given value of α in the interval $\frac{1}{2} < \alpha < 1$, there are infinitely many neutral curves, but for $\delta < 10$ the only neutral curves are those shown in Fig. 2. It is easy to show that all cases with (α, δ) lying below the lower curve are stable. This curve asymptotes to $\alpha = \frac{1}{2}$ for large δ, and $\alpha = 1/\delta$ for δ near 1, lying everywhere above these two lines.

It is important to note that the neutral solutions have a finite frequency. Figure 3 shows the period of the solution along the lower neutral curve ($\sigma_1(\delta)$ obtained from (8) and (9)), expressed as a multiple of the delay. This period is always at least twice as long as the delay, and may be much longer.

4. Numerical solutions

We explored the behavior of the nonlinear system (2) numerically. Three typical solutions are shown in Fig. 4. These are for $\alpha = 0.75$ and $\delta = 2$, 6 and 10 (these points are marked on Fig. 5). For large values of δ (Fig. 4c), the solutions approach a square wave,

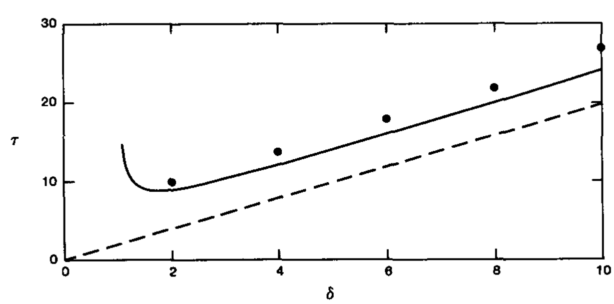

FIG. 3. Period of the neutral solutions to (4) as a function of the delay. The period is everywhere greater than 2δ. Dots indicate numerically determined periods for the full nonlinear model near neutrality.

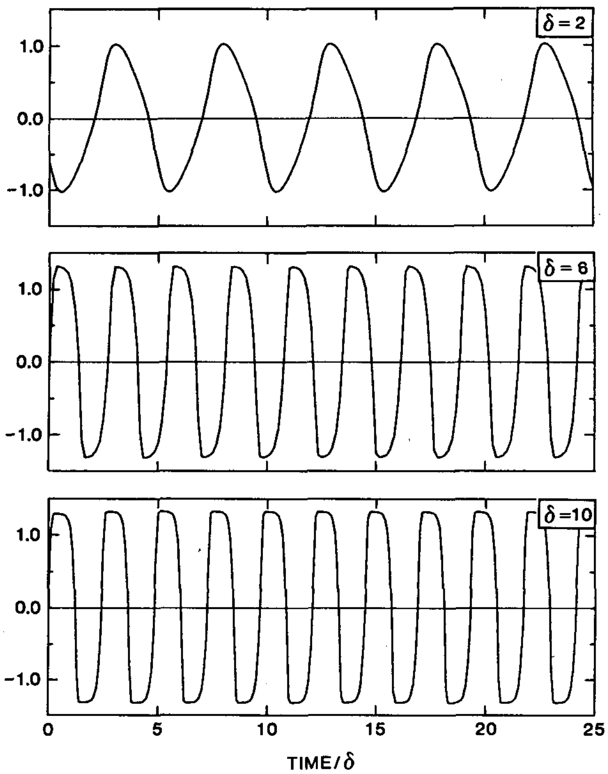

FIG. 4. Behavior of the nonlinear oscillator. (a) $\alpha = 0.75$, $\delta = 2$, (b) $\alpha = 0.75$, $\delta = 6$, and (c) $\alpha = 0.75$, $\delta = 10$. The time axis is scaled in units of the delay.

with a period of twice the delay. For smaller δ the period is longer and solutions more sinusoidal. This is clear from the solution, shown in Fig. 4a, which is near the neutral curve.

Figure 5 shows the periods obtained from many integrations of (2) over the α–δ plane. Contours are only shown for that region where oscillatory solutions were found, which is bounded by the lower neutral curve and $\alpha = 1$. The solid lines give the period in nondimensional time units, while the dashed contours present the same period as multiples of the delay time. The longest oscillations for a given delay are found for the weakest return (smallest α). For $\delta < 6$ all oscillatory solutions have periods in excess of 2.5 times the delay.

The full nonlinear solutions agree resonably well with the linear stability analysis near the neutral curve. Markers on Fig. 3 show the periods of the numerical solutions at several values of δ. For each we use the smallest value of α at which oscillatory behavior was obtained.

Since in (1) the time is scaled by the e-folding time of the linearly growing mode (k^{-1}), a weaker growth for the same delay is equivalent to smaller δ, and therefore longer period oscillations. To fix ideas, consider a case with $k^{-1} = 50$ days and a delay of 400 days, that is $\delta = 8$. If we assume $\alpha = 0.6$, we obtain from Fig. 5 a period of about 2.75 times the delay, or 3 years. If the e-folding time is increased to 100 days while the dimensional delay and α are kept the same, δ is reduced to 4 and the result is a period about 3.5 times as long as the delay, or nearly 4 years.

5. Discussion

The model proposed attempts to account for a strong selection of a time scale of 2 to 4 years by coupled motions in the tropical Pacific. Such selection occurs in coupled numerical models (Schopf and Suarez 1988; Cane and Zebiak 1985), and is at least suggested by the observations (Rasmusson and Carpenter 1982) for ENSO.

The model assumes that the time scale must be related to the propagation times of equatorially trapped oceanic waves in a closed basin. Solutions depend on only two parameters: the ratio of a wave transit time to the e-folding time of the underlying coupled instability (δ); and the *relative* importance of the local and delayed wave effects (α). Oscillatory solutions occur

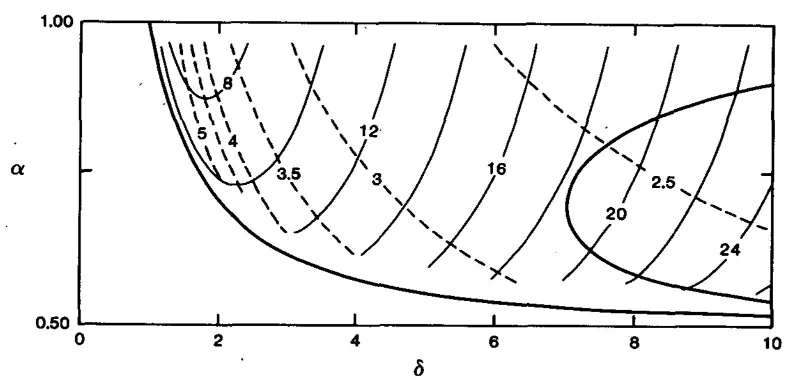

FIG. 5. The fundamental period of the nonlinear oscillator found numerically. The heavy solid lines are the neutral curves of the linear problem, reproduced from Fig. 2. The light solid contours give the period of the oscillation ($2\pi/\sigma_i$), while the dashed contours present the period in multiples of the delay.

if the delayed effects are large enough ($\alpha > 1/2$) *and* the delay is sufficiently long (roughly, $\alpha\delta > 1$).

The period of the oscillation is several times the transit time from the coupled region to the western boundary and back. If we express the period as a multiple of the delay, we find that stronger delayed effects (greater α) or longer delays (greater δ) decrease this multiple. At very long delays (short *e*-folding times) the model flip-flops with a period approaching twice the delay.

Can this model account for the 2–4 year time scale of ENSO? There is substantial room for discussion on this point. The gravest baroclinic mode in the tropical Pacific has a gravity wave speed of roughly 2.5 m s^{-1}. If the center of action for the coupling is near the center of the basin, waves must travel 8000 km or so to the western boundary. The first meridional mode Rossby wave will take roughly 110 days to cover this distance, and the reflected wave will take another 40 days to return. Doubling or quadrupling this 150 day delay then results in a period of less than 2 years. On the other hand, a significant part of the signal may travel in higher meridional mode Rossby waves (which would travel at $1/7$ or $1/11$ the speed of the Kelvin wave), or in the second vertical mode, with a slower gravity wave speed. This choice of a slow gravity wave speed (1.2 to 1.4 m s^{-1}) for El Niño studies has been made by Philander et al. (1984), Anderson and McCreary (1985), and Hirst (1985). These effects can lengthen the delay to a year or more, and result in a period of 2–4 years. Calculations with general circulation models should provide a more conclusive answer to how the influence of wind anomalies in the central Pacific propagates to the western boundary and then back to influence the SST.

Another area where detailed modeling can test the assumptions made here is in showing an appropriate value for α. If the reflected signal is strongly attenuated in transit or is scattered at the western boundary, or if it returns with such a vertical structure as to have a much smaller effect on SST than the locally forced motions in the central region, the effective α may be small enough to preclude oscillatory solutions by this mechanism. One way to estimate α would be to attempt to separate the "local" and "reflected" effects on the SSTs produced by ocean GCMs during wind-burst experiments. Another and more direct approach would be to selectively damp the westward propagating signals in a model that normally exhibits ENSO variability.

Finally, we wish to discuss the drastic assumption of a localized coupled problem. By making this assumption, we can do without a detailed description (partial differential equations) of the information (waves) leaving the region of interest, and replace it with a simple statement of when (δ) and with what effects (α) the information reenters our problem. This very economical statement is obviously an oversimplification of processes acting in ENSO. A number of objections easily come to mind: some degree of coupling will occur over the whole basin; the eastern boundary, whose presence we have glossed over in this discussion, will play some role to cloud the distinction between local and delayed effects; and nonlinear effects may not fit as neatly as we have assumed into the local/delayed description of the wave propagation. Still we contend that (2) may be a useful idealization of the essentials of a time scale selection in the ENSO problem, and may be of more general interest.

In coupled problems we are interested primarily in what happens at the ocean–atmosphere interface. Any other occurrences need to be described only insofar as they affect the ocean surface (unless of course they are the final object of our prediction). For the atmosphere, with its short time scales, we can probably think of information from the surface as being transmitted instantaneously over the domain. But in the oceans, it may often be useful to think of information as being temporarily sequestered, so that it manifests itself with a time delay. Under conditions akin to those presented here, oscillatory solutions may arise.

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