Spiral Bands in a Simulated Hurricane. Part II: Wave Activity Diagnostics

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ABSTRACT

The theory of empirical normal modes (ENMs) was applied in a diagnostic study of the inner spiral bands formed in a simulated hurricane using the high-resolution Pennsylvania State University–National Center for Atmospheric Research (PSU–NCAR) nonhydrostatic mesoscale model version 5 (MM5). The ENM method has the capability to decompose simultaneously wind and thermal fields into dynamically consistent and orthogonal modes with respect to wave activities.

For wavenumber 1 and 2 anomalies, it was found that the leading modes are vortex Rossby waves. These modes explain 70%–80% of the statistical variances in a 24-h period. Gravity waves have small contribution in terms of wave activities.

Analysis of the Eliassen–Palm (EP) flux and its time-mean divergence shows that vortex Rossby waves are generated in the eyewall region where the radial gradient of the basic-state potential vorticity is large. In general, these waves propagate outward in the lower troposphere and inward in the upper troposphere. Consequently, they transport eddy momentum radially inward and outward, respectively. The wave activities also propagate slowly upward inside the eyewall and downward outside. The associated eddy heat transport tends to warm the air in the eye region. The vortex Rossby waves lead to, through wave–mean flow interaction as indicated by the divergence of the EP flux, an acceleration of the mean tangential wind in the lower and middle troposphere inside and outside the eyewall and a deceleration aloft in the eyewall region.

1. Introduction

A hurricane is a highly symmetric vortex so that a 2D symmetric model (e.g., Eliassen 1952; Emanuel 1995) in a radial-height plane can represent approximately its slowly evolving balanced circulation. On the other hand, asymmetries, such as spiral rainbands that evolve in a faster timescale, may lead to a sudden change of hurricane intensity. Understanding the dynamics of the spiral rainbands and their role in intensity change can improve the prediction of rapidly deepening hurricanes.

Spiral rainbands have long been viewed as gravity–inertia waves (e.g., Kurihara 1976; Willoughby 1978a,b). However, by invoking the analogy between the circulation in a hurricane and the general circulation, MacDonald (1968) suggested that spiral rainbands were “Rossby-like” waves possessing a countergradient momentum flux. More formal development of the theory of these waves in hurricanes came only recently. Guinn and Schubert (1993) derived the general conservation equation for wave activities applicable to rotating vortices in the context of shallow-water dynamics. Montgomery and Kallenbach (1997) were the first to coin the term “vortex Rossby waves” and to derive the local dispersion relation for these waves including both azimuthal and radial wave propagation. They were also the first to demonstrate the “countergradient” properties of these waves on a monopolar vortex.

In nature, both gravity waves and vortex Rossby waves should coexist in the hurricane. However, observational analysis (Reasor et al. 2000) and numerical simulations using 3D primitive equation models (e.g., Chen and Yau 2001; Wang 2001, 2002) show that inner spiral rainbands mainly exhibit characteristics of vortex Rossby waves. Due to the intense advective tangential wind, there is no well-defined timescale separation be-
tween the “fast” gravity waves and the “slow” vortex Rossby waves in the hurricane core (Shapiro and Montgomery 1993), and it is still not clear which wave dominates the inner spiral rainbands.

Vortex Rossby waves have only been isolated from gravity waves by the use of filtered models—such as a 2D nondivergence vorticity equation model (e.g., Schubert et al. 1999), an asymmetric balance (AB; Shapiro and Montgomery 1993) barotropic model (Möller and Montgomery 1999), an AB shallow-water model (Montgomery and Kallenbach 1997), a 3D quasigeostrophic model (Montgomery and Enagionio 1998), and a 3D AB model (Möller and Montgomery 2000)—or by the use of a normal-mode shallow-water model (Guinn and Schubert 1993). In the study of Guinn and Schubert, it was found that the band ed features are composed almost entirely of rotational vortex Rossby wave modes. However, the authors also claimed that their unforced model may underestimate the real contribution of the gravitational modes. In Part I of this series, Chen and Yau (2001) performed a high-resolution full-physics numerical simulation of an idealized hurricane. The model results can provide a realistic dataset for investigating the sole effects of vortex Rossby waves and gravity waves on the structure and intensity change of hurricanes if the two kinds of waves can be separated.

To extract wave modes from the dataset with complete physical processes, one can use conventional principal component analysis [e.g., empirical orthogonal function (EOF) and its variants]. However, the modes found by this method may not possess any dynamical meaning, as they are not solutions of the dynamical equations and can only be interpreted statistically. To overcome this problem, we will apply the empirical normal mode (ENM) theory to decompose the wave modes. The ENM method was developed by Brunet (1994). It takes advantage of the conservation law for wave activities, and is capable of decomposing simultaneously wind and thermal fields into dynamically consistent and orthogonal modes. Specifically, the mode space of gravity and Rossby waves can be decomposed in a robust manner into mutually orthogonal subspaces for a small amplitude and stochastically forced system even when there is no frequency separation. The ENM approach also permits a more detailed dynamical characterization of the variability (e.g., discrete versus continuous spectrum mode).

In studies of the dynamics of large-scale Rossby waves, wave activities and their corresponding conservation relations have been used to interpret the development and propagation of wave disturbances, as well as their interactions with the basic state (e.g., Andrews 1983a,b; Held 1985; Killworth and McIntyre 1985; Brunet and Haynes 1996; Magnaudtottir and Haynes 1999; and others). The generalized wave activity conservation law (Andrews 1987; Haynes 1988) can be extended in the context of hurricane disturbances. This conservation law has the form of

\[
\frac{\partial \mathbf{A}}{\partial t} + \nabla \cdot \mathbf{F} = S, \tag{1}
\]

where \(A\) is the wave activity (density), the vector \(\mathbf{F}\) represents a flux of that wave activity, and \(S\) is the source or sink term. Wave activity of this kind has been used to indicate the vortex Rossby wave formation and propagation (Guinn and Schubert 1993; Schubert et al. 1999), to analyze the stability of rapidly rotating vortices (Ren 1999), as well as to study the wave–mean flow interaction (Schubert 1985; Molinari et al. 1995, 1998). When \(A\) is pseudomomentum, \(\mathbf{F}\) is the generalized Eliassen–Palm (EP) flux whose divergence is associated with the wave–mean flow interaction. By the use of (1), the ENM method decomposes wave modes according to their wave activities. The EP flux and its divergence for each mode then provide valuable information on the propagation of the wave modes and on wave–mean flow interaction.

In section 2, we briefly review the generalized wave activity conservation law and the ENM method. In section 3 a description is given of the dataset and the basic state. In section 4 the diagnostic results are presented. Summary and conclusions are in section 5.

2. Wave activity and ENM

a. Generalized wave activity conservation law

To distinguish the wave processes, we usually divide the atmospheric flows into a basic-state part and a wave disturbance part. The latter often satisfies (1) in which \(A\), \(\mathbf{F}\) can be expressed as quadratic functions of the Eulerian wave disturbance (Andrews 1987); \(S\) is associated with forcing, dissipation, or high-order (\(\approx 3\)) terms in the wave amplitude. For small-amplitude conservative (i.e., adiabatic and frictionless) waves, \(S\) is effectively zero and the wave activity is conserved in the global sense (Haynes 1988). Then the flux \(\mathbf{F}\) indicates the propagation of the wave activity, and its divergence indicates the impact of the eddy forcing on the basic state. For finite-amplitude waves, however, the nonlinear wave processes may act as the source and sink terms that make the interpretation of the flux difficult. To avoid this problem, the energy–Casimir methods of Arnl’d were applied to construct wave activity conservation theorems in which the term \(S\) vanishes for all conservative motions (McIntyre and Shepherd 1987; Haynes 1988). The explicit forms of such wave activity conservation relation for the forced and dissipative primitive equations on spherical coordinates have been derived by Haynes (1988). After making some basic assumptions, we will adapt his derivation to the hurricane system. First, we assume that the hurricane is in approximate hydrostatic balance so we can employ the 3D hydrostatic primitive equations in storm-following cylindrical and isentropic coordinates. We verified this assumption by computing the deviation of the vertical perturbation pressure gradient force from the buoyancy...
acceleration (include water loading). The results show that the deviation is less than 5% of each of the individual terms even in the eyewall region where strong convections occur. Second, we choose the azimuthal mean and the time mean of the flow as the basic state. The conservation of two wave activities, namely, pseudomomentum density and pseudoenergy density (pseudomomentum and pseudoenergy, hereafter), arising respectively from the azimuthal invariance and time invariance, can then be found. We obtain the small-amplitude wave activity conservation laws by assuming that the asymmetric circulation is much smaller than the symmetric basic state (see also Shapiro and Montgomery 1993).

The forced primitive equations in storm-following cylindrical and isentropic coordinates \((r, \lambda, \theta)\) are

\[
\begin{align*}
 u_t + \left( \frac{1}{2} u^2 + \frac{1}{2} v^2 + M_\lambda \right) - \eta v &= F - \delta u_y, \\
 v_t + \left( \frac{1}{2} u^2 + \frac{1}{2} v^2 + M_\lambda \right) + \eta u &= G - \delta v_y, \\
 -\frac{r}{\eta} \sigma_y + \frac{1}{r} (\sigma v)_\lambda + \frac{1}{r} (r\sigma)_r &= -(\sigma \theta)_\lambda, \\
 M_\theta &= C_r \left( \frac{p}{p_s} \right)^\kappa, \\
p_y &= -g\sigma, 
\end{align*}
\]

where \(u\) and \(v\) are, respectively, the radial wind and tangential wind relative to the moving hurricane center. Since there is no significant acceleration in the storm motion in the simulation, we assume that the motion of the storm is constant. Note that the storm motion is not apparent in (2) if we neglect the small \(\beta\) effect, because the storm motion (and its acceleration) can be incorporated into the Montgomery function on an \(f\) plane (Shapiro and Montgomery 1993; see also Willoughby 1992, 1994, 1995).

In (2), \(M = C_r T + \Phi\) is the Montgomery function \((T\) is the temperature and \(\Phi\) the geopotential), \(\sigma\) the isentropic density, \(p_s\) a constant reference pressure, \(\eta = f + (rv)/r - u_0/r\) the vertical component of the absolute vorticity, \(\theta\) is the diabatic heating rate and also the vertical velocity in the isentropic coordinates, \((F, G, 0)\) is the body force per unit mass exerted on the flow, and \(\kappa = R/C_r\). The subscripts \(t, r, \lambda, \) and \(\theta\) denote partial derivatives with respect to time, radius, azimuth, and isentrope. By considering an axisymmetric and time-invariant basic state with only the primary circulation \(u_0\) in gradient wind balance, we can find two wave activities that satisfy (1). The condition of axisymmetry leads to a conservation law for the pseudomomentum \(J\):

\[
\begin{align*}
 j_x + \left[ \frac{v_y}{r} j + \frac{\sigma_y}{2} (u'^2 - v'^2) - \frac{R}{2g p_s} \left( \frac{p}{p_s} \right)^\kappa \right]_\lambda &= \frac{1}{r} \left[ -r^2 \sigma_y u' v' \right]_\lambda + \left( \frac{p'}{g} M'_\lambda \right)_\lambda = S_p, \end{align*}
\]

where

\[
J = -r\sigma^2 v' - \frac{r^2 \sigma^2 g q'^2}{2\gamma}. \tag{4}
\]

A zero subscript signifies basic-state quantities, and a prime denotes the anomaly; \(q = \eta/\sigma\) is the potential vorticity (PV); \(\gamma = \partial q/\partial r\) is the radial gradient of the basic state PV; \(S_p\) is the source/sink term for the pseudomomentum. The second term in (4) contains the PV perturbation and the PV gradient and therefore it represents the vortical or Rossby-wave-like contribution \((J_x, J_y)\). The first term is related to the density perturbation and represents the gravitational contribution \((J_x, J_y)\) in general.

The condition for time invariant leads to a conservation law for the pseudoenergy \(\mathcal{A}\):

\[
\begin{align*}
 A_t + \frac{1}{r} \left[ -\frac{u_0 v_y}{r} j + v_0 \sigma_y v'^2 + \sigma_0 v' M' + v_0 \sigma v M' \right]_\lambda &= \frac{1}{r} \left[ -r \sigma_y v_0 u' v' + r \sigma_0 u' M' \right]_\lambda - \frac{1}{r} \left( \frac{M' p'}{g} \right)_\lambda = S_a, \end{align*}
\]

where

\[
\mathcal{A} = -\frac{u_0 v_y}{r} j + \frac{\sigma_0}{2} (u'^2 + v'^2) + \frac{R p_s^{\kappa-1}}{2g p_s} p'^2. \tag{6}
\]

Note that the three terms on the right-hand side of (6) are the Doppler shift term \((A_e + A_a)\), kinetic energy \((K)\), and available potential energy \((P)\), respectively; \(S_a\) denotes the source/sink for the pseudoenergy.

The above equations for \(J\) and \(\mathcal{A}\) bear some resemblance to those in Charron and Brunet (1999), which are cast in a spherical coordinate system.

b. EP flux and wave-mean flow interaction

The EP theorem for hurricanes has been discussed by Willoughby (1978a,b) and Schubert (1985). Molinari et al. (1995, 1998) showed that it can be used to diagnose the interaction of a hurricane and an upper-level trough. Due to the limitation of the dataset, however, Molinari et al. could not resolve the waves in the inner core region. It is also our goal to fill this gap.

When azimuthally averaged (denoted by an overbar), (3) becomes

\[
\bar{\mathcal{J}} = \mathbf{\nabla} \cdot \bar{\mathcal{F}} = \bar{S}_p, \tag{7}
\]

where
\[ \nabla \cdot \mathcal{F} = \frac{1}{r} (-r^2 \sigma u' v'), + \left( \frac{\rho'}{g} \right) \cdot \mathbf{M} \]  

is the divergence of the generalized EP flux \( \mathcal{F} \). Similar to large-scale dynamics, the horizontal component of the EP flux is antiproportional to the azimuthal-mean eddy (angular) momentum transport in the radial direction. When the EP flux is pointing outward, eddies transport momentum radially inward and vice versa. The vertical component of the EP flux, analogous to a \(-u'T'\) term in pressure coordinates, is related with the eddy heat transport in the radial direction. Associated with an upward EP flux, eddies transport heat radially inward and vice versa.

From (2), it is not difficult to connect the EP flux to the time change of the isentropic density-weighted azimuthal-mean tangential wind (or angular momentum) (cf. Schubert 1985; Molinari et al. 1995):

\[ (r \sigma \mathbf{v}), + \left( \frac{1}{r} (r^2 \sigma \mathbf{v}) \right), + r \sigma \mathbf{u} f = \nabla \cdot \tilde{\mathcal{F}} - (r \sigma' \mathbf{v}'), + D, \]  

where

\[ \nabla \cdot \tilde{\mathcal{F}} = \frac{1}{r} (-r^2 (\sigma u') v'), + \left( \frac{\rho'}{g} \right) \cdot \mathbf{M}. \]  

Here \( D \) includes the diabatic heating and friction terms. Note that the primed quantities are the departures from an azimuthally independent but time-varying basic state. The forms of (10) and (8) are slightly different. If we assume that the density perturbation is much smaller than \( \sigma \), and the symmetric component of the radial wind has the same order as the asymmetric component, (10) then reduces to (8). Therefore, (8) can be computed to estimate the wave–mean flow interaction. The EP flux represents effective angular momentum flux (Plumb 1983) and its divergence from the eddy forcing on the mean flow. The vertical component of the EP flux is equivalent to the azimuthal pressure torque exerted on the mean flow. Along with the horizontal component, \( \nabla \cdot \tilde{\mathcal{F}} \) is equal to the net azimuthal pressure force (Andrews et al. 1987). When \( \nabla \cdot \tilde{\mathcal{F}} > 0 \), asymmetric eddies tend to lose locally their own pseudomomentum to accelerate the mean tangential wind. Molinari et al. (1995) showed that the EP flux divergence is also related to the eddy PV flux. Hence, we can use EP maps (i.e., plotting EP flux over its divergence contours) to indicate the wave activity propagation, the eddy momentum, heat and PV flux, and the wave–mean flow interaction (e.g., Edmon et al. 1980, and many others).

c. ENM review

Held (1985) demonstrated that linear modes on shear flows are orthogonal in the sense of pseudomomentum or pseudoenergy. The orthogonality is useful for studying problems dealing with the excitation of neutral modes and Ripa (1981) applied it to study wave–wave interactions. Brunet (1994) developed the ENM theory by combining the EOF analysis and the orthogonality properties of normal modes in the context of wave activities.

The ENM method has been applied to systems with various complexities. Brunet (1994) initially utilized the method to study the variability of the historical National Centers for Environmental Prediction (NCEP) atmospheric dataset within the framework of quasigeostrophic theory. Brunet and Vautard (1996) further extended it to a shallow-water model and demonstrated its dynamical robustness in a study on weakly nonlinear Rossby waves. In an investigation of inertia–gravity wave properties and their impact on the zonal wind, Charron and Brunet (1999) successfully isolated gravity waves from a Geophysical Fluid Dynamics Laboratory (GFDL) SKYHI general circulation model dataset. They applied the generalized ENM method in the context of a 3D primitive equations model in spherical coordinates. Zadra et al. (2002) analyzed the NCEP global analysis dataset with the ENM method. They constructed the spectra of pseudomomentum and pseudoenergy with respect to ENMs and identified Rossby wave quasi modes. In the current study, we will extend the ENM method to the dynamics of a hurricane described by (2).

As previously mentioned, the ENM technique is similar to the EOF method. They are both eigenvalue problems. In the traditional EOF method, the eigenvalues and eigenvectors are associated with a matrix whose elements are simply the covariances of one variable. In contrast, the ENM method embeds a self-adjoint matrix into the “covariance” matrix such that each of its elements is in the form of the wave activity (pseudomomentum in our case). If the model state can be represented by a vector

\[ \mathbf{x}(t) = (u' \, \sigma' \, q')^T, \]  

we need to seek a matrix \( \mathbf{B} \) so that

\[ \mathcal{F} = \mathbf{X}^T \mathbf{B} \mathbf{X}. \]  

Comparing to (4), we obtain

\[ \mathbf{B} = \frac{r}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \frac{\sigma'^2}{\gamma} \end{pmatrix}. \]  

Note that if \( \mathbf{B} \) is not sign-definite, unstable modes may arise. An eigenvalue of the ENM matrix represents how much pseudomomentum an ENM carries. In comparison, an eigenvalue of the EOF can only be interpreted as the variance if the covariance matrix is sign-definite. The eigenvectors of the ENM covariance matrix in (12) provide a basis onto which the model state is projected to obtain ENM modes \( \mathbf{x}_n \), with \( n \) being the ENM mode number. If pseudoenergy is chosen to be the norm, \( u' \)
and \(\rho'\) need to be included into \(X\) and \(B\) becomes a \(5 \times 5\) matrix (see Charron and Brunet 1999). The pseudomomentum \(\left(\mathbf{j}_n\right)\) and the pseudoenergy \(\left(\mathbf{A}_n\right)\) are then computed from \(X_n\). Held (1985) showed that the pseudomomentum and pseudoenergy of one mode are connected by the phase speed of the mode, or angular phase speed in our case,

\[
c_n = \frac{-\mathbf{A}_n}{\mathbf{j}_n},
\]

From (4) and (6), one can easily recognize that the angular phase speed contains the Doppler shift and the intrinsic speed. The latter has two contributions: gravitational and vortical. Since the kinetic energy and available potential energy are positive definite, the intrinsic angular phase speed is positive when the pseudomomentum is negative and vice versa. The period predicted by linear theory is

\[
T_n = \frac{2\pi}{s \left| \mathbf{j}_n / \mathbf{A}_n \right|},
\]

where \(s\) is the azimuthal wavenumber. The details of the ENM theory may be found in Brunet (1994), Brunet and Vautard (1996), Charron and Brunet (1999) and Zadra et al. (2002).

In the context of linear dynamics approximation, the ENM method allows one to extract statistically the normal modes from an observational or numerical dataset. In general, an atmospheric system, especially a hurricane, is nonlinear, dissipative, and forced. Möller and Montgomery (1999) and Montgomery (2000, personal communication) pointed out that linear AB model could capture the main picture of the asymmetric dynamics. From weakly nonlinear shallow-water model simulations, Brunet and Vautard (1996) demonstrated that the principal components associated with ENMs are more monochromatic than those associated with EOFs, and hence ENMs are more dynamically consistent. Charron and Brunet (1999) showed that random forcing and dissipation in a linear model system does not alter the spatial distribution of the ENMs. The noisy signal only broadens slightly the power spectrum of a principal component that peaks around the eigenfrequency of its free wave counterpart. The damping rate can further be measured by the width of the power spectrum (Zadra et al. 2002). In short, ENMs show robustness to noise and dissipation and resemble the linear solutions of the complete model.

3. Dataset

The dataset we will analyze is the output of a numerical simulation of an initially symmetric hurricane reported in Chen and Yau (2001). The simulation is performed with the nonhydrostatic Pennsylvania State University–National Center for Atmospheric Research (PSU–NCAR) Mesoscale Model Version 5 (MM5). With 6-km horizontal resolution, the model simulates explicitly the microphysical processes (see Chen and Yau 2001 and references therein for the details of the model and the simulation). To build good statistics, we output the simulation results every 2 min over a 24-h period. The storm motion is removed from the total horizontal winds using the method of Liu et al. (1999). The model data are then interpolated from terrain-following \(\sigma\) coordinates onto isentropic coordinates upon which PV and isentropic density are calculated. For the sake of simplicity, we exclude the model bottom boundary that is not an isentrope. Since we retain the annular ring structure in the basic-state PV field, which already satisfies the instability criterion (Charney and Stern 1962; Montgomery and Shapiro 1995; Ren 1999), it is not necessary to consider boundary contribution in the context of the instability. The underlying ocean surface may play a significant role in determining the hurricane intensity (e.g., Emanuel 1986). However, this process has a longer timescale and is therefore more important for the slow-evolving symmetric circulation than for the wave processes studied here. There are 15 isentropic levels with 4-K intervals ranging from 310 K (the lowest level that does not touch the ocean surface) to 366 K in the stratosphere (cf. Chen and Yau 2001, their Fig. 1). The basic state is chosen to be the time and azimuthal mean state. Although the simulated hurricane intensities during the 24-h simulation, the time and azimuthal mean state still represents well the slow-evolving symmetric circulation. After the basic state is extracted, the model data are decomposed into different azimuthal wavenumbers. In this diagnostic study we only consider the wavenumber 1 and 2 anomalies, which dominate the asymmetries in the near-core region of a hurricane (cf. Shapiro and Montgomery 1993). Since the wave activities are concentrated in the near core region (<200 km), the analysis domain follows the hurricane center and has a radius of 270 km. To decrease the computational effort, the so-called snapshot method for solving the eigenvalue problem is used (Sirovich and Everson 1992; Zadra et al. 2002). By taking the number of time samples as the degree of freedom, this method allows us to obtain 721 eigenvalues and eigenvectors (i.e., principal components).

The basic states of the tangential wind, the angular velocity, the pressure, the isentropic density, the PV, and the radial gradient of the PV are shown in Fig. 1. The tangential wind is mostly cyclonic in the near-core region. The radius of maximum wind (RMW) is about 50 km and tilts slightly outward with height. The angular velocity decreases monotonically with radius. The rising pressure surface in the center of the hurricane suggests a warm core structure. In general, PV has maximum values in the center of the vortex except in regions above 344 K and below 320 K where the annular PV ring structure is present. Moving outward from the center, the radial gradient of the PV changes sign from positive to negative values at a radius of about 50 km. Note that
\( \gamma = 0 \) may render the wave activity undefined and the ENM method invalid. To avoid such problem, a minimum value of \( \gamma_{\text{min}} = 0.1 \text{ PVU km}^{-1} \) was chosen so that when \( |\gamma| < \gamma_{\text{min}} \), \( \gamma = \text{sgn}(\gamma)\gamma_{\text{min}} \). We tested different values of \( \gamma_{\text{min}} \) and found little sensitivity in the diagnostic results as long as the feature of changing sign is retained.

The generalized Charney–Stern theorem (Charney and Stern 1962) for rapidly rotating vortices (Montgomery and Shapiro 1995) states that the instability depends on the sign of the sum of the radial gradient of interior basic-state PV and a surface contribution that is also related to the gradient of the potential temperature or PV sheet (Bretherton 1966) at the boundaries. The basic state with changing sign of the radial gradient of the basic-state PV could have unstable normal modes (see also Ren 1999; Nolan and Farrell 1999). Schubert et al. (1999) has illustrated how the spiral rainbands and polygonal eyewall form as a result of unstable growth of the discrete vortex Rossby waves. Chen and Yau (2001) show that strong convection is able to maintain the PV ring while it relaxes to a monopolar structure.
in the absence of moist processes. The fact that PV bands always appear regardless of the sign of the PV gradient suggests that unstable modes are not essential for explaining spiral bands. We could expect to observe neutral modes as well as unstable modes (if the dataset is sufficient) in our analysis.

4. Diagnostic results

a. Wave activity spectra

The ENMs are sorted according to their eigenvalues, that is, their pseudomomentum, in descending order. The first mode has the largest and positive pseudomomentum, the last one has the smallest and negative value. The fact that the eigenvalues change signs indicates the possible existence of a vanishing mode (defined as a mode with zero pseudomomentum and pseudoenergy). It can be shown that an exponentially unstable linear wave mode is a vanishing mode (Held 1985).

Wave activity spectra for wavenumber 1 and 2 anomalies are depicted in Fig. 2. Similar distribution can be seen for both wavenumbers. The dips in the pseudomomentum and the pseudoenergy suggest a vanishing mode, which separates the spectra into two regions. To the left of the dip, the ENMs have positive pseudomomentum and therefore a negative intrinsic azimuthal phase speed; that is, they are retrograde waves (relative to the mean tangential wind). To the right of the dip, on the other hand, the ENMs have negative pseudomomentum and propagate progressively. The kinematics of both retrograde and prograde waves have been discussed in Chen and Yau (2001). Upon careful inspection, one can see from Figs. 2b and 2d that the vortical contribution \( \langle J_y \rangle \) to the pseudomomentum is on average an order of magnitude larger than the gravitational contribution \( \langle J_g \rangle \) for small mode numbers \( n < 50 \) and extreme large mode numbers \( n > 717 \). These modes contribute, respectively, 75% and 9% of the variance of the total anomalies. The variance is defined here as the ratio of the absolute value of the pseudomomentum for one mode and the total absolute value of the pseudomomentum for one wavenumber disturbance. We shall call these leading modes vortex Rossby waves. The intermediate modes, with comparable vortical and gravitational contributions to the pseudomomentum, are mixed Rossby–gravity waves. The remaining modes are gravity modes. Because of the importance of the vortical contribution, it is no surprise that wavenumber 1 and 2 inner spiral rainbands in hurricanes, both observed and simulated with full-physics models, show significant vortex Rossby wave characteristics (e.g., Reasor et al. 2000; Chen and Yau 2001; Wang 2001, 2002).

For small mode number ENMs, the pseudoenergy is dominated by the vortical component of the Doppler shift term \( \langle J_y \rangle \), followed by the kinetic energy \( \langle K \rangle \), potential energy \( \langle P \rangle \), and the gravitational Doppler shift term \( \langle J_g \rangle \). Unlike the low-wavenumber planetary Rossby waves, which have an approximate equipartition of kinetic and potential energy (e.g., Zadra et al. 2002), the kinetic energy in these vortex Rossby waves overtakes the potential energy.
It is possible that exponentially unstable growth of the wave modes and nonlinear mixing will occur due to the interaction of the counterpropagating vortex Rossby waves (Schubert et al. 1999). However, the vanishing modes with exact zero pseudomomentum and pseudoenergy are not observed. The time series of the ENMs, with close to minimum absolute value of pseudomomentum and pseudoenergy, do show fast large-amplitude growth (decay) in the first 2 h. But examination of the radius–isentrope plot of pseudomomentum and the estimated phase speed indicates that these ENMs with minimum value of pseudomomentum are fast gravity waves generated during the model spinup, and they propagate out of the analysis domain in 2 h (graphs not shown).

In hurricanes, vortex Rossby waves generally are nonlinear, dissipative, and forced, and it is impossible to find a purely monochromatic ENM. Power spectrum of the time series of an ENM usually contains a pronounced peak at one frequency with some relatively smaller peaks around. The peak frequency can be treated as the ENM’s frequency. When several comparable peaks appear, the average frequency can be computed by using the power spectra density as the weighting function (Brunet and Vautard 1996). Note that this method may overestimate the frequency of a long timescale signal by having too much weight from the high-frequency tail of the spectra. Recognizing that the low-frequency part of the signal is probably more realistic than the high-frequency part, we compute the weighted average period first, then the average frequency. Although this procedure may produce low-frequency bias, the average frequency of a mode is closer to the peak frequency than the previous method. Figure 3 shows the average frequencies of the ENMs for wavenumber 1. The average frequencies of wavenumber 2 ENMs have a similar distribution (graph not shown). In general, ENMs with larger pseudomomentum have longer timescales. Sorting ENMs according to their eigenvalues is equivalent to sorting the modes with different time-scales. The smooth transition between the slow and fast modes suggests that there is no clear separation of the timescales between vortex Rossby waves and gravity waves.

The basic-state shearing flow (cf. Fig. 1a) admits sheared disturbances or continuous spectrum singular modes (Thomson 1887; Farrell 1984). In gridpoint models, a sheared disturbance can only be represented by a superposition over a set of discrete modes whose frequencies lie in the advective interval $[s\Omega_{min}^0, s\Omega_{max}^0]$, where $\Omega_{min}$ and $\Omega_{max}$ are, respectively, the minimum and maximum mean angular velocity $\Omega_{0} = \nu/r$ (Farrell 1982; Montgomery and Lu 1997). In our case, $\Omega_{min}^0 = 0$ (cf. Fig. 1b). The horizontal line in Fig. 3 indicates $\Omega_{0}^*$, Therefore, the leading modes on both sides of the spectra have frequencies typical of continuous spectrum modes, while the remaining ENMs (mostly gravity waves) have frequencies in the discrete mode range. The fact that no low-frequency discrete modes with null total pseudomomentum appear suggests that unstable discrete vortex Rossby waves are not evident. Higher wavenumber anomalies (from 3 to 6) are also analyzed, but no unstable vortex Rossby waves are observed (graph not shown).

b. Characteristics of ENMs

One ENM can only act as a standing wave. To form a propagating wave, we need at least two modes that have similar contributions to the total variance (i.e., degenerate eigenvalues), the same oscillation frequency, and high correlations among their spatial patterns (Zadra et al. 2002). A pair of modes is identified by comparing their time series and the power spectra of the time series, and computing the correlations among their spatial patterns.

Figure 4 depicts the time series for the first pair of the ENMs (modes 1 and 2) of the wavenumber 1 anomaly. The contributions of these two modes to the total wavenumber 1 anomaly are 11.0% and 8.2%, respectively. The power spectrum (Fig. 4c) for both time series has the same peak frequency that corresponds to a 2.4-h period. The lag correlation of these two time series (graph not shown) reveals a maximum value at 34 min (0.57 h); that is, the phase lag between modes 1 and 2 is a quarter of the period indicating that they are in quadrature. The theoretical predictions of the period from (15) for these two modes are, respectively, 3.3 and 4.1 h and are longer than those estimated from the power spectrum. However, the power spectrum in Fig. 4c contains two other peaks: one has a frequency of 3 h and the other of 4 h, resulting in average periods of 4.0 and 6.2 h, respectively.

Since Fig. 2 suggests that for leading modes, the PV term is the most important partition in pseudomomentum and pseudoenergy, we show only the PV spatial patterns for modes 1 and 2 in Fig. 5. Here PV has been weighted by the factor of $|r\sigma^2/2\gamma|^{1/2}$ so that its square
is a part of the pseudomomentum associated with the PV.

After Fourier decomposition in the azimuthal direction, the PV and the other variables become complex. Therefore, both cosine and sine (real and imaginary) components are shown in Fig. 5. In general, PV is confined to the near-core region. Wave activities with large values are concentrated around the eyewall where the basic-state PV gradient is large. Indeed, one typical characteristic of vortex Rossby waves is that they only exist in regions with nonzero basic-state PV gradient (Montgomery and Kallenbach 1997; Chen and Yau 2001; Wang 2002). To form an azimuthally propagating wave, the cosine component of the spatial patterns of one ENM should be equal to or opposite to the sine component of the other ENM (see Zadra et al. 2002 for proof). This is indeed roughly the situation depicted in Fig. 5. The correlations between the two pairs of diagonal panels are −0.44 and 0.57, respectively.

The time series and their power spectra, and the spatial patterns of the weighted PV for the first pair of ENMs (modes 1 and 2) of wavenumber 2 are depicted in Figs. 6 and 7. These two modes contribute 7.4% and 7.1% to the total wavenumber 2 anomaly. The peak oscillation period obtained from the power spectra of the time series (Fig. 6c) is 1.0 h, while the value obtained from (15) is 1.1 h. The two time series have a phase lag of 16 min. The “cross-similarity” relation presented in the weighted PV spatial patterns (Fig. 7) has correlations of 0.90 and −0.84. Figure 7 also shows that the maximum wave activities are located near the eyewall and they extend vertically.
ENMs 1 and 2 of wavenumbers 1 and 2 therefore constitute two retrograde waves. On the other side of the spectra, we can find prograde waves. As an example, Fig. 8 illustrates the time series of the last two modes (720 and 721 with contributions of 2.5% and 4.4%, respectively) for wavenumber 1. The peak time period directly measured from the time series is 1.5 h, consistent with the theoretical value of 1.6 h computed from (15). The time series show growing amplitude during the last 6 h of the simulation. These two modes are not characterized by discrete unstable modes. Instead, they lie in the continuous spectrum and may experience transient growth (Nolan and Farrell 1999; Nolan et al. 2001). The spatial patterns of the weighted PV (Fig. 9) still satisfy the cross-similarity relation with correlations of 0.91 and -0.78. Different from the leading mode on the left side of the spectra, this wave mode has maximum value of PV in the region with $\gamma > 0$ where prograde modes are expected to exist (see also Chen and Yau 2001; Schubert et al. 1999).

Table 1 contains a summary of the six pairs of leading modes we analyzed. The linear wave theory is verified by comparing the periods observed from the time series and those computed from (15). In general, the observed periods match well the predicted values. Although we only show the leading ENMs for wavenumbers 1 and 2, these vortex Rossby wave modes represent quite well the total anomalies. They explain about 30% of the variance of the total asymmetric anomalies.

In terms of wave activities, higher wavenumber anomalies are generally less important, even though their leading modes still show vortex Rossby wave features (graphs not shown). The overall contribution of vortex Rossby waves can be estimated by comparing the magnitude of the two terms that comprise the total pseudomomentum of the anomalies [see (4)]. Figure 10 depicts the vertical profiles of absolute values of the gravitational term and the vortical term in (4). These terms represent 24-h time-averaged quantities integrated horizontally. In general, the vortical term is one order of magnitude larger than the gravitation term. Upon vertical integration, their ra-
ratio becomes 9.14. Note that we used absolute values because there are sign changes in both terms. If absolute values were not used, the gravitational term becomes even smaller relative to the vortical term. Indeed, their ratio then increases to 415.1 since gravity waves do not have a preferred propagation direction while vortex Rossby waves do. Hence, we conclude that, in the inner core region of the hurricane, vortex Rossby waves dominate over gravity waves in terms of wave activity.

c. EP flux and its divergence

For small-amplitude waves, EP flux $\mathcal{F}$ appears in a conservation form (7), which indicates that EP flux can be interpreted as a flux of wave activity. Under the slowly varying basic-state approximation, the group velocity of a linear wave train is $\mathcal{F}/\mathcal{J}$. Moreover, horizontal and vertical components of the EP flux are associated with horizontal eddy momentum and heat fluxes, respectively. The divergence of the EP flux is proportional to the effective eddy forcing exerted on the mean flow [see (9)]. These interesting features of the EP flux and its divergence make the “EP cross section” a very valuable diagnostic tool for the studies of wave processes and wave–mean flow interaction (e.g., Edmon et al. 1980; McIntyre 1982; Schubert 1985; Andrews 1987; Molinari et al. 1995, 1998). In this study, we have used the ENM method to decompose the total anomalies into ENMs, in which the leading modes for low wavenumbers are vortex Rossby wave modes. It is therefore interesting to compute the EP flux and its divergence associated with these individual wave modes. It will help
us to understand the dynamics of vortex Rossby waves and the wave–mean flow interaction solely by vortex Rossby waves.

The EP cross sections for ENMs 1 and 2 of wavenumbers 1 and 2 are depicted in Fig. 11. With 19.2% and 14.5% contributions to the total wavenumber 1 and 2 anomalies, respectively, these two pairs of modes essentially resemble the total anomalies.

In general, the largest fluxes appear in the eyewall region where wave activities (pseudomomentum) are fluxed radially outward in the lower troposphere and inward in the upper troposphere. The vertical propagation is weak. It is mainly downward for wavenumber 1 in the lower eyewall region and for wavenumber 2 in the upper eyewall region, but is upward in the inner side of the lower eyewall region for wavenumber 2. The weak vertical fluxes and the weak horizontal fluxes outside the eyewall indicate that vortex Rossby wave activities merely propagate horizontally and are confined to the near-core region.

The EP flux and its divergence depicted in Fig. 11 are time-mean quantities. Because of periodicity, the time mean of the local tendency in (7) vanishes. The divergence of the EP flux balances the source term to $O(\alpha^2)$. Hence, the divergence of the EP flux can be used to identify the wave nonconservative regions. Figure 11 suggests that the leading vortex Rossby waves of wavenumbers 1 and 2 are generated (damped) in the lower (upper) eyewall region. The wave activity source term $S$ is dominated by the term associated with the PV anomaly $q'$ and PV generation rate $S_{PV}$, that is, $S_{PV} = q'/q$. In regions with $\gamma < 0$, wave activities are generated (damped) when $q'$ and $S_{PV}$ are in phase (out of phase).

In the eyewall, the latent heating is the dominant mechanism for the generation of PV. In Fig. 11 of Chen and Yau (2001), it was shown that at low (upper) levels, the PV anomaly ($q'$) and the PV generation rate due to latent heating ($S_{PV}$) are generally in (out of) phase. This configuration of $q'$ and $S_{PV}$ can be understood by considering a spiral band with positive PV anomaly ($q' > 0$) and its associated profile of diabatic heating. This band extends vertically from the lower to the upper level (cf. Figs. 5 and 7). Strong convection associated with this band produces a maximum of diabatic heating in the middle level. Below (above) this level, the PV generation rate $S_{PV}$ is positive (negative), which then leads to the formation of a wave activity source (sink) in the lower (upper) levels depicted in Fig. 11.
Figure 3 shows that the leading modes of wavenumber 1 and 2 anomalies lie in the continuous spectrum. One of the most important characteristics of the continuous spectrum Rossby-shear waves is the associated critical layer where the phase speed matches the background flow. In large-scale dynamics, the planetary Rossby waves can be absorbed or reflected by the critical layer. Inside the critical layer, Rossby waves may break to alter the mean flow. By analogy, such a situation should also exist in hurricanes. The thick solid lines in Fig. 11 denote one contour of the basic-state angular velocities that match the angular phase speeds for the first mode of wavenumbers 1 and 2, respectively. Therefore, they indicate the critical lines for these two modes. Interestingly, one can see that EP fluxes, which represent the wave activity propagation, are emitted from the source (sink can be thought as a negative source) toward the critical line. Upon reaching the critical line, the EP fluxes reduce significantly, which suggests critical layer absorption of wave activities (see, e.g., Brunet and Haynes 1996).

Radial and vertical components of the EP flux are proportional to the eddy momentum and eddy heat flux in the radial direction [see (9)]. The quasi-horizontal feature of the EP fluxes in Fig. 11 may imply that the vortex Rossby waves are more efficient in the transfer of momentum. Opposite to the direction of the radial component of the EP flux, the leading mode vortex Rossby waves tend to transport eddy momentum inward in the lower troposphere and outward in the upper level, especially around the eyewall. In terms of eddy heat flux, the wavenumber 1 vortex Rossby waves transport heat from the eyewall outward mainly in the lower and middle levels, while the wavenumber 2 vortex Rossby...
waves tend to transport the heat not only outward but also inward into the core to warm up the hurricane center. The total effect of the wavenumbers 1 and 2 leading modes is thus to flux heat both outward and inward from the eyewall (graph not shown). The fact that the inward heat flux has its maximum value in the upper level indicates that the leading modes may produce significant changes in surface pressure (Zhang and Fritsch 1988).

The effect of vortex Rossby waves on the mean flow has been studied extensively in the filtered models (Montgomery and Kallenbach 1997; Montgomery and Enagonio 1998; Möller and Montgomery 1999, 2000; ...
Fig. 10. Time-averaged (over 24 h) and domain-integrated (over radii less than 260 km) absolute values of the pseudomomentum from vortical contribution $J_y$ and gravitational contribution $J_g$ (kg K$^{-2}$ s$^{-1}$).

Schubert et al. 1999; Shapiro 2000) and full-physics models (e.g., Möller and Shapiro 2002). Previous studies suggest that vortex Rossby waves were able to redistribute the momentum, causing acceleration inside the radius of maximum wind and deceleration outside.

The change of the mean tangential wind due to the net eddy effect is proportional to the divergence of the EP flux. Contours in Fig. 11 therefore represent the 24-h time-mean tangential wind changes by the leading modes. Vortex Rossby waves of wavenumber 1 show a dipole structure with strong acceleration in the lower level slightly inside the RMW and deceleration aloft in the eyewall (Fig. 11a). If we assume that the basic-state isentropic density does not change with time, the mean tangential wind acceleration can be estimated. For a maximum value of $10^6$ kg K$^{-1}$ s$^{-2}$ of the EP flux divergence at a radius of 40 km with $\sigma_0 = 160$ kg m$^{-2}$ K$^{-1}$, the acceleration is $0.8$ m s$^{-1}$ h$^{-1}$. While for the minimum value of $-1.8 \times 10^6$ kg K$^{-1}$ s$^{-2}$ of the EP flux divergence at a radius of 50 km with $\sigma_0 = 90$ kg m$^{-2}$ K$^{-1}$, the deceleration is $2.0$ m s$^{-1}$ h$^{-1}$ (cf. Fig. 11).

Hence, the eddies exert significant effects on the redistribution of angular momentum of the hurricane and affect its intensity in the short and long timescales. Vortex Rossby waves of wavenumber 2, on the other hand, tend to accelerate the tangential wind on both side of the RMW, with maximum effect at the lowest level. They also cause deceleration of the wind at the RMW, with maximum effect at the uppermost level. The total tangential wind change, obtained by summing the EP flux divergence of these two wave pairs, is the acceleration in the lower troposphere and deceleration aloft, with maximum acceleration inside the RMW. The interaction between the leading modes of the vortex Rossby waves and the mean flow is therefore the contraction of the eyewall and the intensification of the hurricane.

5. Summary and conclusions

It has long been recognized that hurricane spiral rainbands are in the form of waves superimposed on a strong symmetric circulation. It is still controversial whether they are governed by gravity waves or vortex Rossby waves. Some recent observational and numerical studies suggest that the inner spiral rainbands have characteristics of vortex Rossby waves (Reasor et al. 2000; Chen and Yau 2001; Wang 2001, 2002). In this study, we apply the ENM method to diagnose the inner spiral bands formed in a hurricane simulated using the PSU–NCAR nonhydrostatic primitive equations model MM5, which permits all kinds of waves.

By taking advantage of the wave activity conservation law, the ENM method can extract statistically the normal modes from an observational or numerical dataset in the context of linear dynamics. Each normal mode conserves its wave activity. For a nonlinear and forced system such as a hurricane, ENM can still be used to decompose simultaneously wind and thermal fields into dynamically consistent and orthogonal modes with respect to wave activities.

By assuming the hydrostatic approximation and a time-independent and azimuthally symmetric basic state, we derived two wave activity conservation laws for pseudomomentum density $J$ and pseudoenergy density $A$ in isentropic and storm-following coordinates. Comparison of the two terms in $J$ associated with density perturbation and PV perturbation, respectively, for each ENM allows one to separate vortex Rossby waves from other waves.
FIG. 11. The EP flux and its divergence for ENMs 1 and 2 of (a) wavenumber 1 and (b) wavenumber 2. The horizontal and vertical components of EP flux have units of kg m K$^{-1}$ s$^{-2}$ and kg s$^{-2}$, respectively. The contour intervals for the divergence of the EP flux are $2 \times 10^{-5}$ kg K$^{-1}$ s$^{-2}$, where the positive regions are shaded. The thick-dashed lines indicate the critical lines with the basic-state angular velocities of $7.27 \times 10^{-4}$ s$^{-1}$ in (a) and $8.39 \times 10^{-4}$ s$^{-1}$ in (b). These values correspond to the peak frequencies of each mode.
gravity waves. The ratio of $A$ and $f$ determines the phase speed of a propagating wave.

The wave activity spectra suggest that the leading ENMs with larger absolute value of wave activities are vortex Rossby waves. These leading vortex Rossby wave modes ($n < 50, n > 717$) contain more than 80% of the total wave activities for wavenumber 1 and 2 anomalies. Gravity waves made a small contribution to the spiral bands in terms of wave activities. Both retrograding and prograding vortex Rossby waves relative to the mean tangential wind are found as a result of the reversal of the sign of the radial gradient of the basic-state PV. Although the necessary condition for instability is satisfied, no discrete unstable vortex Rossby waves of leading wavenumbers are found. The principal components for the nearly vanishing modes show unstable features. These modes, however, are fast-propagating gravity waves corresponding to model spinup in the first 2 h. The plot of the average frequencies of the ENMs implies that there is no clear separation between slow and fast modes. The leading vortex Rossby wave modes lie in the continuous spectrum frequency band.

The time series and the spatial patterns of the leading vortex Rossby waves of wavenumbers 1 and 2 are shown. The observed oscillation periods are verified to agree with predictions of linear theory. The EP flux and its divergence show that these vortex Rossby waves are generated (damped) in the middle and lower (upper) troposphere in the eyewall region. In general, these waves propagate outward in the lower level and inward in the upper level. Consequently, they transport eddy momentum radially inward and outward, respectively. The wave activities also propagate slowly upward inside the eyewall and downward outside. The associated eddy heat transport tends to warm the air in the eye region. The fact that EP flux diminishes upon reaching the critical line suggests critical layer absorption of the vortex Rossby waves. The critical layer may thus provide a site for wave–mean flow interaction. The vortex Rossby waves, through wave–mean flow interaction indicated by the divergence of the EP flux, cause acceleration of the mean tangential wind in the lower and middle troposphere inside and outside the eyewall and deceleration aloft in the eyewall region. This tangential wind change acts in the right sense to spin up the hurricane.

This study is limited to a hurricane that intensifies continuously from 948 to 908 hPa during a 24-h period simulation. The EP flux and its divergence may be different for a spindown hurricane. Vortex Rossby waves and their effects on a spindown hurricane will be presented in a future paper.

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