Vortex Rossby Waves in a Numerically Simulated Tropical Cyclone. Part I: Overall Structure, Potential Vorticity, and Kinetic Energy Budgets*

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ABSTRACT

The asymmetric structure in the inner core of a numerically simulated tropical cyclone is analyzed in this study. The simulated tropical cyclone is found to be highly asymmetric in the inner core. In the mid–lower troposphere, the asymmetry in the core is dominated by azimuthal wavenumber-1 and wavenumber-2 vortex Rossby waves. These waves propagate azimuthally upwind against the azimuthal mean cyclonic tangential flow around the eyewall, and thus have a much longer cyclonic rotation period (by a factor of 2) than the period of a parcel moving with the cyclonic mean tangential flow around the circumference. They also propagate outward against the boundary layer inflow of the azimuthal mean cyclone. The waves are only visible within a radius of about 60 km from the cyclone center. Beyond this distance, the radial gradient of potential vorticity (PV) of the azimuthal mean cyclone is too weak to support the vortex Rossby waves. Although the divergent motion remains strong, the geopotential height and wind fields of the vortex Rossby waves are quasi-balanced, with confluent cyclonic (divergent anticyclonic) flow collocated with low (high) perturbation geopotential height. The waves spiral cyclonically inward with maximum amplitudes near the radius of maximum wind (RMW) in the horizontal and tilt radially outward with height. The upward motion of the waves leads cyclonic vorticity in both azimuthal and radial directions by about one-quarter wavelength, implying that convective heating, which is coupled with low-level convergence and upward motion, is the driving force for the vortex Rossby waves.

A PV budget shows that diabatic heating contributes greatly to both the azimuthal mean PV and perturbation PV budgets. The PV tendency associated with diabatic heating is largely balanced by the advective (both horizontal and vertical) flux divergence of the symmetric PV, respectively, due to the asymmetric flow (vortex beta term, similar to the planetary beta term in the large-scale vorticity equation) for the vortex Rossby waves, and due to the symmetric flow for the symmetric cyclone. The vortex Rossby waves transport cyclonic PV from the eyewall to the eye, thus mixing the PV between the eyewall and the eye and spinning up the tangential wind in the eye at the expense of weakening the tangential wind near the RMW. Moreover, the PV tendency due to nonlinear processes associated with the wavenumber-1 vortex Rossby waves is a significant PV source for the wavenumber-2 vortex Rossby waves, indicating a strong wave–wave interaction in the eyewall. An eddy kinetic energy budget indicates that within the RMW, the vortex Rossby waves receive their kinetic energy from the azimuthal mean cyclone through baroclinic conversion and flux divergence of eddy kinetic energy due to the azimuthal mean vortex. Under the eyewall and just outside the RMW in the mid–lower troposphere, the main source for eddy kinetic energy is the eddy potential energy conversion, which is related to the asymmetric diabatic heating associated with moist convection in the eyewall. An interesting finding is that, in both the barotropic and baroclinic conversions, terms related to the radial flow of the azimuthal mean vortex are dominant and contribute to the kinetic energy of the vortex Rossby waves. The horizontal shear of the azimuthal flow of the mean vortex damps eddy kinetic energy, stabilizing the vortex Rossby waves in the mid–lower troposphere. However, both barotropic and baroclinic conversions related to the tangential flow of the azimuthal mean vortex, together with the eddy potential energy conversion, are responsible for the development of asymmetry in the outflow layer.

1. Introduction

Strong tropical cyclones exhibit highly axially symmetric structure in the core (defined as the region within a radius of about 100 km from the cyclone center). Asymmetries in the core, however, are also often observed and are characterized by both polygonal eyewalls and outward-propagating inner spiral rainbands (Simpson 1952; Jordan 1952; Shea and Gray 1973; Lewis and Hawkins 1982; Jorgensen 1984; Black and Marks 1991; Bluestein and Marks 1987; Gall et al. 1998; Kuo et al. 1999; Reasor et al. 2000). Understanding the dynamics of the asymmetric structure is fundamental because the asymmetric structure is related not only to the motion (Neuman and Boyd 1962; Willoughby 1994; Wang and Holland 1996a,b) but also to changes in the structure...
and intensity (Willoughby 1990; Holland and Merrill 1984; Challa et al. 1998) of a tropical cyclone. Earlier views on the dynamics of the asymmetric structure of tropical cyclones were based on the theory of inertia–gravity waves (Abdullah 1966; Kurihara 1976; Willoughby 1978; Lewis and Hawkins 1982), with one exception, which hypothesized that the spiral rainbands resulted from Rossby-type waves (MacDonald 1968). Rossby-type waves have long been known a feature of most fluids with differential rotation (e.g., Rhines 1970). However, it was not recognized until recent years that Rossby-type waves (or potential vorticity waves in general) can exist in the tropical cyclone core and may play important roles in structure and intensity changes (Guinn and Schubert 1993; Montgomery and Kallenbach 1997; Montgomery and Enagónio 1998; Möller and Montgomery 1999, 2000; Kuo et al. 1999; Reasor et al., 2000; Wang 2001).

Building on earlier work, which has identified vortex axisymmetrization as a universal process of smoothly distributed perturbed vortices (Melander et al. 1987; Guinn and Schubert 1993). Montgomery and Kallenbach (1997, henceforth MK97) developed a theoretical Rossby-wave guide model of vortex axisymmetrization. They showed that axisymmetrization of asymmetric perturbations by the strong shearing tangential flow of the mean vortex is accomplished by outward-propagating vortex Rossby waves whose restoring mechanism is associated with the radial gradient of storm vorticity. They suggested that the radially outward-propagating vortex Rossby waves can initiate the inner spiral rainbands and affect the structure and intensity of the mean vortex by interacting with the mean flow. With a three-dimensional quasigeostrophic model, Montgomery and Enagonio (1998) studied tropical cyclogenesis via axisymmetrization of convectively generated potential vorticity (PV) anomalies in the parent vortex core. They showed that the interaction of convectively forced vortex Rossby waves with the mean vortex flow could intensify the mean vortex. This process was also studied recently by Möller and Montgomery (1999, 2000) using both barotropic and baroclinic asymmetric-balance models.

The tropical cyclone is a localized vortex formed and maintained by energy transferred from the ocean surface and released as latent heat in moist convection in the eyewall only tens of kilometers from its center. In the lower troposphere, air parcels flowing inward and spiraling upward in the convective eyewall usually experience a material increase in PV (Schubert et al. 1999). Since this material increase of PV can be very rapid near the eyewall, an annular tower of high PV, with low PV in the central eye region, could be possible if no asymmetries were in the eyewall convection to begin with, and if the PV concentration could continue without mixing of PV in the eye and the eyewall. Barotropic instability near the eyewall can be expected. This type of instability and its potential in causing eyewall breakdown and polygonal eyewall structure have been investigated by Schubert et al. (1999) with a nondivergent barotropic model. They found that when the instabilities grow to finite amplitude, the vorticity in the eyewall region pools into discrete areas, creating the appearance of polygonal eyewalls. They have suggested that the circulations associated with these pools of vorticity are connected to mesovortices in the eyewall of the tropical cyclone, as observed in some real hurricanes (Black and Marks 1991). The vorticity is later redistributed into a nearly monopolar circular vortex through PV mixing and eye contraction.

In addition to barotropic instability, vortex Rossby waves can be forced and driven by convective asymmetries in the eyewall. Such convective asymmetries in the eyewall can be generated by barotropic instabilities in the upper-tropospheric outflow layer (Anthes 1972), by the beta effect (Wang and Holland 1996a), or by environmental flow and flow shears (Shapiro 1983; Wang and Holland 1996b; Bender 1997; Frank and Ritchie 1999). Neutral vortex Rossby waves could also be present near the eyewall where radial PV gradients are large (Montgomery and Lu 1997). Propagation of these forced or neutral waves around the eyewall can produce an apparent cyclonic rotation of the polygonal eyewall in some tropical cyclones (Kuo et al. 1999). Using the reflectivity data from a weather surveillance Doppler radar, Kuo et al. (1999) documented the elliptical eye of Typhoon Herb (1996), which rotated cyclonically with a period of 144 min. They hypothesized that the eye rotation shown in the radar reflectivity was related to the wavenumber-2 vortex Rossby waves that propagated azimuthally around the eyewall.

In an observational study, Reasor et al. (2000) examined the low-wavenumber asymmetric structure and evolution of Hurricane Olivia’s (1994) inner core, which has been observed by airborne dual-Doppler radar with 30-min time resolution for 3.5 h, during which the hurricane was weakening. They found that vertical shear over the hurricane core increased dramatically during the observation period, leading to an azimuthal wavenumber-1 convective asymmetry oriented along the maximum vertical shear vector. The asymmetry in relative vorticity, however, was dominated by an azimuthal wavenumber-2 feature below a height of 3 km, and by a wavenumber-1 pattern above this height. The asymmetry was characterized by a wavenumber-2 discrete vortex Rossby edge wave, which they suggested resulted from barotropic instability in the symmetric vortex. Propagation of the wavenumber-2 vortex Rossby wave around the eyewall offered a physical explanation for the storm eye rotation with a period of 50 min, about twice that for a parcel being advected by the azimuthal flow near the RMW around the storm. Trailing vorticity bands with radial wavelengths of 5–10 km were observed within about 20 km of the hurricane center. These vorticity bands were ascribed to axisymmetrizing vortex Rossby waves and might contribute to the observed rainbands of similar radial wavelengths. As Reasor et al.
(2000) noted, although their analyses revealed many features of the vortex Rossby waves, studies of the detailed process require a higher spatial and temporal resolution dataset, which can be generated by a full-physics numerical model.

In a recent numerical modeling study, Wang (2001) identified vortex Rossby waves in the simulated tropical cyclone core region. He showed that the asymmetries in the core are dominated by low azimuthal wavenumber vortex Rossby waves. Consistent with the findings of Reasor et al. (2000), the vortex Rossby waves have a spiral structure in relative vorticity with maximum amplitudes near the RMW, and propagate upwind around the eyewall relative to the tangential flow of the azimuthal mean cyclone. The waves tilt outward with height and have a coherent structure in the mid–lower troposphere. A salient feature of these waves is their quasi-balanced nature although the divergent flow is strong, consistent with the asymmetric-balance dynamics of hurricanes as first proposed by Shapiro and Montgomery (1993) and later clarified by Montgomery and Franklin (1998). Wang (2001) also revealed that eyewall convection can be enhanced and shifted inward on one side by inflow associated with the vortex Rossby waves in the lower troposphere, while it is suppressed and shifted outward on the other side by outflow. Such a coupling between the vortex Rossby waves and the eyewall convection can result in polygonal eyewalls and eyewall breakdown. In the modeled tropical cyclone, the wavenumber-1 and wavenumber-2 vortex Rossby waves both rotate cyclonically around the eyewall with a period of about 111 min at the mature stage, whereas the rotational period of a parcel advected by the local tangential flow near the RMW of the azimuthal mean cyclone is about 50 min. Thus, the former is about twice the length of the latter, which is consistent with the corresponding ratio found in both Typhoon Herb (1996) by Kuo et al. (1999) and Hurricane Olivia (1994) by Reasor et al. (2000). In addition to propagating azimuthally, wavenumber-1 vortex Rossby waves propagate radially outward. Their outward propagation is believed to be responsible for initiating inner spiral rainbands, as proposed by MK97 and observed in Hurricane Olivia by Reasor et al. (2000) and the polygonal eyewall structure as observed in Typhoon Herb (1996) by Kuo et al. (1999).

Although these recent studies have revealed many characteristics of vortex Rossby waves and emphasized their potential importance in tropical cyclone dynamics, their PV dynamics and energetics and their role in the life cycle of a tropical cyclone are still poorly understood. In particular, their potential interaction with the moist convection in the cyclone core is crucial, but such interaction has not been investigated in the previous studies. For example, asymmetric PV anomalies can be related to asymmetries in convection, while circulations associated with the PV asymmetries could further modify convection in the eyewall. These processes are nonlinear and complex. Although some of the fundamental mechanisms have been revealed in the context of inviscid nondivergent barotropic models, potential feedbacks can be neither examined nor thoroughly understood unless a full-physics model is used.

This study investigates the complex interactions and feedbacks between the symmetric vortex, the asymmetric vortex Rossby waves, and the eyewall convection in a tropical cyclone simulated by a three-dimensional tropical cyclone model developed by Wang (1999, 2001). This paper discusses only the characteristics of the vortex Rossby waves and their PV dynamics and energetics. The complex interactions and feedbacks between the vortex Rossby waves and the eyewall convection and their role in structure and intensity changes of the simulated tropical cyclone will be addressed in Part II (Wang 2002) of this series. The next section briefly describes the tropical cyclone model. Sections 3 and 4 present the overall structure of the simulated tropical cyclone and characteristics of the vortex Rossby waves, respectively. Sections 5 and 6 provide the PV and kinetic energy budgets of the vortex Rossby waves, respectively. The last section summarizes major findings.

2. The tropical cyclone model

The tropical cyclone model used in this study is the triply nested, movable-mesh primitive equation model (TCM3) developed by Wang (1999). A detailed description of this numerical model and its performance and capability of simulating scale interactions in tropical cyclones can be found in Wang (1999, 2001). Here only some major features of the model will be discussed. The model is a hydrostatic, primitive equation model formulated in Cartesian coordinates in the horizontal either on an f plane or a β plane, with σ (pressure normalized by the surface pressure) coordinate in the vertical. The model consists of 20 vertical layers from σ = 0 to σ = 1 with high resolution within the boundary layer. The model domain is triply nested with the two interior meshes being movable so that the model tropical cyclone can always be located near their centers. The outermost mesh is fixed with open lateral boundary condition in the north and south boundaries, and a cyclic boundary condition in the east–west direction. The outermost domain is sufficiently large so that the influence of the lateral boundary conditions on the evolution of the model tropical cyclone is minimized.

A two-way nesting strategy is used for time integration, which is accomplished by the second-order leapfrog scheme with intermittent use of the Euler-backward scheme to suppress high frequency numerical noise and computational modes. The finite-difference scheme is second order and conserves both mass and energy (Lilly 1964). The model physics include an E – ε turbulence closure scheme for subgrid-scale vertical mixing above the surface layer (Detering and Etling 1985), a modified Monin–Obukhov scheme for the surface flux calculation (Fairall et al. 1996), an explicit treatment of mixed ice-
phase cloud microphysics (Rutledge and Hobbs 1984; Reisner et al. 1998; Wang 1999, 2001), and a second-order horizontal diffusion with a deformation-dependent diffusion coefficient. The model prognostic variables then consist of both zonal and meridional winds, surface pressure, temperature, turbulence kinetic energy and its dissipation rate, and mixing ratios of water vapor, cloud water, rainwater, cloud ice, snow, and graupel.

In the numerical simulation presented here, a grid size of 5 km is used for the innermost mesh, which covers an area of 540 km by 540 km and has the model tropical cyclone located near its center. Grid sizes of 15 km with a domain of 1620 km by 1620 km and 45 km with a domain of 7200 km by 5400 km are used, respectively, for the intermediate and outermost meshes. The model is initialized with a cyclonic vortex that has a maximum azimuthal wind of 25 m s\(^{-1}\) at the surface, which decreases gradually to zero at about 100 hPa, at a radius of 100 km. Given the wind fields, the mass and thermodynamic fields are obtained based on a nonlinear balance equation so that the vortex is in both hydrostatic and gradient balances (Wang 1995, 2001). The initial water vapor mixing ratio and the environmental sounding are assumed horizontally homogeneous and have the vertical profile of the January monthly mean at Willis Island, northeast of Australia (Holland 1997). This is representative of tropical ocean conditions. The sea surface temperature is fixed at 29°C and an \(f\) plane with a constant Coriolis parameter at 18°S is assumed. Since our focus is on internal dynamics in this study, a quiescent environment is assumed to exclude any external forcing on the simulated tropical cyclone. Note that although there is no environmental flow and the \(f\) plane is assumed, the simulated tropical cyclone is not stationary and exhibits a cyclonic looping motion with a radius of several tens of kilometers due to the development of asymmetric structure of the model tropical cyclones. If an environmental flow is considered, a trochoidal track can be expected.

3. Overall structure of the simulated tropical cyclone

The detailed structure of the simulated tropical cyclone can be found in Wang (2001). Thus, only those features not presented in Wang (2001) or important to this study will be discussed. The overall symmetric structure of the simulated tropical cyclone at the mature stage after 90 h of simulation is shown in Fig. 1. The maximum tangential wind of the azimuthal mean cyclone (Fig. 1a) is located about 30 km from the cyclone center near the surface and tilts outward with height, with a low-level wind maximum in the boundary layer about 500 m above the sea surface. Such a low-level wind maximum in the core is unique to real tropical cyclones (Keppert 2001; Keppert and Wang 2001). Strong inflow occurs in the boundary layer below 800 hPa with maximum inflow of about 26 m s\(^{-1}\) just outside the radius of maximum tangential wind (Fig. 1b). Broad outflow happens in the upper troposphere, outside the eyewall (defined as the radius of maximum wind which tilts outward with height, Fig. 1a) with its roots at lower levels inside the eyewall (Fig. 1b). Weak inflow and outflow across the eye and the eyewall show that mixing occurs between the eye and the eyewall. There is a strong updraft in the eyewall, which tilts outward with height (Fig. 1c). Weak descending motion at low levels just outside the eyewall indicates the presence of downdraffs associated with the eyewall convection. The descending motion in the eye with its maximum around the inner edge of the eyewall originates from the returning flow at the top of the eyewall (Fig. 1b) and is enhanced by the sublimative cooling of ice particles detrained from the eyewall. The motion in the radial-vertical plane of the mean cyclone is mainly along the absolute angular momentum surface (Fig. 1d), consistent with the theory of Emanuel (1991). The cyclone has a warm core structure with a maximum temperature anomaly of about 18°C at about 250–300 hPa (Fig. 1e). This warming is consistent with the thermal wind balance required for the outward tilt of the maximum tangential wind with height. The upper-level warming extends to large radii due to the outward advection of warm air by the outflow. A distinctive feature of the simulated tropical cyclone is the homogeneous distribution of the azimuthal mean PV in the core with an off-centered maximum in the mid–lower troposphere (Fig. 1f). The PV maximum just inside the RMW below about 800 hPa is a unique feature and is due to concentration of PV associated with convective heating in the eyewall.

Figure 2 shows a plan view of the model-estimated radar reflectivity at the surface (a) at 86 h 30 min in the simulated tropical cyclone and its vertical cross section (b) through line A–B in (a). The eyewall is encircled by high radar reflectivity with an opened shape to the northwest and connected to an inner spiral rainband (Fig. 2a). Although the eye is nearly precipitation free, there are some regions with relatively low reflectivity, indicating light precipitation there. In addition to several small inner rainbands just outside the eyewall, two primary spiral rainbands are located between 80 and 180 km from the cyclone center. The high radar reflectivity in the eyewall tilts outward with height and is located below the melting level at about 500 hPa (Fig. 2b). Just outside the eyewall, low radar reflectivity is related to stratiform precipitation in the mid–upper troposphere and to shallow convection in the boundary layer; and high reflectivity is related to deep convection in the convective rainbands (Fig. 2b). Relatively low reflectivity in the eye seen in Fig. 2a extends to about 700 hPa, sometimes above 500 hPa (Fig. 2b), and is accompanied by light precipitation in the eye, a feature usually observed in strong tropical cyclones (e.g., Black et al. 1972; Marks 1985). Note that the simulated tropical cyclone is highly asymmetric, even in the inner core.
Fig. 1. The overall axially symmetric structure of the numerically simulated tropical cyclone at the mature stage after 90 h of model integration. Shown are height-radius cross-sections of the azimuthal mean of (a) tangential and (b) radial winds (m s$^{-1}$, inflow positive), (c) vertical motion (m s$^{-1}$), (d) angular momentum (m$^2$ s$^{-1}$), (e) temperature anomalies (K) from the undisturbed environment, and (f) potential vorticity (PVU = 10$^{-5}$ K m$^2$ kg$^{-1}$ s$^{-1}$).

(Fig. 2). Wang (2001) has already shown that asymmetry near the core in the mid–lower troposphere is dominated by low azimuthal wavenumber vortex Rossby waves. A detailed analysis of the characteristics of these waves is the subject of the next section.

4. Characteristics of the vortex Rossby waves

Figure 3 shows the total asymmetric geopotential height and horizontal wind fields (a), and their wavenumber-1 (b) and wavenumber-2 (c) components, together with the residual (d) after subtracting both wavenumber-1 and wavenumber-2 components from the total asymmetric fields at 850 hPa at 86 h 30 min of simulation. The asymmetry in both geopotential height and horizontal winds is dominated by low azimuthal wavenumber (one or two) components in the core within a radius of about 90 km, with the maximum amplitude near the RMW where a large radial PV gradient exists (Fig. 1f). This finding is consistent with the results of Smith and Montgomery (1995), who demonstrated that only low wavenumber disturbances can exist near the tropical cyclone core where strong shearing damps high wavenumber disturbances effectively through the axi-symmetrization process. Wang (2001) has demonstrated that although the divergent flow remains strong, the asymmetric geopotential height and the asymmetric horizontal wind fields are quasi balanced for both wavenumber-1 and wavenumber-2 waves (Fig. 3). The divergent anticyclonic flow is collocated with the positive perturbation geopotential height, and the confluent cyclonic flow is collocated with the negative perturbation.
Fig. 2. Horizontal plan view of (a) the simulated radar reflectivity (in dBZ) at the surface in the cyclone inner region (400 km by 400 km with circles shown every 30 km from the cyclone center) and (b) its zonal-vertical cross section along A–B in (a), at 80 h 30 min of simulation.

geopotential height (Fig. 3b). Wang (2001) has identified that these waves are forced vortex Rossby waves and are similar to the free vortex Rossby waves discussed by Montgomery and Lu (1997) in a shallow water model in their quasi-balanced structure and their azimuthal and radial propagations.

Although the wavenumber-2 waves have a stronger divergent motion in the present case, their phase speeds are still within the “slow” regime\(^1\) as defined by Montgomery and Lu (1997). In this regime, perturbation vorticity and geopotential height are anticorrelated (in the Northern Hemisphere) and exhibit quasi-balanced structure. Note also that this is a Southern Hemisphere (18°S) tropical cyclone so that cyclonic (anticyclonic) is clockwise (counterclockwise) with negative (positive) PV.

\(^1\) Here slow regime is referred to waves with frequencies less than or equal to the advective frequency, as defined in Montgomery and Lu (1997).
Fig. 3. (a) Total asymmetric geopotential height (m$^2$ s$^{-2}$, in color) and horizontal wind (arrows) fields and (b) their wavenumber-1 and (c) wavenumber-2 components, together with the (d) residual after subtracting both wavenumber-1 and wavenumber-2 components from the total asymmetric fields, at 850 hPa at 86 h 30 min of simulation. The domain shown in each panel is 180 km by 180 km. Circles shown every 30 km from the cyclone center.

gomery and Lu (1997), and the negative (positive) perturbation geopotential height leads the cyclonic (anticyclonic) perturbation winds. These disturbances are vortex Rossby-type waves rather than inertia–gravity waves because the latter belong to the “fast” regime and have a frequency between the Brunt–Väisälä frequency and the local inertia frequency, implying that inertia–gravity waves must either propagate upwind so rapidly that they are nearly stationary or propagate downwind much faster than the azimuthal wind of the mean cyclone (Willoughby 1977). The waves shown in Fig. 2 also differ in structure from the inertia–gravity waves whose negative (positive) perturbation geopotential height is usually far upstream of the cyclonic (an-
Fig. 4. (left panels) Wavenumber-1 and (right panels) wavenumber-2 components of vertical relative vorticity (upper panels) and horizontal divergence (lower panels) at the same time and same vertical level as shown in Fig. 3. The domain shown in each panel is 240 km by 240 km. Shaded are regions with anticyclonic vorticity or divergence (solid contours). Dashed contours are for cyclonic vorticity or convergence both in unit of $10^{-4}$ s$^{-1}$.

ticyclonic) perturbation winds (Montgomery and Lu 1997).

Since vortex Rossby waves are vorticity (more generally, PV) waves, we will focus, like Reasor et al. (2000), on the vorticity structure of the waves and their corresponding divergence (or vertical motion) fields. Figure 4 shows simultaneously the wavenumber-1 and wavenumber-2 vertical relative vorticity and horizontal divergence fields at the same time and same pressure level as given in Fig. 3, except for a larger domain. We can see that in the vorticity as well as in the divergence fields, the disturbances spiral cyclonically inward with maximum amplitudes near the RMW. This structure of the waves indicates an outward propagation in addition to a cyclonic rotation around the eyewall, similar to the vortex Rossby waves discussed by MK97.

To give a three-dimensional view of the vortex Rossby waves in our simulated tropical cyclone core, we show in Fig. 5 the vertical structure of the wavenumber-1 (Figs. 5a–d) and wavenumber-2 (Figs. 5e–h) disturbances in both an azimuthal–vertical plane around 40-km radius (circle shown in Fig. 4) and in a radial–vertical plane from the cyclone center to 100-km radius along the line shown in Fig. 4. The wave-
Fig. 5. The vertical structure of the (a)–(d) wavenumber-1 and (e)–(h) wavenumber-2 disturbances in both an azimuthal–vertical plane around 40-km radius circles in Fig. 5 (left panels) and in a radial–vertical plane from the cyclone center to 100-km radius along the line shown in Fig. 4 (right panels). (a), (b), (e), and (f) Relative vorticity and (c), (d), (g), and (h) vertical motion. Regions with anticyclonic (positive) vorticity and upward motion are shaded. Contour intervals are $10^{-2}$ s^{-1} for relative vorticity and 0.2 m s^{-1} for vertical motion.

Number-1 waves tilt downwind azimuthally with height below about 400 hPa and then upwind above in relative vorticity (Fig. 5a), but they consistently tilt downwind in vertical motion (Fig. 5c). Radially, the waves tilt consistently outward with height with a coherent structure below about 500 hPa in relative vorticity (Fig. 5b), but extend throughout the troposphere in vertical motion (Fig. 5d). A salient feature of the waves in the mid–lower troposphere is that the upward motion leads cyclonic vorticity in both azimuthal (Figs. 5a,c) and radial (Figs. 5b,d) directions by about one-quarter wavelength. This implies that convective heating,
which is coupled with low-level convergence and upward motion, is the driving force for the waves. On the other hand, as already demonstrated by Wang (2001), the circulation associated with the waves (Fig. 3) plays an important role enhancing (suppressing) convection between an upstream (downstream) cyclonic gyre and a downstream (upstream) anticyclonic gyre associated with the waves. This is due to the fact that the boundary layer inflow is enhanced by the inflow between an upstream cyclonic gyre and a downstream anticyclonic gyre, but suppressed by the wave outflow between a downstream cyclonic gyre and an upstream anticyclonic gyre. The vortex Rossby waves and the convective asymmetries in the eyewall are, therefore, a well-coupled system. Such a coupling is crucial because it causes changes in both structure and intensity of the simulated tropical cyclone. A detailed study of these processes will be given in Part II.

Overall, the wavenumber-2 vortex Rossby waves have a much more complicated vertical structure than the wavenumber-1 waves (Figs. 5e–h). The wavenumber-2 waves tilt upwind in the azimuth with height (Fig. 5e) and they tilt radially outward below about 700 hPa and then aligned in the vertical up to about 400 hPa (Fig. 5f) in relative vorticity. The vertical motion both azimuthally (Fig. 5g) and radially (Fig. 5h) shows a baroclinic structure, implying strong mid-level divergence or convergence. Similar to the wavenumber-1 waves, the wavenumber-2 waves also have a consistent vorticity–divergence coupling in the lower troposphere with vertical motion leading vorticity, implying that the waves are coupled with convection in the eyewall.

Although vortex Rossby waves mostly have a persistent vertical structure resembling that shown in Fig. 5, they can vary occasionally. A typical example in which stronger wavenumber-2 disturbances are dominant is shown in Figs. 6 and 7 at 119 h 30 min of simulation. In this case, the wavenumber-2 waves are increasingly trapped in the eyewall (Fig. 6). The wavenumber-1 waves consistently tilt upwind azimuthally (Figs. 7a,c) and outward radially (Figs. 7b,d) with height in both vorticity and vertical motion, instead of a downwind tilt in the mid–lower troposphere seen in Fig. 5. The wavenumber-2 waves now tilt slightly downwind azimuthally with height in the lower troposphere and then upwind above (Fig. 7e), and tilt radially outward with height in the lower troposphere (Fig. 7f) in relative vorticity. There is a coherent vertical motion in the mid–lower troposphere (below about 300 hPa) both azimuthally (Fig. 7g) and vertically (Fig. 7h). Further analyses for different times show that the dominant waves mostly tilt downwind azimuthally in the lower troposphere and then upwind with increasing height, and they consistently tilt radially outward. As already seen from Fig. 5, the vertical motion leads the vorticity in the waves by about one-quarter wavelength. This feature is robust and does not change with time or wavenumbers in the lower troposphere.

Note also that the waves described above are mostly trapped in the mid–lower troposphere, as opposed to inertia–gravity waves whose intrinsic frequency is so high that the vertical group velocity is so large that the waves loose a lot of energy through propagation into the stratosphere (Willoughby 1977). Although the vortex Rossby waves also propagate vertically, there is a stagnation height at which the vertical group-velocity becomes zero [see Eq. (3.5) in Möller and Montgomery 2000]. For an upward-propagating vortex Rossby wave, this stagnation height decreases with increasing azimuthal wavenumber (Möller and Montgomery 2000). From Figs. 5 and 7, we can clearly see that while both wavenumber-1 and wavenumber-2 vortex Rossby waves are mostly trapped in the mid–lower troposphere, wavenumber-1 waves have a more coherent vertical structure than wavenumber-2 waves. This again is consistent with the behavior of vortex Rossby waves, but different from the behavior of inertia–gravity waves.

As already shown by Wang (2001), the wavenumber-1 vortex Rossby waves in the simulated tropical cyclone rotate cyclonically around the eyewall and propagate outward. Figures 8a and 8b show, respectively, the azimuthal time Hovmöller of the wavenumber-1 relative vorticity around a radius of 30 km from the cyclone center (a) and the radius–time Hovmöller of the wavenumber-1 relative vorticity from the cyclone center to 100 km to the east (b) at 850 hPa from 84 to 108 h of simulation. The waves take about 111 min to complete one circle around the eyewall. Since the period of a parcel moving around the eyewall with the local mean tangential flow of the primary cyclone is about 48 min (estimated based on 65 m s$^{-1}$ tangential wind near the eyewall), the waves propagate upwind against the tangential flow of the primary cyclone around the eyewall at a speed of about 36 m s$^{-1}$. Even though this is about three times the phase speed of planetary Rossby waves in the midlatitude troposphere, it is within the slow regime for the intense tropical cyclone core and consistent with the large radial PV gradients near the eyewall (Fig. 1f). The wavenumber-1 waves move outward at a speed of about 4–5 m s$^{-1}$ (right panel in Fig. 8). Because this movement is against the boundary layer inflow of the azimuthal mean vortex, it implies an even larger outward propagation of the waves.

No waves are seen beyond about 70 km from the cyclone center (right panel in Fig. 8). The wave amplitude decreases very quickly beyond a radius of about 50 km where the radial gradients of PV become quite small. MK97 studied theoretically the radial propagation of vortex Rossby waves and showed the existence of a stagnation radius at which the radial group velocity of a wave packet becomes zero. In our simulated tropical cyclone, this stagnation radius occurs near 70 km from the cyclone center, while inertia–gravity waves do not have such a feature, demonstrating further that the waves we identified are vortex Rossby waves.
The wavenumber-2 vortex Rossby waves propagate upwind against the azimuthal mean tangential flow around the eyewall with a similar phase speed to that of the wavenumber-1 waves (left panel in Fig. 9). As shown in Fig. 9, the wavenumber-2 waves are quite active between 86 and 100 h but inactive between 100 and 106 h. Their outward propagation (right panel in Fig. 9) is not as clear as for the wavenumber-1 waves and is much faster (10–20 m s$^{-1}$) (right panel in Fig. 8). This seems to be in conflict with the linear theory of free vortex Rossby waves, which predicts a reduced radial phase propagation for increased azimuthal wavenumbers [see Eq. (15), in MK97]. Since the wavenumber-2 waves in our simulated tropical cyclone have a larger radial wavelength (smaller wavenumber) than the wavenumber-1 waves (see Figs. 4 and 6), they could propagate faster radially than the wavenumber-1 waves. In addition, the wavenumber-2 waves are mostly trapped near the eyewall and are generally less strong than the wavenumber-1 waves.

Although the vortex Rossby waves in our simulated tropical cyclone are similar in many aspects to the linear free vortex Rossby waves discussed by MK97 and Möller and Montgomery (2000), there are also differences. First, the waves in our simulated tropical cyclone are
well coupled with nonconservative processes such as convection. Second, the linear free-wave solution derived by MK97 was obtained for a barotropic vortex whose vertical vorticity is monopolar with a maximum value at the vortex center, while the radial PV in the core of the simulated tropical cyclone is not monopolar and has a local maximum just inside of the eyewall in the lower troposphere, consistent with observations in real tropical cyclones (Shapiro and Franklin 1995). Third, the vertical propagation of the vortex Rossby waves could be much more complicated than that predicted by the linear theory given in Möller and Montgomery (2000) since the waves change their azimuthal tilt with height (Figs. 5 and 7) and tilt radially outward instead of inward with height. This complex structure may originate from those effects not included in the linear theory, such as the effect of vertical shear of both radial and azimuthal winds in the azimuthal mean vortex, the effect of diabatic heating, etc. Finally, the wavenumber-1 and wavenumber-2 vortex Rossby waves in
the simulated tropical cyclone propagate around the eyewall with roughly the same azimuthal phase speed. This behavior is unlike the Kelvin dispersion relation for two-dimensional nondivergent vortex Rossby edge waves recently discussed by Kuo et al. (1999), which predicts a stationary nature of the wavenumber-1 edge waves. Note that the simple Kelvin dispersion relation [see Eq. (2.1a) in Kuo et al. 1999] was based on nondivergent barotropic equations and a basic vorticity structure of Rankine vortex, while our simulated tropical cyclone has a complex radial and vertical structure with strong diabatic and frictional effects. Therefore, we should not expect perfect agreement with previous theories of linear free-vortex Rossby waves although the linear dynamics help understand most of the wave behaviors in the simulated tropical cyclone.
5. PV budget

Since the vortex Rossby waves are a unique feature of the simulated tropical cyclone, we are interested in examining the major PV and kinetic energy sources of the waves. In this section, a PV budget is carried out for the vortex Rossby waves in order to understand their PV dynamics in the simulated tropical cyclone.

The PV tendency equation in cylindrical and pressure coordinates translating with the tropical cyclone can be written as (see Wu and Wang 2000)

$$\frac{dP}{dt} = -\nabla \cdot (V \cdot P - gQ \zeta_\theta + g \nabla \theta \times F),$$  \hspace{1cm} (1)

where $P$ is PV, which can be written as (Hoskins et al. 1985)

$$P = -g \zeta \cdot \nabla \theta = -g(fk + \nabla \times V) \cdot \nabla \theta,$$  \hspace{1cm} (2)

where $k$ is the unit vector in the vertical with positive upward; $f$ the Coriolis parameter; $g$ the gravitational acceleration; $Q$ the diabatic heating rate (in Kelvin per second); $F$ the friction, $\theta$ the potential temperature; $t$
the time; \( \zeta \) the three-dimensional absolute vorticity vector; and
\[
\mathbf{V} = u \mathbf{r} + v \mathbf{\lambda}, \quad \mathbf{V}_3 = u \mathbf{r} + v \mathbf{\lambda} - \omega \mathbf{k},
\]
\[
\nabla_3(\cdot) = \frac{\partial (\cdot)}{\partial r} + \mathbf{r} \frac{\partial (\cdot)}{\partial \lambda} - k \frac{\partial (\cdot)}{\partial p}, \tag{3}
\]
where \( u, v \) are the radial and azimuthal winds, respectively, relative to the moving cyclone; \( \omega \) is the vertical \( p \) velocity; \( r, \lambda, \) and \( p \) are the radius, azimuth, and pressure, respectively; \( \mathbf{r} \) and \( \mathbf{\lambda} \) the unit vectors in radial and azimuthal directions, respectively.

Let \( A = \mathbf{\bar{A}} + A' \), here the quantities with overbar and with prime are, respectively, the azimuthal mean on pressure surface and the deviation from the azimuthal mean with \( A \) being \( u, v, \omega, P, Q, \) or \( \mathbf{F} \). Substituting the partitioned variables into (1) and performing an azimuthal mean, we get the PV tendency equation for the azimuthal mean cyclone given by
\[
\frac{\partial \mathbf{P}'}{\partial t} = -\nabla_3 \cdot (\nabla_3 \mathbf{\nabla} - g Q \mathbf{\bar{\zeta}} + g \nabla_3 \mathbf{\theta} \times \mathbf{F}) + \nabla_3 \cdot (\mathbf{V} P' - g Q' \mathbf{\bar{\zeta}} + g \nabla_3 \mathbf{\theta} \times \mathbf{F}) \tag{4}.
\]
Terms in the bracket on the rhs of (4) present PV fluxes due to the following processes: the mean cyclone circulation; diabatic heating, friction (including horizontal and vertical diffusions), and fluxes (last three terms) associated with nonlinear eddy processes, including asymmetric components of motion, diabatic heating, and friction.

Subtracting the azimuthal mean PV tendency equation (4) from Eq. (1), and noting the definition of partitioning, we can obtain a PV tendency equation for the asymmetric motion given by
\[
\frac{\partial \mathbf{P}'}{\partial t} = -\nabla_3 \cdot (\nabla_3 \mathbf{\nabla} - g Q \mathbf{\bar{\zeta}} + g \nabla_3 \mathbf{\theta} \times \mathbf{F}) + \nabla_3 \cdot (\mathbf{V} P' - g Q' \mathbf{\bar{\zeta}} + g \nabla_3 \mathbf{\theta} \times \mathbf{F}) + g (\nabla_3 \mathbf{\theta} \times \mathbf{F}) - g \mathbf{\nabla}_3 \mathbf{\theta} \times \mathbf{F}, \tag{5}
\]
where
\[
\mathbf{J}_E = (\mathbf{V} P' - \mathbf{V}_3 P') - g (Q' \mathbf{\bar{\zeta}} - Q \mathbf{\bar{\zeta}}) + g (\nabla_3 \mathbf{\theta} \times \mathbf{F} - \nabla_3 \mathbf{\theta} \times \mathbf{F}), \tag{6}
\]
is the PV flux solely due to nonlinear eddy processes (NONL), which will be shown to be small (at least 1 order smaller than the leading terms). Terms in the bracket on the rhs of (5) are, respectively, flux of asymmetric PV due to the mean cyclone motion (horizontal and vertical advection of asymmetric PV by the symmetric motion, LADV); flux of mean PV due to the asymmetric motion (horizontal and vertical advection of symmetric PV by the asymmetric motion, BETA); PV fluxes due to asymmetric (DIABE) and symmetric (DIABM) components of diabatic heating; and PV fluxes due to symmetric (FRICM) and asymmetric (FRICE) components of friction and diffusion. Note that the second term (BETA) on the rhs of (5) is due to the radial and vertical PV gradients of the mean cyclone, to which we refer as the vortex beta term, similar to the planetary \( \beta \) term in the large-scale vorticity equation.

Since the vortex Rossby waves are weak in the upper troposphere (Figs. 5 and 7), we will analyze the PV budget in the lower troposphere. Figure 10 shows a projection of each term on the rhs of the perturbation PV equation (5) onto wavenumber-1, together with the wavenumber-1 PV (Fig. 10a) and the total wavenumber-1 PV tendency (Fig. 10i) estimated from the rhs of (5) at 86 h 30 min of simulation at 850 hPa. The flux divergence of perturbation PV due to the azimuthal mean cyclone (Fig. 10b), namely, the linear advection of perturbation PV by the azimuthal mean cyclone flow, contributes to a cyclonic rotation of the waves. The vortex beta term (Fig. 10c) has a sign nearly opposite to that of the linear advection term (Fig. 10b), and thus slows down the cyclonic rotation of the waves caused by the linear advection, or alternatively, contributes to an upwind propagation of the waves relative to the mean tangential flow. The term associated with the asymmetric diabatic heating (Fig. 10d) is the leading term in the perturbation PV equation and largely balances the vortex beta term. Thus it plays a role in reducing the upwind propagation of the waves that would otherwise occur as a result of the vortex beta term alone. Therefore, both diabatic heating and vortex beta terms are primary PV sources for the vortex Rossby waves. We would emphasize here that the asymmetric convective heating with its maximum at middle levels is very effective in material PV concentration in the lower troposphere because of the strong vorticity in the eyewall of the azimuthal mean cyclone, that serves as a basic state for the vortex Rossby waves.

Contribution by symmetric diabatic heating to the wavenumber-1 PV budget (Fig. 10e) is relatively small (about 25% of that by asymmetric diabatic heating), but it is about 60% of the sum of the vortex beta term and the term associated with the asymmetric diabatic heating (not shown). Importantly, its distribution (Fig. 10e) is nearly in phase with the wavenumber-1 PV distribution given in Fig. 10a. This indicates that diabatic heating associated with the symmetric cyclone is also a PV source of the waves. This is so because the PV tendency associated with the symmetric diabatic heating is related to the perturbation relative vorticity [third term on the rhs of Eq. (5)]. In the lower troposphere, in the region of cyclonic (anticyclonic) vorticity of the waves, cyclonic (anticyclonic) PV will be concentrated due to symmetric diabatic heating, that has a maximum at middle troposphere, such as is the case for our simulated tropical cyclone, and for real tropical cyclones. This also implies that stronger tropical cyclones may support stronger vortex Rossby waves near their eyewalls.

The PV tendency due to friction associated with the mean cyclone (Fig. 10f) is quite small compared to the other terms in the perturbation PV equation (5). However, the PV tendency due to friction associated with
Fig. 10. The wavenumber-1 component of (a) PV and (b)–(h) PV tendencies due to terms on the rhs of the perturbation PV equation (6) and the total local PV tendency (i) estimated from all the terms on the rhs at 86 h 30 min of simulation at 850 hPa. (b) Horizontal and vertical advection of perturbation PV by the symmetric cyclone flow (LADV); (c) horizontal and vertical advection of symmetric PV by the asymmetric flow (BETA); (d) PV flux divergence due to asymmetric diabatic heating (DIABE); (e) PV flux divergence due to the symmetric diabatic heating (DIABM); (f) nonconservative PV flux divergence due to symmetric friction (FRICM); (g) PV flux divergence due to asymmetric friction (FRICE); (h) PV tendency due to all the nonlinear eddy processes (NONL). The domain shown in each panel is 120 km by 120 km and the circles are every 30 km from the cyclone center. Contour interval is 1 PVU in (a) and 1 PVU day$^{-1}$ in the rest with positive (anticyclonic) PV or PV tendency shaded.

The asymmetric wave motion (Fig. 10g) is about 15% of the leading term (Figs. 10d) and about 60% of that due to diabatic heating associated with the symmetric cyclone (Fig. 10e). Thus this friction is a major sink for the PV tendency of the waves. Finally, the PV tendency associated with all the nonlinear processes due to the eddy themselves (Fig. 10h), as indicated earlier, is negligible for wavenumber-1 waves. In the mid–upper troposphere, the PV balance in the wavenumber-1 waves are mainly among three terms: linear advection, vortex beta term, and diabatic heating, while both friction and nonlinear processes contribute little to the total perturbation PV tendency (not shown).

The PV balance in the wavenumber-2 waves (Fig. 11) is similar to that in the wavenumber-1 waves except that the PV tendency due to nonlinear processes (Fig. 11h)
is larger and comparable to the contribution by diabatic heating (Fig. 11d). The nonlinear term (Fig. 11h), which represents a wave–wave interaction, acts as a PV source of the wavenumber-2 waves, especially in the eyewall region. This indicates that the wavenumber-2 waves may obtain their energy from waves with different wavenumbers. Our calculation shows that the nonlinear term in the PV tendency equation (5) projected onto wave-number-2 waves results largely (over 95%) from the nonlinear interaction with the wavenumber-1 waves (Fig. 12). Note also that the nonlinear processes, as well as the friction, contribute to a considerable spiraled structure in wavenumber-2 PV tendency outside the RMW in the mid–lower troposphere (Figs. 11f,g,h).

Schubert et al. (1999) have already shown that eddies (vorticity or PV waves) developed in the eyewall can mix the vorticity (or PV) between the eye and the eyewall. It would therefore be interesting to examine in the simulated tropical cyclone the role of the vortex Rossby waves in the PV budget of the azimuthal mean vortex. Figure 13 shows such a PV budget for three vertical levels at 86 h 30 min, the same time as shown in Figs. 10 and 11. Terms shown in Fig. 13 represent the following [see Eq. (4)]: 1) flux divergence of symmetric PV due to both radial and vertical advection by the azimuthal mean flow (FLUX); 2) diabatic heating associated with the symmetric cyclone (DIAB); 3) friction (including diffusion) related to the symmetric cyclone.
(FRIC); 4) eddy PV flux divergence (EDDY, contribution by asymmetric motion to the mean vortex); and 5) diabatic heating associated with the asymmetric motion (EDHT). The friction term associated with the asymmetric motion [last term in (4)] is quite small and, therefore, is not shown in Fig. 13.

Throughout the troposphere (Fig. 13), both diabatic heating and advective PV flux divergence are the leading terms, and largely balanced. In the lower troposphere, diabatic heating of the symmetric cyclone is a major source for cyclonic (negative) PV in the eyewall (between 15- and 40-km radii), consistent with strong convection in the eyewall in the simulated tropical cyclone. Within a radius of 5 km from the cyclone center, diabatic heating of the symmetric cyclone makes a small contribution, which is due to some shallow convection developed in the eye of the modeled tropical cyclone (Fig. 2). Between 5- and 15-km radii, there is an anticyclonic PV tendency, which is related to a downward gradient of diabatic heating that is the result of cooling above due to melting of snow and graupel near the inner edge of the eyewall and warming below due to sensible heat flux from the sea surface. This diabatic heating, with a maximum near the sea surface favors the concentration of anticyclonic PV in the lower troposphere. This may be one of the major processes in the simulated tropical cyclone responsible for the relatively low cyclonic PV in the eye in the lower troposphere (see Fig. 9d in Wang 2001). Further analysis shows that diabatic heating associated with the horizontal heating gradient transports cyclonic PV inward from the eyewall to the eye (not shown), and thus partly offsets the anticyclonic PV tendency associated with the vertical heating gradient in the eye. The advective PV flux divergence transports cyclonic PV upward and thus reduces both cyclonic PV in the mid–lower troposphere and anticyclonic PV in the upper troposphere (thin solid curve, Fig. 13). The friction (including horizontal diffusion) term (dotted curve, Fig. 13) is small compared to other leading terms, and is negligible in the mid–upper troposphere, while it plays a role in transporting cyclonic PV outward across the eyewall in the lower troposphere (Fig. 13a). The asymmetric eddy motion redistributes PV in such a way that it transports cyclonic PV inward from the eyewall to the eye in the lower troposphere. Although the nonlinear eddy term is only about 25% of the leading terms, it is dynamically important in mixing PV between the eye and the eyewall, in agreement with the findings of Schubert et al. (1999). This inward transport of PV also spins up the tangential wind in the eye of the mean vortex (MK97). Note that the diabatic heating associated with the asymmetric motion contributes to a concentration of cyclonic PV under the eyewall (Fig. 13a).

In summary, the above PV budget suggests that diabatic heating is the major PV source in both the vortex Rossby waves and the symmetric cyclone. The nonlinear wave–wave interaction contributes significantly to the PV budget of the wavenumber-2 vortex Rossby waves. The vortex beta effect (the vortex beta term in the perturbation PV equation) contributes to an upwind propagation of the vortex Rossby waves relative to the azimuthal mean tangential flow, while the diabatic heating reduces such an upwind propagation. Although the waves are mainly forced and maintained by the mean cyclone, which provides an environment with radial PV gradients, they contribute to the PV budget of the mean vortex by wave–mean flow interaction as already demonstrated in simple barotropic models (e.g., MK97; Möller and Montgomery 1999).

6. Kinetic energy budget

In order to examine the kinetic energy sources of the vortex Rossby waves, we have performed an eddy kinetic energy budget. The azimuthally averaged eddy kinetic energy equation can be written as (see appendix)

\[
\frac{\partial K'}{\partial t} = \text{FDM} + \text{FDE} + \text{BTC} + \text{BCC} + \text{PTC} + \text{DISS},
\]

where

\[
\text{FDM} = - \frac{\partial (r \pi K')}{\partial r} - \frac{\partial (\pi K')}{\partial p},
\]

flux divergence of \( K' \) due to azimuthal mean vortex;

\[
\text{FDE} = - \frac{\partial \left( ru' \psi v' + v'^2 \right)}{2 \partial r} - \frac{\partial \left( \psi u'^2 + v'^2 \right)}{2 \partial p},
\]

flux divergence of \( K' \) due to eddies;
BTC = $\frac{\nabla}{r} u'v' - \frac{\partial u'}{\partial r} v' - \frac{\partial v'}{\partial r} u' - \frac{\nabla}{r} u'D'_z$,

energy conversion from azimuthal mean vortex by barotropic processes;

BCC = $-\frac{\partial}{\partial p} u' \omega' - \frac{\partial}{\partial p} v' \omega'$,

energy conversion from azimuthal mean vortex by baroclinic processes;

PTC = $-\frac{u'}{r} \frac{\partial \Phi'}{\partial r} - \frac{v'}{r} \frac{\partial \Phi'}{\partial \lambda}$

transfer of eddy potential energy into eddy kinetic energy;

DISS = $\bar{u}'F'_z + \bar{v}'F'_z + \bar{u}'D'_z + \bar{v}'D'_z$,

dissipation of eddy kinetic energy due to both friction and diffusion;

$K' = \frac{1}{2}(u'^2 + v'^2)$ is the azimuthal mean eddy kinetic energy; and $\Phi'$ the asymmetric geopotential.

Figure 14 shows the radial–vertical cross section of both kinetic energy of the azimuthal mean cyclone (Fig. 14a) and kinetic energy of the asymmetric disturbances (Fig. 14b) at 86 h 30 min of simulation. Distribution of the kinetic energy for the azimuthal mean cyclone (Fig. 14a) is similar to the tangential flow shown in Fig. 1a. The eddy kinetic energy (Fig. 14b) has a relatively complex vertical structure with three maxima in the lower, middle, and upper troposphere, respectively. The maximum in eddy kinetic energy in the lower troposphere is near the RMW, and thus is responsible for the vortex Rossby waves. The maximum eddy kinetic energy in the upper troposphere appears to result from barotropic instability of the outflow layer, as discussed by Anthes (1972). The maximum in eddy kinetic energy is only 1%–2% of the kinetic energy of the azimuthal mean cyclone, and thus can be inferred from the PV budget in the last section, the eddies can be described by linear dynamics, especially in the lower troposphere.

The eddy kinetic energy budget at 86 h 30 min of simulation is shown in Fig. 15. The six panels correspond to the six terms on the rhs of the eddy kinetic energy equation (7). In the mid–lower troposphere, flux divergence of the eddy kinetic energy due to both the azimuthal mean vortex (FDM, Fig. 15a) and the eddies themselves (FDE, Fig. 15b) transports the eddy kinetic energy inward from outside the RMW. In the upper troposphere, the azimuthal mean vortex (FDM) transports eddy kinetic energy upward (Fig. 15a), while the eddy transport (FDE) is of secondary importance (Fig. 15b). Consistent with the findings of Anthes (1972) and Flatau and Stevens (1989), the barotropic conversion of mean kinetic energy to eddy kinetic energy (BTC) is a major source of eddy kinetic energy in the upper troposphere and contributes largely to the development of upper-level asymmetries (cf. Fig. 14b with Fig. 15c). In the mid–lower troposphere, however, the eddies (that is, the vortex Rossby waves) give their kinetic energy to the mean vortex through barotropic conversion (BTC, Fig. 15c), and thus such a conversion damps the eddies and represents an upscale-energy cascade mechanism for tropical cyclones as first proposed by Montgomery and Enagonio (1998). However, in the boundary layer within the RMW, the mean vortex feeds kinetic energy to the eddies by barotropic conversion (Fig. 15c). This is mainly due to the barotropic conversion associated with the radial inflow convergence of the mean cyclone to the eyewall (BTCR = $-\frac{\partial}{\partial p} (\bar{u}' \omega')$ and $-\frac{\partial}{\partial p} (\bar{u}' \omega')$, Fig. 16a), rather than that associated with the mean azimuthal flow (BTC = $-\frac{\partial}{\partial p} \bar{u}' \omega'$, Fig. 16b). The latter is a major sink for eddy kinetic energy in the mid–lower troposphere (Fig. 16b), indicating that the convergent inflow, not the instability of nondivergent azimuthal flow, helps remain the vortex Rossby waves in the lower troposphere. The role of the confluent inflow, in causing the energy growth of vortex Rossby waves found here, has some similarities to the Rossby wave energy accumulation mechanism in a confluent basic zonal flow as discussed by Webster and Chang (1988). Another important kinetic energy source for vortex Rossby waves is the baroclinic conversion (BCC), which contributes to the eddy kinetic energy in the eyewall throughout the troposphere but with a maximum in the lower troposphere (Fig. 15d). As opposed to the barotropic conversion discussed above, the two components in the baroclinic conversion related to the radial (BCCR = $-\frac{\partial}{\partial p} (\bar{u}' \omega')$ and $-\frac{\partial}{\partial p} (\bar{u}' \omega')$, Fig. 16c) and azimuthal (BCCA = $-\frac{\partial}{\partial p} \bar{u}' \omega'$, Fig. 16d) flows of the mean cyclone are both sources of eddy kinetic energy in the lower troposphere between 20 and 40 km radii. The baroclinic conversion related to the vertical shear of the radial flow (Fig. 16c), however, is about three times the magnitude of that related to the vertical shear of the azimuthal flow (Fig. 16d). The energy conversion from eddy potential energy is the main source for the eddy kinetic energy (Fig. 15e). The eddy potential energy conversion is related to asymmetric diabatic heating associated with moist convection in the eyewall. This demonstrates that vortex Rossby waves are coupled with, and maintained by eyewall convection, as already seen from PV budget discussed in the last section. Note, however, that within the RMW in the mid–lower troposphere, eddy potential energy conversion is a sink for the eddy kinetic energy and balances other eddy kinetic energy sources. Frictional dissipation and horizontal diffusion (DISS) are mainly sinks for eddy kinetic energy (Fig. 15f).

The above energy budget demonstrates that the vortex Rossby waves receive their kinetic energy mainly from the mean vortex through both baroclinic conversion and flux divergence of eddy kinetic energy flux due to the azimuthal mean flow within the RMW, and from the transfer of eddy potential energy under the eyewall and just outside the RMW. Frictional dissipation and hori-
Fig. 13. The PV budget for the azimuthal mean vortex at three vertical levels of (a) 850 hPa, (b) 500 hPa, and (c) 200 hPa at 86 h 30 min of simulation. Different curves correspond to PV tendencies (with unit of PVU day$^{-1}$) due to different terms on the rhs of the azimuthal mean PV tendency equation (5). See text for further explanation.

Horizontal shear of the azimuthal flow of the mean vortex both act as energy sinks in the lower troposphere. An interesting finding is that in both the barotropic and baroclinic conversions, the effects related to the radial flow of the mean vortex dominate and act as energy sources for vortex Rossby waves. This has some similarity to the Rossby wave energy accumulation mechanism in a confluent basic zonal flow as discussed by Webster and Chang (1988). The damping of eddy kinetic energy that occurs in the lower troposphere as a result of horizontal shear of azimuthal flow in the mean vortex is a manifestation of the axisymmetrization processes as discussed by MK97 and Carr and Williams (1989) using barotropic models. However, in the upper troposphere, both barotropic and baroclinic conversions related to the azimuthal mean flow, together with the eddy
potential energy conversion, are responsible for the development of asymmetric structure in the outflow layer, in agreement with the findings of Anthes (1972) and Flatau and Stevens (1989).

7. Conclusions

This study has analyzed the highly asymmetric inner core of a numerically simulated tropical cyclone. The asymmetry is found to be dominated by low-wave-number vortex Rossby waves that are trapped in the mid–lower troposphere within a radius of about 60 km from the cyclone center. This occurs because the vortex Rossby waves are supported by the radial PV gradient of the azimuthal mean cyclone within which they are embedded. Beyond a radius of about 60 km, the radial PV gradient of the azimuthal mean vortex is too weak to support the vortex Rossby waves. In contrast to large-scale, quasigeostrophic motion, in which the divergence is about one order smaller than the vorticity, the perturbation vorticity and divergence in the vortex Rossby waves have similar amplitudes even in the eyewall. This finding agrees with the observational assessment by Montgomery and Franklin (1998). The geopotential
height and wind fields of the vortex Rossby waves are quasi balanced with confluent cyclonic (divergent anticyclonic) flow collocated with negative (positive) geopotential height perturbation.

The vortex Rossby waves spiral cyclonically inward in the horizontal with maximum amplitudes near the radius of maximum winds, and they tilt radially outward with height. In the lower troposphere, both horizontal convergence and upward motion in the waves consistently lead cyclonic vorticity in the azimuthal as well as in the radial directions by about one-quarter wavelength. Since the low-level convergence and upward motion are always coupled with convection, diabatic heating associated with the eyewall convection, especially its asymmetric component, is a major driving force for the vortex Rossby waves. The vortex Rossby waves propagate upwind relative to the cyclonic azimuthal flow around the eyewall of the mean cyclone and thus have a cyclonic rotation period about twice the period of a parcel moving with the cyclonic mean tangential flow around the circumference. In additional to the azimuthal propagation, the vortex Rossby waves propagate radially outward against the boundary layer inflow of the azimuthal mean cyclone.
A PV budget for the vortex Rossby waves shows that diabatic heating is the major PV source for both the vortex Rossby waves and the symmetric cyclone. The PV tendency in the vortex Rossby waves due to diabatic heating is largely balanced by the advective (both horizontal and vertical) flux divergence of the symmetric PV due to the asymmetric motion (similar to the planetary beta term in the large-scale vorticity equation). In contrast, in the symmetric cyclone, the PV tendency due to diabatic heating is largely balanced by the flux divergence of symmetric PV due to the symmetric motion. The vortex Rossby waves, through wave–mean flow interaction, transport cyclonic PV from the eyewall to the eye, thus mixing PV between the eyewall and the eye and spinning up the tangential wind in the eye at the expense of weakening the tangential wind near the RMW. Such an inward transport of PV by the vortex Rossby waves is also found in previous studies using barotropic models (Schubert et al. 1999; Montgomery and Kallenbach 1997). Moreover, the PV tendency due to nonlinear processes associated with the wave number-1 vortex Rossby waves is found to be a significant PV source for the wave number-2 vortex Rossby waves, indicating that strong wave–wave interaction occurs in the tropical cyclone eyewall.

An eddy kinetic energy budget shows that within the radius of maximum wind, the vortex Rossby waves receive their kinetic energy from the azimuthal mean cyclone through both baroclinic conversion and flux divergence of eddy kinetic energy due to the azimuthal mean flow. Under the eyewall and just outside the radius of maximum wind in the mid–lower troposphere, the main source for eddy kinetic energy is the eddy potential energy conversion, which is related to the asymmetric diabatic heating associated with moist convection in the eyewall. In both the barotropic and baroclinic conversions, conversions related to the radial flow of the azimuthal mean vortex dominate those related to the tangential flow and act as energy sources for vortex Rossby waves. This energy conversion possesses some similarity to the Rossby wave energy accumulation mechanism in a confluent basic zonal flow as discussed by Webster and Chang (1988). In the lower troposphere, the horizontal shear of the azimuthal flow in the mean vortex damps eddy kinetic energy, stabilizing the vortex Rossby waves. In the upper troposphere, however, both barotropic and baroclinic conversions related to the azimuthal flow of the mean vortex, together with the eddy potential energy conversion, are responsible for the development of asymmetric structure in the outflow layer, consistent with previous findings by Anthes (1972).

The relative inflow and outflow associated with the
vortex Rossby waves is of dynamical importance. The inflow in the lower troposphere enhances convergence and upward motion in the eyewall and strengthens convection there, while outflow contributes to divergence with reduced upward motion and weakens the convection. Such a coupling between the waves and the asymmetries in eyewall convection is essential to understanding how the vortex Rossby waves cause structure and intensity changes of real tropical cyclones. Since the vortex Rossby waves rotate cyclonically around the eye of the cyclone, weakening the outflow can be strong enough to shift the waves outward and cause a breakdown of the vortex Rossby waves. When the waves are strong and connected to an outer rainband, the outflow, the waves of the cyclone can be strong enough to shift the eyewall outward and can cause a breakdown of the cyclone, weakening the cyclone. Although this current investigation of partitioning in (A1), one can get the azimuthal mean of the resultant equation and using the definition in (A2) and using the same procedure as above, we can obtain the kinetic energy equation

\[
\frac{\partial K}{\partial t} + \frac{\partial (r \bar{u} K)}{\partial r} + \frac{\partial \bar{v} K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} = - \frac{u \partial \bar{v}}{\partial \bar{r}} - \frac{v \partial \bar{v}}{\partial \bar{r}} + uF_r + vF_{\bar{r}} + uD_r,
\]

(A4)

Multiplying the first equation in (A2) by \(u\) and the second equation by \(v\), and using the continuity equation (A3), we can obtain the kinetic energy equation

\[
\frac{\partial K}{\partial t} + \frac{\partial (r \bar{u} K)}{\partial r} + \frac{\partial \bar{v} K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} = - \frac{u \partial \bar{v}}{\partial \bar{r}} - \frac{v \partial \bar{v}}{\partial \bar{r}} + uF_r + vF_{\bar{r}} + uD_r,
\]

(A4)

Multiplying the first equation in (A2) by \(u\) and the second equation by \(v\), and using the continuity equation (A3), we can obtain the kinetic energy equation

\[
\frac{\partial K}{\partial t} + \frac{\partial (r \bar{u} K)}{\partial r} + \frac{\partial \bar{v} K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} = - \frac{u \partial \bar{v}}{\partial \bar{r}} - \frac{v \partial \bar{v}}{\partial \bar{r}} + uF_r + vF_{\bar{r}} + uD_r,
\]

(A4)

where \(K = \frac{1}{2}(u^2 + v^2)\) is the kinetic energy.

Taking an azimuthal mean for the momentum equations in (A2) and using the same procedure as above, we obtain the kinetic energy equation for the azimuthal mean vortex given by

\[
\frac{\partial K}{\partial t} + \frac{\partial (r \bar{u} K)}{\partial r} + \frac{\partial \bar{v} K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} = - \frac{u \partial \bar{v}}{\partial \bar{r}} - \frac{v \partial \bar{v}}{\partial \bar{r}} + uF_r + vF_{\bar{r}} + uD_r,
\]

(A4)

where \(K = \frac{1}{2}(u^2 + v^2)\) is the kinetic energy.

Taking an azimuthal mean for the momentum equations in (A2) and using the same procedure as above, we obtain the kinetic energy equation for the azimuthal mean vortex given by

\[
\frac{\partial K}{\partial t} + \frac{\partial (r \bar{u} K)}{\partial r} + \frac{\partial \bar{v} K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial K}{\partial \bar{r}} = - \frac{u \partial \bar{v}}{\partial \bar{r}} - \frac{v \partial \bar{v}}{\partial \bar{r}} + uF_r + vF_{\bar{r}} + uD_r,
\]

(A4)

Subtracting (A5) from (A4), then taking an azimuthal mean of the resultant equation and using the definition of partitioning in (A1), one can get the azimuthally averaged eddy kinetic energy equation (7) in section 6.

REFERENCES


Anthes, R. A., 1972: The development of asymmetries in a three-


