Interactions between Tropical Convection and Its Environment: An Energetics Analysis of a 2D Cloud Resolving Simulation

XIAOFAN LI,† C.-H. SUI, AND K.-M. LAU

NASA Goddard Space Flight Center, Greenbelt, Maryland

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ABSTRACT

The phase relation between the perturbation kinetic energy ($K'$) associated with the tropical convection and the horizontal-mean moist available potential energy ($\mathcal{P}$) associated with environmental conditions is investigated by an energetics analysis of a numerical experiment. This experiment is performed using a 2D cloud resolving model forced by the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) derived vertical velocity. The imposed upward motion leads to a decrease of $\mathcal{P}$ through the associated vertical advective cooling, and to an increase of $K'$ through cloud-related processes, feeding the convection. The maximum $K'$ and its maximum growth rate lags and leads, respectively, the maximum imposed large-scale upward motion by about 1–2 h, indicating that convection is phase locked with large-scale forcing. The dominant life cycle of the simulated convection is about 9 h, whereas the timescales of the imposed large-scale forcing are longer than the diurnal cycle.

In the convective events, the maximum growth of $K'$ leads the maximum decay of the perturbation moist available potential energy ($P'$) by about 3 h through vertical heat transport by perturbation circulation, and perturbation cloud heating. The maximum decay of $P'$ leads the maximum decay of $\mathcal{P}$ by about 1 h through the perturbation radiative processes, the horizontal-mean cloud heating, and the large-scale vertical advective cooling. Therefore, maximum gain of $K'$ occurs about 4–5 h before maximum decay of $\mathcal{P}$.

1. Introduction

Tropical convection occurs as a result of the release of unstable energy of its environment. The large-scale environment provides favorable thermal and moisture conditions for occurrence and development of convection, on one hand. On the other hand, it is adjusted by redistribution of vertical thermal, moisture, and momentum structures induced by the convection. Such interaction allows us to use environmental conditions to estimate the properties of the convection such as the precipitation. Since the environmental timescales (a few days and longer) are much longer than the convective timescales (a few hours and shorter), the rate of production of available potential energy by the large-scale processes is nearly balanced by the rate of consumption of the available potential energy by the convection (Manabe and Strickler 1964). This quasi-equilibrium concept is the basic premise of the cumulus parameterization scheme proposed by Arakawa and Schubert (1974). The decrease of convective available potential energy (CAPE) that measures the thermal and moisture conditions of the environment often coincides with the development of convection so that the CAPE and rain rate are negatively correlated (e.g., Thompson et al. 1979; Cheng and Yanai 1989; Wang and Randall 1994; Xu and Randall 1998). The phase relation between the CAPE and rainfall must be related to the coupling between environmental dynamic and thermodynamic fields (Cheng and Yanai 1989).

The phases of CAPE and rainfall could be different because time is needed for development of clouds. This phase difference may be included as relaxing the quasi-equilibrium assumption in cumulus parameterization (e.g., Betts and Miller 1986; Randall and Pan 1993). The minimum CAPE occurs a few hours after maximum rainfall. Such a phase lag is also demonstrated by Xu and Randall (1998) in their 2D cloud resolving simulations. Xu and Randall (1998) interpreted the maximum phase lag as the adjustment time scale from disequilibrium to equilibrium states in the presence of time-varying large-scale forcing.

Lorenz (1955) first introduced a concept of available potential energy of dry atmosphere that represents a portion of the potential energy that can be transferred into the kinetic energy. Lorenz (1978, 1979) extended this concept to the moist atmosphere by considering the
moist-adiabatic processes. Randall and Wang (1992) and Wang and Randall (1994) further argued that the vertical component of the moist available potential energy is a generalization of the CAPE. In this study, the physical processes responsible for such a phase relation are examined through the analysis of energy conversion processes between available potential energy and kinetic energy in a 2D cloud resolving simulation. We first establish the phase relation between available potential energy and kinetic energy, and use a set of energetics equations (section 2) to examine the essential physical processes determining the phase relation (section 3). The phase relation is discussed in section 4, and the conclusion is given in section 5.

2. Formulations for model, energetics, and CAPE

a. Model

The cloud resolving model was originally developed by Soong and Ogura (1980), Soong and Tao (1980), and Tao and Simpson (1993), for studying deep convective response to the specified large-scale forcing. A 2D version of the model used by Sui et al. (1998) and modified by Li et al. (1999) is used in this study. The governing equations with an anelastic approximation can be expressed as follows:

\[
\frac{\partial u'}{\partial t} + \frac{1}{\overline{\rho}} \frac{\partial}{\partial z}(\overline{\rho} u' w') = 0, \tag{1}
\]

\[
\frac{\partial u'}{\partial t} = -\frac{\partial}{\partial x}(u' \overline{w'} + \overline{u'} u' + u' u')
- \frac{1}{\overline{\rho}} \frac{\partial}{\partial z}(\overline{w'} u' + \overline{w'} u' + \overline{u'} u' - \overline{w'} u')
- \overline{c_p} \frac{\partial}{\partial x} (\overline{\theta'} - D_u - D_v), \tag{2}
\]

\[
\frac{\partial w'}{\partial t} = -\frac{1}{\overline{\rho}} \frac{\partial}{\partial z}(u' \overline{w'} + \overline{u'} w' + u' w')
- \overline{c_p} \frac{\partial}{\partial z} (\overline{\theta'} - D_u - D_v), \tag{3}
\]

\[
\frac{\partial \theta'}{\partial t} = -\frac{\partial(u' \theta')}{\partial x} - \overline{\rho} \frac{\partial \theta'}{\partial x} - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z}(\overline{w'} \theta')
- \overline{w'} \frac{\partial \theta'}{\partial z} - \overline{w'} \frac{\partial \theta}{\partial z} + Q_{\alpha} + Q_{\beta}, \tag{4}
\]

Here, \( u \) and \( w \) are zonal, and vertical wind components; \( \theta \) and \( q \) are potential temperature and specific humidity, respectively; \( C = (q_s, q_r, q_i, q_s, q_r, q_i) \); \( q_s, q_r, q_i \) are the mixing ratios of cloud water, rain, cloud ice, snow, and graupel, respectively; \( \overline{\rho} \) is a mean air density that is a function of height only; \( w_{ry} \) is a terminal velocity that is zero for cloud water and ice; \( \pi = (p/p_o)^r \); \( R \) is the gas constant, \( c_p \) is the specific heat of dry air at constant pressure \( p \) and \( p_o = 1000 \text{ mb} \); \( c, e, d, s \) denote condensation, evaporation, deposition, and sublimation, respectively; \( Q_{\alpha} = L_{(c - e)} + L_{(d - s)} + L_{(f - m)} \) denotes the net latent heat release through phase changes among different cloud species, where \( f \) and \( m \) are fusion and melting, respectively; \( L_c, L_s, L_f \) are heat coefficients due to phase changes; \( Q_R \) is the radiative heating rate due to convergence of net flux of solar and infrared radiative fluxes; \( S_c \) is source and sink of cloud species determined by microphysical processes; \( D_u, D_v, D_s, D_i \) and \( D_c \) are dissipation terms; overbar (\( \overline{\quad} \)) denotes a zonal mean; subscription \( s \) denotes an initial value, which does not vary with time; superscript * denotes imposed observed variables in the model.

Soong and Ogura (1980) developed this cloud resolving model based on the observed scale separation evidence that the timescale of the large-scale processes is much larger than the timescale of the life cycle of an individual convective cloud. Similar approach was adopted in Xu and Krueger (1991). The large-scale vertical velocity imposed in the model serves as the major forcing. In such a model setup, the large-scale (horizontal mean) thermodynamic states are adjusted not only by responding to the imposed large-scale dynamic forcing but also by interacting with the convection. The adjustment of the mean thermodynamic stability distribution due to the convection in the semiprognostic approach for the large-scale environment has been demonstrated to simulate the mean thermodynamic states more reasonably (Li et al. 1999). Recently, Mapes (1997) argued that deep convection and its large-scale environment interact each other so that it may be improper to impose a vertical profile of large-scale forcing in the cloud resolving simulation. Nevertheless, the cloud resolving model is a useful tool to study one-way interaction of clouds to the “large-scale forcing,” which
may serve as the guidance to further study interaction between convection and large-scale environment.

The experiment analyzed in this study is conducted with the model forced by zonally uniform vertical velocity, zonal wind, and horizontal advects, which are derived by Sui et al. (1997) based on the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) observations within the intensive flux array (IFA) region at a time of 6 h. Hourly sea surface temperature at the Improved Meteorological (IMET) surface mooring buoy (1.75°S, 156°E) (Weller and Anderson 1996) is also imposed in the model. The model is integrated from 0400 LT 18 December to 0400 LT 25 December 1992. The horizontal domain is 768 km. A grid mesh of 1.5 km and a 12-s step are used in model integrations. More discussion of the model and its responses to prescribed TOGA COARE forcing are reported in Li et al. (1999).

b. Energetics equations

Lorenz (1955) defined the available potential energy of the dry atmosphere as the difference between actual total enthalpy and the minimum total enthalpy that could be achieved by rearranging the mass under the adiabatic flow. The dry enthalpy per unit mass is defined as the product of the temperature and the specific heat at constant pressure. In the absence of energy sources and sinks, the total kinetic energy and total enthalpy are conserved during adiabatic expansion. In the moist atmosphere, latent heat energy should be included in the energy conservation. The latent heat energy per unit mass is defined as the product of the specific humidity and the latent heat of vaporization at 0°C. In the absence of energy sources and sinks, the total kinetic energy and total enthalpy and latent heat energy are conserved during dry and subsequent saturated adiabatic expansion. Therefore, the moist available potential energy is defined as the difference between the actual moist potential energy (sum of the enthalpy and latent heat energy) and the minimum moist potential energy that could be achieved by rearranging the mass under moist-adiabatic processes. Zonal-mean and perturbation moist available potential energy are, respectively, defined by

\[ \bar{P} = \frac{1}{2c_p} \left( \frac{\Gamma}{\bar{h}} - h_0^2 \right), \]

\[ P' = \frac{1}{2c_r} \left( \frac{\Gamma}{(h_0^2)} \right), \]

where \( h = c_r T + L_q \); \( T \) is temperature; \( \Gamma = -\kappa \theta(pT)^{-1} (\partial \theta/\partial p) + (L_q/c_r \pi)(\partial q_\omega/\partial p) \), which is a parameter related to static stability; the angle bracket implies a vertical integration:

\[ \langle () \rangle = \int_{z_b}^{z_r} \bar{p}(z) \, dz. \]

Here \( z_b \) and \( z_r \) are the heights of bottom and top of the model, respectively. (7a) is derived after some approximations similar to Lorenz (1955) and Peixoto and Oort (1992). \( h_0 \) is a constant reference state here, and is calculated from the initial observed sounding. The perturbation kinetic energy is defined by

\[ K' = \frac{(u')^2 + (w')^2}{2}, \]  

An equation for the perturbation kinetic energy (\( K' \)) can be derived by multiplying (2) by \( u' \) and (3) by \( w' \) and applying the zonal mean and the vertical integration on the resulting equation:

\[ \frac{\partial K'}{\partial t} = C(\bar{K}, K') + C(P', K') + G_e(K'), \]

where

\[ C(\bar{K}, K') = -\left( \frac{u'w'}{\bar{z}} \right) - \left( \frac{w'w'}{\bar{z}} \right), \]

\[ C(P', K') = \left( \frac{g^2 w' T}{\bar{T}} \right), \]

\[ G_e(K') = \langle 0.61 g w' q_s \rangle, \]

\[ G_e(K') = -\langle g w' q_s \rangle. \]

Here, \( C(\bar{K}, K') \) is the conversion between kinetic energy through covariance between perturbation zonal wind and vertical velocity under vertical shear of imposed horizontal-mean zonal wind, and between perturbation vertical velocities under vertical shear of imposed horizontal-mean velocities. \( C(P', K') \) is the conversion between \( P' \) and \( K' \) through covariance between perturbation vertical velocity and temperature. \( G_e(K') \) and \( G_e(K') \) are the generation terms of \( K' \) through covariance between perturbation vertical velocity and specific humidity, and between perturbation vertical velocity and cloud mixing ratio, respectively.

To derive the equations for the zonal-mean and perturbation moist available potential energy, the following equation is formed by multiplying (4) by \( c_r \) and (5) by \( L_q \) and adding the resulting equations:

\[ \frac{\partial \bar{h}}{\partial t} = -\frac{\partial (u' h')}{\partial x} - \bar{u} \frac{\partial \bar{h}'}{\partial x} - \frac{c_r \pi}{\bar{p}} \pi \frac{\partial \bar{h}'}{\partial \bar{z}} - c_r \bar{w} \frac{\partial \bar{h}'}{\partial \bar{z}} - \frac{L_q}{\bar{p}} \frac{\partial \bar{q}_s}{\partial \bar{z}} - L_q \frac{\partial \bar{q}_s}{\partial \bar{z}} - c_r \bar{w} \frac{\partial \bar{h}'}{\partial \bar{z}} - \frac{\partial \bar{h}_s}{\partial x} - \frac{\partial \bar{q}_s}{\partial \bar{x}} - L_q \frac{\partial \bar{q}_s}{\partial \bar{x}} \]

\[ - L_q \bar{w} \frac{\partial \bar{q}_s}{\partial \bar{z}} + L_q (d - s + f - m) + Q_s \]

The equations for the zonal-mean moist available potential energy (\( \bar{P} \)) and the perturbation moist available potential energy (\( P' \)) can be derived by multiplying (10)
by $c_p \Gamma (\bar{h} - h_b)$, and by $c_p \Gamma h'$, and applying the zonal mean and the vertical integration on the resulting equations. Thus the zonal mean equation is
\[
\frac{\partial \bar{P}}{\partial t} = C(P', \bar{P}) + G_{\text{sh}}(\bar{P}) + G_{\text{m}}(\bar{P})
+ C_s(\bar{K}, \bar{P}) + C_c(\bar{K}, \bar{P}),
\]
(11)
where
\[
C(P', \bar{P}) = -\frac{\Gamma}{c_p} (\bar{h} - h_b)
\times \left( c_p \pi \frac{\partial}{\partial z} \bar{P} \frac{w'}{P} + L_v \frac{\partial}{\partial z} \bar{P} \frac{w' \theta'}{P} \right),
\]
\[
G_{\text{sh}}(\bar{P}) = \frac{\Gamma}{c_p} Q_{\text{sh}} (\bar{h} - h_b),
\]
\[
G_{\text{m}}(\bar{P}) = \frac{\Gamma}{c_p} L_s (d - s + f - m),
\]
\[
C_s(\bar{K}, \bar{P}) = -\frac{\Gamma}{c_p} (\bar{h} - h_b) \pi \left( c_p \pi \frac{\partial \theta'}{\partial x} + L_v \frac{\partial \theta'}{\partial x} \right),
\]
\[
C_c(\bar{K}, \bar{P}) = -\frac{\Gamma}{c_p} (\bar{h} - h_b) \pi \left( c_p \pi \frac{\partial \theta'}{\partial x} + L_v \frac{\partial \theta'}{\partial x} \right),
\]
and the perturbation equation is
\[
\frac{\partial P'}{\partial t} = -C(P', \bar{P}) - C(P', K') + G_{\text{sh}}(P')
+ G_{\text{m}}(P') + G(P'),
\]
(12)
where
\[
G_{\text{sh}}(P') = \frac{\Gamma}{c_p} Q_{\text{sh}} (\bar{h} - h_b),
\]
\[
G_{\text{m}}(P') = \frac{\Gamma}{c_p} L_s (d - s + f - m),
\]
\[
G(P') = -\frac{g L_v}{c_p T_b} \frac{w'}{q_u'} - \frac{\Gamma}{c_p} \frac{\partial}{\partial z} \left( \bar{h} - h_b \right) \bar{P} (h' w')
- \frac{g \Gamma}{c_p T_b} \left( \frac{\bar{T}}{T_b} - 1 \right) h' w'
- \frac{g \Gamma}{c_p T_b} (\bar{h} - h_b) T' w'
- \frac{g \Gamma}{c_p} \frac{\partial}{\partial z} \left( \frac{\bar{w}' + w'}{\bar{P}} (h')^2 \right)
- \frac{g \Gamma}{c_p} \frac{\partial}{\partial z} \left( \frac{\bar{w}' + w'}{\bar{P}} T' h' \right).
\]
Here, $C(P', \bar{P})$ is the conversion between $P'$ and $\bar{P}$ through covariance between $\bar{h} - h_b$ and convergence of vertical flux of potential temperature and moisture. Terms $G_{\text{sh}}(\bar{P})$ and $G_{\text{m}}(\bar{P})$ represent the generation of $\bar{P}$ through covariances between $\bar{h} - h_b$ and horizontal-mean radiative heating, and between $\bar{h} - h_b$ and horizontal-mean heating due to phase change of the cloud contents, respectively. Next, $C_s(\bar{K}, \bar{P})$ and $C_c(\bar{K}, \bar{P})$ are the conversion between $\bar{K}$ and $\bar{P}$ through covariances between $\bar{h} - h_b$ and imposed horizontal temperature and moisture advections, and between $\bar{h} - h_b$ and horizontal-mean vertical temperature and moisture advections by imposed vertical velocity, respectively. Terms $G_{\text{sh}}(P')$ and $G_{\text{m}}(P')$ represent the generation of $P'$ through covariances between $h'$ and perturbation radiative heating, and between $h'$ and perturbation heating due to phase changes of the cloud contents, respectively. The term $G(P')$ is the generation of $P'$.

The CAPE calculation

The CAPE can be calculated by
\[
\text{CAPE} = g \int_{z_{\text{LFC}}}^{z_e} \frac{\theta_{\text{pfc}}(z) - \theta_{\text{env}}(z)}{\theta_{\text{env}}(z)} \, dz.
\]
Here $\theta_{\text{pfc}}$ is the potential temperature of an air parcel lifted from $z_{\text{b}}$ to $z_{\text{f}}$ while not mixing with its environment ($\theta_{\text{env}}$). The air parcel is lifted dry adiabatically until it becomes saturated and then is lifted moist adiabatically thereafter. The level of free convection (LFC) is the height where $\theta_{\text{pfc}} > \theta_{\text{env}}$. $z_e$ is the level where $\theta_{\text{pfc}} = \theta_{\text{env}}$.

The CAPE is calculated for a pseudoadiabatic process and a reversible moist adiabatic process, respectively, in this study. In the pseudoadiabatic process, an air parcel is lifted adiabatically while all condensed water drops out from the parcel. In the reversible moist adiabatic process, an air parcel is lifted adiabatically while all condensed water is kept in the parcel. Following Xu and Emanuel (1989), the virtual temperatures ($T_{\text{vpa}}$) for the pseudoadiabatic process and ($T_{\text{vce}}$) for the reversible moist-adiabatic process are, respectively, expressed by
\[
T_{\text{vpa}} = T_r \frac{1 + q_s(T_r)/0.622}{1 + q_s},
\]
\[
T_{\text{vce}} = T_r \frac{1 + q_s(T_r)/0.622}{1 + q_s(T_p)},
\]
where $T_r$ is the temperature of a pseudoadiabatically displaced air parcel, $q_s$ is the saturation specific humidity, and $q_s$ is the total water content of the air parcel. CAPE for the pseudoadiabatic process and CAPE for
the reversible moist-adiabatic process are calculated by using (14) and (15), respectively.

3. Results

Figure 1 shows the time evolution of vertical distribution of the large-scale vertical velocity and zonal wind during 19–25 December 1992 that are imposed in the model. Strong upward motions with maxima of 15–25 mb h\(^{-1}\) occur on the late of 20 December, and during the early mornings of 23 and 25 December between 400 and 500 mb. The latter two maxima are quasi-2-day oscillations (Takayabu et al. 1996) in the convective phase of an intraseasonal oscillation during TOGA COARE. Two less intense upward motion centers appear during the nights of 19 and 21 December. The occurrence of maximum upward motion at each night is consistent with the diurnal signals observed by Sui et al. (1997). The large-scale zonal wind in the lower troposphere (below 700 mb) are westerly that strengthens to 10 m s\(^{-1}\) around 23 December. The midtroposphere has an easterly–westerly wind oscillation with maximum easterly wind of \(-10\) m s\(^{-1}\) at 500 mb on 20 December. The upper troposphere (above 250 mb) is dominated by easterly winds. As mentioned previously, the model is also forced by the observed horizontal temperature and moisture advections (not shown), which have much smaller amplitudes than the vertical advections respectively.

Figure 2a shows lag correlation coefficients between zonal-mean CAPE and rain rate. Positive lag hour denotes that CAPE leads rain rate. The maximum lag correlation coefficients between zonal-mean CAPE and rain rate indicate that the CAPE reaches maximum about 3–4 h before the maximum rain rate. The minimum lag correlation coefficients indicate that the CAPE reaches minimum about 2 h after the maximum rainfall. Both maximum and minimum are above 99% confidence level. The phase difference between maximum and minimum correlation coefficients is about 5 h. Since a significant spectral peak appears at 9 h by the power spectrum analysis of the hourly rain rate (not shown), the phase difference is about half of the lifetime of the simulated convection. Figure 2a also show that the lag correlation coefficients for CAPE\(_{\text{re}}\) and CAPE\(_{\text{pa}}\) are similar.

Since the model is forced by imposed vertical velocity, the relationship between energy and imposed vertical velocity is first analyzed. Figure 2b shows lag correlation coefficients between \(\overline{F}\) and \(\langle \pi^\nu \rangle\) (solid line), and between \(K'\) and \(\langle \pi^\nu \rangle\) (dashed line). Statistically significant lag correlation coefficients display that maximum \(K'\) lags imposed upward motion (positive \(\langle \pi^\nu \rangle\)) by 1–2 h whereas minimum \(\overline{F}\) lags upward motion by...
about 6 hours. This suggests that the $K'$ leads $\mathbf{P}$ by about 4–5 h, which is about the half of the lifetime of the simulated convection. The statistically significant relationship between $\overline{\partial P}/\partial t$ and $\langle \pi^\infty \rangle$ (solid line) and between $\partial K'/\partial t$ and $\langle \pi^\infty \rangle$ (dashed line) can be also shown by lag correlation coefficients in Fig. 3a. The imposed upward motion leads minimum $\overline{\partial P}/\partial t$ (maximum decrease of horizontal-mean moist available potential energy) by 3 h, whereas it lags maximum $\partial K'/\partial t$ (maximum increase of perturbation kinetic energy) by 1–2 h. Thus, minimum $\overline{\partial P}/\partial t$ lags maximum $\partial K'/\partial t$ by 4–5 h. The occurrence of maximum imposed upward motion and maximum $\partial K'/\partial t$ and $K'$ within 3 h indicates that convection is phase locked with the imposed large-scale upward motion. The negative lag correlation coefficient between $\overline{\partial P}/\partial t$ and $\langle \pi^\infty \rangle$ in Fig. 3 means that the imposed large-scale downward motion leads maximum $\overline{\partial P}/\partial t$ by about 3 h. Thus, the imposed downward motion results in a buildup of $\mathbf{P}$, and provides a favorable environmental conditions for occurrence of convection.

The phase relation between $\overline{\partial P}/\partial t$ and $\partial K'/\partial t$ is also linked by local change of perturbation moist available potential energy $\partial P'/\partial t$. Minimum $\overline{\partial P}/\partial t$ lags minimum $\partial P'/\partial t$ by about 1 h, and minimum $\overline{\partial P}/\partial t$ lags maximum $\partial K'/\partial t$ by about 3 h (Fig. 3b), so that minimum $\overline{\partial P}/\partial t$ lags maximum $\partial K'/\partial t$ by about 4 h. This is a statistically significant phase relation consistent with that shown in Fig. 2b, although there are two other lag correlation coefficients between $\partial P'/\partial t$ and $\partial K'/\partial t$ that are above the 99% confidence level. The two minimum lag correlation coefficients are 9 h apart, indicative of the dominant life cycle of the model convective events. The maximum lag correlation coefficient appears between the two minimum lag correlation coefficients, indicating that $\partial P'/\partial t$ reaches maximum about 1 h before maximum $\partial K'/\partial t$.

To further examine the dominant physical processes determining the phase relations, the lag correlation between each term of $\overline{\partial P}/\partial t$ [Eq. (11)] and $\partial P'/\partial t$, and between $\overline{\partial P}/\partial t$ and each term of $\partial P'/\partial t$ [Eq. (12)], and the lag correlation between each term of $\overline{\partial P}/\partial t$ [Eq. (12)] and $\partial K'/\partial t$, and between $\overline{\partial P}/\partial t$ and each term of $\partial K'/\partial t$ [Eq. (9)] are plotted respectively in Figs. 4 and 5. Figure 4a shows that only the zero-hour lag correlation coefficient between $C_0(\overline{K}, \overline{P})$ and $\partial P'/\partial t$ is a major component that contributes to the maximum positive zero-hour lag correlation between $\overline{\partial P}/\partial t$ and $\partial P'/\partial t$. The term $C_0(\overline{K}, \overline{P})$ is related to the vertical temperature and moisture advections [see expansion following (11)]. Further analysis shows that the lag correlation coefficient between vertical temperature advection and imposed vertical velocity has the same sign as those between the sum of the vertical temperature and moisture advections and imposed vertical velocity, whereas
the lag correlation coefficient between vertical moisture advection and imposed vertical velocity has the opposite sign (not shown). This indicates that vertical temperature advection determines the conversion term $C_v(K, \bar{P})$. The imposed upward (downward) motion causes the vertical advective cooling (warming), which results in the loss (gain) of horizontal-mean moist available potential energy through the conversion term $C_v(K, \bar{P})$. Figure 4a also shows that maximum zero-hour lag correlation coefficient between $G_{cv}(\bar{P})$ and $\partial P'/\partial t$ is slightly less than that between $C_v(K, \bar{P})$ and $\partial P'/\partial t$. The conversion $G_{cv}(\bar{P})$ carries convective signals. As shown in Fig. 4a, the maximum lag correlation is about 9 h apart.

Figure 4b shows that the lag correlation coefficients between $C(\bar{P}, P')$ and $\partial P'/\partial t$, and between $G_v(P')$ and $\partial P'/\partial t$, and between $G(P')$ and $\partial P'/\partial t$ are above 99% confidence level. The terms $C(\bar{P}, P')$ and $G(P')$ have the same order of magnitude (not shown), but the opposite signs (Fig. 4b) so that they cancel each other in large part. In addition, the amplitude of the term $C(K', P')$ is smaller than those of the terms $C(\bar{P}, P')$ and $G(P')$. As a result, the lag correlation coefficient between $C(\bar{P}, P') + G(P') + C(K', P')$ and $\partial P'/\partial t$ becomes statistically insignificant. This suggests that the vertical perturbation advection processes do not play important roles in determining phase of $\partial P'/\partial t$. Therefore, the term $G_v(P')$ causes $\partial P'/\partial t$ to $\partial \bar{P}'/\partial t$ by about 1 h. The radiative cooling with positive $h'$ and radiative warming with negative $h'$ cause the decrease of perturbation available potential energy through the conversion term $G_{cv}(P')$.

Figure 5a shows three maximum negative lag correlation coefficients and one maximum positive lag correlation coefficient which are above 99% confidence level. Again, the small amplitude of the contribution of $C(K', P')$ to $\partial P'/\partial t$, and cancelation between $C(\bar{P}, P')$ and $G(P')$ make the vertical perturbation advection processes less important in determining the phase of $\partial P'/\partial t$. The term $G_{cv}(P')$ plays a crucial role in controlling the phase of $\partial P'/\partial t$ as shown in Fig. 5a where the lag correlation coefficient between $G_{cv}(P')$ and $\partial K'/\partial t$ has the maximum negative value at $-2$ h, and is statistically significant. This suggests that the maximum loss of perturbation moist available potential energy and the maximum gain of perturbation kinetic energy are linked by the term $G_{cv}(P')$. The heating released by deposition and fusion with positive $h'$ causes the loss of perturbation moist available potential energy. Maximum $G_{cv}(P')$ occurs about 3 h after the maximum $\partial K'/\partial t$. It is important to notice that the maximum $K'$ also occurs about 3 h after the maximum $\partial K'/\partial t$ (Figs. 2b and 3a), indicating the minimum $G_{cv}(P')$ coincides with the strongest convection. Thus, the 3-h phase difference between $\partial P'/\partial t$ and $\partial K'/\partial t$ is the time for convection to develop to the greatest strength.

Figure 5b shows that the lag correlation coefficients between $C(P', K')$ and $\partial P'/\partial t$ and between $G_{cv}(K')$ and
\( \frac{\partial P'}{\partial t} \) and \( \frac{\partial K'}{\partial t} \) are similar, and their maximum negative values at \(-3\) h are barely above 99% confidence level, which contribute to maximum negative lag correlation coefficient between \( \frac{\partial P'}{\partial t} \) and \( \frac{\partial K'}{\partial t} \) at \(-3\) h (Fig. 3b). The terms \( G_{\text{cn}}(K') \) and \( C(K, K') \) have the opposite signs with the terms \( C(P', K') \) and \( G_{\text{cn}}(K') \) (Fig. 5b), the magnitude of \( C(P', K') \) is larger than the other three terms (not shown) so that the term \( C(P', K') \) determines \( \frac{\partial K'}{\partial t} \). Covariance between the temperature and vertical velocity perturbations determines the local change of the perturbation kinetic energy. The thermally direct circulation of the upward motion with the higher temperature and the downward motion with the lower temperature converts the perturbation moist available potential energy to the perturbation kinetic energy, feeding the convection.

4. Discussion

Figure 6 summarizes the phase relations between the convection and its environment. The imposed large-scale downward motion yields a growth of \( \tilde{P} \) by the associated vertical advective warming \( [C(K, \tilde{P}) > 0] \), building the favorable environment for occurrence of convection. The near-simultaneous occurrence of maximum \( \frac{\partial K'}{\partial t} \), \( K' \), and imposed large-scale upward motion implies that convection is phase locked with the large-scale forcing. The life cycle of the simulated convective events (about 9 h) is much shorter than the timescales of imposed large-scale forcing (longer than the diurnal cycle). In the convective events, maximum \( \frac{\partial K'}{\partial t} \) leads maximum \( \frac{\partial P'}{\partial t} \) by about 3 h through perturbation cloud heating \( [G_{\text{cn}}(P')] \) and the vertical heat transport by perturbation circulations \( [C(P', K')] \). Maximum \( \frac{\partial K'}{\partial t} \) also leads maximum \( K' \) by about 3 h, indicating that 3 h is the time required by convection to reach the maximum strength. Minimum \( \frac{\partial P'}{\partial t} \) leads minimum \( \frac{\partial \tilde{P}}{\partial t} \) by about 1 h through perturbation radiative processes \( [G_{\text{rad}}(P')] \) and the horizontal-mean cloud heating \( [G_{\text{cn}}(\tilde{P})] \), and the large-scale vertical advective cooling. Consequently, maximum \( \frac{\partial K'}{\partial t} \) leads minimum \( \frac{\partial \tilde{P}}{\partial t} \) by 4–5 h (about the half of the convective lifetime).

The phase difference between perturbation kinetic energy associated with convection and its environment associated with horizontal-mean moist available potential energy indicates that the generation of environmental unstable energy by large-scale processes is not simultaneously balanced by its destruction by convection. The minimum horizontal-mean moist available potential energy occurs 4–5 h after maximum perturbation kinetic energy, and the phase lag is about the half of the convective lifetime. This provides concrete evidence for the adjustment from disequilibrium to equilibrium states...
proposed by Xu and Randall (1998). The results show that the convective lifetime is related to the cloud microphysical processes, convective–radiative interactions, and dynamic–thermodynamic coupling inside the convective system. These suggest that convective lifetime (as well as the phase lag) may depend on characteristics of convection (cloud type). When the phase lag is included in cumulus parameterization, the convective lifetime is introduced in the general circulation simulations. The scale interaction may have accumulated effects on the large-scale variability ranging from diurnal to interannual timescales. However, the lifetime of convection may depend on the environmental conditions. More experiments with different environmental conditions are needed to establish the lifetime of convection, and the phase relation to the environment.

Lipps and Hemler (1986) showed that 2D simulation develops deep convection earlier and has larger values of the volume-mean kinetic energy than the 3D simulation. The 3D simulation may change the conversion term \( C(K, K') \). The 2D simulation here shows that the conversion term \( C(P', K') \) has the dominant contribution to the growth of \( K' \). Thus, it is worth to apply the similar energetics analysis in the 3D cloud resolving simulation and compare with the 2D simulation.

5. Conclusions

Energetics analysis has been carried out with a 2D cloud resolving simulation to determine the physical processes responsible for the phase difference between convection and its environment. The cloud resolving model is forced by imposed time-varying horizontal-mean vertical velocity, zonal wind, and horizontal advectons derived from the TOGA COARE dataset for a 6-day period. The imposed vertical velocity serves as a major external forcing in this particular model setup. Lag correlation analysis shows that the maximum perturbation kinetic energy associated with the simulated convective events and its maximum growth rate leads to the maximum imposed large-scale upward motion by about 1–2 h, respectively, indicating that the convection is phase locked with the imposed large-scale forcing. The imposed large-scale vertical velocity has the timescales longer than the diurnal cycle, whereas the simulated convective events have the dominant lifetime of about 9 h. The imposed large-scale upward motion decreases the horizontal-mean moist available potential energy by the associated vertical advective cooling, providing the favorable environment for convection development.

The maximum latent heating and vertical heat transport by perturbation circulations cause maximum growth of perturbation kinetic energy to lead maximum loss of perturbation available potential energy by about 3 h. The maximum vertical advective cooling, the horizontal-mean cloud related heating, and perturbation radiative processes cause maximum loss of perturbation moist available potential energy to lead maximum loss of the horizontal-mean moist available potential energy by about 1 h. Consequently, the maximum gain of perturbation kinetic energy leads the maximum loss of horizontal-mean moist available potential energy by about 4–5 h (about the half of the lifetime of the simulated convection).

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APPENDIX

Relations Used in Derivation of Energetics Equations

The following relations are derived to obtain (9):

\[
\begin{align*}
&- \left< u' \frac{\partial}{\partial x} (u' \Pi' + \Pi' u' + u' u') - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho (w' \Pi' + \Pi' w' + w' u' - w' w')} \right> \\
&= - \left< w' \frac{\partial}{\partial x} (u' w' + \Pi' w' + u' w') - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho (w' w' + \Pi' w' + w' w' - w' w')} \right> \\
&= - \left< \frac{(\Pi' + u') \frac{\partial}{\partial x} [(u')^2 + (w')] + (\Pi' + w') \frac{\partial}{\partial z} [(u')^2 + (w')] }{2} - \left< u' w' \frac{\partial \Pi'}{\partial z} - \left< w' w' \frac{\partial w'}{\partial z} \right> \right> \\
&= - \left< u' w' \frac{\partial \Pi'}{\partial z} - \left< w' w' \frac{\partial w'}{\partial z} \right> \right> \\
&= \left< c_p u' \frac{\partial}{\partial x} (\Pi') + w' \frac{\partial}{\partial z} (\Pi' \Pi') \right> = 0, \\
\end{align*}
\]

(A1)

(A2)
where mass continuity and zero vertical velocity at top and bottom of the model atmosphere are applied. The following relations are derived to obtain (12):

\[- \frac{1}{\rho^c} \left( \frac{\partial (u' h')}{\partial x} + \bar{w'} \frac{\partial h'}{\partial x} + \frac{\rho^c \bar{v} \partial \theta'}{\rho \partial z} \right) + \frac{\rho^c \bar{w} \partial \theta'}{\rho \partial z} + L_v \frac{\partial \theta'}{\partial z} \]

\[= - \frac{1}{\rho^c} \left( \bar{w} \frac{\partial \bar{w}'}{\partial x} + \left( \bar{w} \frac{\partial \theta'}{\partial x} + \bar{L}_v \frac{\partial \bar{w}'}{\partial z} \right) \right) \]

\[= - \frac{1}{\rho^c} \left( \bar{w} \frac{\partial \bar{w}'}{\partial x} + \left( \bar{w} \frac{\partial \theta'}{\partial x} + \bar{L}_v \frac{\partial \bar{w}'}{\partial z} \right) \right) \]

\[= - \frac{1}{\rho^c} \left( \bar{w} \frac{\partial \bar{w}'}{\partial x} + \left( \bar{w} \frac{\partial \theta'}{\partial x} + \bar{L}_v \frac{\partial \bar{w}'}{\partial z} \right) \right) \]

\[\text{REFERENCES} \]


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