A Fast, Accurate Method of Computing Near-Surface Longwave Fluxes and Cooling Rates in the Atmosphere

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ABSTRACT

As radiation plays a key role in the determination of the near-surface thermal environment, great accuracy is required in the computation of radiative fluxes, especially because a small error in the fluxes can lead to large errors in estimated cooling rates. A new code that employs a novel numerical scheme for making precise estimates of longwave fluxes and cooling rates near the surface of the earth, for arbitrary surface emissivities, is presented here. The code is a development of the infrared band model of Chou, Ridgway, and Yan. Unacceptable oscillations found near the surface in the cooling rates provided by this code have been overcome through the new numerical scheme. The new code gives results in excellent agreement with available results from Intercomparison of Radiation Codes used in Climate Models (ICRCCM) test cases and with the line-by-line calculations of Clough, Iacono, and Moncet. The code has no restriction on the number of grid points, yields fluxes accurate to a prescribed tolerance, and permits a discontinuity in temperature at the surface (although this is not used in the results presented). The computing times are comparable (for given accuracy) to those demanded by current codes in use elsewhere. It is found that, as surface emissivity $\varepsilon_s$ departs from unity, the cooling rate rises dramatically near the surface, reaching values as high as nearly 40 K day$^{-1}$ at $\varepsilon_s = 0.8$ in the midlatitude summer atmosphere, and the effect of the surface is noticeable at heights of up to about 1 km. An analysis of spectral distribution shows that, when the surface is not radiatively black, the major contributions to near-surface cooling rates (due to water vapor) come from the two wavenumber bands, 340–540 cm$^{-1}$ and 1215–1380 cm$^{-1}$ (located on either side of the atmospheric window), in which both absorption and radiative flux are significantly high.

1. Introduction

Infrared radiation plays a key role in the determination of the near-surface thermal environment, especially during night. Several workers (Ramdas and Malurkar 1932; Ramanathan and Ramdas 1935; Lake 1956; Goody 1964; Coantic and Seguin 1971; Kondratyev 1972) have pointed out the importance of accurate computation of near-surface radiative fluxes. A small error in the relatively large fluxes can lead to appreciable errors in the estimated cooling rates, especially near the surface where the radiative calculations have to be over thin atmospheric layers (Tjemkes and Duynkerke 1989; Quinet and Vanderborgh 1996).

The objectives of the present work are to study longwave radiative transfer near the ground in different bands for arbitrary surface emissivities, and to generate a code that provides precise estimates of fluxes and cooling rates near the surface.

Most previous studies on near-surface radiative transfer use a broadband flux emissivity approach (see, e.g., Coantic and Seguin 1971; Coantic and Simonin 1984; André et al. 1978; Garratt and Brost 1981; Estournel and Guedalia 1985; Vasudeva Murthy et al. 1993, hereafter VSN; Narasimha 1997; Ragothaman et al. 2001 2002). This is sufficiently accurate for many applications, but cannot provide an estimate of the contribution of different longwave bands to radiative cooling, which would be important when one considers the effect of...
different constituents of atmospheric air or needs to use band-dependent surface emissivities.

This drawback of flux-emissivity schemes can be overcome by band models, in which the absorption properties of lines, with parameters such as line strength, separation, and position, are averaged over prescribed bands, and appropriate transmission functions are expressed as a function of pseudo line parameters (mean line intensity, mean half-width, and mean line spacing). Integration over frequency ($\nu$) is replaced by a summation over the finite number of bands or spectral intervals that span the longwave absorption spectrum.

Tjemkes and Duynkerke (1989) have, in their calculations of a nocturnal boundary layer, used a narrow-band model in which the infrared spectrum between 3.6 and 100 $\mu$m is divided into 178 bands, and solved the transfer equations given by Rodgers and Walsh (1966) for the mean upwelling and downwelling fluxes in each of the bands. The solution extends over a domain limited at the top by the location where the wind is set equal to the geostrophic value. The "surface" level in the model is within the vegetation layer, and has a temperature different from that of the surface itself; the differential is calculated using a scheme proposed by Holtslag and de Bruin (1988).

The present work takes as its starting point the very useful code developed by Chou et al. (1993). This code is based on a band model that comprises 11 bands for water vapor and 5 bands for CO$_2$ absorption, using an appropriate one-parameter scaling approximation in each case. The Planck-weighted diffuse transmittance is then reduced to a function depending only on the scaled absorber amount. This transmittance is then fitted with an exponential sum in each of the bands (11 in total). By this method, Chou et al. (1993) can calculate the flux integrals very rapidly compared to commonly used band models because the evaluation of each transmission function requires only $n$ exponential operations for each value of $k$ (absorption coefficient), where $n$ is the number of atmospheric layers, whereas it would require $n^{3/2}$ operations in a conventional band model. Also the coefficients for the exponential sum fit are given recursively, which makes the evaluation even faster. These considerable advantages of the Chou code are utilized in the further development reported here.

One difficulty we experienced with the Chou code when we used it for near-surface computations—for which it was not really designed—was that the evaluation of fluxes and cooling rates near the surface was not only insufficiently precise but also highly oscillatory. The reason for this is presumably that the integrals are evaluated using a numerical scheme that is unstable near the surface, because of a sharp boundary layer there (as explained in the next paragraph) caused by the huge absorption coefficients in some bands. In fact, evaluating integrals of functions that have large gradients in special regions requires special care. The present work is directed toward tackling this issue.

The main departure of the present code from Chou’s rests on the recognition that, if the transmission functions are given by exponential sums, then the corresponding flux integral can be evaluated by solving an ordinary differential equation (ODE). To illustrate this idea, consider evaluating the integral:

$$y(z) = \int_0^z e^{-(z-x)/\delta} \, dx; \quad (1.1)$$

then $y$ satisfies the ODE:

$$\frac{dy}{dz} = \delta^{-1}(\delta - y); \quad z > 0,$$

$$y(0) = 0. \quad (1.2)$$

Now, evaluating the integral by quadrature causes problems if $\delta$ is small, because of a thin layer of large gradient in the integrand near $x = z$. But the equivalent ODE can be solved very efficiently using readily available software that not only incorporates adaptive gridding but also generates solutions to a given user-specified tolerance. It is known that uniform grids are not only highly inefficient but are often incapable of solving singularly perturbed ODEs, such as (1.2), (see, e.g., Farrel et al. 2000). In fact, Shishkin (1997) has proved that if a singularly perturbed heat equation were to be discretized on a uniform mesh, then the error (i.e., the difference between the exact and numerical solutions) will not tend to zero even if the mesh size tends to zero. Such a phenomenon has also been computationally demonstrated by Sundaresan et al. (1999) in their work on lid-driven cavity flow at high Reynolds number [see Fedoseyev (2001) on this point].

In summary the parameterization of Chou et al. (1993) provides a speed-up over conventional band models; we enhance it further by evaluating the flux integrals more precisely, especially at very low altitudes, using the ODE strategy mentioned above. This is possible mainly due to the exponential structure of the transmission functions.

Compared to the method of Tjemkes and Duynkerke (1989) the present code has the advantages that it uses fewer bands and goes all the way right from the surface to a height of 100 km.

A preliminary account of this work has appeared in Varghese et al. (2001). All the calculations for the results presented have been carried out on a Compaq AlphaServer ES40 in double precision.

2. Preliminary numerical experiments

As part of a continuing program on understanding the nocturnal near-surface thermal environment (VSN; Narasimha 1997; Ragothaman et al. 2001, 2002), preliminary experiments were carried out with the band model of Chou et al. (1993, but using the 1997 version of the code). This code has performed well in intercomparison.
3. Chou’s one-parameter scaling approximation of the absorption coefficient

Chou and his coworkers have demonstrated that, as the absorption in regions near a line center saturates quickly, an accurate treatment of it in such regions is not critical for longwave radiative transfer calculations. Based on this argument, Chou et al. (1993) proposed the one-parameter scaling

\[ k_s(p, T) = k_s(p_r, T_r)(p/p_r)^m f(T, T_r), \]  

(3.1)

for the absorption coefficient \( k_s \), at frequency \( \nu \), here, \( p, T \) are pressure and temperature at level \( z \), while \( p_r, T_r \) are respective reference values, and \( f \) is a scaling factor for temperature. The absorption coefficient is thus scaled up from its value at a reference pressure and temperature. The exponent \( m \) is a positive number typically close to but less than unity. For water vapor, \( p_r, T_r \) are chosen respectively to be 500 mb and 250 K, which are characteristic of the middle regions of the troposphere. The diffuse transmittance of an atmospheric layer at level \( z \) with water vapor absorber amount \( u \) is

\[ \tau_s(u) = 2 \int_0^1 \exp \left[ \frac{-k_s(p_r, T_r)u}{\mu} \right] \mu \, d\mu, \]  

(3.2)

where

\[ u(z) = \int_0^z [p(z')/p_r]^{-m} f(T(z'), T_r)p_r(z') \, dz', \]  

(3.3)

\( \rho_r \) being the density of water vapor and \( \mu \) the cosine of the angle between the beam and the vertical.

To include the water vapor continuum effect, a scaled absorber amount

\[ w = eu \exp \left[ 1800 \left( \frac{1}{T} - \frac{1}{296} \right) \right] \]  

(3.4)

is used; here, \( e \) is the water vapor partial pressure in atmospheres. Equation (3.4) is based on the observation that the continuum absorption increases with increasing water vapor partial pressure but with decreasing temperature. For more details on the above parameterizations (based on Roberts et al. 1976), we refer to Chou et al. (1993). We follow this parameterization because of its excellent performance in intercomparison tests of the Ellingson type (Ellingson et al. 1991), and also, as we shall demonstrate, the more elaborate line-by-line calculations using the water vapor continuum model of Clough et al. (1992), which includes contributions from foreign continuum and a modified self-continuum (Clough and Iacono 1995).

The upwelling and downwelling radiative fluxes at frequency \( \nu \) are given, respectively, by

\[ F^\uparrow_\nu = G_\nu + \int_0^u \pi B_s(T) \frac{d\tau}{du'} (u - u') \, du', \]  

(3.5)

\[ F^\downarrow_\nu = -\int_u^\infty \pi B_s(T) \frac{d\tau}{du'} (u' - u) \, du', \]  

(3.6)

where \( B_s(T) \) is the Planck function:

\[ B_s(T) = \frac{2h\nu^3}{c^2 \exp[(h\nu/KT) - 1]} \]  

(3.7)

in standard notation; \( K \) being the Boltzmann constant. Following Paltridge and Platt (1976), the term \( G_\nu \) in (3.5), namely, the contribution of radiation emitted by
the surface and the reflected component of the downwelling radiation received at the surface, is taken to be

$$G_v = [e_v \pi B_v(T_s) + (1 - e_v)F_v(0)]\tau_v,$$  \hspace{1cm} (3.8)

where $e_v$ is the emissivity of the surface and $T_s$ is the ground temperature. Strictly speaking, $(3.8)$ is a function of the frequency $\nu$, but we shall assume it to be a constant in the present work for the sake of simplicity; the present code permits incorporation of a functional dependence on $\nu$ for $e_v$.

The longwave spectrum is divided into 11 bands in Chou's model. The spectrally integrated flux in band $j$ is denoted by

$$F_{j}^{\downarrow} = \int_{\nu_j}^{\nu_{j+1}} F_{j}^{\downarrow}(\nu) d\nu,$$

where $\nu_j$ and $\nu_{j+1}$ are the starting and ending wavenumbers of band $j$. From (3.6), the downwelling flux in band $j$ is

$$F_{j}^{\downarrow} = -\int_{\nu_j}^{\nu_{j+1}} \pi B_{j}(T') \tau_{j}'(u' - u) \, du' \, du,$$

where $\tau_{j}'(u)$ is the derivative of the transmittance function $\tau_j$ with respect to $u$. The above double integral can be reduced to the single integral

$$F_{j}^{\downarrow} = -\int_{\nu_j}^{\nu_{j+1}} \pi B_{j}(T') \tau_{j}'(u' - u) \, du' \hspace{1cm} (3.9)$$

by defining

$$B_{j}(T') = \int_{\nu_j}^{\nu_{j+1}} B_{j}(T') \, dv,$$

$$\tau_{j}(u) = \frac{1}{B_{j}(T') \nu_j} \int_{\nu_j}^{\nu_{j+1}} B_{j}(T') \tau_{j}(u) \, dv \hspace{1cm} (3.10)$$

Note that $\tau_j$ is a function of $\nu$ and $T$ also, the temperature dependence appearing both through $B_j$ and $\tau_j$. However, Chou and Suarez (1994) have demonstrated that it suffices to consider the temperature dependence only in $B_j(T')$, and that the argument in $\tau$ can be replaced by an appropriate constant reference value $T_0$ (taken by them to be 250 K). With this assumption (which we shall presently assess), the fluxes in band $j$ can be written as

$$F_{j}^{\downarrow} = G_j - \int_{\nu_j}^{\nu_{j+1}} \pi B_{j}(T') \tau_{j}'(u' - u) \, du', \hspace{1cm} (3.11)$$

$$F_{j}^{\downarrow} = -\int_{\nu_j}^{\nu_{j+1}} \pi B_{j}(T') \tau_{j}'(u' - u) \, du', \hspace{1cm} (3.12)$$

$$G_j = [e_v \pi B_v(T_s) + (1 - e_v)F_v(0)]\tau_j(u), \hspace{1cm} (3.13)$$

where we have found it useful to introduce the approximation

$$\int_{\nu_j}^{\nu_{j+1}} F_{j}^{\downarrow}(\nu) \tau_j(u) \, d\nu = F_{j}^{\downarrow}(0)\tau_j(u). \hspace{1cm} (3.14)$$

This approximation has the advantage of avoiding the use of another transmittance

$$\tau_{j}(u) = \frac{\int_{\nu_j}^{\nu_{j+1}} F_{j}^{\downarrow}(0) \tau_j(u) \, d\nu}{F_{j}^{\downarrow}(0)}, \hspace{1cm}$$

more precisely, the approximation is that

$$\frac{\int_{\nu_j}^{\nu_{j+1}} F_{j}^{\downarrow}(0) \tau_j(u) \, d\nu}{F_{j}^{\downarrow}(0)} \approx \frac{\int_{\nu_j}^{\nu_{j+1}} B_{j}(T_0) \tau_j(u) \, d\nu}{\int_{\nu_j}^{\nu_{j+1}} B_{j}(T_0)} \hspace{1cm} (3.15)$$

We have found that this approximation, which is roughly equivalent to assuming that $F_{j}^{\downarrow}(0)$ is Planckian as in (3.10), does not introduce any significant error (see appendix A).

Chou and Arking (1980) have shown that the diffuse transmittance $2E_{j}(k, u)$ can be satisfactorily approximated by $\exp(-dk_j u)$, where $d=5.3$ is called the diffusivity factor. Comparing the cooling rates obtained from line-by-line calculations with and without the diffuse approximation, Chou et al. (1993) report that the error induced by this approximation is less than 0.05°C day$^{-1}$. Based on this approximation, an exponential sum fit was made to the transmission function in band $j$, writing

$$\tau_{j}(u) = \sum_{i=1}^{m} c_i \tau_i; \hspace{1cm} \tau_i = \exp(-dk_i u). \hspace{1cm} (3.16)$$

where

$$\frac{du}{dz} = \left(\frac{p}{p_v}\right)^m f_j(T, T_v) p_v, \hspace{1cm} (3.17)$$

$$f_j(T, T_v) = 1 + a_j (T - T_v) + b_j (T - T_v)^2, \hspace{1cm} (3.18)$$

and the coefficients $c_i$ (derived from line-by-line calculations) satisfy

$$\sum_{i=1}^{m} c_i = 1.$$

Table 1 (reproduced from Chou et al. 1993, 2001) gives the values of $k_j, c_i, a_j,$ and $b_j$ for the 11 bands considered.

When multiple absorbing species are present in a band, the total diffuse transmittance is approximated by multiplying together the individual transmittances. For example, when the effect of carbon dioxide is also taken into account (see section 9) we write

$$\tau_{j}(u) = \tau_{j}^w(u) \cdot \tau_{j}^{CO_{2}}(u),$$

where $\tau_{j}^w(u)$ and $\tau_{j}^{CO_{2}}(u)$ are the transmittance for water vapor and carbon dioxide, respectively, for band $j$. 


Given the temperature distribution as a function of $z$, the integrals in (3.11) and (3.12) can be evaluated from a quadrature rule. It is not clear what kind of quadrature was used in Chou et al. (1993) to calculate the fluxes, but it seems plausible that the oscillations seen in the cooling rates reported in section 2 could be due to the numerics associated with the quadrature.

4. Present approach

Our proposal here is to reformulate the flux integrals (3.11) and (3.12) as solutions of certain ordinary differential equations. There are two main reasons for adopting this approach. The first is that the exponential sum fit proposed in (3.16) facilitates it, because each band integral will be the sum of several subintegrals with an exponential kernel. This means that the derivatives of those integrals can be expressed in terms of the integral itself. This is not the case with other broadband emissivity formulations in which the kernel is logarithmic (VSN). Secondly the absorption coefficient $k_j$ in the exponentials (3.16) can be extremely large (see Table 1), and so result in a thin radiative sublayer near the surface, generally extending to heights of less than a centimeter and characterized by extremely sharp temperature gradients (of order several degrees per centimeter).

Contributions to the integrals at altitudes less than 1 km come mainly from the large near-surface values, and this is why high resolution is needed there to evaluate the integrals accurately. However, the high resolution, which involves the use of a large number of grid points, usually leads to round-off error propagation and subsequently to possibly violent numerical instabilities. On the other hand, when $k_j$ is not large we do not require high resolution. To incorporate this kind of adaptivity it is convenient to convert the integrals into ODEs whenever possible (as it is in the present case, because of the exponential sum fitting that gives rise to exponential kernels), and then to solve them numerically with an appropriate ODE code.

This procedure is extremely efficient because present-day ODE solution codes have reached a mature capability stage; that is, the output from the code can be quantified very well in terms of both confidence and accuracy (Oberkampf and Blottner 1998). Note that, in the radiation problem under consideration, we have several parameters like the factor $\delta$ in the example (1.2) presented in section 1; these take the form of $1/k_j$; some of them extremely small values. In order to have a method that will automatically take into account this variation in $k_j$ we need mesh adaptivity. The direct evaluation of the integral using numerical quadrature poses problems (like the oscillations seen in Fig. 1) because of the sharp variation of the integrand near the upper limit $z$ (when $\delta \ll 1$ in the example [(1.1) and (1.2)]. On the other hand, the ODE can be numerically solved accurately for both small and moderate values of $\delta$ using a

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<th>Band (cm⁻¹)</th>
<th>0</th>
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modern code to solve stiff ODEs. As we shall explain below, such a code incorporates adaptivity in an efficient manner.

Accordingly, instead of evaluating the integrals using numerical quadrature, we solve here the equivalent ODEs by a readily available code that contains strategies to control error by adapting the time-stepping procedure. To derive the ODEs corresponding to the integrals (3.11) and (3.12), we first note from (3.16) that

\[
\tau_j'(u) = -d \sum_{i=1}^{m_i} c_i k_i \tau_i',
\]

(4.1)

where the prime denotes derivative with respect to \( u \). Using (4.1) in (3.12), we can write

\[
F^\dagger_j = \sum_{i=1}^{m_i} F^\dagger_{ji},
\]

(4.2)

with

\[
F^\dagger_{ji} = dc_i k_i \int_0^u \pi B_j(T') \tau_i(u' - u) \, du'.
\]

(4.3)

In a similar way we can decompose \( F^\dagger_j \) and \( G_j \) as

\[
F^\dagger_j = \sum_{i=1}^{m_i} F^\dagger_{ji}, \quad G_j = \sum_{i=1}^{m_i} G_{ji},
\]

(4.4)

where

\[
F^\dagger_{ji} = G_{ji} + dc_i k_i \int_0^u \pi B_j(T') \tau_i(u - u') \, du'.
\]

(4.5)

\[
G_{ji} = [c_e \pi B_j(T_e) + (1 - c_e) F^\dagger_j(0)] c_i \tau_i(u).
\]

(4.6)

Setting

\[
A_j = dk_i (p/p_e)^\alpha f_j(T, T) \rho_n,
\]

(4.7)

we can easily show (appendix B) that the ODEs satisfied by \( F^\dagger_{ji} \) are

\[
\frac{dF^\dagger_{ji}}{dz} = -A_j [c_e \pi B_j(T) - F^\dagger_j],
\]

(4.8)

\[
F^\dagger_{ji}(0) = 0;
\]

\[
\frac{dF^\dagger_j}{dz} = A_j [c_e \pi B_j(T) - F^\dagger_j],
\]

(4.9)

\[
F^\dagger_j(0) = c_e [c_e \pi B_j(T_e) + (1 - c_e) F^\dagger_j(0)].
\]

For computational purposes, we need to impose the top boundary condition in (4.8) at a finite but large height \( L \). Therefore, instead of (4.8) we shall consider

\[
\frac{dF^\dagger_j}{dz} = -A_j [c_e \pi B_j(T) - F^\dagger_j], \quad z \in (0, L),
\]

(4.10)

The cooling rate is given by

\[
\frac{1}{\rho c_p} \sum_{j=1}^{11} \left( \frac{dF^\dagger_j}{dz} - \frac{dF^\dagger_{ji}}{dz} \right),
\]

where \( F^\dagger_j \) and \( F^\dagger_{ji} \) are defined in (4.2) and (4.4), \( \rho_c \) is the density of air, and \( c_p \) its specific heat at constant pressure.

Now \( F^\dagger_j(L) \) may be prescribed using (4.8) in (5.5) below. To solve (4.9) and (4.10), we use the code of Sommeijer et al. (1998). This code is based on the explicit Runge–Kutta formulas of the Chebyshev type proposed by Van der Houwen and Sommeijer (see the above reference). These formulas have good stability bounds and are ideal for large, mildly stiff systems because of their explicit nature. The code is written in such a way that it automatically selects, at each vertical level \( z \), the most efficient stable formula as well as the most efficient step size. Furthermore, it is possible to evaluate the formulas in just a few vectors of storage, making the code not only very fast but also simple to use. For more details we refer to Sommeijer et al. (1998).

Finally the blackbody fraction \( B_j(T) \) in (3.10) is evaluated in the following way. Define

\[
\overline{F}(\xi) = \frac{1}{\sigma T^4} \int_0^\xi \pi B_j(T) \, dv,
\]

then

\[
B_j(T) = \left[ \overline{F} \left( \frac{C_2 V_{j+1}}{T} \right) - \overline{F} \left( \frac{C_2 V_j}{T} \right) \right] \sigma T^4,
\]

(4.11)

where

\[
T = \frac{T_s}{T} \approx \frac{T_s}{T_e}.
\]
Fig. 3. Comparison of computed cooling rates for an isothermal atmosphere with $\varepsilon_g = 0.8$, water vapor absorption only.

$$C_2 = \frac{hc}{K}$$

and $h$, $c$, and $K$ are the constants appearing in the Planck function. Based on the approximation mentioned in section 3, we can replace $T$ by $T_0$ in $F$, and take

$$B_j(T) = \left[ F\left( \frac{C_i \nu_{j+1}}{T_0} \right) - F\left( \frac{C_i \nu_j}{T_0} \right) \right] \sigma T^4,$$

where the $F$ functions are computed using the method given by Lawson (1997).

5. Verification and validation

To test the code we construct some exact solutions in this section. Apart from serving to verify the code, these solutions also address the crucial $z$ test that Ellingson et al. (1991) suggest: "A trivial but important result to check is a model’s output for an isothermal atmosphere. Although the importance of these type tests are known to many modelers, it appears that they are not always done in practice." We construct here an exact solution that allows for a surface that is not radiatively black ($\varepsilon_g \neq 1$), a case not considered by Ellingson.

We therefore consider an (artificial) isothermal atmosphere:

$$T(z) = T_1. \quad (5.1)$$

Furthermore, it is convenient to ignore (for the purpose of deriving the exact solution) the vertical variation of $p$ in (3.17); for this purpose we can either set $p(z) = p_0$, where $p_0$ is the surface value, or set $m = 0$ in (3.17). (In either case the solution is the same to within a numerical multiplier.) Consequently, (4.7) simplifies to

$$A_j = a_j\rho_n, \quad (5.2)$$

where

$$a_j = dk_jf(T_1, T_s) \quad (5.3)$$

is a constant. Under these conditions (4.9) and (4.10) can be easily solved to yield

$$F_j^1 = \varepsilon_jc_j\sigma T_1^4 - (1 - \varepsilon_j)[\varepsilon_jc_j\sigma T_1^4 - c_jF_j^1(0)]\tau_j(z), \quad (5.4)$$

$$F_j^1 = F_j^1(L)\tau_j(u_s - u) + \varepsilon_jc_j\sigma T_1^4[1 - \tau_j(L - z)], \quad (5.5)$$

where $c_j$ is defined in (3.16) and given in Table 1, and

$$\varepsilon_j = F\left( \frac{C_i \nu_{j+1}}{T_0} \right) - F\left( \frac{C_i \nu_j}{T_0} \right).$$

Note that (5.4) is different from the expression of Ellingson et al. (1991), in that the upwelling fluxes here are not constant as we have taken into account the nonblack ($\varepsilon_g \neq 1$) effects of the surface as well. For large $k_j$, we note that $\tau_j$ [see (3.16)] will have a large gradient at $z = 0$. Also $[a_j\rho_n]^{-1}$ is a length scale defined for the $j$th band. We emphasize here that the approximations (5.1)–(5.2) are introduced only to verify the code; for validation with other temperature profiles corresponding to different atmospheres we remove these assumptions. Nevertheless the exact solutions illustrate the effects of a nonblack ground, in particular the associated strong cooling rates.
6. Code implementation

As the present radiation code has no restriction on the number of grid points and yields fluxes accurate to a prescribed tolerance limit, it consequently also provides cooling rates to a desired accuracy. In all the computations reported here, cooling rates have been computed over the entire atmosphere (surface to 100 km), using a total of 771 grid points with the resolution in different layers as shown in Table 2.

A typical result for the variation in the error in the downwelling flux at the surface and at 100 m for different prescribed tolerance levels is shown in Fig. 2. This shows the flexible capability of the present numerics to cope with the different accuracies that may be demanded in different applications.

The computing times for executing the current program are comparable (for given accuracy) to those demanded by other standard current codes (such as, e.g., that of Chou et al. 1993). The time taken by the present code for various tolerance levels is also shown in Fig. 2. In the tolerance limit ranging from $10^{-3}$ to $10^{-6}$ W m$^{-2}$, the time taken for execution of the code hardly varies, but when the tolerance limit is further reduced, the execution time shoots up very rapidly. Figure 2 suggests that an acceptable tolerance level to prescribe is $10^{-6}$ W m$^{-2}$, the value that has therefore been adopted in all the computations reported here.

We first validate the code by comparing it with the exact solution given in the previous section. Figure 3 shows the variation of cooling rate with height (up to 100 km) for $e_g = 0.8$ in an isothermal atmosphere at 300 K, using line absorption only. The ordinate has been plotted on a quasi-logarithmic scale, following Narasimha (1983), using the expression $y = \log_{10}(1 + aC)$, where $C$ is the cooling rate (in kelvins per day) and $a$ is an arbitrary constant that is taken here as 100. (This scale is linear near the origin but becomes asymptotically logarithmic at large $z$, and has the advantage of permitting explicit display of the $z = 0$ point.) The comparison of the exact and computational results from the present code agree very well at all levels, so the $z$ test is found positive and thus provides strong verification for the code. Figure 3 also presents the cooling rates from the code of Chou et al. (1993), which agrees with the exact solution from approximately 100 m and above.

A final validation of this code comes from a comparison with the results of Clough et al. (1992). Clough and his coworkers present a line-by-line integration with
Fig. 5. Effect of ground emissivity on cooling rates computed by the present code: MLS atmosphere, water vapor absorption only.

7. Spectral density of cooling rate at different altitudes

We now present results of a study carried out to understand the influence of ground emissivity on cooling rates across the longwave spectrum in the entire atmosphere (surface to 100 km).

Two cases, namely, $g = 1.0$ and $0.8$, are compared. Although the value of $g$ for most natural surfaces like snow, water, and vegetation are close to unity (Wilber et al. 1999), a value of 0.8 for bare soil is not uncommon. See for instance Paltridge and Platt (1976), Becker et al. (1981), Garrat and Brost (1981), and Moriyami and Arai (1995). Also (as already mentioned in VSN) most of these compilations refer to vertical emissivities appropriate to remote measurements by vertically oriented radiometers on spacecraft. The quantity relevant to us is the global or flux emissivity, which may be much less. Work on engineering surfaces has established that the global (also called hemispherical) emissivity is much less than the normal emissivity (see, e.g., Siegel and Howell 2002, chapter 2). Although no corresponding result is available for natural surfaces, a similar factor will presumably apply. More work is of course needed to establish these arguments. But our main aim in taking $g = 0.8$ is to highlight the dramatic differences in near-surface cooling rates between $g = 1.0$ and 0.8. For purposes of illustration, we adopt the midlatitude summer atmosphere (MLS; Ellingson et al. 1991).

Figure 5 shows the results for this atmosphere, including both line and continuum contributions. It is seen that a decrease in $g$ from 1.0 to 0.8 increases the surface cooling rate from 4.47 to 37.54 K day$^{-1}$, and that the effect of lower values of surface emissivity extends to a height of almost 1 km. This confirms the estimates made by Ragothaman et al. (2001), using the flux emissivity scheme, of the unsuspectedly large extent of the influence of surface conditions on radiative cooling.

Figure 6 shows the cooling rate spectrum (due to water vapor alone) for clear-sky MLS conditions. Once again, it is seen that the difference between $g = 1.0$ and 0.8, that is, the ground emissivity effect, is generally observed all the way up to about 1 km, but it is dramatically large near the surface. Band 2 (wavenumber range 340–540 cm$^{-1}$, or wavelength range 29.4 to 18.5
Fig. 6. Influence of change in ground emissivity from 1.0 to 0.8 on spectral distribution of cooling rate ([K day$^{-1}$ cm$^{-1}$] x 10$^{-3}$) for MLS atmosphere; water vapor absorption only.

μm) is the single largest contributor to near-surface cooling (>50 x 10$^{-3}$ K day$^{-1}$ cm$^{-1}$), followed closely by band 9 (1215–1380 cm$^{-1}$, 8.2 to 7.2 μm, contribution 45 x 10$^{-3}$ K day$^{-1}$ cm$^{-1}$). The cooling in these bands is approximately 50 times higher than that in the weakest band; note that they are on either side of the atmospheric window region (8–12 μm). There is a strong gradient of the cooling rate over the first few centimeters above the surface, constituting what VSN have called the emissivity sublayer; but the gradient rapidly decreases with height, and at higher levels (100 m to 1 km) it is the window region that contributes most to the cooling.

To understand further the basic mechanisms underlying the drastic changes in near-surface cooling rate indicated by the present calculations when ground emissivity is less than unity, we first note that they can, in principle, be due to changes in either the fluxes themselves or the flux gradients or both. In order to investigate these factors, we now explore the spectral distribution of the fluxes and the cooling rates, which are...
Fig. 7. Influence of ground emissivity in spectral distribution of upward flux; MLS atmosphere, water vapor absorption only.

presented in Figs. 6–13. Note that the ordinate in these diagrams is the average spectral density in each band (K day$^{-1}$ cm$^{-1}$ for cooling rate and W m$^{-2}$ cm$^{-1}$ for fluxes), obtained by dividing the total contribution from the band by its width.

For a given temperature distribution there can be no difference in the downwelling flux between $\varepsilon_g = 1.0$ and 0.8, so we may confine our attention to the upwelling flux. Figure 7 shows the spectral distribution of the upwelling fluxes at the surface for the MLS atmosphere, in the two cases $\varepsilon_g = 0.8$ and 1.0. In general, the upwelling flux exhibits only small differences: the largest is in band 6 (where the flux at $\varepsilon_g = 0.8$ differs by less than 10% of the value of 0.34 W m$^{-2}$ cm$^{-1}$ at $\varepsilon_g = 1.0$). The differences are even less significant at higher altitudes, and drop to 0.065 W m$^{-2}$ cm$^{-1}$ at $z = 10$ km.

The computed differences in cooling rates must therefore be largely attributed to changes in flux gradients, which determine the cooling rate. That this is so is demonstrated in Fig. 8, which replots the data of Fig. 6 in more quantitative terms at selected heights for both $\varepsilon_g = 1.0$ and 0.8; the total cooling rate is also shown alongside the data. It is immediately clear again that the highest contributions to the surface cooling rate at $\varepsilon_g = 0.8$ come from bands 2 and 9; the values here, respectively, 13.6 and 7.78 K day$^{-1}$ are 42 and 31 times higher than at $\varepsilon_g = 1.0$. (The spectral density of the cooling rate in the same bands is 68.0 $\times$ 10$^{-3}$ and 47.2 $\times$ 10$^{-3}$ K day$^{-1}$ cm$^{-1}$ at the surface, higher than at $\varepsilon_g = 1.0$ by the same factors as above.) Note that, when $\varepsilon_g = 1.0$, the spectral distribution of the cooling rate remains largely similar till a height of about 1 km, being centered around bands 4 to 6, but at 10 km the contributions to cooling rate shift to longer waves. With $\varepsilon_g = 0.8$, there are spectacularly high cooling rates in bands 2 and 9 at the surface; by $z = 1$ m, there is already a remarkable change, the greatest spectral density now being in band 5 (720–800 cm$^{-1}$, or wavelength range 13.89–12.5 $\mu$m), although bands 2 and 9 continue to be prominent. Beyond 10 m, the shape of the cooling rate spectral distribution remains roughly similar at both $\varepsilon_g = 0.8$ and 1.0, although the magnitudes are higher at the lower emissivity.

To understand why different bands play major roles at different heights, we display in Fig. 9 the spectral density of the cooling rate (K day$^{-1}$ cm$^{-1}$) at $\varepsilon_g = 0.8$ along with the spectrum of blackbody radiation as well as the transmission function at four selected heights, $z = 0$, 1 m, 10 m, and 10 km. [The details of the transmission function are given in section 3, (3.16)–(3.18).] Near the surface, the lowest values of the transmission function occur in bands 1 and 10, but the ground radiation in these bands is small. In bands 2 and 9, on the other hand, the transmission function is lower (0.58 and 0.84) but the radiation flux is higher. The spectral density of radiation is highest around bands 3 through 5, but here the transmission is nearly unity, and there is little absorption. It is therefore clear that bands 2 and 9 play a special role because they have sufficient absorption at frequencies where the ground radiation flux is significant; it is the combination of these two factors that results in the dramatically high cooling rates. We may note that band 2 is in the rotation spectrum of the H$_2$O molecule, and band 9 in the rotation–vibration spectrum at the edge of the continuum window.

Similar reasons operate at greater heights. By $z = 10$ m, absorption in bands 2 and 9 has saturated, and the contribution to the cooling rate comes from around band 5. At $z = 10$ km the air temperature is lower and the radiation spectrum has shifted to lower wavenumbers; so has the absorption, which is also higher at lower wave-
numbers. The net result is that the cooling now comes from bands 1 and 2, as may be seen from Fig. 9.

Let us briefly recapitulate the present findings on the longwave radiative cooling rate. First of all, the surface cooling rate is sensitive to the ground emissivity, going up from nearly 5 K day$^{-1}$ at $e_g = 1.0$ to nearly 40 K day$^{-1}$ at $e_g = 0.8$ in the MLS atmosphere. The effect of the surface emissivity drops with height, becoming

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**Figure 8.** Spectral distribution of cooling rate at five levels for $e_g = 1.0$ and $e_g = 0.8$, showing also the gross cooling rate at each level; MLS atmosphere, water vapor absorption only.
totally negligible only at and beyond heights of order 1 km.

The spectral distribution of the cooling rate varies with altitude and ground emissivity. At $\varepsilon_g = 1.0$, the largest contributions to the cooling rate come from bands 4, 5, and 6 (wavenumber 620 to 980 cm$^{-1}$, wavelength 16.1 to 10.2 $\mu$m) up to $z = 1$ km, but the contributing bands slowly shift to longer waves as altitude increases. This shift becomes substantial at higher altitudes; for example, at 10 km, bands 1 and 2 (wavenumber below 540 cm$^{-1}$, wavelength greater than 18.5 $\mu$m) play the major role.

At $\varepsilon_g = 0.8$ the picture is quite different. The biggest contributions near the surface are now bands 2 and 9 (wavenumber 340 to 540 cm$^{-1}$, wavelength 29.4 to 18.5 $\mu$m; wavenumber 1215 to 1380 cm$^{-1}$, wavelength 8.2 to 7.2 $\mu$m), on either side of the atmospheric window. From 10 m on, however, the spectral distribution begins to resemble that at $\varepsilon_g = 1.0$, although the total cooling rate remains higher up to $z = 1$ km.

The identity of the bands contributing most to the cooling rate depend on a balance between flux and absorption; if either is too low, little cooling occurs. As altitude increases and temperature falls, the flux spectrum peaks at longer wavelengths, where absorption still remains significant; consequently, the contribution to cooling also moves to the longer waves.

8. Source-wise decomposition of cooling rate

To understand the energetics of near-surface long-wave cooling, we first recall (from section 7) that the
downwelling flux $F^↓$ plays no significant role, as its gradient in $z$ near the surface is negligible. The upwelling flux can be conveniently decomposed into three parts:

$$F^↑ = F^↑_{\text{ae}} + F^↑_{\text{ge}} + F^↑_{\text{gr}},$$

where

$$F^↑_{\text{ae}} = \int_0^u \pi B_c(T') \tau_j^j (u - u') \, du',$$  \hspace{1cm} (8.2a)

$$F^↑_{\text{ge}} = \epsilon_j \pi B_c(T) \tau_j^j (u),$$  \hspace{1cm} (8.2b)

$$F^↑_{\text{gr}} = (1 - \epsilon_j) F^↓(0) \tau_j^j (u).$$  \hspace{1cm} (8.2c)

These terms may, respectively, be called the fluxes due to air emission (ae), ground emission (ge), and ground reflection (gr); note that each of these includes the appropriate transmission factor.

The cooling depends on the $z$ derivative of the flux; we find it convenient to write

$$C = \frac{d}{dz} F^↑ = C_{\text{ae}} + \epsilon_x C^{(1)}_{\text{ge}} + (1 - \epsilon_x) C^{(0)}_{\text{gr}},$$

decomposing $C$ also into three components and showing explicitly the dependence on $\epsilon_x$. Thus, $C_{\text{ae}}$ is the cooling due to ground emission at $\epsilon_x = 1$, and $C^{(0)}_{\text{gr}}$ is that due to ground reflection at $\epsilon_x = 0$; note that $C_{\text{ae}}$, which is the cooling due to air emission, is independent of $\epsilon_x$.

Figure 10 shows the spectral distribution of $C_{\text{ae}}$ at the surface and at heights of $z = 1, 10, 100$ m. The contributions here are much larger than the cooling rates shown in Fig. 8 at $\epsilon_x = 0.8$, and even more so than those at $\epsilon_x = 1.0$; furthermore they are largest in bands 1, 2, 9, and 10, on either side of the atmospheric window. However, they are not affected by a change in $\epsilon_x$ and so will not be able to explain its effect.

To illustrate the dependence of cooling on $\epsilon_x$, we again consider the two cases $\epsilon_x = 1.0$ and $\epsilon_x = 0.8$ and note that

$$C^{(1)} = C_{\text{ae}} + C^{(1)}_{\text{ge}},$$

$$C^{(0.8)} = C_{\text{ae}} + 0.8 C^{(1)}_{\text{ge}} + 0.2 C^{(0)}_{\text{gr}},$$

(where the superscripts denote the value of $\epsilon_x$). The difference between the two cases is therefore

$$\Delta C = C^{(0.8)} - C^{(1)} = -0.2 C^{(1)}_{\text{ge}} + 0.2 C^{(0)}_{\text{gr}}.$$  \hspace{1cm} (8.5)

We first show in Fig. 11 the spectral distribution of the ground emission term in (8.4a). Because of the linearity in $\epsilon_x$, each of the gradients at $\epsilon_x = 0.8$ and 1.0 may be obtained simply by multiplying the cooling rate shown in Fig. 11 by $\epsilon_x$. The contributions to cooling rate are once again largest in bands 1, 2, 9, and 10 at the lower altitudes, but by $z = 100$ m the contributions come from bands 2, 3, and 4. Compared to the values seen in Fig. 8, the cooling rate shown in Fig. 11 is substantial.

Next, Fig. 12 shows the ground reflection term in (8.5). Interestingly, it is immediately seen that the differentials in the gradient of $F^↑_{\text{gr}}(\Delta C_{\text{gr}})$ closely match 0.2 times those of $F^↓_{\text{gr}}(\Delta C_{\text{gr}})$ displayed in Fig. 11. In fact, in the case of $\epsilon_x = 1.0$, the ground emission gradient (8.2b) is almost completely cancelled out by the air emission term (8.2a) (Fig. 10).

Finally, we show the contribution of both ground terms together, as a differential between values at the
two emissivities, in Fig. 13. It is clear that bands 2 and 9 play the major role in determining the high cooling rates at $e_g = 0.8$.

The explanation of the dramatically high surface cooling rates that emerges from the above analysis is thus the following. When $e_g = 1.0$, the large gradients of air emission and ground emission nearly cancel each other out, leaving a relatively small cooling rate that receives dominant contributions from bands 4 to 6. At $e_g = 0.8$ the ground emission is lower relative to the value at $e_g = 1.0$, so the balance that prevailed between air and ground emission at $e_g = 1.0$ is lost; but in addition, the ground reflection is a substantial contributor to the cooling. Thus, the predominant cause of the
higher cooling when ground is not radiatively black is lower ground emission and higher ground reflection.

Figures 11–13 confirm that the high surface cooling rates, when ground emissivity is below unity, are to be attributed chiefly to absorption in bands on either side of the atmospheric window. It follows that emissivities in these bands can play a crucial role in determining the near-surface longwave radiative environment. The spectral dependence of emissivity therefore merits careful investigation.

Clough et al. (1992) conclude from their study that the cooling at high altitudes in the MLS atmosphere (considering water vapor only) arises from the strongly absorbing regions of the water vapor spectrum; in the lower atmosphere, the weaker regions play the dominant role. This result broadly agrees with our own conclusion at altitudes greater than a few kilometers as can be seen from Fig. 8. Below about $z = 1$ km, as we have seen, the contribution to the cooling rate depends strongly on the ground emissivity. Although no specific statement is made by Clough et al. (1992) about $e_g$, it appears as if they take $e_g = 1$, so the contribution to the upwelling flux from the component of the downwelling flux reflected from ground, $C_{gr}$ in our notation, is presumably not taken into account. However, our work shows that this contribution plays an important role in near-surface cooling when $e_g < 1$.

The present computations show that the net surface cooling rate in the MLS atmosphere is given by $C_0 = 4.47 + 165.4(1 - e_g)$ K day$^{-1}$, indicating that the cooling rate more than doubles when $e_g$ falls from 1 to 0.97.

9. Carbon dioxide

The effect of other atmospheric constituents on longwave radiative transfer can be handled by the present code in exactly the same way as described above for water vapor. Thus, the effect of carbon dioxide (CO$_2$), the second most important greenhouse gas in the atmosphere, has been included in the present code; the diffuse transmittance of CO$_2$ is computed in a way very similar to that of water vapor line absorption, where the absorption coefficient is scaled from its value at reference pressure and temperature [see Eqs. (3.1), (3.16)–(3.18)]. The values of the constants associated with the CO$_2$ transmission functions are again taken from Chou et al. (1993, 2001); CO$_2$ is mainly absorbed in the 15-$\mu$m (540–800 cm$^{-1}$) band. In Chou’s model, this band has been divided into central and wing regions (540–620 cm$^{-1}$, 620–720 cm$^{-1}$, and 720–800 cm$^{-1}$ constituting three subbands). Two other weak CO$_2$ absorption bands (800–980 cm$^{-1}$ and 980–1100 cm$^{-1}$) are located in the water vapor window region. The total diffuse transmittance for individual bands is given by

$$\tau = \tau_{wl} \cdot \tau_{wc} \cdot \tau_{CO_2},$$

the product of the respective transmittances for water vapor line (WL), water vapor continuum (WC), and carbon dioxide (CO$_2$). The rest of the calculations are carried out using exactly the same coding scheme as described above in sections 4–6.

Comparison of present results for the fluxes with Intercomparison of Radiation Codes used in Climate Models (ICRCCM) results for an isothermal atmosphere at
300 K with 300 ppmv of CO₂ only shows good agreement (generally better than 1%), the largest difference being about 3 W m⁻² in the downflux at the tropopause. For the MLS, the addition of CO₂ results in a general decrease in the cooling rate everywhere in the atmosphere, the differences at the surface being 3.2 and 1.21 K day⁻¹ for ground emissivity of 0.8 and 1.0, respectively.

10. Conclusions

We have presented here a radiation code that employs new numerics to enable precise computation of longwave radiation fluxes and cooling rates, all the way from the surface to a height of 100 km. We have furthermore presented an analysis of the spectral distribution of the cooling rate, and find that the greatest contributions to it near the surface come from bands on either side of the atmospheric window. At different heights different bands contribute to the cooling rate, depending on the balance between the flux and absorption at each altitude.

The present code enables us to investigate in considerable detail the spectral energetics of longwave radiative transfer that result in the extremely high cooling rates observed near ground. This should be of use in exploring the basic problem of interaction between radiation and turbulence on the one hand, and (with the inclusion of effects of vegetation) the elucidation of the meteorological conditions that influence the health of agricultural crops—in particular, the occurrence and possible control of frost.

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APPENDIX A

Basis for Approximation of Fluxes in (3.15)

From (3.6) and (3.12) we obtain

\[
\int_{\nu_j}^{\nu_{j+1}} \frac{F^\pm_j(0) \tau_j(u) \, d\nu}{F^\pm_j(0)} = -\int_{\nu_j}^{\nu_{j+1}} d\nu \tau_j(u) \int_u^{u_{j+1}} \pi B_j(T') [d\tau_j(u')/du'] \, du' \\
- \int_0^{u_{j+1}} \pi B_j(T') \tau_j(u') \, du'.
\]

Dividing the numerator and denominator above by \(\pi B_j(T')\), and replacing \(T'\) in the Planckian with \(T_j\) based on the argument given in the paragraph below (3.10), we obtain

\[
\int_{\nu_j}^{\nu_{j+1}} \frac{F^\pm_j(0) \tau_j(u) \, d\nu}{F^\pm_j(0)} = \int_{\nu_j}^{\nu_{j+1}} \pi B_j(T_j) \tau_j(u) [\tau_j(u_{j+1}) - \tau_j(u)] \, d\nu \\
= \frac{\pi B_j(T_j) [\tau_j(u_{j+1}) - \tau_j(u)]}{\pi B_j(T_j) [\tau_j(u_{j+1}) - \tau_j(0)]}.
\]

Now, from (3.2) and (3.16) it follows that \(\tau_j(u_{j+1}) = 0\) \(\approx \tau_j(u_{j+1})\), and \(\tau_j(z(0)) = 1 = \tau_j(0)\). Consequently, we obtain the approximate result (3.15):

\[
\int_{\nu_j}^{\nu_{j+1}} \frac{F^\pm_j(0) \tau_j(u) \, d\nu}{F^\pm_j(0)} \approx \int_{\nu_j}^{\nu_{j+1}} \pi B_j(T_j) \tau_j(u) \, d\nu \\
= \int_{\nu_j}^{\nu_{j+1}} \pi B_j(T_j) \, d\nu.
\]

This in turn yields

\[
\frac{dF^\pm_j}{dz} = dk \frac{du}{dz} [F^\pm_j - c \pi B_j(T_j)],
\]

and from (3.17) we have

APPENDIX B

ODEs Satisfied by the Fluxes

Let us consider the downwelling flux \(F^\downarrow\) without subscripts in (4.3). Using the expression for \(\tau_j\) given in (3.16), we obtain

\[
F^\downarrow = cd k \int_u^{u_{j+1}} \pi B_j(T') e^{-dk(u' - u)} \, du'.
\]

Treating \(F^\downarrow\) and \(T'\) as functions of \(u\), we get

\[
\frac{dF^\downarrow}{du} = cd^2 k^2 \int_u^{u_{j+1}} \pi B_j(T') e^{-dk(u' - u)} \, du' - cd k \pi B_j(T)
= dk [F^\downarrow - c \pi B_j(T)].
\]

This in turn yields

\[
\frac{dF^\downarrow}{dz} = dk \frac{du}{dz} [F^\downarrow - c \pi B_j(T)],
\]
\[
\frac{dF}{dz} = -A[c\pi B(T) - F^4],
\]

which is Eq. (4.8) with \( A \) defined in (4.7). The equation for upwelling flux is obtained similarly.

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