Multiparameter Radar Modeling and Observations of Melting Ice

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ABSTRACT

This paper uses a microphysically detailed graupel and hail melting model, described by Rasmussen and Heymsfield, which is coupled to a radar model that computes multiparameter variables such as differential reflectivity, linear depolarization ratio, the specific propagation differential phase shift and X-band specific attenuation. The microphysical model is initialized with two different summer-time sounding profiles (Colorado and Alabama). Sensitivity studies are performed with respect to particle shape and orientation distributions. The hail melting model is also initialized with a summertime sounding from the Munich, FRG area, and C-band differential reflectivity is computed for application to radar data from the DFVLR radar. A simple spherical hail melting model is also used to study the effects of absorption and scattering on the X-band attenuation. NCAR CP-2 radar measurements from the MIST (Microburst and Severe Thunderstorm) project and from CINDE (Convective Initiation and Downburst Experiment) are used to illustrate the usefulness of multiparameter data in studying the melting of ice in convective storms.

1. Introduction

Downdrafts that cause strongly divergent and damaging winds near the surface pose a serious problem for aviation safety leading to many studies of the downburst phenomena, Fujita (1985). Wet downbursts are accompanied by an intense precipitation shaft. Srivastava (1987) has used a numerical model to study intense downdrafts forced by the melting and evaporation of precipitation. He found that the melting and evaporation of precipitation and precipitation loading below cloud base were sufficient to produce wet downbursts. Cooling due to ice melting was found to be concentrated in a narrow layer of the atmosphere. He also found that, “As the stability of the lapse rate is increased, higher precipitation contents, precipitation in the form of ice, and relatively lower concentrations of small precipitation particles are required to force an intense downdraft”.

Multiparameter radar measurements based on dual-polarization and dual-frequency techniques (in addition to conventional Doppler parameters) can play an important role in the remote sensing of the microphysics of the precipitation downdraft. Wakimoto and Bringi (1988) and Tuttle et al. (1989) have used the NCAR CP-2 multiparameter radar as the central tool in studying the microphysical evolution of an intense microburst-producing storm during the summer of 1986 near Huntsville, Alabama. They found that the differential reflectivity (Z_{DR}) and the specific attenuation at X-band (A_X) were important measureables related to the microphysical evolution of the cloud. In particular, the vertical profiles of Z_{DR} and A_X within the storm core could be related to the precipitation content, type and phase, well before the strongly divergent winds appeared at the surface.

Both dual-polarized and dual-frequency techniques are now fairly well established and can yield significantly more microphysical information than the conventional reflectivity factor. Hall et al. (1980, 1984) were the first to show that Z_{DR} could be used to differentiate between rain and ice, while Bringi et al. (1986a) used a coupled graupel melting and radar model together with aircraft 2D-PMS data to show that the vertical profile of Z_{DR} was an excellent indicator of the onset and progression of melting ice into raindrops. Hail detection using the Z_{DR} technique is well documented by Bringi et al. (1984, 1986b) and Illingworth et al. (1987). The measurement of attenuation using dual-frequency methods is documented in Eccles and Mueller (1973), Eccles (1979), and Tuttle and Rinehart (1983). In rainfall and at long wavelengths Z_{DR} is an excellent estimator of the reflectivity-weighted mean axis ratio of the raindrops filling the radar resolution volume, while the X-band specific attenuation is related to approximately the fourth moment of the raindrop size distribution, Jameson (1983a,b). Recently, the specific propagation differential phase shift (K_{DR}) in rainfall has been measured using an algorithm derived by Mueller (1984) at the National Severe Storms Laboratory (NSSL), Sachidananda and Zrnić (1986, 1987) and by Golestani et al. (1989) using the CP-2 radar. This parameter is related to nearly the 4th mo-

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ment of the raindrop size distribution, at long wavelengths, Jameson (1985). The above mentioned multiparameter observables show marked differences in the ice and melting ice regions because of the differences in shape, orientation (i.e., fall mode) and dielectric constant of the ice and melting ice particles as compared to raindrops. Thus profiles of these observables as a function of height below the 0°C level should be a sensitive indicator of the melting process, which potentially could be important in the study of microphysical factors responsible for the forcing of intense downdrafts.

In this paper we use the graupel and hail melting model developed by Rasmussen et al. (1984) and Rasmussen and Heymsfield (1987) and couple it to a radar model to calculate the vertical profiles of multiparameter observables such as reflectivity, \(Z_{DR}\), \(A_X\), LDR and \(K_P\). These parameters are chosen to correspond to the measurement capabilities of the NCAR CP-2 radar. The detailed microphysical melting model of Rasmussen and Heymsfield (1987) provides information on particle size, density, fraction of ice melted, temperature and axis ratio (assuming oblate shape). The input parameters are the sounding data and the initial particle type and density. We have considered only graupel and small hail with sizes < 12 mm, since for these sizes shedding of the meltwater does not occur. In the case of hail we assume that the initial shapes are spherical, and that upon melting the water forms a liquid coat around the ice core, the shape being oblate at all times. For graupel, the melt water initially percolates into the particle and soaks it. Upon complete soaking, additional melting proceeds as in the hail case. The initial shape for graupel is assumed to be conical as in Brungi et al. (1986a), and becomes oblate when the mean density equals 0.9 g cm\(^{-3}\). The key data on particle orientation is not provided by the model but wind-tunnel observations show that particle stability increases with melting. Thus, we have assumed that the particle orientation distribution is Gaussian with mean = 0° and standard deviation (\(\sigma\)) linearly decreasing with height below the 0°C level and reaching a value of 5°–10° for raindrops. The melting model results are sensitive to the initial particle diameter, the temperature profile, the relative humidity profile (especially below cloud base) and vertical air motions which are assumed to be zero in this study. Sounding data from both Colorado and Alabama (summer time convection) have been used to study differences in melting profiles for warm and cold-based clouds. The radar model used is based on computing the principal plane backscatter matrix elements of homogeneous and twolayer bodies using the T-matrix method, Brungi and Seliga (1977). The integration over shape, size and orientation distributions is described by Brungi et al. (1986a). The effects of variable dielectric constants (\(\epsilon_r\)) for different particle types based on their composition and temperature are important; we have used mixture formulas as proposed by Bohren and Barten (1981) to calculate \(\epsilon\), using the melting model results.

Our paper is organized as follows: we first present results of sensitivity studies on the vertical profiles of \(Z_{DR}\) and LDR with respect to particle shapes and orientation distributions followed by a comparison of graupel and hail particle melting under two different sounding profiles, viz., Colorado and Alabama. The hail melting model is also run using a summer time sounding from the Munich, West Germany area in order to compute multiparameter radar profiles at C-band for application to interpretation of \(Z_{DR}\) data obtained with the DFVLR polarimetric radar. Schroth et al. (1988). To understand the effects of hail melting on the X-band specific attenuation, a simple (spherical) ice model is used to show the contribution of scattering and absorption to the X-band attenuation. Finally, we use the model results as a basis for interpreting CP-2 radar data taken from the MIST project, (Dodge et al. 1986) and the Convective Initiation and Downburst Experiment (CINDE, Wilson et al. 1988) conducted near Denver during the summer of 1987.

2. Radar model

The scattering geometry adapted from Holt (1984) is shown in Fig. 1. The particle symmetry axis \(\vec{N}\) is assumed to be located in the \(YZ\)-plane \(\phi = \pi/2\), so that \(\theta\) describes the polar canting angle. The angle \(\alpha\) is the canting angle in the polarization plane. For simplicity we assume that the incident wave with amplitudes \([E^v_{inc}, E^b_{inc}]\) propagates along the negative \(X\)-axis so that \(\psi = 0\). The principal plane backscatter matrix elements \(S_{11}, S_{22}\) are defined by

\[
\begin{bmatrix}
E_b^v \\
E_b^b
\end{bmatrix} =
\begin{bmatrix}
S_{11} & 0 \\
0 & S_{22}
\end{bmatrix}
\begin{bmatrix}
E_{inc}^v \\
E_{inc}^b
\end{bmatrix}
\]  

(1)

FIG. 1. Scattering geometry adapted from Holt (1984). \(\phi\) is the polar canting angle while \(\alpha\) is the canting angle in the polarization plane. We assume \(\phi = \pi/2, \psi = 0\) in the calculations.
where the backscatter amplitudes are $E_b^v, E_b^h$ and the geometry is such that $\bar{N}$ is aligned along the Z-axis. Let $N(D)$ be the particle size distribution ($D$ being an equivalent spherical diameter), and $p(\theta)$ be the probability distribution function of the angle $\theta$ (assumed Gaussian with $\bar{\theta} \sim 0^\circ$, standard deviation $\sigma$). The multiparameter radar observables can be defined as follows, Holt (1984):

\[
Z_{vv} = |C|^2 \int N(D) dD \times \int |S_{11} \cos^2 \theta + S_{22} \sin^2 \theta|^2 p(\theta) d\theta \tag{2a}
\]

\[
Z_{hh} = |C|^2 \int N(D) dD \times \int |S_{11} \sin^2 \theta + S_{22} \cos^2 \theta|^2 p(\theta) d\theta \tag{2b}
\]

\[
Z_{hv} = |C|^2 \int N(D) dD \times \int |S_{11} - S_{22}|^2 \cos^2 \theta \sin^2 \theta p(\theta) d\theta \tag{3}
\]

where $|C|^2 = \lambda^4 |K|^2 / \pi^2$ and $K = (\epsilon_2 - 1)/(\epsilon_2 + 2)$, $\epsilon_2$ being the dielectric constant and $\lambda$ the wavelength.

The differential reflectivity is defined as,

\[
Z_{DR} = 10 \log(Z_{hv}/Z_{vv}) \tag{4}
\]

and the linear depolarization ratio is defined as

\[
LDR = 10 \log(Z_{hv}/Z_{hh}). \tag{5}
\]

The particle orientation model used here is a one-dimensional approximation to the more realistic two-dimensional orientation model $p(\theta, \phi)$ described by Metcalf (1988). We assume $\phi = \pi/2$ for simplicity whereas a uniform distribution for $\phi$ in the interval $(0, 2\pi)$ is more realistic. Holt (1984) has shown that the very little error is incurred between the two $p$ distributions for raindrops which are strongly oriented along the vertical direction, i.e., $\sigma \ll 5^\circ$. We have investigated the effects of using the more rigorous two-dimensional description for $p(\theta, \phi)$ assuming a uniform $p$-distribution and a Gaussian distribution for $p(\theta)$ including integration of the form shown in Eqs. (2) and (3) with $d\theta$ replaced by the elemental solid angle $d\Omega = \sin \theta d\theta d\phi$.

Our computations show that for $\sigma < 15^\circ$, there is no effect on $Z_{DR}$ relative to the one-dimensional $p(\theta)$ with $\phi = \pi/2$. For $15^\circ \leq \sigma \leq 45^\circ$, $Z_{DR}$ is reduced by about 0.2 to 0.3 dB. For LDR, the values are reduced by 2 to 3 dB. We do not consider $\sigma > 45^\circ$ in this paper.

The propagation model is based on Oguchi (1983). Assume that the propagation path is filled with scatterers as shown in Fig. 1. Let the principal plane forward scattered amplitudes be $f_{11}$ and $f_{22}$ corresponding to the same geometry used to define $S_{11}, S_{22}$. Along the incidence direction (assumed to be the negative X-axis) the differential equation satisfied by the coherent wave electric field components $E^v, E^h$ is

\[
\begin{bmatrix}
\frac{dE^v}{dx} \\
\frac{dE^h}{dx}
\end{bmatrix} =
\begin{bmatrix}
M_{vv} & M_{vh} \\
M_{hv} & M_{hh}
\end{bmatrix}
\begin{bmatrix}
E^v \\
E^h
\end{bmatrix} 
\tag{6}
\]

where

\[
M_{vv} = jk_0 + j(2\pi/k_0) \int N(D) dD \times \int [f_{11} \cos^2 \theta + f_{22} \sin^2 \theta] p(\theta) d\theta \tag{7a}
\]

\[
M_{hh} = jk_0 + j(2\pi/k_0) \int N(D) dD \times \int [f_{11} \sin^2 \theta + f_{22} \cos^2 \theta] p(\theta) d\theta \tag{7b}
\]

\[
M_{vh} = M_{hv} = jk_0 + j(2\pi/k_0) \int N(D) dD \times \int (f_{11} - f_{22}) \sin \theta \cos \theta p(\theta) d\theta \tag{7c}
\]

and

\[
k_0 = 2\pi/\lambda_0
\]

The solution to Eq. (6) is given by the usual procedure of solving simultaneous linear differential equations that use eigenvalues and eigenvectors of the matrix $\mathbf{M}$, Oguchi (1983). The transmission matrix $\mathbf{T}$ can be written as,

\[
\mathbf{T} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\tag{8}
\]

whose elements are given by Oguchi (1983) in terms of the eigenvalues of the matrix $\mathbf{M}$. The absolute attenuation at V or H polarization is given by $|T_{11}|$ or $|T_{22}|$, respectively, while the differential phase shift between H and V states is given by arg($T_{22} - T_{11}$). By suitable normalization the X-band specific attention is denoted by $A_X$, dB km$^{-1}$ while the S-band specific differential phase shift is denoted by $K_{DP}$, deg km$^{-1}$.

3. Model computations

a. Sensitivity to shape and orientation

Graupel melting studies were previously considered by Bringi et al. (1986a) who assumed conical shapes and a Gaussian distribution for $p(\theta)$ with $\bar{\theta} = 0^\circ$, and $\sigma$ decreasing linearly with height from $\sigma = 45^\circ$ at $z = 3.3$ km to $\sigma = 10^\circ$ at $z = 2.1$ km. Note that all heights denoted by $z$ are above mean sea level. The initial graupel density varies with size based on Knight.
and Heymsfield (1983). The sounding data were obtained from aircraft measurements taken on 28 May 1983 during the MAYPOLE '83 (May Polarization Experiment) field program. The maximum graupel size was 8 mm and the initial size spectrum was obtained from 2D-PMS precipitation probe images at $z = 3.3$ km through a convective cloud; see Bracing et al. (1986a) for details concerning this case.

Sensitivity of the vertical profiles of $Z_{HH}$, $Z_{DR}$ and LDR to the particle shape assumption is now considered by assuming the initial graupel shape to be spherical with diameter equal to the equivolumic spherical diameter of the cones. All other parameters are kept the same. Figure 2a shows the reflectivity profile which is insensitive to the particle shape assumption, as expected. Figure 2b shows the sensitivity results for $Z_{DR}$ and LDR. The dashed curves refer to initial spherical shapes for the graupel while the solid curves refer to initial conical shapes. For conical graupel in the melting stages the shape is assumed to depend on both water fraction (i.e., mass of ice melted) and mean particle density ($\rho$). If the water fraction $< 30\%$ and $\rho < 0.9$ g cm$^{-3}$ then the shape is maintained conical. If $\rho > 0.9$ then the shape is oblate spheroidal. If the water fraction $> 30\%$ and $\rho < 0.9$ the conical graupel is assumed to have a thin water coat, i.e., the shape is spherical (with homogeneous or with concentric water coat) if $\rho < 0.9$ g cm$^{-3}$, and oblate if $\rho > 0.9$. In Fig. 2b we note that the $Z_{DR}$ profiles are quite different showing shape sensitivity as expected. The maximum difference occurs at $z = 2.5$ km where $Z_{DR}$ (conical) $\sim 1.1$ dB while $Z_{DR}$ (sphere) $\sim 0$ dB. More significantly, the slope of $Z_{DR}$ with height differs considerably between the two cases. The LDR profiles also differ substantially, the LDR (cone) values exceeding the LDR (sphere) values by up to 12-15 dB. At the lowest level where complete melting has occurred, the $Z_{DR}$ and LDR curves converge, as expected. As shown by Bracing et al. (1986a) the slope of $Z_{DR}$ with height as given by the cone shapes is in much better agreement with radar measurements than the spherical shape assumption.

Next, we consider sensitivity to the parameter $\sigma$ which is the standard deviation of the canting angle distribution. Two linearly decreasing $\sigma$ profiles are considered for the conical graupel case. This approximation is based on the fact that melt water stabilizes particle orientation for $D \leq 12$ mm, Rasmussen et al. (1984). The maximum value for $\sigma$ at 3.3 km height is chosen as 45° and 30° for the two cases; the smallest value for $\sigma$ is 10° at the lowest height. Figure 3 shows the sensitivity results for $Z_{DR}$ and LDR. The $Z_{DR}$ curves do not differ significantly, the maximum difference being $\sim 0.5$ dB at 2-3 km. The LDR curves also do not show much sensitivity for $z < 3$ km, the maximum difference being $\sim 2$ dB.

b. Sensitivity to sounding profiles

As mentioned by Rasmussen and Heymsfield (1987) the melting model is most sensitive to initial particle density and the sounding profile, especially the relative humidity below cloud base. We now consider conical graupel melting under two different sounding profiles.
one taken from Colorado during MAYPOLE '83 (the 28 May case study reported by Bringi et al. 1986a), and one taken from the MIST project. Figure 4 shows sounding data for the MIST case. The 0°C height, the cloud base height, and cloud base temperature are, respectively, 4.5 km, 1.9 km and 12.5°C. The MAYPOLE '83 temperature and relative humidity profiles were derived from in situ aircraft measurements, see Table 1 of Bringi et al. (1986a). Thus, we consider the sensitivity of typical-sized graupel particles of the same initial density as they melt under two different sounding profiles and the corresponding effects on the $\sigma_{\text{DR}}$ profiles, where $\sigma_{\text{DR}}$ now refers to $Z_{\text{DR}}$ for the monodisperse case, i.e., no integration is performed over the size distribution, see Eqs. (2, 4). Figures 5a, b show the vertical profiles for two initial sizes of graupel, viz., 6 and 10 mm. The 6 mm case does not show much sensitivity to the sounding profile. In each case the particle melts completely by $z \sim 2.5$ km with the final $\sigma_{\text{DR}} \sim 2.5$ dB. The 10 mm case shows considerable sensitivity to the sounding profiles. Complete melting occurs about 500 m higher in the MIST sounding as compared to the MAYPOLE '83 sounding. In the height interval $2.25 < z < 3.0$ km the slope of $\sigma_{\text{DR}}$ versus height is also different between the two cases, e.g., at $z = 2.5$ km the difference in $\sigma_{\text{DR}}$ is around 4 dB which is very significant. The small difference in $\sigma_{\text{DR}}$ between the two soundings after complete melting has occurred is due to temperature differences and its effect on $\epsilon_r$. We can conclude that the slope of $Z_{\text{DR}}$ with height below the 0°C level would be sensitive to the sounding profiles, especially for the larger graupel particles and also for small hailstones.

c. Sensitivity to particle type

We now consider differences in graupel and small hail melting using the MIST sounding of 17 June 1986 shown in Fig. 4. As mentioned previously the initial graupel density and shape (conical) assumptions are different from that assumed for small hail (dia < 12 mm) which has high density and is assumed spherical in shape. Figures 6a, b show profiles of density and water fraction for typical-sized graupel and hail particles, respectively. In Fig. 6a the initial graupel sizes are 6 and 10 mm, with initial density $\sim 0.4$ g cm$^{-3}$. Both particles melt completely by $z = 1.5$ km. In Fig. 6b

FIG. 5. Height profile of $\sigma_{\text{DR}}$ (monodisperse $Z_{\text{DR}}$) for a melting graupel particle of (a) 6 mm initial size and (b) 10 mm initial size under two different soundings, viz., MIST (Alabama) and MAYPOLE (Colorado).
similar profiles are shown for initial hail sizes of 6 and 10 mm with initial density $\sim 0.9 \text{ g cm}^3$. While the 6 mm particle melts completely by $z = 2.5 \text{ km}$, the 10 mm particle has only partially melted with 80% of mass melted at $z = 1.5 \text{ km}$. Figures 6a, b are based on the melting model of Rasmussen and Heymsfield (1987). In Fig. 7 we show the hail shape and composition used in the radar model at the different heights for the 6 and 10 mm hail sizes, respectively. Note that the inner ice core is spherical and concentric with the outer oblate shell of melt water. Complete melting has occurred for the 6 mm particle; the 10 mm particle has a significant ice core as shown in Fig. 6b. The ice core diameter is calculated based on the mass of ice melted at a given height. For conical graupel particles, the shape and composition during melting are identical to that described by Bringi et al. (1986a). Figure 8 shows profiles of $\sigma_{\text{DR}}$ for the graupel and hail particles. The low density graupel particles are completely melted after a fall of about 2 to 2.5 km below the 0°C level, whereas the high density hail particles are nearly completely melted after a fall of about 2.5 to 3.5 km below the 0°C level. Moreover, the smaller sized low density graupel shrinks considerably in size upon melting, e.g., the initial 6 mm size shrinks to 4 mm with $\sigma_{\text{DR}} = 2.4 \text{ dB}$ whereas particles less than 2 mm in size are evaporated. In contrast, the 6 mm hail particle shrinks to 5.8 mm upon complete melting giving a $\sigma_{\text{DR}}$ of 4 dB. The 10 mm graupel shrinks to 7.5 mm upon complete melting (with $\sigma_{\text{DR}} = 4.9 \text{ dB}$), while 10 mm hail shrinks to 8 mm with an inner ice core of diameter $= 1.21 \text{ mm}$. The corresponding $\sigma_{\text{DR}}$ is 5.3 dB.

In order to test the sensitivity of the single particle results to integration over an exponential particle size distribution, we assume an initial two-parameter distribution of the form $N(D) = N_0 \exp[-3.67D/D_0]$ where $N_0 = 2000 \text{ m}^{-3} \text{ mm}^{-1}$, $D_0 = 2.7 \text{ mm}$, and $D_{\text{max}} = 12 \text{ mm}$. Figure 9a shows the reflectivity profile for graupel and hail while Fig. 9b shows $Z_{\text{DR}}$. The initial increase in the reflectivity is due to a thin water coat on the melting ice particles. Subsequent reduction in graupel reflectivity is due to shrinkage in size. Figure 9b shows that the graupel $Z_{\text{DR}}$ exceeds the hail $Z_{\text{DR}}$ for $z > 1 \text{ km}$. The conical graupel melts quickly and assumes
an equilibrium oblate shape, while the spherical hail melts slowly with an inner ice core, the resultant shape being not as oblate.

Figure 9c shows computed profiles of X-band specific attenuation for the graupel and hail cases. The hail attenuation exceeds the graupel attenuation by 0.3 to 0.5 dB km⁻¹, mainly reflecting the higher hail reflectivity and the presence of an ice core in the melting hail particles.

In Fig. 9d we show the S-band specific propagation differential phase shift $K_{DP}$ as a function of height for the hail melting model. The rapid increase in $K_{DP}$ upon melting to a peak value of $\sim 2^\circ$ km⁻¹ at $z = 2$ km is striking. Even at $z = 3.5$ km, $K_{DP} = 0.8^\circ$ km⁻¹ while $Z_{DR} \sim 0$ dB and $A_X \sim 0.8$ dB km⁻¹. By comparing the $A_X$ and $K_{DP}$ profiles for the hail model in Fig. 9d we see that both parameters are sensitive to the onset of melting to a higher degree than $Z_{DR}$.

d. Hail melting profiles at C-band

It is well known that resonance effects start to appear at C-band, especially for oblate raindrops of size 5–7 mm, Holt and Evans (1977). Similar resonance effects also appear for melting hailstones. We have run the hail melting model using a typical summertime sounding from the Munich, West Germany area, see Fig. 10. The 0°C height, cloud base height and cloud base temperature are, respectively, 4.3 km, 1.65 km and 12.5°C. Figure 11a shows $\sigma_{DR}$ versus height for an initial hail size of 7 mm. The melting hail shapes are similar to those shown in Fig. 7. The peak $\sigma_{DR}$ of 8 dB at $z = 1.5$ km is noteworthy. Figure 11a also shows $\sigma_{DR}$ for a raindrop model, assuming there is no ice core during melting, the shape being oblate spheroidal at all times with size and axis ratio equal to that of the hail melting case. Figure 11a clearly shows the effects of resonance.
due to the presence of the ice core. Figure 11b shows a profile of $\sigma_{DR}$ and the inner ice core diameter versus height for an initial hail size of 7 mm. The ice core diameter ($D_{ice}$) is obtained from the hail melting model. At $z = 1.5$ km, the ice core diameter $\sim 0.46$ cm and the axis ratio of the oblate particle as estimated by the melting model is 0.58. Figure 11c shows $\sigma_{DR}$ as a function of the inner ice core diameter, the axis ratio of the melting particle being fixed at 0.58. The ice core is concentric with the outer oblate water shell, and is completely contained within it. Figure 11c shows again the resonance effects at C-band due to the inner ice core. Calculations show that the peak $\sigma_{DR}$ due to an inner ice core occurs for initial hail sizes in the range 6.5–8.0 mm. For initial hail sizes $< 6$ mm and for those in the range 8.5–11 mm there is no observed peak in $\sigma_{DR}$, rather the $\sigma_{DR}$ values increase smoothly to the maximum value (at the lowest height) similar to the profiles shown in Fig. 8.

An English and Cheng (1984) size distribution giving $Z_{HH} \sim 40–45$ dBZ has been chosen for the initial hail size spectrum initialized at $z = 4$ km. The integrated value of $Z_{DR}$ as a function of height is shown in Fig. 12 for $D_{max} = 7.5$ mm and 9 mm. As expected, the peak $Z_{DR} \sim 6.5$ dB occurs at $z \sim 1.5$ km when $D_{max} = 7.5$ mm. When $D_{max}$ is increased to 9 mm, Fig. 12 shows that the $Z_{DR}$ profile no longer shows a peak, rather the $Z_{DR}$ increases to 5 dB near the surface rather smoothly, similar to the $Z_{DR}$ profiles at S-band. These computations show that it may be possible to detect small hail ($D_{max} < 8$ mm) above the melting level by
In this section we use a simplified radar model to compute the X-band absorption from melting ice spheres, Sivhola and Kong (1988). At long wavelengths the effective medium approximation can be used to measuring the vertical profile of C-band $Z_{DR}$ and noting the occurrence of any large values, say 6–8 dB, below the melting level. We have observed such high $Z_{DR}$ values (6–8 dB) with the C-band polarimetric DFVLR radar in the main precipitation shaft of severe thunderstorms, Meischner et al. (1988). (Model runs initialized with graupel particles do not show this resonance behavior). If the maximum hail size exceeds 12–15 mm, then the wind-tunnel data of Rasmussen et al. (1984) indicates that the melt water would be shed off the particle, leading to the conventional $Z_{DR}$ hail signature, Bringi et al. (1984, 1986b).

e. Simplified hail melting model

Measurements of X-band specific attenuation, $A_X$, in downburst-producing storms during the COHMEX project showed that the vertical profile of $A_X$ generally reached a maximum value near the melting level with a sharp decrease below this level, i.e., an attenuation bright band could be detected, see Tuttle et al. (1989). Microwave cavity absorption measurements of melting ice particles by Hansman (1986) showed that the X-band absorption reached a maximum value when 10% (by volume) of the ice had melted. It is well known that this bright band effect disappears when the particle size-to-wavelength ratio is no longer small compared to unity, i.e., it is a Rayleigh limit behavior. From a remote sensing viewpoint, the vertical profile of $A_X$ can be used to detect this bright band feature, and if it exists some deduction can be made regarding the mass-weighted mean diameter of the ice particles relative to the X-band wavelength.

FIG. 12. Height profile of C-band $Z_{DR}$ for maximum hail diameters of 7.5 and 9 mm. Particle size distribution is exponential, English and Cheng (1984).

FIG. 13. (a) X-band absorption loss, (b) X-band total attenuation (scattering plus absorption) with $D_{max} = 10$ mm, and (c) X-band total attenuation with varying $D_{max}$ as a function of melt index, $\nu$, $\nu = 0$ implies ice while $\nu = 1.0$ implies a fully melted raindrop. Marshall-Palmer (1948) size distribution is assumed.
calculate the attenuation due to absorption. Melting ice spheres are treated as two-layer bodies with an inner ice core surrounded by a concentric layer of melt water. The polarizability of such simple two-layer bodies is well known, Bohren and Huffman (1983). We have assumed a Marshall–Palmer (1948) distribution \( N(D) = 8000 \exp(-\Delta D) \) with \( \Delta = 4.1R^{-0.21} \) where \( R \) is the rainrate in mm h\(^{-1} \) and \( D \) is in mm. Figure 13a shows the absorption loss, dB km\(^{-1} \), at X-band as a function of melt index with rain rate (50–200 mm h\(^{-1} \)) as a parameter. Note that the size spectrum is held constant through the melting process. The melt index \( \nu \) is defined as the volume fraction of melt water which is assumed identical for all sizes. The absorption loss peaks at \( \nu = 0.1 \) for the rain rates considered. The absorption for \( \nu = 1 \) (fully melted case) is about a factor of 8 less than the maximum absorption. In Fig. 13b we plot the X-band attenuation (due to both absorption and scattering) as a function of using the Mie theory as

\[
A_X = 0.4343 \int_D \sigma_{\text{ext}}(D)N(D)dD \text{ [dB km}^{-1} \text{]} 
\]

where \( N(D) \) is the Marshall–Palmer size spectrum and \( \sigma_{\text{ext}}(D) \) is the extinction cross section of the two-layer melting ice sphere in cm\(^2\). The inner ice core diameter is expressed as \( D(1-\nu)^{1/3} \) where \( D \) is the overall particle diameter. Figure 13b shows that the maximum attenuation no longer occurs at \( \nu = 0.1 \), rather the \( A_X \) increases rapidly in the range \( 0 < \nu < 0.1 \), and then it saturates in the region \( 0.2 \leq \nu \leq 1.0 \). Comparing Figs. 13a, b we can see that the absorption bright band is a Rayleigh limit effect, i.e., the ice spheres must be small compared to \( \lambda = 3.2 \) cm. In order to show this effect more clearly we have truncated the size spectrum at various values of \( D_{\text{max}} \), with \( D_{\text{max}} \) chosen successively as 1, 3, 5 and 7 mm. The rainrate is conserved at 50 mm h\(^{-1} \) at each \( D_{\text{max}} \) by scaling the \( N_0 \) parameter of the Marshall–Palmer distribution. Fig. 13c shows a plot of \( A_X \) (due to both absorption and scattering) versus melt index with \( D_{\text{max}} \) as a parameter. The bright band feature is enhanced as \( D_{\text{max}} \) is reduced indicating that it is a Rayleigh limit behavior. If the ice spheres are large compared to \( \lambda \) then the scattering loss dominates the total attenuation for \( \nu \gg 0.1 \). We are, of course, assuming a steady state situation in this melting model, as well as in the Rasmussen–Heymsfield melting model considered earlier. Thus, care must be used in translating these deductions to the interpretation of radar observations where unsteady conditions are generally prevalent. In any case, the bright band profile of radar-measured \( A_X \) below the melting level may be used to qualitatively infer the relative abundance of smaller ice particles (say, \( D \approx 3–5 \) mm) as compared to the larger hail.

4. Mutliparameter radar observations

The multiparameter CP-2 radar operated by the National Center for Atmospheric Research (NCAR) is capable of simultaneously measuring \( Z_{\text{HH}}, Z_{\text{DR}} \) and mean Doppler velocity \( \bar{V} \) at S-band. At S-band \( K_{\text{DP}} \) can be estimated if the radar is operated in a time-series mode. The CP-2 X-band radar system can measure reflectivity and LDR. X-band specific attenuation and Hail Signal (HS) can be estimated using the dual-frequency method, Tuttle and Rinehart (1983). The measurement accuracies of \( Z_{\text{HH}}, Z_{\text{DR}} \) and \( A_X \) are estimated to be \( \pm 1 \) dB, \( \pm 0.2 \) dB and \( \pm 0.5 \) dB, respectively. Vertical profiles of these multiparameter observables through storm cores can provide important information about ice melting processes. For example, Figs. 14a, b show profiles of \( Z_{\text{HH}} \) and \( A_X \) versus height through a storm core which occurred on 1 July 1986 of the MIST project. See Wakimoto and Bringi (1988) for a description of the MIST radar network. This storm produced a

![Figure 14](http://journals.ametsoc.org/jas/article-pdf/47/5/549/3425483/1520-0469(1990)047_0549_mrmaoo_2_0_co_2.pdf)

**FIG. 14.** Height profiles of CP-2 radar measured (a) reflectivity and (b) \( A_X \) for the July 1st microburst. Each data point represents the maximum value of the observable over a rectangular box surrounding the storm core. The microburst was detected near the surface at 1624 local time.
downburst as determined by the divergent Doppler velocity signature near the surface at 1624 local time. The peak differential Doppler velocity along the radial direction was about 26 m s\(^{-1}\) over a 4 km distance. The storm core was located northwest of the radar at a range of 16 km. Figure 14a shows the \(Z_{HH}\) profile at two times separated by 12 minutes. Each data point represents the maximum value of \(Z_{HH}\) occurring at a given height over the storm core, Mohr et al. (1986). Between the times 1603 and 1615 the maximum \(Z_{HH}\) has increased from 57 to 63 dBZ. Figure 14b shows the profile of \(A_X\). The increase in maximum \(A_X\) from 2.4 dB km\(^{-1}\) at 1603 to 4.2 dB km\(^{-1}\) at 1615 is noteworthy. At 1615, it is also interesting that \(A_X\) peaks at \(z = 4\) km and falls off rapidly with decrease in height below this level to 1.7 dB km\(^{-1}\) near the surface. This bright band feature in \(A_X\) may be due to small melting hail as discussed in Section 3e.

Another case we have considered is the downburst which occurred on 19 July 1986 of the MIST project. Figures 15a, b show the \(Z_{HH}\) and \(A_X\) profiles through the storm core at 1809 and 1816 local time. Again, each data point corresponds to the maximum value of the radar observable at the particular height. In Fig. 15a the peak \(Z_{HH}\) of 59 dBZ occurs in the height interval 4–5 km. The \(A_X\) has increased from a maximum of 2.3 dB km\(^{-1}\) to 3.6 dB km\(^{-1}\) during the 7 minute time interval. At 1816, the \(A_X\) bright band feature is clearly evident. The downburst occurred at 1820, the storm core being located northwest of the radar at a range of 10 km. The peak differential Doppler velocity was about 26 m s\(^{-1}\) over a 3 km distance.

In both the 1 and 19 July cases there were no occurrences of either the \(Z_{DR}\) hail signature or the dual-frequency hail signal below \(z = 4\) km. \(Z_{DR}\) values near the surface were in the range 3–5 dB. Based on the model calculations presented earlier it is inferred that the \(A_X\) bright band is due to melting ice with typical sizes in the range 3–5 mm. The strong cooling produced by the melting ice phase probably played an important role in driving the downburst in these two cases, Srivastava (1987). Tuttle et al. (1989) have studied the 20 July microburst case of MIST in detail using the CP-2 data to infer the bulk microphysical evolution of the storm. They also observed an \(A_X\) bright-band feature prior to the peak differential velocity at the surface. In their case the maximum \(A_X\) of 4.5 dB km\(^{-1}\) occurred just below the melting level.

We now present color images of multiparameter radar data for the downburst which occurred on 14 July of the MIST project. The color images were obtained using the Research Data and Support System (RDSS) software developed by NCAR. Figures 16a–f show range height indicator (RHI) profiles at IS04 of S-band reflectivity (\(Z_{HH}\)), X-band reflectivity (\(Z_X\)), \(Z_{DR}\), mean Doppler velocity (\(V\)), X-band specific attenuation (\(A_X\)) and attenuation-corrected hail signal (HS), respectively.

The RHI data were taken just prior to the occurrence at 1510 of the peak velocity differential at the surface. Comparing Figs. 16a, b it is clearly evident that \(Z_X\) is significantly affected by path attenuation in the range interval 22–25 km and in the height interval 4–5 km (above the surface), which is near the 0°C level. The 60 dBZ reflectivity core is centered at a range of 20 km and height around 4–5 km. Figure 16c depicts the vertical structure of \(Z_{DR}\) which clearly shows the onset and progression of melting ice into raindrops. \(Z_{DR}\) values for the most part are in the range 1.5–2.5 dB except near the surface at a range of 24 km where \(Z_{DR} \sim 4\) dB. Figure 16d shows the mean Doppler velocity profile. The storm core was located NNW of the radar at a range of 20 km. Radial velocity convergence is clearly seen at the range of 22 km and in the height interval 2–8 km. The peak differential velocity of 25 m s\(^{-1}\) was observed near the surface at 1510 (about 6 minutes after
Fig. 16. RHI color profiles of (a) S-band reflectivity, dBZ, (b) X-band reflectivity, dBZ, (c) \( Z_{DR} \), dB, (d) mean Doppler velocity m s\(^{-1}\), (e) X-band attenuation, dB km\(^{-1}\), and (f) hail signal, dB, taken by the CP-2 radar on 14 July of the MIST project. These data were taken about 6 minutes prior to the detection of a microburst near the surface.
Fig. 17. As in Fig. 16 except the radar data were taken on 24 July 1987 of the CINDE project.
the RHI profiles) along the same azimuthal angle as the RHI images, and at range of 21 km. In Fig. 15d the surface divergence has not reached a maximum; however, a hint of the radial divergence signature may be observed at a range of 17 km. Figure 16e shows the $A_X$ profile obtained by first differencing the $S$ and X-band reflectivities along each beam, and then obtaining the finite difference approximation to the range derivative over a 1 km path. These computations were performed using the RDSS software. The $A_X$ has been subsequently smoothed to reduce statistical fluctuations. An extensive core of high $A_X$ can be seen between 2–5 km height. $A_X$ bright bands can be observed at different heights in the range interval 20–24 km. Figure 16f shows the hail signal after attenuation correction, Tuttle and Rinehart (1983). No hail signal is noted in the region of interest, i.e., less than 4 km height. Thus, the hail sizes are less than about 10 mm in regions where $HS < 3$ dB. A small region of $HS > 3$ dB is noted in Fig. 16f in the height interval 5–6 km. In the region of high $A_X$, the $Z_{DR}$ values range from 14.2–2.4 dB, with $Z_{HH}$ of 50–55 dBZ. These data are consistent with the interpretation that the $A_X$ bright band is caused by melting of high concentrations of smaller-sized hail particles.

In Figs. 17a-f we show RHI color images of CP-2 radar data taken on 24 July 1987 during CINDE (Convective Initiation and Downburst Experiment). The RHI is taken through the core of a severe storm that developed 50 km east of the radar (which was located near Boulder, CO). Figure 17a shows the $Z_{HH}$ profile while Fig. 17b shows the $Z_X$ profile. Two 65 dBZ reflectivity cores may be observed at ranges of 48 and 54 km. The X-band signal is severely attenuated. Figure 17c depicts the vertical structure of $Z_{DR}$, the melting level being clearly delineated. Maximum $Z_{DR} \approx 3.4$ dB occurs near the surface. At a range of 53–55 km, a depression in the $Z_{DR}$ melting level can be observed. This feature is related to the 65 dBZ reflectivity core at 54 km range, see Fig. 17a. Figure 17d shows the $P'$ profile where the surface (0–2 km height) divergence signature can be easily recognized at a range of 48 km. Low to midlevel radial convergence can also be detected at 48 km range and in the height interval 3–7 km. Figure 17e shows the $A_X$ profile, the attenuation core being centered at a range of 48 km. A peak $A_X$ of about 5 dB km$^{-1}$ is noted within the center of the attenuation core, corresponding to a bright band. The HS profile is shown in Fig. 17f where $HS < 3$ dB within the $A_X$ core centered at 48 km. $HS > 3$ dB occurs aloft and corresponds to the 65 dBZ core centered at 54 km, see Fig. 17a, and the $Z_X$ core of 30 dBZ, see Fig. 17b. This hail signal is well correlated with the $Z_{DR}$ melting level depression noted in Fig. 17c. The $A_X$ core and bright band centered at 48 km is a region of small, melting hail ($D \leq 10$ mm), and corresponds to the center of the surface divergence as detected by the Doppler mean velocity, see Fig. 17d. These data are consistent with the interpretation discussed in Section 3c.

5. Conclusions

This paper has considered in detail the multiparameter radar modeling of graupel and small hail melting into raindrops. A microphysically detailed graupel/hail melting model developed by Rasmussen and Heymsfield (1987) has been coupled to a radar model previously described by Bringi et al. (1986a,b). Sensitivity studies have been performed with respect to the following parameters: (a) graupel shape (conical versus spherical), (b) graupel orientation distribution, and (c) graupel melting profiles under two different soundings, one from Colorado and the other from Alabama. We also consider differences between graupel and small hail (dia $\leq 12$ mm) melting processes and the corresponding effects on $Z_{DR}$ and X-band specific attenuation, $A_X$. Polarimetric observables are sensitive to hydrometeor shape, orientation and dielectric constant. Differential reflectivity was found to be sensitive to the shape assumption but relatively insensitive to the standard deviation ($\sigma$) profile of the canting angle (assuming mean canting angle $\approx 0$). The slope of $Z_{DR}$ versus height is sensitive to both sounding profiles as well as the initial particle type and density, i.e., graupel or small hail. $A_X$ is sensitive to the above factors but is independent of shape, as expected. The radar model was used to predict the profiles of reflectivity, $Z_{DR}$ and $A_X$ for graupel and small hail.

Computations of $Z_{DR}$ for small hail melting at C-band showed strong resonance effects leading to very high values $\approx 7–9$ dB when the particle consists of an inner ice core surrounded by an oblate shell of water. Measurements with the C-band DFVLR polarimetric radar frequently showed very high values of $Z_{DR}$ below the melting level and within the main precipitation shaft of convective storms, Meischner et al. (1988). The calculations presented here can be used to interpret such high C-band $Z_{DR}$ values as caused by small, melting hail with maximum diameters $\leq 8$ mm.

A simple, spherical ice melting model has been used to compute the contributions of scattering and absorption to the X-band specific attenuation. This simple model predicts an $A_X$ bright band occurring at the melting level if absorption is the principal mechanism, i.e., if the particles are small compared to $\lambda = 3.2$ cm. We have observed $A_X$ bright bands in a number of convective storms during the MIST project and these data are reported here. We infer that the measured $A_X$ bright band corresponds to strong cooling within a narrow layer of the atmosphere (caused by small ice melting processes). This is conducive to formation of strong downdrafts, and possibly downbursts, Srivastava (1987). The MIST measurements discussed here were all cases where a downburst was inferred either through
surface damage reports, or through observation of a strong surface divergence signature (using radial Doppler velocity data).

Color radar images of reflectivity at S- and X-bands, $Z_{DR}$, mean Doppler velocity, X-band specific attenuation and Hail Signal have been presented for two downburst cases, one from the MIST project and one from CINDE. These RHI images clearly show the great utility of combining $Z_{DR}$ and $A_X$ data in real time with conventional Doppler data to increase our understanding of microphysical processes that occur in strong precipitating downdrafts.

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