An Investigation of Turbulence Generation Mechanisms above Deep Convection

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ABSTRACT

An investigation of the generation of turbulence above deep convection is presented. This investigation is motivated by an encounter between a commercial passenger aircraft and severe turbulence above a developing thunderstorm near Dickinson, North Dakota, on 10 July 1997. Very high-resolution two- and three-dimensional numerical simulations are used to investigate the possible causes of the turbulence encounter. These simulations explicitly resolve the convection and the turbulence-causing instabilities. The configurations of the models are consistent with the meteorological conditions surrounding the event.

The turbulence generated in the numerical simulations can be placed into two general categories. The first category includes turbulence that remains local to the cloud top, and the second category includes turbulence that propagates away from the convection and owes its existence to the breakdown of convectively generated gravity waves. In both the two- and three-dimensional calculations, the local turbulence develops rapidly and occupies a layer about 1 km deep above the top of convective updrafts after their initial overshoot into the stratosphere. This local turbulence is generated by the highly nonlinear interactions between the overshooting convective updrafts and the tropopause. Gravity wave breakdown is only present in the two-dimensional calculation and occurs in a layer about 3 km deep and 30 km long. This gravity wave breakdown is attributed to an interaction between the gravity waves and a critical level induced by the background wind shear and cloud-induced wind perturbations above cloud top.

1. Introduction

Atmospheric turbulence poses a significant risk to the aviation community. On average, tens of commercial airline passengers are injured each year over the continental United States during turbulence-related aviation incidents. Such turbulence has a variety of sources including wind shear–induced Kelvin–Helmholtz instabilities (e.g., Sekioka 1970; Werne and Fritts 1999), mountain-wave breaking (e.g., Clark et al. 2000), and mountain-wave rotors (e.g., Doyle and Durran 2002). Convective clouds are also an important source of turbulence (e.g., Prophet 1970; Keller et al. 1983; Pantley and Lester 1990), and gaining a better understanding of convectively induced turbulence (CIT) generation is the focus of this study.

Developing convective clouds cause turbulent motion within those clouds and in the surrounding clear air. In-cloud turbulence generally owes its existence to moist instability within convective updrafts and the subsequent mixing of cloudy air. Out-of-cloud turbulence, however, is poorly understood. Increasing our knowledge of out-of-cloud turbulence generation is important to understand how clouds interact with their environment and is also very important for aviation safety. Aviation safety is the main motivation of this study, which will examine the generation of turbulence in the regions at and above the tops of deep convective clouds.

The Federal Aviation Administration (FAA) has established guidelines for CIT avoidance (FAA 2002). These guidelines include avoiding thunderstorms by 20 nautical miles (37 km) horizontally, and flying above thunderstorms by at least 1000 feet for every 10 kn (5 m s\(^{-1}\)) of cloud-top wind speed. However, the unpredictable nature of developing convection is such that thunderstorms are often unable to be avoided as per the FAA guidelines. Also, it is not clear whether these guidelines would safely avoid turbulence in all cases. In particular, one such incident that motivates this study is an encounter between a commercial airliner and severe out-of-cloud turbulence on 10 July 1997 near Dickinson, North Dakota.

There are a number of mechanisms that may be responsible for out-of-cloud turbulence generation. These include the enhancement of the background wind shear by penetrating convection (e.g., Bedard and Cunningham 1991), cloud-induced flow deformation at the cloud boundary (e.g., Grabowski and Clark 1991), as well as
convectively generated gravity waves (e.g., Lane et al. 2001) and their subsequent breakdown. Nonetheless, despite the relative importance of CIT there has been very little progress made in understanding these sources. Probably the two most limiting factors are the paucity of high-resolution in situ turbulence data, and computational power, which limits numerical model resolution. To explicitly model turbulence generation, with a focus on aviation safety, a number of temporal and spatial scales must be resolved. These scales range from the largest scales that generate the turbulence-causing instabilities, down to the turbulent scales that affect aircraft (50–1000 m). To our knowledge, only two published studies have explicitly modeled (and resolved) turbulence with a focus on the scales that affect aviation (Clark et al. 1997, 2000).

One focus of this study is the role of convectively generated gravity waves in generating out-of-cloud turbulence. Although recent numerical modeling studies (e.g., Piani et al. 2000; Lane et al. 2001; Lane and Reeder 2001; Piani and Durran 2001; Beres et al. 2002) explicitly resolved the generation of gravity waves by deep convection, their model resolutions were too coarse (∼1 km) to resolve turbulence-causing instabilities outside the clouds. The gravity waves generated in such models typically possess horizontal scales of approximately 10 km upward, which are too large to significantly affect aircraft. Some instability, such as wave breaking, is required to cause the cascade of energy down to the appropriate spatial scales. In the two-dimensional calculations presented herein, gravity wave breaking and the subsequent turbulence generation occur and are explicitly resolved by the numerical model. CIT however, is an aviation term encompassing all convectively induced motions that induce a turbulent (not smooth) response in an aircraft. Large aircraft, such as the Boeing 757, are affected by horizontal scales of motion between approximately 50 and 1000 m via the complicated physics of aircraft response. Thus, the flows that induce CIT may be laminar or turbulent (in a fluid dynamics sense). Evaluation of aircraft response to different small-scale flows is beyond the scope of this study. However, our qualitative discussion of turbulence generation does focus on scales of motion that would affect aviation. All discussions of turbulence\(^1\) in this paper concern resolved motions on scales less than about 1 km, down to the relevant grid scale.

This study has three central aims. The first aim is to document the details of the Dickinson turbulence encounter. The second is to use two- and three-dimensional cloud-resolving models to examine possible mechanisms that may have caused the Dickinson encounter. The final aim is to examine the generation of the turbulence represented by the model calculations. The modeled turbulence will be placed into two categories: turbulence local to the cloud top due to the interaction between the convective updrafts and the tropopause, and turbulence that propagates away from the cloud due to breaking gravity waves. Moreover, while motivated by the Dickinson encounter, this study forms a test case for future fundamental research into turbulence generation mechanisms at cloud top and the conditions that favor the breaking of (convectively generated) gravity waves aloft. As well as being important for aviation safety, these processes are extremely important in understanding both cloud-top dynamics and the role of convectively generated gravity waves in the middle-atmospheric momentum budget (see Hamilton 1997, and the references therein).

The remainder of the paper is organized as follows. In section 2, the details of the Dickinson encounter are presented, along with the available observations. The numerical model and simulations are described in section 3. First, the results from a two-dimensional model calculation are presented, which explicitly resolves turbulence local to the cloud top as well as gravity wave breaking aloft. Second, a more realistic three-dimensional cloud model calculation that receives external forcing from a regional-scale model is presented. The mechanisms causing the turbulence in the two- and three-dimensional calculations are examined in section 4, and these results are discussed and related to the Dickinson encounter in section 5. Finally, in section 6 some conclusions are presented and related to aviation safety.

2. Case investigation and observations

On 10 July 1997 a commercial passenger jet (Boeing 757) encountered severe turbulence near Dickinson, North Dakota, en route from Seattle to New York (NTSB 1997). The aircraft was negotiating a number of scattered thunderstorms yet passed directly over a rapidly developing deep convective cloud. While passing over this cloud, the aircraft suffered perturbation accelerations between approximately 9.9 and \(-17.5\) m s\(^{-2}\) (+1.01 and −1.75 perturbation g’s), in a period of about 10 s. Consequently, 20 passengers and two flight attendants suffered minor injuries, and the aircraft had to be rerouted to Denver. The encounter occurred at 2141 UTC at approximately 46.4°N, 103.6°W.

The aircraft was flying at 217 mb (37 000 feet),\(^2\) with an approximately eastward heading. Using a sounding taken at Bismarck, North Dakota (described below), 217 mb corresponds to about 10.9 km above ground level (AGL). The encountered convective cell formed between two more mature thunderstorms that were approximately 110 km apart. The aircraft apparently experienced turbulence while out of cloud and directly

\(^1\) Unless otherwise noted, the motions described herein as turbulent are turbulent in a fluid dynamics sense.

\(^2\) Commercial aircraft define their altitude by measuring pressure; the assigned altitude is defined using the corresponding standard atmospheric height above sea level.
above this center convective cell as it overshot into the relatively undisturbed air at the aircraft’s flight level. In a passenger’s statement, it was noted that the cloud was still climbing vertically when the aircraft “skimmed” over its top.

Using the technique described by Wingrove and Bach (1994), Boeing engineers approximated the atmospheric vertical velocity encountered by the aircraft (Fig. 1). Wingrove and Bach’s technique incorporates numerous quantities from the digital flight recorder, such as aircraft velocity components, angle of attack, and aircraft accelerations (g) loading) into their analysis. Nonetheless, the uncertainty in these vertical velocity values may be as high as 30%, and therefore the value of this trace may only be qualitative. The vertical velocity trace shows two narrow peaks in the upward motion, followed by a smaller negative perturbation. The limited flight data of this incident, as well as the uncertainty in the derived vertical velocity, adds significant uncertainty to the details of the encounter.

The radar reflectivity measured by a radar situated in Bowman, North Dakota (approximately 50 km southeast of the incident), is shown in Fig. 2 at four times surrounding the turbulence encounter. These images show the radar reflectivity at 11 km AGL, that is, at the approximate height of the turbulence encounter. At 2135 UTC (Fig. 2a) a deep cell exists about 50 km to the southwest of the radar, while a smaller radar echo is present about 80 km to the northwest. At 2138 UTC (Fig. 2b), a cell penetrates the 11-km height level at approximately \((x, y) = (-25, 45)\) km (in radar coordinates), between the two cells present at 2135. (Note, however, that for some reason the northernmost cell is not observed by the radar at 2138 UTC at 11 km AGL.) By 2142 UTC (Fig. 2c), all three cells have expanded in the horizontal and the center cell at \((x, y) = (-25, 45)\) km is about 30 km to the south of the northernmost cell and 80 km to the north of the southernmost cell. It is this center cell that was encountered by the aircraft at 2141. Later in the evolution of the convection, 2150 UTC (Fig. 2d), the cells are expanding outward and eventually merge. This time evolution of the radar reflectivity illustrates that the southernmost cell exists for some time before the development of the northernmost cell, which only shortly precedes the overshoot of the middle cell. This sequence is consistent with the pilot’s statement (NTSB 1997).

The three convective cells developed in the warm air to the east of an eastward-moving cold front. Surface analyses suggest that the convective development was forced by this frontal system along a prefrontal trough that was about 50–100 km ahead of the front. The speed of movement of the front was very slow, and it reached Dickinson at about 0000 UTC. The fact that the convection was forced by the synoptic-scale flow makes the explicit modeling of the synoptic-scale flow and subsequent convection (considered later) viable. If this were a purely convective case (i.e., not forced by a synoptic-scale system), attempts to reproduce the configuration of convective development would have been more difficult due to the unpredictable nature of convective initiation.

There are some slight inconsistencies between the radar data and the National Transportation and Safety Board (NTSB 1997) report. In particular, it was reported that the cloud top was below the flight level at the time of the incident. However, the Bowman radar detects a cloud (reflectivity of 20–25 dBZ) at the appropriate location and height, 3 min before the encounter (2138 UTC, Fig. 2b). This inconsistency adds some uncertainty to the timing of the event, and it is possible that the turbulence encounter occurred in a 5-min window following the initial overshoot of the convective cloud through the aircraft’s flight level. Also, there is some uncertainty concerning the distance between the aircraft and the cloud top when the turbulence was encountered.

The closest radiosonde sounding to the Dickinson encounter was released from Bismarck, North Dakota (46.77°N, 100.75°W), at 0000 UTC 11 July 1997. This sounding was approximately 2 h after the turbulence encounter, and approximately 200 km to the east of the encounter. The wind, water vapor mixing ratio, and potential temperature from this sounding are shown in Fig. 3. The tropopause is at approximately 11 km (AGL), almost exactly the flight level of the aircraft. The sounding features a moist well-mixed layer approximately 1.8 km deep. Parcels lifted from this layer possess approximately 3600 J kg\(^{-1}\) of convective available potential energy (CAPE), 11 J kg\(^{-1}\) of convective inhibition (CIN), a level of free convection of 3.2 km AGL, and a level of neutral buoyancy marginally above the tropopause. The large CAPE and low CIN make the environment highly favorable for deep convective development. At low levels, there is an 18 m s\(^{-1}\) southeasterly jet approximately 2 km deep. Above 2 km the wind profile is fairly unidirectional, with a maximum wind speed of approximately 22 m s\(^{-1}\) (at approximately 12
km) from the 215° direction. In this direction, the mean upper-tropospheric shear is approximately 2 m s⁻¹ km⁻¹ (2 × 10⁻³ s⁻¹), and the mean lower-stratospheric shear is approximately −3.5 m s⁻¹ km⁻¹ (−3.5 × 10⁻³ s⁻¹).

3. Numerical model simulations

The numerical model used throughout this study has been documented in detail by Clark (1977, 1979), Clark and Farley (1984), and Clark and Hall (1991). The model is nonhydrostatic, anelastic, can be configured in either two or three spatial dimensions, is written in terrain-following coordinates, and is capable of two-way interactive grid nesting. Also, the model contains an explicit treatment of moist processes via a combination of the Kessler (1969) warm rain and the Koenig–Murray (1976) ice parameterizations. The subgrid is closed using the first-order Smagorinsky–Lilly (Smagorinsky 1963; Lilly 1962) scheme. Finally, in the model interior, a sixth-order spatial filter is used (with fourth-order and second-order filters active at the boundaries); the sixth-order filter strongly damps motions with spatial scales less than approximately 4Δx, 4Δy.¹

¹ It is this filter scale that we assume to be the smallest resolvable scale of the model.
**a. Two-dimensional modeling**

In this section, an idealized two-dimensional model calculation is presented. Such two-dimensional calculations are useful because they provide theoretical and physical guidance for (more computationally intensive) three-dimensional calculations. Also, in two dimensions higher resolutions can be attained in comparison to three-dimensional calculations. This calculation examines the dynamics of a penetrating convective updraft, and the local response to that updraft. Also, the stratospheric gravity waves, which are a response to the developing convection, are examined.

**1) TWO-DIMENSIONAL MODEL CONFIGURATION AND INITIALIZATION**

The two-dimensional model is configured with 4000 points in the horizontal ($x$) direction and 500 points in the vertical ($z$) directions. The horizontal grid has constant spacing of 50 m. In the vertical, the grid spacing is 50 m below 18 km, 150 m above 22 km, and is matched using a hyperbolic-tangent profile in between. Consequently, the grid is 36 km high and 200 km wide. The time step is 0.5 s. The model uses open lateral boundaries, free-slip upper and lower boundaries, and the uppermost 100 points (15 km) feature a Rayleigh-friction sponge to absorb vertically propagating disturbances without reflection. In this configuration, with a single domain, this experiment is denoted as experiment 2D-1.

The two-dimensional model is initialized using a modified version of the 0000 UTC 11 July Bismarck sounding (detailed in section 2). The background thermodynamic fields are identical to those observed by the sounding. The model wind is the 215° component of the observed wind. This direction was chosen as it is the approximate direction of maximum wind (and vertical shear). A Galilean transform is performed by subtracting 8 m s⁻¹, the approximate midtropospheric wind, from the 215° wind. This transformation ensures that the mature convection is located in the approximate center of the domain throughout the model integration. The resultant wind profile used to initialize the two-dimensional model is shown in Fig. 4.

Convection is initiated using a localized surface heating of Gaussian shape, defined by $250 \exp\left[-(x - 100)^2/225\right]$ W m⁻², where $x$ is the horizontal coordinate in kilometers, and $x = 100$ km is the (horizontal) center of the domain. The heating is maintained throughout the simulation and is of sufficient strength to eventually overcome the convective inhibition. Nevertheless, the heating is weak enough so that convective development is not too rapid, allowing the atmosphere to realistically adjust to the external forcing. As a result, the convection takes approximately 70 min to deepen to the tropopause. Such an initialization is preferable to a “bubble” initialization, which artificially chooses an initially unbalanced state. Such solutions do not necessarily behave like the real atmosphere, because such initial states are not realistic. Subsequently, convection deepens rapidly
from the initial imbalance, and it is unclear whether turbulent and gravity wave structures are an artifact of initialization or realistic dynamics. (Note that in the three-dimensional calculation considered later, the convection is forced by the evolving synoptic-scale flow and, therefore, an artificial initialization is unnecessary.)

Figure 5 shows two representative times for the modeled two-dimensional convection, at 80 and 110 min after model initialization. At early times, the convection develops as a cloudy mass of midtropospheric air, with a number of isolated updrafts penetrating to the upper troposphere. By approximately 100 min the convective system is mature and has reached a quasi-steady state, with a pronounced anvil extending downshear. Convective updrafts with peak vertical velocities greater than 20 m s$^{-1}$ are maintained after approximately 60 min. The peak convective updraft of approximately 50 m s$^{-1}$ occurs at approximately 80 min. Of interest for this study is the small-scale structure of the isentropes in the clear air overlying the mature convection (Fig. 5b). This turbulent motion is due to breaking gravity waves, which will be discussed in more detail later in this section.

2) LOCAL RESPONSE

The Dickinson turbulence encounter occurred when an isolated convective updraft penetrated the relatively undisturbed tropopause region. In this section, the local response to an isolated penetrating convective updraft is examined. To do this, a higher-resolution domain (domain 2) is nested within domain 1 of experiment 2D-1. Domain 2 has 16.667-m spatial grid spacing, is 50 km wide, and spans the region $80 < x < 130$ km. It is 7 km high and elevated, with its lowest level at 8 km AGL, and therefore it extends to 15 km AGL. Domain 2 has a time step of 0.1667 s. It is integrated forward in time for 15 min from time $t = 75$ until $t = 90$ min. This calculation with two domains is denoted as experiment 2D-2. The remaining discussion of the two-dimensional local response is limited to the atmospheric response to the first convective updraft that penetrates the tropopause region. It is this first updraft that is most relevant in understanding the turbulence encounter. The first overshoot occurs at approximately 80 min, at $x = 110$ km. Note that the initial updraft is the rightmost penetrating tower in Fig. 5a.

Figure 6 shows three representative times during the evolution of the first updraft that penetrates the tropopause region. In these plots, the eddy-mixing coefficient of momentum $K_m$ is also shown, to highlight regions of small-scale instability and, therefore, likely regions of small-scale turbulence. (Note that $K_m$ is nonzero only when the local Richardson number is negative and, therefore, the flow is convectively unstable. This convective instability can be forced by turbulent or laminar motions on the resolved scale. Thus, $K_m$ is used as a proxy for small-scale turbulent mixing, but we do not necessarily assume that the closure allows a true representation of the smallest-scale motion.) Note also that the horizontal axes of all three plots are different and are chosen to ensure that the penetrating convective updraft remains in the approximate center of the plot. Incidentally, as opposed to Fig. 5, these plots are transposed horizontally at 15 m s$^{-1}$, the approximate background wind speed at 11.5 km.

Figure 6a is shortly after the initial updraft has penetrated the tropopause, and the majority of the vertical motion above and within the cloud is still upward. Nevertheless, the cloudy air above approximately 10.5 km is negatively buoyant, and the net buoyancy force on the cloud top is directed downward, acting to decelerate the penetrating updraft. At this time, the flow above the convective updraft is stable and laminar, with no evidence of instability or turbulence. There is, however, evidence that gravity waves are being generated and are propagating vertically. These waves are evident as perturbations in the potential temperature field above the cloud that have a horizontal wavelength of approximately 2 km.

Figure 6b shows the penetrating updraft approximately 5 min after overshooting into the lower strato-
sphere. At this time, the downward buoyancy force was sufficient to overcome the vertical momentum of the updraft, and the cloudy air has reversed direction. In fact, the cloud top at 85 min is approximately 0.5 km below the cloud top at 80 min (Fig. 6a). Associated with the updraft reversal, the stable isentropes generally descend toward the cloud top from their background height. In contrast to the previous time, the motion above the updraft is no longer smooth and stable. The gravity waves aloft continue to propagate both vertically...
Fig. 6. Contours of potential temperature at 1-K intervals for domain 2 of experiment 2D-2 at (a) 80, (b) 85, and (c) 90 min. Also shown is the 0.05 g kg\(^{-2}\) total cloud loading mixing ratio contour (thick line), and regions where \(K_M\) (outside cloud) exceeds 0.05 m\(^2\)s\(^{-1}\) (shaded); here \(Z\) is height AGL.

and horizontally, and what appears to be one phase of the gravity waves (those with phase lines tilting in the negative direction) is becoming unstable and breaking in the region \(11 \leq z \leq 11.5\) km, \(x = 113\) km. Closest to the cloudy air there is well-resolved overturning of the isentropes, which in turn is generating smaller-scale structures on the 100-m scale. Cloudy air is also being drawn upward from the remains of the updraft, into the overturning region, generating a narrow protruding tongue of cloud. In addition to the overturning region, the isentropes are strongly compressed along the edge of the cloud \((111 < x < 113\) km), indicating a region of strong flow deformation. In this region, \(K_M\) is nonzero just outside the cloudy air, indicating the onset of convective instability.

Five minutes after the onset of out-of-cloud turbulence (Fig. 6c), the turbulent region has extended both vertically and horizontally. Subsequently, there exists a region approximately 1 km wide, extending 1.5 km above the cloudy air, of turbulent mixing and overturning. In addition to the out-of-cloud overturning, there are unstable eddies of approximately 200-m spatial scale propagating along the edge of the cloudy air. At the left edge of the plot another convective updraft is penetrating the tropopause, which would be expected to initiate a similar sequence of local turbulence generation.

Figure 7 focuses on the flow properties in the turbulent region at 90 min. (Note that the axes in Figs. 6 and 7 are different.) The vertical velocity (Fig. 7a) possesses two distinct scales of motion. The first has a horizontal scale of about 2 km and is related to the propagating gravity waves due to the oscillation of the convective updraft about its level of neutral buoyancy (LNB) [see Lane et al. (2001) for a discussion of this mechanism]. This is evident as mean descent above the convective updraft, and ascent laterally adjacent to it. The second, smaller scale is less than 500 m in the horizontal and is due to the turbulent overturning above cloud top. In this region, the vertical velocity perturbations have magnitudes of approximately 4 m s\(^{-1}\) and up to 8 m s\(^{-1}\) closer to the cloud. The horizontal velocity (Fig. 7b) is composed of small-scale perturbations due to the overturning, in addition to strongly enhanced (vertical and horizontal) wind shear close to the cloud. In particular, along the cloud edge at \(116 < x < 117\) km, the vertical shear exceeds 50 m s\(^{-1}\) km\(^{-1}\) (0.05 s\(^{-1}\)), equating to a Richardson number less than 0.25. Consequently, a shearing instability occurs at the cloud interface, causing the unstable Kelvin–Helmholtz eddies that are seen in Fig. 6c. [This instability is the cloud–interfacial instability described by Grabowski and Clark (1991).] Finally, the resolved turbulent kinetic energy (TKE) \((u'^{2} + w'^{2})/2\) is shown in Fig. 7c. (Here, \(u'\) is defined as the perturbations from the domain-averaged horizontal velocity, and \(w\) is the true vertical velocity.)

The resolved TKE field has a number of detailed structures. In particular, the peak TKE in the overturning region, and also in the strongly sheared region at the cloud interface, exceeds 40 m\(^2\)s\(^{-2}\). These TKE values occur on spatial scales less than 500 m.

To examine the horizontal and temporal variability of the out-of-cloud vertical velocity, a number of horizontal traces are shown for six different times in Fig. 8. This plot is constructed by plotting the vertical velocity at \(z = 11.5\) km (AGL) for the different times, with each time displaced along the ordinate. The abscissa is also shifted according to the motion of the convective updraft. A height of 11.5 km is chosen to ensure that the plots depict vertical motion out of cloud; 11.5 km (AGL) is approximately 500 m above the flight level of the aircraft on 10 July 1997. Figure 8 shows that as time...
Fig. 7. Contours of (a) vertical velocity at 1 m s$^{-1}$ intervals (with negative values dashed), (b) horizontal velocity at 1 m s$^{-1}$ intervals, and (c) resolved TKE at 4 m$^{2}$ s$^{-2}$ intervals, at 90 min for domain 2 of experiment 2D-2. Also shown is the 0.05 g kg$^{-1}$ total cloud loading mixing ratio contour (thick); here $Z$ is height AGL.

progresses and the above-cloud turbulence develops, the horizontal structure of the vertical velocity becomes more detailed. Along this constant height and time, the vertical velocity varies by up to 10 m s$^{-1}$ within less than 500 m. Moreover, many of the traces (e.g., 89 min) exhibit similar qualitative behavior to that encountered by the aircraft on 10 July (see Fig. 1). However, these similarities must be interpreted with care because it is
impossible to determine whether the modeled cloud evolves in the same way as the cloud encountered during the turbulence incident.

To investigate the resolution dependence of the results illustrated in Figs. 6 and 7, model solutions completed without the high-resolution domain (experiment 2D-1) are considered. These results possess 50-m spatial grid spacings as described earlier. Figure 9 shows the model solution at 85 min, for comparison to Fig. 6b. As expected, Fig. 6b shows more detail in the smallest scale both in and out of cloud. Inside cloud the gradients in potential temperature are much larger in Fig. 6b than Fig. 9. Also, Fig. 6b features a well-defined cloud boundary with highly compressed isentropes, whereas the cloud boundary in Fig. 9 is more diffuse. Outside cloud, the regions of instability in experiment 2D-1 are not as well resolved as in experiment 2D-2. In particular, in Fig. 6b the region of instability directly above the updraft is represented as a coherent horizontal vortex with strong horizontal gradients in potential temperature, whereas in Fig. 9 the region of instability appears more like a buoyant bubble. Nonetheless, in the two models, regions of turbulent mixing (indicated by non-zero $K_M$) coincide and do not diverge significantly in the 15 min that domain 2 is available. This suggests that, in this case, 50-m horizontal grid spacing is probably adequate to identify regions of turbulent mixing.

In this section, the local response to a relatively isolated penetrating convective updraft has been examined. It has been shown that a local instability occurs near the cloud top, causing turbulent mixing. Also, the updraft caused an enhancement of the background wind shear at the cloud interface and, subsequently, cloud-interfacial Kelvin–Helmholtz waves were generated. All of these processes remain local to the cloud top.

3) STRATOSPHERIC GRAVITY WAVE BREAKING

In this section, the stratospheric gravity wave response to the convection is examined to determine the vertical and spatial extent of turbulence due to gravity wave breaking.

Figure 10 shows the vertical velocity from experiment 2D-1 in the lower stratosphere above the mature convective system. The vertical velocity shows gravity waves above the cloud with approximately 1–2 m s$^{-1}$ amplitude. Those gravity waves in the region $x > 110$ km have a coherent phase structure and a horizontal wavelength that ranges from approximately 6 km near the cloud top to approximately 10 km at $z = 20$ km. However, these waves appear to be trapped or evanescent because the phase lines of the waves are approximately vertical and the amplitude of the waves decreases with height. In the region $x < 110$ km, the...
gravity waves phase lines tilt in the negative direction with height, indicating vertical propagation. However, there are fewer complete (horizontal) wavelengths in this region, and the waves appear to have a less coherent structure. In addition to these smooth gravity waves there exists some smaller-scale (<1 km) horizontal structures in the vertical velocity field in the region below approximately \( z = 14 \) km, directly above the cloud. This small-scale motion is of interest to the current study.

A close-up view of the potential temperature field, at three representative times, in the region of the small-scale motion is shown in Fig. 11. Figure 11a shows wavelike perturbations in the potential temperature, which appear to be propagating in both directions with respect to the cloud. However, some of the waves propagating in the negative direction are convectively unstable, marked by the shaded region in the plot, and are beginning to break. The resolved-scale overturning develops into a turbulent breakdown of the gravity waves (Fig. 11b), and by 120 min (Fig. 11c), a region of wave breaking and turbulence extends approximately 3 km above the cloud top, and 30 km in the horizontal. This wave breaking above the convection appears to be an important source of out-of-cloud turbulence. In particular, there is an instability inducing a turbulent cascade of energy from the 6-km horizontal scale of the gravity waves, down to the limits of resolution of the model. The mechanism causing the wave breaking will be examined later in the paper.

b. Three-dimensional modeling

In the previous section, the two-dimensional response to penetrating convection was investigated. In that case it was shown that the convection produced local instabilities directly above the penetrating updrafts, and wave breaking in the stratosphere above the convection. In this section, the local response to the penetrating convection is examined in a more realistic three-dimensional framework. In particular, the atmospheric response at the approximate height of the aircraft flight level is investigated to further examine the possible mechanisms that may have caused the Dickinson encounter. First, the regional-scale model and its configuration are described, as well as the method in which the cloud-scale model is nested within the regional-scale model. Second, the development of the resolved-scale convection is presented. Third, the small-scale instability associated with the penetrating convection is examined on the highest-resolution grid possible.

1) REGIONAL-SCALE MODEL AND CLOUD-SCALE GRID REFINEMENT

To provide a more realistic evolving background flow and convective initiation, a regional-scale forecast model is used to force the cloud model. The forecast model used is the Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model Generation 5 (MM5; Dudhia 1993). MM5 is initialized with the 0000 UTC 10 July 1997 National Centers for En-
environmental Prediction (NCEP) Global Tropospheric Analysis dataset and run for 48 simulation hours. MM5 is configured using 35 vertical levels, and three (two-way) nested domains, with horizontal grid spacings of 27, 9, and 3 km, respectively. The locations of these domains are chosen to encompass the majority of the cloud development over Montana and North Dakota. However, only data from the outermost (27 km) domain are used to force the cloud model, which is located to minimize cloud at its lateral boundaries. The forecast produced by MM5 does a reasonable job in reproducing the convection observed by the radar (Fig. 2). However, this simulation produces convection to the east of the observations, and about an hour too late. Nonetheless, despite the shortfalls of the forecast, the configuration of the convection is similar to that observed, and the convection is initialized by a realistic forcing. These properties make it appropriate to use the forecast as forcing for the cloud model.

The MM5 simulation is used as initial and boundary conditions for the Clark model via a one-way nesting procedure. To do this, the MM5 data is converted from sigma to height coordinates and interpolated onto the Clark model outermost domain, and adjusted to ensure mass continuity. The Clark model is initialized using the fields from MM5, and the boundary values from the subsequent (one hourly) data are used as forcing for the Clark model outermost boundary. Boundary forcing at intermediate times are calculated from a linear interpolation of the one-hourly fields. See Clark et al. (2000) for a more detailed discussion of the procedures involved in nesting the cloud model within a larger-scale model.

During the three-dimensional calculation, seven levels of grid refinement are used to examine the small-scale features of the convection, while still integrating the synoptic-scale flow responsible for the convective initiation and development. This process uses the two-way nesting procedure used in the two-dimensional calculations previously considered. The details of the seven domains are described in Table 1. The horizontal distribution of these grids is shown in Fig. 12. Also, the uppermost 10 km of domain 1 feature a Rayleigh-friction sponge. Both domains 1 and 2 are initialized with the 2000 UTC fields from MM5’s domain 1 (with 27-km grid spacing). Subsequently, the higher-resolution domains are initialized within the cloud model at appropriate times.

Domain 1, with its 9-km horizontal grid spacing, is too coarse to be appropriate for explicit cloud modeling. However, domain 2 is also used from initialization (but has a grid spacing that is reasonable for explicit cloud modeling), and virtually all clouds form within it. The role of domain 1 is primarily to control the larger-scale (dry) flow, and for that purpose its use is appropriate.

2) NESTED CLOUD MODEL, 1-KM DEVELOPMENT

Figure 13 shows four representative times during the development of the convection in domain 3 of the cloud model. These cross sections are at a height of 10 km (AGL) and, therefore, indicate convective cells that have deepened to 1 km below the tropopause. These four plots show the convection on domain 3 to consist of a number of large scattered cells. In particular, a large convective cell develops in the northern part of the domain (at approximately \( y = 350 \) km), and a larger mass of convection, composed of a number of deep cells, forms in the southern part of the domain (\( y < 250 \) km). In between these two regions of convective activity a
relatively isolated convective cell deepens to the tropopause at \((x, y) = (400, 290) \text{ km}\) at about 2100 UTC. Later in the evolution, the convection in the domain merges to form a north–south oriented convective line.

The vertical and meridional distribution of the convection are shown at two times and two corresponding longitudes in Fig. 14. This figure shows that the center cell overshoots the tropopause at about 2100 between the two more mature cells. By 2130, the middle cell continues to penetrate into the lower stratosphere, and strong vertically propagating gravity waves are evident as perturbations in the potential temperature directly above the convective cell. Similarly, gravity waves are generated by the two adjacent cells and are particularly evident above the northern cell at 2100 (Fig. 14a, \(y = 350 \text{ km}\)).

Although the middle cloud develops at approximately the correct latitude in comparison to the turbulence encounter, it is approximately 80 km to the west and approximately 45 min before the observed event. (This timing and location are different from that produced by MM5, because the convection in the cloud model develops more rapidly, possibly a result of the higher resolution and the more appropriate scales for explicit microphysics.) Nevertheless, the spatial character and evolution of the convection is in good qualitative agreement with the radar observations (Fig. 2). The center cell is relatively isolated, forms between two larger cells, overshoots into a relatively undisturbed environment, and undergoes realistic (forced) initiation. Therefore, it is appropriate to focus on the response of the atmosphere to this overshooting updraft. Moreover, it is extremely difficult to simulate the timing and location of convective development in such situations, and this level of agreement between the observations and the model simulation is considered acceptable for this study.

In domain 3 the model produces peak CAPE values, to the east of the convection, of about 2200 J kg\(^{-1}\). These values are about 1400 J kg\(^{-1}\) less than those observed by the Bismarck sounding. This difference is mostly because the surface water vapor in the model is about 4 g kg\(^{-1}\) less than in the sounding. The reduced water vapor is due to the different timing and location of the sounding and the location of convective development in the model. The different CAPE values cause the peak updrafts in the three-dimensional calculation to be weaker than the two-dimensional calculation.

3) CENTER CLOUD DEVELOPMENT AND TURBULENCE

Figure 15 shows four representative times during the development of the center cloud in domain 5, experiment 3D-5. These figures show that after the initial overshoot of the convective updraft (Figs. 15a,b), the cloudy air expands downstream. As the cloud develops, strong gradients in cloud loading mixing ratio are confined to the upstream side of the cloud (Figs. 15b,c), while downstream the cloud becomes more diffuse. The remainder of the discussion focuses mainly on the flow...
in domain 6 (experiment 3D-6), which is located as shown in Fig. 15a.

In a series of cross sections that are (almost) aligned with the background wind, Fig. 16 shows the temporal development of the center cloud as it overshoots the tropopause region. (These cross sections are chosen to pass through the most turbulent region at 2108 UTC.) Early in the updraft’s evolution (while it is still moving upward), the flow above and around the penetrating updraft is smooth (Fig. 16a). Strong gradients in potential temperature form at the cloud boundary and are most intense when the cloud reaches its maximum height (Fig. 16b). At this stage, small regions of instability exist at the cloud edge, and after the updraft reversal (Fig. 16c) a region of turbulent mixing exists in the clear air above the cloud. The region of turbulence is approximately 3 km wide and 800 m deep.

To elucidate the three-dimensional structure of the turbulent region shown in Fig. 16c, zonal, meridional, and northwest to southeast cross sections through the turbulent region are shown in Fig. 17. These three cross sections show that the turbulent region is localized in the horizontal, encompassing a region where the isentropes aloft are perturbed. These plots show the presence of a gravity wave of approximately 3-km horizontal wavelength. In the upward phase of this gravity wave ($y = 278$ km in Fig. 17b and $YY = -0.5$ km in Fig. 17c), a tongue of cloudy air protrudes about 500 m above the mean cloud top. Figures 17b,c show a great deal of similarity to Fig. 6b, where a similar protruding
Fig. 14. Meridional cross sections of potential temperature at 2-K intervals for domain 3 of experiment 3D-3. Regions where the total cloud loading mixing ratio exceeds 0.1 g kg\(^{-1}\) are shaded. Plots are valid at (a) 2100 UTC at \(x = 402\) km and (b) 2130 UTC at \(x = 422\) km; here Z is height AGL.

tongue of cloudy air existed directly adjacent to the turbulent region. The turbulent region here may be generated in a similar fashion to that seen in the two-dimensional calculation; however, three-dimensional effects and the lower resolution used here make the turbulence generation mechanisms less clear. Like Fig. 7, in this region of turbulence there exist strong perturbations and horizontal gradients in all model fields (not shown), on scales as small as 200 m.

To examine the time evolution of this turbulent region, and the effects of model resolution, Fig. 18 shows three closely spaced times derived from domain 7 (experiment 3D-7). These three times show that the turbulent layer at \(y = 277\) km forms within a period of about 1 min, in what appears to be the downward phase of a gravity wave. (However, this feature may not necessarily be a gravity wave, just cloud-induced perturbations.) In the turbulent region, the increased resolution has introduced more coherent smaller-scale structures in the flow (Fig. 18c). Also, in this domain, a shearing instability at the cloud interface is becoming better resolved (Figs. 18a,b), with small Richardson number present along the upstream cloud boundary. However, the coherent eddies seen in the two-dimensional run are not present.

4. Turbulence generation mechanisms

In the two- and three-dimensional model simulations it was shown that turbulent motion can develop in the clear air immediately above developing convective updrafts. Also, in the two-dimensional model, convectively generated gravity waves propagated and broke above the convective system. In this section, the possible mechanisms leading to the turbulence are considered. First, the mechanisms underlying the lower-stratospheric gravity wave breaking are considered in detail. Next, these ideas and others are used to explain the
possible mechanisms causing the local turbulence generation.

a. Stratospheric wave breaking

In the two-dimensional simulations, gravity waves broke in a 3-km-deep layer above the developing convection. Before breaking, these waves appeared to possess a horizontal wavelength of approximately 6 km and were propagating downshear (upstream). Those waves propagating upshear (downstream) possessed similar horizontal wavelengths; however, these waves remained smooth and laminar (see Fig. 10). The reason why the downshear branch of the waves became unstable and broke, but the upshear branch did not, can be explained first by examining the character of the waves using spectral analysis, and second by evaluating the effects of the sheared wind on the spectrum of propagating waves.

1) SPECTRAL ANALYSIS

To better understand the propagation characteristics of the gravity waves generated in the two-dimensional calculation, spectral analyses of the vertical velocity are completed. The vertical velocity (for domain 1, experiment 2D-1) at \( z = 13 \) km (AGL) is stored at 30-s intervals from \( t = 75 \) min to \( t = 120 \) min (\( z = 13 \) km is chosen because it is high enough that convective updrafts do not penetrate this level, but not so high that all of the waves have already broken and been removed.

\footnote{In the lower stratosphere the background shear vector points in the negative \( x \) direction, and therefore downshear propagation represents waves with negative (intrinsic) phase speeds.}
Fig. 16. Contours of potential temperature at 1-K intervals for domain 6 of experiment 3D-6 at (a) 2100, (b) 2104, and (c) 2108 UTC. Also shown is the 0.1 g kg$^{-1}$ total cloud loading mixing ratio contour (thick line), and regions where $K_M$ (outside cloud) exceeds 0.05 m$^2$ s$^{-1}$ (shaded). The horizontal axis (XX) points toward the northeast (45°), and XX = 0 corresponds to the point $(x, y) = (405.7, 276.9)$ km; here $Z$ is the height AGL.

Fig. 17. Contours of potential temperature at 1-K intervals for domain 6 of experiment 3D-6 at 2108 UTC. Model fields and contours are the same as Fig. 16: (a) a zonal cross section through $y = 276.9$; (b) a meridional cross section through $x = 405.7$; and (c) the horizontal axis (YY) points toward the southeast (135°) and YY = 0 corresponds to the point $(x, y) = (405.7, 276.9)$ km; here $Z$ is height AGL.

from the spectrum). This time–space section is used to construct a two-dimensional (frequency–horizontal wavenumber) amplitude spectrum. The amplitude spectrum is multiplied by its complex conjugate to give the power, which is then smoothed over nine adjacent frequency–wavenumber bins. The normalized power spectrum of the vertical velocity is shown in Fig. 19. Note, for ease of interpretation, that the horizontal wavenumber ($k$) is assumed to be strictly positive, and therefore positive (negative) frequencies ($\omega$) correspond to waves with positive (negative) horizontal phase speeds ($c = \omega/k$).

Figure 19 shows four regions with significant power. Two of these are at wavelengths greater than 12 km ($k < 0.5 \times 10^{-3}$ m$^{-1}$), and two are at wavelengths of approximately 6 km ($k = 10^{-3}$ m$^{-1}$). It is these two peaks at $k \approx 10^{-3}$ m$^{-1}$ that are of particular interest, as it is waves with this approximate wavelength that appear to break in the simulation. The strongest of these two regions of significant power possesses a ground-based frequency of approximately 0.025 s$^{-1}$ and the second, slightly less energetic region possesses a ground-based frequency of approximately −0.005 s$^{-1}$. Thus, there is an asymmetry about $\omega = 0$.

Of particular importance in interpreting power spectra
above convection is the correct choice of reference frame [e.g., see the discussion in Lane and Reeder (2001)]. For example, Fig. 19 is presented in the ground-based reference frame, and the frequency corresponds to the frequency of the waves with respect to a stationary observer. This is the appropriate reference frame for mountain waves, for example, where the wave source is obviously stationary. However, convective clouds are usually moving sources of gravity waves, with their translation speed controlled primarily by the background flow. Therefore, the interpretation of the power spectra of convectively generated gravity waves should incorporate the horizontal translation of the cloud.

Changing to a reference frame translating with a constant speed $U_R$ causes an apparent frequency shift of a wave with horizontal wavenumber $k$, such that the frequency in the new reference frame, $\omega_R$, is

$$\omega_R = \omega - U_R k.$$  

(1)

It follows that

$$C_R = c - U_R,$$

where $C_R = \omega_R / k$ is the phase speed in the new reference frame and $c$ is the ground-based phase speed. Note that, at the wave source, $\omega_R$ is equal to the intrinsic frequency and, likewise, $C_R$ is the intrinsic phase speed.

Following Fovell et al. (1992), Lane et al. (2001) discussed the generation of gravity waves by the oscillation of convective updrafts about their LNB. Relevant to this discussion, Lane et al. had three important conclusions. The first was that gravity waves generated by this mechanism should possess an intrinsic frequency (at the source) equal to the local Brunt–Väisälä frequency. The second conclusion was that such waves should possess the same amount of wave action regardless of their (horizontal) propagation direction (with respect to their moving source). The third conclusion was that the appropriate reference frame (or source velocity) was the wind speed at the LNB, which corresponds to the approximate horizontal translation speed of the convective updrafts at that height.

In the case considered here, the cloud properties in Fig. 6 suggest that the LNB is at approximately 10.5 km AGL. Furthermore, the model wind speed at this height is approximately 10 m s$^{-1}$ (see Fig. 4). As mentioned earlier, the two (ground based) frequency peaks at ($k = 10^{-3}$ m$^{-1}$) are at $\omega = -0.005$ s$^{-1}$ and $\omega = 0.025$ s$^{-1}$. Using Eq. (1) it can be shown that in a reference frame with $U_R = 10$ m s$^{-1}$ (the wind speed at the LNB) the corresponding frequency peaks are at $\omega_R = \pm 0.015$ s$^{-1}$. These peaks are symmetric about $\omega_R = 0$. The power spectrum in the $U_R = 10$ m s$^{-1}$ reference frame is shown in Fig. 20.

Figure 20 is remarkably symmetric about $\omega_R = 0$, for $k > 0.5 \times 10^{-3}$ s$^{-1}$. Also, the Brunt–Väisälä frequency at 10.5 km, as measured from the Bismarck sounding, is equal to 0.0146 s$^{-1}$; almost exactly equal to the peaks in $\omega_R$. Therefore, these results are highly consistent with Lane et al.’s (2001) hypothesized source mechanism and suggest that those waves with $k > 0.5 \times 10^{-3}$ s$^{-1}$ are generated by the oscillation of the convective updrafts about their LNB. These results also show that moderate wind shear has little effect on the source of the highest-frequency gravity waves.

There are, however, marginal differences in amplitude between the $\omega_R > 0$ and the $\omega_R < 0$ parts of the spectrum. These amplitude differences are probably due to a combination of wave breaking and wave refraction. The spectrum is calculated at $z = 13$ km where the background wind speed is approximately 8 m s$^{-1}$. Therefore, the gravity waves propagating in the negative direction, $\omega_R < 0$, are propagating downstream with respect to their source and, therefore, their vertical ve-
However, waves with \( k < 0.5 \times 10^{-3} \text{ s}^{-1} \) are asymmetric about \( \omega_p = 0 \), and their generation is not adequately explained by Lane et al.'s (2001) mechanism. These waves may be asymmetric about \( \omega_p = 0 \) for three main reasons. First, these longer wavelength, lower-frequency waves may be generated in the midtroposphere and therefore \( U_R = 10 \text{ m s}^{-1} \) may not be the correct choice of reference frame. Second, these Fourier components may be nonlinear cloud-induced circulations, which do not necessarily behave like gravity waves. Or third, these disturbances are gravity waves generated by a different mechanism to that described by Lane et al. and therefore need not necessarily be symmetric.

In this case it is clear that the correct reference frame, for the small-scale high-frequency gravity wave generation, translates with the wind at the LNB. However, in some recent studies, Piani and Durran (2001) and Beres et al. (2002) completed their analysis in a frame of reference moving with the squall-line propagation speed (or equivalently the speed of the gust front). However, the speed of the gust front represents the propagation of the envelope of the convection, rather than the motion of the individual convective cells that compose that envelope. (As is generally the case, in their simulations the convective cells propagated backward away from the gust front.) Therefore, as is the case here, the squall-line propagation speed may differ from the relevant source reference frame by 10–20 m s\(^{-1}\). Nonetheless, due to the evolving flow field in convective systems, it may not be appropriate to assign any single value to the source reference frame for the entire lifetime of the convection, nor for all parts of the wave spectrum.

2) WAVE INSTABILITY AND TRAPPING

In Fig. 11, it can be seen that only those waves that propagate in the negative direction with respect to the
cloud top (i.e., $\omega_g < 0$) and have horizontal wavelengths of approximately 6 km become unstable and break. However, as discussed above, Fig. 20 shows little bias in the spectrum of those waves of 6-km wavelength to either propagation direction. This prompts the question, why is it the negative branch of the waves that is breaking?

Linear gravity wave theory defines a critical level as the point at which the phase speed of a gravity wave becomes equal to the local flow speed, that is,

$$U = c,$$

or in a translating reference frame,

$$U - U_r = C_g. \quad (2)$$

As is well known, if a wave encounters a critical level during its vertical propagation, it will break or dissipate in some way (e.g., Fritts 1982). (This dissipation may occur a finite distance below the critical level, giving rise to the terminology “critical layer.”) At the wave source, $U - U_r$ equals zero. Equation (2) is equivalent to stating that in sheared conditions a wave propagating downshear (with respect to the moving source) will encounter a critical level if the change in wind speed is equal to the source-relative phase speed $C_g$.

Figure 20 shows that the peak power in the 6-km wavelength waves occurs at $\omega_g = \pm 0.015 \text{ s}^{-1}$, or equivalently, $C_g = \pm 15 \text{ m s}^{-1}$. Therefore, if locally $U - U_r = 15 \text{ m s}^{-1}$, gravity waves propagating into this region may encounter a critical level and break. Given that $U_r = 10 \text{ m s}^{-1}$, a critical level will occur if the model wind speed $U = -5$ or $25 \text{ m s}^{-1}$. Above the cloud the shear is negative (see Fig. 4) and, therefore, the negative branch of the gravity waves ($\omega_g < 0$) is propagating downshear with respect to the moving source. Furthermore, the mean model wind speed equals $-5 \text{ m s}^{-1}$ at a height of 18 km. Thus, the negative branch of the gravity waves encounters a critical level in the stratosphere about 7.5 km above the source. This simple description explains the occurrence of the breaking of one branch of the gravity waves yet overestimates the height of the breaking.
Although Fig. 20 shows that peak power occurs at \( C_R = \pm 15 \text{ m s}^{-1} \), the spectrum also shows significant power in the regions where \( C_R = \pm (15 \pm 5) \text{ m s}^{-1} \) (and \( k = 10^{-3} \text{ m}^{-1} \)). Therefore, the downshear propagating waves will encounter a critical level if \( C_R \) is as little as \(-7 \text{ m s}^{-1} \) (or \( U = 3 \text{ m s}^{-1} \)), that is, beginning at about \( z = 15 \text{ km} \) (cf. Fig. 4). This estimate is still higher than the height of the breaking seen in Fig. 11, for two main reasons. First, nonlinear cloud-induced circulations cause the \( 3 \text{ m s}^{-1} \) isopleth to descend above the cloud to about 13–14 km, reducing the height of the critical level. Second, nonlinear effects cause the waves to become “self-critical”; that is, the sum of the wave perturbations \( u' \) and the background wind \( U \) induce a critical level, \( U \pm u' = c \) (see Fritts 1984, for details). Therefore, any wave (with nonzero amplitude) will become self-critical some finite distance below the critical level defined by the background wind profile.

Finally, consider the upshear \( (\omega_0 > 0) \) branch of the gravity waves. In Fig. 10 these waves appear trapped because their amplitude decreases with height and their phase lines are vertical (above about \( z = 14 \text{ km AGL} \)). Waves become trapped when the vertical wavenumber changes from real to imaginary. A simplified form of the two-dimensional dispersion relation for nonhydrostatic gravity waves is

\[
m^2 = \frac{N^2}{(U - c)^2} - k^2 = \left[ \frac{N^2}{(\omega - Uk)^2} - 1 \right] k^2, \tag{3}
\]

where \( m \) is the vertical wavenumber and \( N \) is the Brunt–Väisälä frequency. Equation (3) shows that waves are trapped when the magnitude of the intrinsic frequency, \( |\omega| = |\omega - Uk| \), exceeds the Brunt–Väisälä frequency \( N \). The decreasing wind speed above cloud top increases the intrinsic frequency (of the \( \omega > 0 \) part of the spectrum) until eventually it exceeds the local Brunt–Väisälä frequency and the waves become trapped. This is a similar process to that explained by Lane and Clark (2002), who showed that the trapping of upshear waves and the critical level dissipation of downshear waves readily occurs for gravity waves generated by the convective boundary layer.

The three-dimensional calculation, however, does not feature the coherent gravity wave breaking that is evident in the two-dimensional calculation. Indeed, gravity waves are generated by the developing convection (e.g., see Fig. 14b); however, these waves do not break. This is because the three-dimensional model winds at the location of convective development are different from those observed in the Bismarck sounding and, hence, the two-dimensional model. Although the respective flow directions and profile shapes are comparable, the peak wind in the three-dimensional model (and therefore the vertical shear) is approximately half that observed in the Bismarck sounding. This may be because of the differences in time or position of the sounding and the convective development, or may be due to inaccuracies in MM5’s modeled flow. The reduced shear in the lower stratosphere removes the critical level that causes the breaking in the two-dimensional model. As a test of this explanation, another two-dimensional model calculation was completed, denoted 2D-RS. This calculation is the same as experiment 2D-1, except that it is initialized with the wind profile shown as the dashed curve in Fig. 4. This wind profile has exactly half of the wind shear (above \( z = 6 \text{ km AGL} \)) as that used to initialize experiments 2D-1 and 2D-2, but the same wind below \( z = 6 \text{ km} \) to maintain a similar degree of convective organization. (This profile with reduced shear is similar to the mean wind in domain 4 of the three-dimensional model.) The resultant convection is similar to Fig. 5; however, as expected, the occurrence of wave breaking above the cloud is reduced (Fig. 21). Gravity waves do exist above the cloud with similar horizontal wavelengths to those seen earlier, but these waves are not saturated and are able to propagate vertically. There is, however, isolated breaking of a gravity wave with a small (<1 km) horizontal scale, and some small-scale mixing directly adjacent to the convective updrafts. This small-scale mixing is similar to that seen in three-dimensional calculation. These results add confidence to the previous arguments that the wave breaking above the cloud is induced by interactions with a critical layer.

b. Local response

In both the two- and three-dimensional calculations, turbulent overturning and instability are evident in the clear air just above the overshooting convective updraft (Figs. 6, 16). Close to the cloud, the flow barely resembles the background wind used to initialize the model, and nonlinear effects are significant. In particular, consider Fig. 7b, where wind perturbations of up to 10 m s\(^{-1}\) are present. Such strongly perturbed flow makes the use of linear theory inappropriate to describe any instability. Nonetheless, such turbulence is common to all of the high-resolution calculations considered in this study. Therefore, this local turbulent response appears to be an important source of out-of-cloud turbulence,
and in this section possible causes of this turbulence are examined.

In the highest-resolution two-dimensional calculation (experiment 2D-2), the turbulence just above the convective updraft appears to be associated with a gravity wave response to the penetrating convective updraft. In particular, in Fig. 6 the overturning region appears to be the breaking of one phase of a gravity wave propagating in the negative direction. This wave has a horizontal wavelength of approximately 2 km. Consider the arguments concerning wave breaking due to encounters with a critical layer presented in the previous section. Say, for example, that such a wave with a 2-km horizontal wavelength \((k = 3 \times 10^{-3} \text{ m}^{-1})\) had an intrinsic source frequency \(\nu_0\) of 0.02 s\(^{-1}\) \((=N \text{ at } z = 12 \text{ km AGL})\). The phase speed \((\nu = \omega_0/k)\) of such a wave would be approximately 6 m s\(^{-1}\). This phase speed represents the upper bound on the magnitude of the phase speed \((\nu = k \delta N)\) expected for a gravity wave trapped. In reality the true phase speed may be less than half this amount. Therefore, it seems likely that such a wave may encounter a critical level near the updraft top, where cloud-induced wind perturbations are as large as 10 m s\(^{-1}\). These cloud-induced wind perturbations may induce a critical level and be responsible for the turbulence adjacent to the updraft.

The three-dimensional calculation, however, does not clearly show a relationship between a breaking gravity wave and the turbulence directly above the updraft. This could be because the turbulence is not generated by a breaking gravity wave. Or the differences may be a result of the larger grid spacing that is used in the three-dimensional calculation. As was shown in Fig. 9, the two-dimensional calculation with 50-m grid spacing (experiment 2D-1) does not show a coherent horizontal vortex or breaking wave at the top of the updraft. Rather, there is a mass of unstable air at the updraft top, similar to that seen in the three-dimensional calculation. Therefore, it is possible that the turbulence generated in the three-dimensional calculation is a result of a poorly resolved gravity wave instability.

In both the two- and three-dimensional calculations, the local turbulence near the cloud top is generated later in the updraft’s evolution. While the updraft overshoots the tropopause region, the flow remains relatively laminar. It is not until the cloud boundary has reversed direction and has been forced downward that the turbulence develops. In both the two- and three-dimensional calculations, the turbulent motion is located in a region in space that was once occupied by the updraft. This suggests another possible turbulence-generating mechanism associated with the dry response to the updraft reversal. The strong horizontal gradients in potential temperature, induced by the convective overshoot into the lower stratosphere (e.g., Fig. 16b), will generate vorticity that may be strong enough to induce turbulent mixing in the clear air above the cloud. Further investigations into the mechanisms responsible for this local turbulence generation are required and are a topic of current research.

5. Discussion

This study examined the generation of turbulence by deep convection that evolved in a similar environment to the cloud encountered by the aircraft on 10 July 1997. In this encounter, the aircraft passed directly over a developing updraft at the approximate level of the tropopause. It was reported that the aircraft was not in cloud, but extremely close to the cloud top. It was also reported that the updraft was still moving vertically when it was encountered; however, the radar data possibly contradicts that statement. In this section we briefly discuss which, if any, of the turbulence generation mechanisms discussed earlier may have actually caused the turbulence encountered by the aircraft.

Both the two- and three-dimensional modeling studies produced a turbulent layer, directly above the cloud top, about 1 km deep. It seems reasonable that such a layer could also have been present above the cloud top encountered by the aircraft. However, in all of the modeling cases, this turbulence was generated approximately 5 min after the updraft penetrated the tropopause region. Furthermore, this turbulence occurred while the cloud top was moving downward due to the buoyancy reversal, not while the cloud was still moving upward. Therefore, it is possible that the modeled turbulence occurs too late in the updraft’s evolution to explain the turbulence encountered by the aircraft. Also, any coherent wave breaking, as was seen in the two-dimensional calculation, would probably also occur too late in the updraft’s evolution, and probably too high above the cloud top. Nonetheless, the above turbulence mechanisms cannot be discounted because of the significant amount of uncertainty surrounding the encounter, in particular, the uncertainty in the timing of the incident with respect to the evolution of the convective updraft.

Moreover, when an aircraft experiences “turbulence” it is not necessarily due to turbulent flow. A large aircraft such as the Boeing 757 is affected primarily by horizontal scales of motion between 50 and 1000 m; such motion may be laminar in a dynamical sense and may still strongly affect the motion of an aircraft. Consider zonal cross sections of the vertical velocity at early stages during the updraft overshoot (while it is still moving upward), from the three-dimensional model (Fig. 22). (Note that zonal cross sections are chosen here because the aircraft was moving essentially eastward.) Figure 22a shows that during the upward movement of the
Fig. 22. Zonal cross section of vertical velocity at 2101 UTC for domain 6 of experiment 3D-6. Contour interval is 1 m s$^{-1}$ with negative values dashed. Also shown are the 0.1, 1, and 2 g kg$^{-1}$ total cloud loading mixing ratio contours: (a) through $y = 273.14$ km and (b) through $y = 272.64$ km; here $Z$ is height AGL.

Penetrating updraft, strong vertical velocity on about a 1-km horizontal scale is present in the clear air above the cloud. This upward motion above the cloud is due to the convective updraft displacing stable air upward and laterally during its penetration. (This upward motion is almost always simulated in cloud models above upward-moving convective updrafts in a stably stratified environment.) The maximum out-of-cloud vertical velocity is at the cloud boundary (total cloud loading mixing ratio equal to 0.1 g kg$^{-1}$) and is equal to approximately 6 m s$^{-1}$. However, the definition of the “cloud boundary” is somewhat arbitrary and, as can be seen in Fig. 22, there exists a layer about 100 m thick of relatively dilute cloudy air (total cloud loading mixing ratio less than 1 g kg$^{-1}$). It is possible that the aircraft was in this region. In that case the maximum vertical velocity is approximately 8 m s$^{-1}$, which increases with depth into the cloudy air.

Of interest is the vertical velocity distribution in Fig. 22b. Like Fig. 22a, this figure shows strong vertical velocities outside the cloudy air. However, in Fig. 22b, the vertical velocities are not monotonic and at a given height near the cloud top feature two peaks of local maxima. This vertical velocity distribution will produce a vertical velocity trace that is qualitatively similar to Fig. 1. Also, in both Figs. 22a,b, there is a negative vertical velocity on the eastward side of the cloud. Such a negative perturbation is seen in Fig. 1 and is a common feature of the downshear side of penetrating convective updrafts. However, because of the spatial and temporal

penetrating updraft, strong vertical velocity on about a 1-km horizontal scale is present in the clear air above the cloud. This upward motion above the cloud is due to the convective updraft displacing stable air upward and laterally during its penetration. (This upward motion is almost always simulated in cloud models above upward-moving convective updrafts in a stably stratified environment.) The maximum out-of-cloud vertical velocity is at the cloud boundary (total cloud loading mixing ratio equal to 0.1 g kg$^{-1}$) and is equal to approximately 6 m s$^{-1}$. However, the definition of the “cloud boundary” is somewhat arbitrary and, as can be seen in Fig. 22, there exists a layer about 100 m thick of relatively dilute cloudy air (total cloud loading mixing ratio less than 1 g kg$^{-1}$). It is possible that the aircraft was in this region. In that case the maximum vertical velocity is approximately 8 m s$^{-1}$, which increases with depth into the cloudy air.

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variability of the vertical velocity at cloud top (e.g., Figs. 22a,b are only 400 m apart), there is some uncertainty in comparing the features of the vertical velocity trace derived from the aircraft’s flight recorder (Fig. 1) to any specific feature in the model.

6. Summary and conclusions

Realistic two- and three-dimensional simulations of an encounter between a commercial airliner and severe CIT over Dickinson, North Dakota, on 10 July 1997 have been presented. The turbulence was encountered while the aircraft passed directly over the top of a developing convective cloud. There is, however, significant uncertainty concerning the height of the aircraft above cloud top and the timing of the encounter relative to the development of the convective updraft. With these uncertainties in mind, possible sources of the turbulence encountered were examined.

The two-dimensional model used a background environment derived from the closest radiosonde sounding to the turbulence event. This calculation resolved horizontal scales (4Δx) as small as 66 m close to the cloud, and 200 m away from the cloud. In these calculations turbulence was generated in a layer about 1–2 km deep directly above the cloud top, which appeared to be associated with a propagating small-scale gravity wave. In addition to this instability, Kelvin–Helmholtz-like billows were seen to form along the cloud interface due to the strong deformation of the flow by the cloud. It was also shown that the convection generated gravity waves, with horizontal wavelengths of about 6 km, through the same mechanism as described by Lane et al. (2001). Part of the spectrum of these waves broke in a layer that was approximately 4 km deep and 30 km wide.

The three-dimensional cloud model was initialized and its boundaries were forced using a regional-scale forecast of the meteorological conditions surrounding the turbulence encounter. The cloud model contained up to seven levels of grid refinement with the smallest resolvable scale equal to 111 m in the horizontal (4Δx, 4Δy). These calculations produced convection that formed in a similar configuration to that observed; however, the convection formed about 80 km to the west and 45 min before the actual turbulence encounter. Like the two-dimensional calculations, the three-dimensional model produced turbulence in a layer about 1 km deep above the cloud top. It was hypothesized that this turbulence was generated by either a breaking small-scale gravity wave, vorticity generation at the cloud top, or a more complicated nonlinear response during the updraft reversal. The understanding of these turbulence-generating mechanisms are a topic of current research.

The cause of the turbulence encounter was attributed to one of two mechanisms. The first, which occurs early in the convective updraft’s penetration of the aircrafts flight level, was caused by "smooth" upward motion of about 5–10 m s⁻¹ with a 1-km horizontal scale above the cloud top. This upward motion was directly linked to the convective updraft within the cloudy air and was due to the upward displacement of stable air above the penetrating turret. However, it was noted that there were some inconsistencies between the radar data and the incident report, and it is possible that the turbulence encounter occurred sometime later in the evolution of the penetrating convective cloud. Thus, the second possible cause of the turbulence encountered by the aircraft may have been generated by small-scale turbulent mixing at the cloud top. A turbulent layer at the cloud top was seen in both the two- and three-dimensional calculations. However, the amount of uncertainty in the details of the timing and position (with respect to the cloud top) of the encounter adds significant uncertainty to our conclusions.

Although probably not responsible for the Dickinson encounter, breaking gravity waves were seen to cause turbulence in the two-dimensional calculation in a layer that extended up to 3 km above cloud top and 30 km in the horizontal. This breaking was attributed to a critical level caused by a combination of the background wind shear and cloud-induced wind perturbations. Furthermore, it was shown that a critical level may be encountered, and subsequent wave breaking and turbulence generated, provided a change in wind speed above the cloud top is as little as 7 m s⁻¹. This result has motivated continuing fundamental research into the conditions that favor small-scale gravity wave breaking, in the upper troposphere and lower stratosphere, above convection. The role of background wind shear and stratification in controlling the wave breaking will be reported in a future paper.

The understanding of CIT-generating mechanisms is extremely important for commercial and other high-altitude aircraft flying above developing convection. Given the results reported here, it seems possible that the FAA guidelines may not be adequate to avoid such turbulence in all cases. However, only one turbulence encounter has been investigated here. Future work is required to increase the understanding of how clouds generate out-of-cloud turbulence. Such research will eventually improve current turbulence avoidance strategies.

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