Simulation of the Partial Reflection by the Critical Latitude with a Linear Model. Part II: Stationary Wave Responses to Total Forcing

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ABSTRACT

The total response of a linear two-layer stationary wave model to all the forcing functions (mountains, diabatic heating, nonlinear eddy fluxes) was calculated using various reflection coefficients for the critical latitude (CL). The reflectivity was regulated by modifying the y-derivative of the absolute vorticity near the CL. The vertical resolution of the model is so low that no CL reflection coefficient produced anything better than a mediocre simulation. A two-layer model, however, proved to be considerably better than a single-layer model. The best simulations were obtained with low CL reflectivities.

1. Introduction

In linear models, stationary eddies are obtained as a response to forcing by orography, zonally varying diabatic heating, and nonlinear momentum and heat fluxes by transient and stationary eddies. Linearized equations are formally applicable everywhere except at the critical latitude (CL). At this latitude the basic-state zonal velocity vanishes, making the inviscid linear Rossby-wave equation singular. As discussed in Ruosteenoja (1989a, hereafter referred to as Part I), there is disagreement about whether the waves are predominantly absorbed or reflected by the CL.

This tentative work endeavors to simulate the stationary waves in the atmosphere with a two-layer steady-state primitive equation model, and to compare some of the results with those given by a single-layer model. The vertical resolution of the models certainly inadequate for obtaining quantitatively reliable results. This is especially so in low latitudes, while in middle and high latitudes the two-layer model may be at least qualitatively reasonable (Panetta et al. 1987). The linearization error is likewise largest in tropical and subtropical latitudes, where the basic-state zonal flow is weak. Accordingly, our domain consists of two areas: 1) the extratropics north of ~30°N, where the model results are at least qualitatively interesting, and 2) low latitudes—containing the critical layer—where the model is distinctly unrealistic. In order to obtain a proper simulation over area 1, area 2 must be parameterized so that the wave component reflected by the CL is correct. The CL reflectivity in the model is regulated by modifying the gradient of the basic-state absolute vorticity ($\beta_{\text{EFF}}$), as discussed in Part I. The stationary wave responses are then obtained with various CL reflectivities, and the quality of the simulation is evaluated in each case.

In addition to the wintertime climatological stationary waves, the waves in a single month (February 1979) have been studied. For February 1979, we used the level IIIb initialized “final” FGGE analyses prepared by the European Centre for Medium Range Weather Forecasts. The wintertime climatological flow studied consists of that of the five winters from 1968/69 to 1972/73, which are part of the circulation statistics compiled by Oort (1983). Both datasets are global, but only the Northern Hemisphere is studied in this work.

2. Model and forcing functions

a. Model description

The details of the two-layer primitive-equation model used have been discussed in previous papers (Ruosteenoja 1989b and references) and thus only the main features are repeated here. The model applies the momentum and continuity equations to model levels 1 (400 mb) and 3 (800 mb) and the thermodynamic equation to level 2 (600 mb). In single-layer simulations, only the 400-mb level

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is considered, and there is no thermodynamic equation. In the zonal direction each independent variable is expanded in a Fourier series, with a truncation point of 8. In the meridional direction, a grid point representation with 800 points between the pole and the equator is used; according to Ruosteenoja (1988), this ensures an adequate horizontal resolution.

Lindzen and Fox-Rabinovitz (1989) suggested that models having a resolution that is sparse in the vertical and excessive in the horizontal may produce small-scale noise associated with inadequately resolved vertical modes. Our linear simulations indeed manifested small-scale features (apparent in the vorticity and eddy zonal velocity fields) associated with the CL, but elsewhere the response was dominated by the large scale. In addition, the response of the present two-layer model converges to a "correct" solution as the meridional grid gets denser; for example, 400 and 800 grid points yielded fairly similar responses, unlike sparser grids. If artifacts due to excessive horizontal resolution were to occur, errors would be severest when the grid length is very small, in contrast to our experience. On the other hand, any linear model with an inadequately resolved critical layer yields responses with a substantial numerical error (Ruosteenoja 1988). Outside the critical layer a resolution as high as that in the present model is unnecessary, but innocuous. Of course, our findings do not preclude the possibility that excessive horizontal resolution might cause problems in time-dependent models.

\[ w = U_{900} \frac{\partial h}{a \cos \phi \partial \lambda} \tag{1} \]

and the complete version

\[ w = \mathbf{V} \cdot \nabla h \tag{2} \]

where \( w \) is the \( z \) coordinate vertical velocity, \( U_{900} \) the zonal mean westerly wind at 900 mb, \( h \) the height of topography, \( a \) the earth radius, \( \phi \) the latitude, and \( \mathbf{V} \) the observed time-mean wind at a height \( h \). The vertical velocity is transformed into pressure coordinates by

\[ \omega_4 = -g p w / RT \tag{3} \]

where \( p = 900 \) mb and \( T = 282 \) K in the traditional version, whereas the time-mean values of \( p \) and \( T \) at height \( h \) are used in the complete version. Here \( \omega_4 \) is used as a lower boundary condition in both models. Since the complete version did not produce any better simulations than the traditional one, we mainly deal with the traditional version.

The diabatic heating \( q(\lambda, \phi, p, t) \) for February 1979 was calculated from daily FGGE analyses by Fortelius (1989) as a residual term in the thermodynamic equation. The climatological circulation statistics do not contain any heating fields. Thus, we used the 50-1000 mb average heating given by Hoskins et al. (1989, p. 64). (In that paper, no midtropospheric heating was given.)

The transient and standing eddy forcings consist of mechanical and thermal parts, i.e. convergences of momentum and heat fluxes [for expressions, see (9)-(10) in Ruosteenoja (1989)]. Mechanical stationary and transient eddy forcings were calculated at model levels 1 and 3 (in the one-level simulations at 400 mb only). Thermal eddy forcings (as well as the diabatic heating) at level 2 are obtained by taking an average over 400 mb-800 mb.

In order for the model equations to be formally complete, forcing terms associated with tendencies of eddy velocity components and temperature must be included. The responses to these forcing terms proved to be fairly insignificant. Tendency forcing terms are omitted when studying winter-time climatology.
3. Model responses with various CL reflectivities

Part I introduced a method of regulating the CL reflectivity by modifying the basic state in the critical layer:

\[ \tilde{U} = U + \kappa \Delta U_0 e^{-\mu (\phi - \phi_{CL})^2} \]  

where \( U \) and \( \tilde{U} \) are the original and modified basic-state zonal velocities, \( \phi_{CL} \) the CL of the modified flow and \( \mu \) a parameter defining the width of the region where \( U \) is considerably modified. \( \Delta U_0 = -a^2 \beta_{EFF}/(2\mu) \), \( a \) being the radius of the earth, and \( \beta_{EFF} \) the \( y \)-derivative of the absolute vorticity of the unmodified flow. (In this study \( \mu = 65.7 \), and thus \( \Delta U_0 \sim -4 \text{ m s}^{-1} \).) The parameter \( \kappa \) defines the strength of modification of \( U \), \( \kappa = 1 \) implying that the modified \( \beta_{EFF} = 0 \) at the CL. In theory, \( \kappa < 1 \) implies partial absorption by the CL, \( \kappa = 1 \) perfect reflection, and \( \kappa > 1 \) overreflection. When friction is taken into consideration, a slightly larger value than \( \kappa = 1 \) is required for total reflection. The relationship between \( \kappa \) and the CL reflectivity for the two basic states is shown in Part I (Fig. 11a) and in Ruosteenoja (1989b, Fig. 17). Qualitatively, the relationship is similar in both cases.

In two-layer simulations, the basic state is modified at the upper level only. In the lower troposphere the minimum of \( \beta_{EFF} \) is weak and, moreover, located distinctly in the easterlies. Therefore, modifying \( \beta_{EFF} \) at the lower level would result in an unrealistic distribution of \( U_3 \). This holds both for the climatological (see Fig. 8 in Part I) and the February 1979 basic states.

\( a. \) Responses to total forcing with various values of \( \kappa \)

The model response to the total forcing (mountains, heating, transient and stationary eddy flux convergence, and in February 1979 also the tendencies of eddy velocity and temperature) was calculated with nine values of \( \kappa \), ranging from 0.0 to 1.6. Spatial correlations with the observed mean \( z^* \) are given in Fig. 1. In two-layer simulations, the correlation is almost constant when \( \kappa \leq 0.8 \), and decreases with larger values. This holds both for climatology and the single month studied. \( \kappa \approx 0.8 \) corresponds to a reflectivity of \( \sim 0.3 \ldots 0.4 \). Unexpectedly, the response to climatological forcing seems to improve again with a highly reflecting CL.

In general, the single-level simulation gives distinctly lower correlations with the observations than the two-layer model (Fig. 1). The behavior of the correlation coefficient as a function of \( \kappa \) is totally different in the two cases of February 1979 and climatological flow. Both the single- and two-layer models produced amplitudes that were too large, but the single-layer responses were \( \sim 1.5 \) times larger than the two-layer ones. The results indicate that the two-layer model, though still

![Fig. 1. Spatial correlation between the observed and model-simulated z* as a function of \( \kappa \): (a) February 1979, (b) wintertime climatological flow. The solid curve compares the two-layer simulation with the observed 600 mb z*, the dashed one that of the single-layer simulation with the 400-mb z*.

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inadequate, is superior to the single-level model.

In the climatological experiments, the complete mountain forcing (2) gave results nearly as good as the linearized forcing (1). In February 1979, by contrast, the complete mountain forcing proved to be bad, the highest correlations with observations being ~0.2. The error in the complete mountain forcing seems to originate in high latitudes (Greenland and Alaska), where the \( \omega_4 \) derived from the mean wind analysis was unrealistically strong.

Examples of \( z^* \) responses obtained with some values of \( \kappa \), together with the observed \( z^* \), are given in Fig. 2. As long as the CL reflectivity is low, the patterns of \( z^* \) are insensitive to variations in \( \kappa \). In fact, all the response patterns with \( \kappa \leq 0.8 \) have a spatial correlation higher than 0.94 with the response obtained with \( \kappa = 0 \). Thus, the stationary wave pattern seems to be almost independent of whether the CL reflectivity is, say, 0.2 or 0.4.

With a reflectivity greater than ~0.5, the response turned out to be sensitive to the value of the reflection coefficient. In February 1979, the pattern with \( \kappa = 1.2 \) (Fig. 2c) has a spatial correlation of 0.66, and patterns with \( \kappa \geq 1.4 \) (overreflection) nearly zero correlation with the response obtained with an unmodified CL (Fig. 2a). In addition, an overreflecting CL produces unrealistically high amplitudes of the response. The sensitivity of the climatological response to the reflectivity is somewhat weaker.

b. Importance of individual forcing functions

The two-layer responses to individual forcings obtained with an unmodified critical layer are given in Table 1. Both transient and standing eddy forcings appear to be important, especially the mechanical parts related to eddy momentum fluxes (compare with Opsteegh and Vernekar 1982 and Valdes and Hoskins 1989). The responses to nonlinear transient eddy forcing seem to even exceed those to “primary” forcings, i.e., mountains and diabatic heating. When a short period such as one month is studied, the standing eddy forcing is likewise essential. This is due to the slowly varying part of the transient eddy spectrum appearing as quasi-stationary waves.

A striking feature of Table 1 is that responses to mechanical forcings (mountains and eddy momentum fluxes) clearly exceed those to thermal forcings (diabatic heating and eddy heat fluxes). This is presumably an artifact of two-layer models, since in multilayer models (Held 1983; Chen and Trenberth 1988; Nigam et al. 1988; Valdes and Hoskins 1989) the effect of diabatic heating, for instance, is by no means negligible. A large part of the near-field response to a thermal forcing is obviously contained in baroclinic vertical modes. Hoskins and Karoly (1981) and Valdes and Hoskins (1989) indeed found that in a multilayer simulation the thermally forced near-field response is highly baroclinic. From the baroclinic modes the wave energy transfers into the equivalent barotropic external mode, which is the only mode that can propagate horizontally. As the baroclinic vertical modes are poorly resolved in a two-layer model, the exited propagating external mode may become weak. Of course, a two-layer model wins over a single-layer model in that in the latter no thermal forcing can be included at all.

It is encouraging that the response to the total forcing has a correlation with observations higher than the response to virtually any individual forcing. There is only one exception, namely the response to the climatological diabatic heating, but the amplitude of this response is small. Correspondingly, leaving out any individual forcing factor (mountains, heating, or total TE, SE, or tendency forcing) from the total forcing almost invariably impaired the agreement with observations. Admittedly the correlations between our model responses and the observations are only moderate, and the amplitude of the total response is too large.

4. Discussion and conclusions

Two-layer linear stationary wave simulations with various CL reflection coefficients produced the highest spatial correlations with the observations when the reflectivity was low (~0.2–0.4). This holds both for the wintertime climatological waves and the single month considered. As long as the reflectivity is kept low, the model response is not sensitive to the exact value of the reflection coefficient. When the reflectivity exceeds 0.4, the correlation with observations rapidly deteriorates.

In all of the experiments, the agreement between the two-layer model response and the observed \( z^* \) was only mediocre, the highest spatial correlations obtained being about 0.54. Single-layer simulations produced still lower correlations. Nigam et al. (1988) seemed to have obtained a far better linear simulation. One explanation is that their forcing functions, inferred from a GCM simulation, were accurate. In addition, in our model the vertical resolution is low and the upper boundary condition unrealistic. The uppermost level simulated by the model is 400 mb, while the strongest basic-state velocities and eddy momentum fluxes occur at 200–300 mb. Moreover, in the real atmosphere the longest planetary waves propagate into the stratosphere, while this is not possible in a two-layer model. However, even the simplest baroclinic model appeared to be superior to the barotropic one. It will therefore be highly interesting to repeat
Fig. 2. The pattern of eddy geopotential height ($z^*$) at 600 mb in February 1979, obtained as a response to the total forcing (including the traditional version of mountain forcing) with $\kappa = 0$ (a), $\kappa = 0.8$ (b), $\kappa = 1.2$ (c), and $\kappa = 1.6$ (d). The observed pattern of $z^*$ is given in panel (e). Contour interval is 80 m.
Table 1. Midtropospheric (600 mb) responses of the two-layer model to various forcing functions: rms amplitudes of $z^*$ (in meters) and correlations between the observed and simulated $z^*$, both averaged over the Northern Hemisphere.

<table>
<thead>
<tr>
<th>Forcing</th>
<th>February 1979</th>
<th>Climatological flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude</td>
<td>Correlation</td>
</tr>
<tr>
<td>Mountain, traditional $\omega_4$</td>
<td>34.6</td>
<td>0.18</td>
</tr>
<tr>
<td>Diabatic heating</td>
<td>14.2</td>
<td>0.39</td>
</tr>
<tr>
<td>Mechanical TE</td>
<td>81.8</td>
<td>0.36</td>
</tr>
<tr>
<td>Thermal TE</td>
<td>29.1</td>
<td>0.30</td>
</tr>
<tr>
<td>Mechanical SE</td>
<td>59.1</td>
<td>-0.11</td>
</tr>
<tr>
<td>Thermal SE</td>
<td>24.2</td>
<td>-0.06</td>
</tr>
<tr>
<td>Velocity tendency</td>
<td>26.7</td>
<td>0.23</td>
</tr>
<tr>
<td>Thermal tendency</td>
<td>6.5</td>
<td>-0.07</td>
</tr>
<tr>
<td>Total</td>
<td>89.0</td>
<td>0.53</td>
</tr>
<tr>
<td>Observed</td>
<td>50.7</td>
<td>—</td>
</tr>
</tbody>
</table>

the present simulations with a higher vertical resolution.

When interpreting the present results obtained with a linear model, one must bear in mind that modifying $\beta_{\text{EFF}}$ is not adequate for simulating all the features of the nonlinear critical layer. First, because of the nonlinear wave-wave interaction, the reflected wave may have an energy spectrum different from that of the incident one, and there may be phase shifts not predicted by the model. Second, in modifying the CL reflectivity, the theoretically most relevant quantity is the meridional gradient of potential vorticity. In models with low vertical resolution, however, it is easiest to actively change only the meridional structure of the flow, while keeping the basic state in thermal wind balance.

The present paper indicates that a linear two-layer model is inadequate for quantitative simulation of stationary waves in the real atmosphere, even if the reflection by the CL is parameterized by modifying the basic state. Thus the conclusion that a low reflectivity gives the best agreement with observations remains to be confirmed by the use of better vertical resolution.

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**REFERENCES**


