Surface Layer Turbulence Measurements during a Frontal Passage

M. Piper*
Program in Atmospheric and Oceanic Sciences, University of Colorado, Boulder, Colorado

Julie K. Lundquist
Atmospheric Science Division, Lawrence Livermore National Laboratory, Livermore, California

(Manuscript received 21 January 2004, in final form 2 March 2004)

ABSTRACT

Very little is known about the nature of turbulence in the transition zone of a synoptic-scale cold front, especially at the dissipative scales. Lacking this knowledge, accurate models of surface frontogenesis are compromised. To address this problem, high-frequency measurements from sonic and hot-wire anemometers are used to analyze the finescale turbulence in the atmospheric surface layer (ASL) within a cold front observed in the MICROFRONTS field experiment. To quantify the turbulence in the front, velocity spectra and dissipation rates are calculated as functions of time and stability in the ASL. The normalized first and second moments of the one-dimensional velocity spectrum conform to the scaling suggested by Kolmogorov’s equilibrium hypotheses, even during the intense turbulence associated with the frontal passage. The spectra compare well with other data collected at high Re, in the ASL, but not as well with a recent model of the dissipative range of turbulence. Dissipation rate $\varepsilon$ is calculated with one direct and two indirect techniques. The calculations from the different techniques compare well with one another and, when nondimensionalized, with a historical expression for dissipation rate as a function of ASL stability. The magnitude of the dissipation rate increases by an order of magnitude to a maximum value of $\approx 1.2 \, m^2 \, s^{-3}$ during the frontal passage compared to prefrontal values of $\approx 0.05 \, m^2 \, s^{-3}$; the latter is typical for a slightly stable nighttime boundary layer over land. These results can be used in assessing the effects of turbulence in traditional semigeostrophic models of frontal collapse. The dissipation rate calculations may be of particular use to modelers.

1. Introduction

What physical mechanism counteracts the intensification of temperature and velocity gradients across a developing front? Emanuel (1985) offers two current schools of thought on the subject: 1) a minimum crossfrontal scale may be reached when frontogenetical forcing is balanced by turbulent mixing and dissipation within the frontal zone (Hoskins and Bretherton 1972; Williams 1974), or 2) as advanced by Orlanski and Ross (1984) and Orlanski et al. (1985), numerical simulations employing the primitive equations suggest that frontal zone contraction may be limited by ageostrophic motions that develop at the frontal boundary and reduce vorticity generation. However, these numerical simulations, which have been extended, for example, by Gall et al. (1987), Garner (1989), and Snyder et al. (1993), show that in the absence of dissipation, frontal collapse proceeds to the grid size of the simulation, which may be at scales as small as $O(km)$. The key missing ingredient for these studies appears to be actual measurements of turbulence in a frontal zone.

A survey of the literature reveals that there have been studies of turbulence in the boundary layer during frontal passages (Young and Johnson 1984; Shapiro 1984; Shapiro et al. 1985; Taylor et al. 1993); however, these investigations are focused more on describing the mean vertical structure and circulation in the boundary layer during a frontal passage. Very little is said about the characteristics of turbulence at small time scales and high wavenumbers during these frontal passages. Some recent investigations have begun to quantify turbulence and dissipation in frontal zones. Frank (1994) examines the turbulence structure of two fronts that passed a 200-m instrumented tower near Karlsruhe, Germany. In addition to showing the mean vertical structure of the fronts as they pass the tower, Frank demonstrates that there is an order of magnitude or more increase in turbulent kinetic energy across the frontal zone. Blumen and Piper (1999) reported turbulence statistics, including dissipation rate measurements, from the MICRO-
The MICROFRONTS field experiment was conducted from 1 March 1995 through 30 March 1995 at a site near De Graff, Kansas, approximately 75 km northeast of Wichita. The field site was situated in gently rolling farmland in eastern Kansas, with a homogeneous fetch to the northwest. The Atmosphere–Surface Turbulent Exchange Research (ASTER) facility, operated by the National Center for Atmospheric Research (NCAR) Atmospheric Technology Division, was deployed at MICROFRONTS to collect turbulence data. The ASTER setup consisted of two 10-m-tower groups, separated by 300 m along a southwest–northeast line, with turbulence instrumentation mounted at several levels, as shown in Table 1. In addition to the standard instrumentation in ASTER, a constant-temperature hot-wire anemometer was deployed at 3 m on the south tower to make fine-scale wind measurements and to directly measure dissipation rate $\epsilon$. This hot wire, a TSI model 1210-T1.5 with a 1.3-mm-long, 4-$\mu$m-diameter tungsten wire, was oriented vertically to measure the horizontal component of the wind, and aligned at 315$^\circ$ from north to coincide with the climatologically preferred direction of frontal passages at the site.

The ASTER sonic anemometers on the south tower were used to compute turbulence statistics for the three
velocity components. The data from the sonic anemometers are important not only because they can be used to calculate dissipation rate, but also because they can be used to demonstrate some evidence of local isotropy, which is necessary to justify the one-dimensional measurements from the hot-wire anemometer.

According to Champagne (1978), the existence of a $-5/3$ slope in a velocity spectrum is a relatively weak indicator of local isotropy. A stronger statement of isotropy and the existence of an inertial subrange is given by the 4:3 ratio between the amplitudes of the transverse ($v$)-to-streamwise ($u$) and the normal ($w$)-to-streamwise ($u$) component spectra ($S$) within inertial subrange frequencies. Figure 1 shows the ratio between the $v$ and $u$ spectra and the $w$ and $u$ spectra at 3 and 10 m for an hour of 10 samples per second data beginning at 0000 UTC 20 March 1995, about 2.5 h before the frontal passage. The surface layer was slightly stable during this period. In this case, the 10-m data approach and achieve the isotropic 4:3 ratio near $f = 1$ Hz. The value of $S_v/S_u$ at 3 m almost achieves the 4:3 ratio before the Nyquist frequency at 5 Hz. The $S_w/S_u$ ratio does not achieve the 4:3 ratio within the bandwidth of the sonic anemometer indicating that, at 3 m, the turbulence might still be feeling the effects of the ground. Kaimal (1986) suggests that the lowest height at which a sonic anemometer can be operated without subjecting the inertial subrange to flow distortion is $z_{min} = 8\pi d = 3.8$ m for a path length $d = 15$ cm. This figure, which compares well with Kaimal et al. (1972, Fig. 22), suggests that an inertial subrange probably exists within frequencies measured by the sonic anemometer at 10 m, but its existence might be questionable in the frequency range of the sonic anemometer at 3 m.

Owing to the long run times of the hot wire in the MICROFRONTS experiment (continuous operation for several days), an in situ calibration of the hot-wire anemometer using wind speed data from the collocated sonic anemometer at 3 m was needed. The calibration was performed using King’s law

$$E^2 = a + bU^n$$

(Hinze 1975; Bruun 1995), where $E$ is hot-wire voltage, $U$ is the wind speed, $a$ and $b$ are empirical coefficients to be determined, and $n = 0.45$ provides a good fit over the range of Reynolds numbers normally encountered in surface layer turbulence (Friehe 1986). Wind speeds from the 3-m sonic anemometer and the 9600 sample per second hot-wire anemometer voltages were block averaged into nonoverlapping 5-s intervals. A linear least squares fit was performed on (1), using 5-min blocks of the 5-s average $E^2$ and $U^{0.45}$ values, resulting in 5-min-average values of the coefficients $a$ and $b$. King’s law (1) can then be inverted to obtain wind speeds from the hot-wire anemometer. This technique was successfully used by Oncley et al. (1996) and as a check on the hot wires used by Champagne (1978).

For direct dissipation calculations, all scales that experience dissipation must be resolved. These scales include eddies at the Kolmogorov microscale $\eta$, which is given by

$$\eta = (\nu^3/\epsilon)^{1/4},$$

where $\nu$ is kinematic molecular viscosity. During the frontal passage, $\eta$ reached a minimum value of approximately 0.25 mm. The frequency required to resolve eddies on that scale, also known as the Kolmogorov frequency,

$$f_K = \frac{U}{2\pi\eta}$$

(Champagne 1978), is a function both of the Kolmogorov microscale and wind speed $U$. As wind speeds during the frontal passage reached 12 m s$^{-1}$, the Kolmogorov frequency $f_K = 7600$ Hz exceeded the Nyquist frequency of 4800 Hz for the hot wire. Therefore, not all scales experiencing dissipation during the frontal passage could be resolved. A rough guideline for the highest wind speeds that can be used for direct dissipation calculations with this hot-wire anemometer can be found by inverting (3) for $f_K = 4800$ Hz. The highest wind speed that allows resolution of a Kolmogorov microscale of 0.25 mm with a hot-wire anemometer sampling at 9600 Hz is about 7 m s$^{-1}$. This threshold is noted on the first panel of Fig. 2 to emphasize that direct dissipation calculations are inaccurate for wind speeds exceeding this threshold.
b. The cold front

A dry Arctic cold front passed the MICROFRONTS site at approximately 0237 UTC (0312 LST) 20 March 1995, 2 h after local sunset at 1839 LST. The parameters for this front are listed in Table 2. They include the beginning and end times for the frontal passage; the frontal velocity \( U_f \), as calculated by isochrone analysis of the frontal passage at surface stations in western and central Kansas; the wind shift \( \Delta D \) from before to after the front; the temperature change \( \Delta T \); the mixing ratio change \( \Delta q \); the time for the frontal zone to pass the site \( t_D \); and the calculated frontal width \( \Delta q \). The frontal zone is defined by the beginning and the end of the wind shift associated with the frontal passage, a criterion used, for example, by Taylor et al. (1993).

Time series spanning the period 0000–0600 UTC 20 March 1995 are shown in Fig. 2. The 6-h time period was chosen because it allows time for the front to completely pass the instrumented tower, with time on either side to view the state of the surface layer. The top two panels show wind speed and wind direction from the 10-m south tower sonic anemometer, at a rate of 10 samples per second. The next panel shows dry bulb temperature from the 10-m south tower platinum resistance thermometer, also at 10 samples per second. The last panel shows two surface layer scaling parameters: the local friction velocity \( u_\theta \), given by

\[
u_\theta^2 = \frac{\overline{w^2} + \overline{w^2}}{L} \tag{4}\]

(Stull 1988; Garratt 1992), where the momentum fluxes \( \overline{w^2} \) and \( \overline{w^2} \) are computed from the 3-m south tower sonic anemometer, and the Monin–Obukhov scaling parameter \( \zeta = z/L \), where \( L \) is a local Obukhov length, given by

\[L = -\frac{u_\theta^2 \Theta}{kg_w \theta_e}. \tag{5}\]

Here, \( k \) is the von Kármán constant, with a value of 0.365, \( g \) is the gravitational acceleration constant, \( \Theta \) is the mean potential temperature between 2 and 10 m, and \( \theta_e \) is the virtual potential temperature flux calculated from the 3-m sonic anemometer. These scaling parameters are calculated using 900-s averaging intervals, centered on the time of the frontal passage. The dotted lines delimit the frontal zone, as listed in Table 2.

Note, visually, the sharp increase in wind speed variance with the passing of the front. After the front passes, the wind speed and speed variance decay to near-prefrontal values. The wind has a southwesterly component in the prefrontal period. The temperature trace in the third panel shows that there was a \( 2^\circ C \) rise in temperature starting at 0200 UTC, possibly due to advection of warmer air from the southwest or increased mixing in the surface layer. After the frontal passage, temperature decreased steadily due to radiational cooling and cold air advection. The wind shift and temperature drop were not coincident in this front, with the temperature drop at 10 m lagging the wind shift by about 180 s. This is also observed in Taylor et al. (1993) and Shapiro et al. (1985), suggesting that this front may have an elevated head, like a density current.

### Table 2. Parameters for the 20 Mar 1995 cold front.

<table>
<thead>
<tr>
<th>Begin (UTC)</th>
<th>End (UTC)</th>
<th>( U_f ) (m s(^{-1}))</th>
<th>( \Delta D ) (°)</th>
<th>( \Delta T ) (°C)</th>
<th>( \Delta q ) (g kg(^{-1}))</th>
<th>( \Delta t ) (s)</th>
<th>( \Delta ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0237</td>
<td>0312</td>
<td>12</td>
<td>70</td>
<td>-2.5</td>
<td>-3</td>
<td>2100</td>
<td>25 200</td>
</tr>
</tbody>
</table>

3. Energy and dissipation spectra

According to the universal equilibrium hypothesis of Kolmogorov (1941a, 1962), the dissipative scales of turbulence are statistically independent of the large, energy-containing scales in a fully developed turbulent flow. Under the equilibrium hypothesis, the shape of the normalized spectrum is posited to be a universal function of Reynolds number. The bulk of the data supporting the equilibrium hypothesis were collected in controlled wind tunnel experiments (Comte-Bellot and Corrsin 1965; Champagne 1978; Saddoughi and Veeravalli 1994) or under narrowly specified conditions in the atmospheric boundary layer (Williams and Paulson 1977; Champagne et al. 1977; Champagne 1978; Oncley et al. 1996). The data presented here, on the other hand, were collected in the atmospheric boundary layer during the passage of a sharply defined synoptic-scale cold
One-dimensional kinetic energy spectra are calculated as functions of time using data from the sonic and hot-wire anemometers at intervals before, during, and after the frontal passage. These spectra are nondimensionalized with Kolmogorov scales to test for the collapse of the spectra into a dimensionless function that can be compared with the results from the authors cited above.

a. Theoretical and experimental background

Taylor’s hypothesis must be invoked to convert the frequency spectra calculated from the sonic and hot-wire anemometer data to the wavenumber domain used in turbulence theory. The appropriate form of Taylor’s hypothesis is given by

\[ f S_u(f) = \kappa_1 F_{11}(\kappa_1), \]

where \( f \) is frequency, \( S_u(f) \) is the frequency spectrum of the streamwise velocity component, \( F_{11} \) is the one-dimensional streamwise wavenumber spectrum (which is measured by the hot-wire anemometer), and \( \kappa_1 \) is the streamwise wavenumber, related to \( f \) by

\[ \kappa_1 = \frac{2\pi f}{U}, \]

where \( U \) is the advection velocity (Tennekes and Lumley 1972).

Taylor’s hypothesis is an approximation. It is valid only when the turbulence intensity in a flow is small, or to first order,

\[ \frac{u}{U} \ll 0.1, \]

where \( u \) is a characteristic velocity of the turbulent fluctuations (Tennekes and Lumley 1972). This condition is often not met for tower-based turbulence measurements in the atmospheric boundary layer (Wyngaard 1986), including during this frontal passage. For high-intensity turbulence, when (8) is not valid, Lumley (1965) developed a model wherein the advection velocity \( U \) in (7) is not constant, but is instead composed of mean and fluctuating components. Wyngaard and Clifford (1977) used Lumley’s model to derive a correction for the streamwise wavenumber spectrum \( F_{11} \). In the inertial range, the correction is

\[ F_{11}^{\prime} = F_{11} \left(1 - \frac{\overline{u^2}}{9U^2} + \frac{2\overline{v^2}}{3U^2} + \frac{2\overline{w^2}}{3U^2}\right), \]

where \( F_{11}^{\prime} \) denotes the measured wavenumber spectrum; \( F_{11} \) is the true spectrum; and the wind speed variances \( \overline{u^2}, \overline{v^2}, \) and \( \overline{w^2} \) are computed from time series (Champagne 1978). Champagne (1978) took Lumley’s model a step further by actually solving for the true streamwise wavenumber spectrum \( F_{11} \) as a function of \( F_{11}^{\prime} \) and wavenumber \( \kappa_1 \), thereby providing a means of correcting the measured spectrum in both the inertial and dissipative ranges. Wyngaard and Clifford (1977) also give the correction for the streamwise derivative variance

\[ \left(\frac{\partial u}{\partial x}\right)^2 = \left(\frac{\partial u}{\partial x}\right)^2 \left(1 + \frac{\overline{u^2}}{U^2} + 2\frac{\overline{v^2}}{U^2} + 2\frac{\overline{w^2}}{U^2}\right). \]

This correction is used for the direct calculation of dissipation rate (see section 4). This result was also derived independently by Heskestad (1965), under slightly different assumptions.

The scaling relations for the three-dimensional energy spectrum obtained from Kolmogorov’s hypotheses hold for the one-dimensional spectrum \( F_{11} \) (Hinze 1975). When normalized by Kolmogorov scales, \( F_{11} \) takes the form

\[ F_{11}(\kappa_1) = e^{14/5} \varphi_{11}(\eta \kappa_1) \]

in the equilibrium range, where \( \eta \) is given by (2) and \( \varphi_{11}(\eta \kappa_1) \) is a universal function of \( Re_\kappa \). In the inertial subrange, dimensional analysis and isotropy lead to the \(-5/3\) law of spectral decay (Hinze 1975)

\[ F_{11}(\kappa_1) = \alpha_i \varepsilon^{2/3} \kappa_1^{-5/3}, \]

where \( \alpha_i \) is the streamwise Kolmogorov constant. This result is used for the inertial dissipation calculation of dissipation rate (see section 4).

To clearly present changes in the energy spectrum brought about by the frontal passage, spectra from three periods are selected for display:

- an hour before the front, in a developing stable surface layer;
- during the frontal passage;
- an hour after the completion of the frontal passage, when a stable surface layer is redeveloping; \( z/L \) is increasing.

The selection of these periods is arbitrary, since there is no theory to guide at what time relative to the front they should be picked. Table 3 summarizes the information used in computing the three spectra. The periods are denoted before (B), during (F), and after (A) the frontal passage. For each period, Table 3 displays mean wind speed \( U \) and the component fluctuation wind speeds \( \sigma_u, \sigma_v, \) and \( \sigma_w \), computed from the sonic anemometer. Means are taken over the duration of the 900-s period. Mean dissipation rate \( \varepsilon \) (discussed in section 4), the Kolmogorov microscale \( \eta \), the Taylor microscale \( \lambda \), and the turbulence Reynolds number \( Re_\kappa \) are calculated from the high-rate hot-wire data. The Taylor microscale is define here as (Tennekes and Lumley 1972)

\[ \lambda = \frac{\sigma_u}{\sigma_{u_{rms}}}, \]

where \( \sigma_u \) and \( \sigma_{u_{rms}} \) are the root-mean-square values of the streamwise velocity and the streamwise velocity derivative, respectively. Using the Taylor microscale, a turbulence Reynolds number can be calculated:
Table 3. Selection times and turbulence parameters for the three spectra chosen to describe the 20 Mar 1995 front. The statistics are computed at a height of 3 m. See the text for additional details.

<table>
<thead>
<tr>
<th>Period</th>
<th>Time (UTC)</th>
<th>$U$ (m s$^{-1}$)</th>
<th>$\sigma_u$ (m s$^{-1}$)</th>
<th>$\sigma_v$ (m s$^{-1}$)</th>
<th>$\sigma_w$ (m s$^{-1}$)</th>
<th>$\epsilon$ (m$^2$ s$^{-3}$)</th>
<th>$\eta$ (mm)</th>
<th>$\lambda$ (m)</th>
<th>$Re_\lambda$</th>
<th>$F_{\eta l}/F_{\eta 0}$ (IR)</th>
<th>$F_{\eta l}/F_{\eta 0}$ (DR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0130–0145</td>
<td>3.2</td>
<td>0.55</td>
<td>0.36</td>
<td>0.27</td>
<td>0.027</td>
<td>0.61</td>
<td>0.057</td>
<td>2000</td>
<td>1.010</td>
<td>1.070</td>
</tr>
<tr>
<td>F</td>
<td>0245–0300</td>
<td>10.8</td>
<td>2.36</td>
<td>1.52</td>
<td>0.90</td>
<td>0.760</td>
<td>0.27</td>
<td>0.051</td>
<td>8000</td>
<td>1.013</td>
<td>1.106</td>
</tr>
<tr>
<td>A</td>
<td>0415–0430</td>
<td>6.3</td>
<td>1.33</td>
<td>0.98</td>
<td>0.54</td>
<td>0.105</td>
<td>0.42</td>
<td>0.055</td>
<td>5000</td>
<td>1.016</td>
<td>1.107</td>
</tr>
</tbody>
</table>

\[ Re_\lambda = \frac{\sigma_u \lambda}{\nu}. \] (14)

Here, the kinematic molecular viscosity $\nu$ is considered to be $1.5 \times 10^{-5}$ m$^2$ s$^{-1}$. In the final two columns, two corrections for high-intensity turbulence are calculated, the correction for the one-dimensional spectrum $F_{\eta l}$ in the inertial subrange and the correction for the mean square streamwise velocity derivative $(\partial u/\partial x)^2$, which has most of its contributions in the dissipative range. Both corrections are from Wyngaard and Clifford (1977).

b. Kolmogorov spectrum: First moment

Figure 3a shows the three spectra from before, during, and after the frontal passage on 20 March 1995 plotted as dimensionless Kolmogorov spectra $\varphi_{11}(\eta \kappa_1)$. The spectra appear to collapse, as suggested by the universal equilibrium theory, despite the different conditions under which they were measured. Here, three consecutive 300-s wind speed spectra from the hot-wire anemometer in the B, F, and A periods are smoothed by block averaging the spectral estimates in 75 nonoverlapping bands spaced over the logarithm of frequency. This technique is discussed in Blackman and Tukey (1959) and used, for example, in Williams and Paulson (1977). The wind speed spectra are then converted to wavenumber spectra using (6) and made dimensionless using mean values of dissipation rate $\epsilon$ and kinematic viscosity $\nu$ from the corresponding time intervals according to (11). Note that the value of $\epsilon$ calculated using the inertial dissipation technique with the hot-wire anemometer is used, since the direct technique was unusable in high wind speed conditions, as explained in sections 2 and 4. The dotted line shows a negative five-thirds slope indicative of an inertial subrange in the spectrum, according to (12). The noise floor of the hot-wire anemometer, manifested as a flattening of the spectrum at high frequencies, is apparent in the spectra taken in the B and A periods.

Figure 3b shows the compensated Kolmogorov spectrum function $\tilde{\varphi}_{11} = (\eta \kappa_1)^{\alpha_1} \varphi_{11}$ calculated from the hot-wire anemometer data in the B, F, and A periods of the front. Fifteen compensated spectra computed over consecutive 60-s intervals are interpolated with a cubic spline function to a set of logarithmically spaced abscissas, corresponding to the plotted values of $\eta \kappa_1$. The inertial range correction (9) has been applied to these spectra. The dotted line gives a value of 0.53 for the streamwise Kolmogorov constant $\alpha_1$ for these data. This value compares well with others in the literature (Wyngaard and Coté 1971; Williams and Paulson 1977; Champagne 1978; Dyer and Hicks 1982; Oncley et al. 1996).

The advantage of the compensated Kolmogorov spectra is that it shows the inertial subrange as a horizontal line, without the distortion of a logarithmic ordinate. The Reynolds number dependence of the three curves is seen as an elongation of the constant level of the inertial subrange. The constant level is small for the B period, but it is longer in the F and A periods, consistent with the larger $Re_\lambda$ in these regions. This follows from the increasing separation of the energy-containing and dissipative scales with increasing $Re_\lambda$. Despite the large $Re_\lambda$, the three spectra achieve a nearly constant value over barely 2 decades of normalized wavenumber.

Several authors have commented on the “bump” that
appears in the compensated Kolmogorov spectrum $\psi_{11}$ at normalized wavenumber $\eta_{11} = 0.05$. This phenomenon is seen, for example, in the compensated spectra presented by Saddoughi and Veeravalli (1994, Fig. 13), Mestayer (1982, Fig. 10b), and Gotoh et al. (2002). But it may or may not be present in the spectra from Champagne (1978, Fig. 25) or Williams and Paulson (1977, Fig. 6). Sreenivasan and Antonia (1997) call this the bottleneck phenomenon. It is hypothesized that it represents a buildup of excess energy in the inertial range that is waiting to be dissipated. Notice that a secondary bump appears at the low-frequency end of the constant-$\eta_{11}$ range in the three spectra. This secondary bump is also seen in the transverse spectrum in Saddoughi and Veeravalli (1994, Fig. 13) and in the streamwise spectrum in Williams and Paulson (1977, Fig. 6).

c. Kolmogorov spectrum: Second moment

Figure 4 shows the Kolmogorov-normalized second-moment spectrum $(\eta_{11})^2 \varphi_{11}$ in the B, F, and A periods. The spectra are normalized with Kolmogorov scales and plotted versus normalized frequency. When dimensionless and integrated, the result is the mean dissipation rate $\epsilon$ for each period. Three consecutive 300-s spectra, corrected with the formula provided by Champagne (1978), are composited to form the curves shown in each 900-s interval. Mean values are displayed with symbols, with 95% confidence intervals, based on the variation of $\epsilon$ and $F_{11}$, plotted as crosshairs on the means. The development of the error analysis is covered in Piper (2001). Despite the differences in the spectral amplitudes, $Re_\lambda$ and $\epsilon$, the spectra collapse. Note, however, that the spectrum taken during the F period does not achieve $\eta_{11} = 1.0$, consistent with the bandwidth limitation of the hot-wire anemometer discussed in section 2.

The eleventh-order polynomial curve fit to the Kolmogorov spectrum supplied by Champagne (1978, his Table 4) is also plotted. It was computed using corrected spectra taken in the atmospheric boundary layer in the Minnesota experiment (Champagne et al. 1977) at $Re_\lambda = 7000$. The measured spectra here compare favorably with the Champagne data, although there is a discrepancy at the high-wavenumber end of the inertial subrange. The measured spectra also compare well against data from lower $Re_\lambda$; see, for example, Fig. 9 from Saddoughi and Veeravalli (1994). Additionally, a theoretical model prediction for the spectrum from Pope (2000) is presented. The model spectrum appears to agree well with the spectra from the present study in the inertial subrange $(\eta_{11} < 0.05)$ and at the higher end of the dissipative range $(\eta_{11} > 0.5)$, but there is disagreement throughout the rest of the dissipative range.

The value of $\eta_{11}$ at which the three spectra reach their maximum value is approximately $\eta_{11} = 0.08$. This compares fairly well with observational data from Grant et al. (1962), Champagne et al. (1977), Williams and Paulson (1977), Champagne (1978), and Saddoughi and Veeravalli (1994). In those studies, the maximum in the second moment spectrum is found consistently at $\eta_{11} = 0.1$. Also, the theoretical model of Pope (2000) gives the maximum at $\eta_{11} = 0.1$, although it is apparent that the modeled values do not match the Champagne data or the current data very well for $\eta_{11} > 0.05$. The maximum value of $(\eta_{11})^2 \varphi_{11}$ is between 0.20 and 0.22 for the spectra displayed in Fig. 4. These values compare well with the values given by the authors above (between 0.21 and 0.22), although there is noticeably more scatter here than in the data presented in Champagne (1978, Fig. 22).

4. Dissipation rate

Dissipation rate was measured directly with the hotwire anemometer and calculated, using two inertial range techniques, with data collected from the sonic and

![Figure 4. Kolmogorov second-moment spectra $(\eta_{11})^2 \varphi_{11}$, plotted vs dimensionless wavenumber $\eta_{11}$ on linear axes. Data are from the hot-wire anemometer before (Δ), during (□), and after (○) the frontal passage. Table 3 gives the time and $Re_\lambda$ of each spectrum. Crosshairs give 95% confidence intervals for the data values. The solid line gives the curve fit supplied by Champagne (1978) for the normalized spectrum at $Re_\lambda = 7000$. The dashed line shows the theoretical model spectrum from Pope (2000).](image-url)
hot-wire anemometers. For reference, Table 4 provides a list of the calculations. A quantitative comparison between the estimated and directly calculated values of $\epsilon$ provides a test of the relative value of the inertial range estimates under varying turbulence and stability conditions.

a. The direct dissipation technique

Under the assumption of local isotropy, since there is no preferred direction, the tensorial form of the mean turbulent kinetic energy dissipation rate $\epsilon$ reduces to a form that includes only a single velocity derivative (Tennekes and Lumley 1972):

$$\epsilon = 15\nu \frac{\partial U}{\partial t}^2, \quad (15)$$

where $\nu$ is the kinematic viscosity, $x$ is in the streamwise direction, and $u$ is the velocity component in that direction.

Applying Taylor’s hypothesis with (10), the Heskestad–Lumley correction for high-intensity turbulence, to (15) yields

$$\epsilon = \frac{15\nu}{U^2} \left( \frac{\partial U}{\partial t} \right)^2 \left( 1 + \frac{w^2}{U^2} + 2 \frac{u^2 + w^2}{U^2} \right)^{-1}. \quad (16)$$

Because velocity derivative terms are involved, the correct instrument to use is the hot-wire anemometer, since it is one of the few instruments capable of resolving the sharp gradients that occur at the smallest scales of turbulent motion. Equation (16) uses the square of the fluctuating wind speed derivative. A derivative series is constructed by differentiating the hot-wire wind speed series $U$. The derivative series is low-pass filtered at the noise floor of the hot-wire anemometer. The square of the derivative series is then calculated over nonoverlapping 60-s intervals. The terms $u^2$, $w^2$, $U^2$, and $\partial U/\partial t$ are computed with a 900-s average from the sonic anemometer. The time series of these terms are then interpolated to 60-s intervals for use in (16). The kinematic viscosity $\nu$ varies weakly as a function of temperature, so its value is indexed at 60-s intervals with the temperature measured by the hygrothermometer collocated with the hot-wire anemometer at 3 m. The Heskestad–Lumley correction factor in (16) ranged between 5% and 10%, which is comparable in magnitude to that given by Wyngaard and Coté (1971). Direct calculations of $\epsilon$ are not possible at all times during the frontal passage, as discussed in section 2. The values of $\epsilon$ calculated when wind speeds are above 7 m s$^{-1}$ are still displayed, yet it is noted that they are incorrect because of this systematic error.

b. The inertial dissipation technique

As described, for example, in Champagne et al. (1977), if $S_u(f)$ is the frequency spectrum of velocity component $u$, in the inertial range and $\alpha_i$ is the Kolmogorov constant for the velocity component, the dissipation rate is given by

$$\epsilon = \frac{2\pi}{U} \left[ \frac{f^{5/3} S_u(f)}{\alpha_i} \right]^{3/2}. \quad (17)$$

This technique can be used by any sensor that has a sufficiently high-frequency response to measure velocities in the inertial range (Oncley et al. 1996). Here, wind speed data from the sonic and hot-wire anemometers are used to estimate $\epsilon$ with this technique.

Streamwise velocity spectra $S_u$ were computed from the hot-wire anemometer data as a function of frequency $f$ in consecutive 60-s intervals using a fast Fourier transform (FFT). The spectra were smoothed by block averaging spectral amplitudes in 75 nonoverlapping logarithmically distributed bands, giving approximately 10 bands per decade of the spectrum. Before computing the FFT, the 60-s sample was tapered using a Hamming window. This technique is recommended by Kaimal and Kristensen (1991) to reduce the overestimation of the amplitude of the inertial subrange when using an averaging interval less than the integral time scale of the turbulence being measured. To compensate for the Hamming window, the resulting spectra were multiplied by a factor of 2.52. The average value of $f^{5/3} S_u(f)$ was then computed over the frequency band $5 < f < 30$ Hz. By examination of several hours of frequency spectra, this frequency band was determined empirically to contain a subset of the frequencies where power-law behavior develops in the streamwise velocity spectrum. Mean wind speed $U$ is computed over 900-s intervals, then interpolated to the 60-s intervals. Based on an analysis similar to what produced Fig. 3, a Kolmogorov constant of $\alpha_i = 0.53$ was chosen for use in (17). This value falls within the distribution of Kolmogorov constants reported in the literature (Wyngaard and Coté 1971; Williams and Paulson 1977; Champagne 1978; Dyer and Hicks 1982; Oncley et al. 1996).

Sonic anemometers measure the $u$, $v$, and $w$ components of the wind, with the spectrum of each component obeying (17). Only the streamwise velocity component $u$ is used here, however, because it achieves an inertial subrange first (Kaimal et al. 1972; also see Fig. 1) and because it is contaminated least by probe-induced distortion for our wind data. Spectra of the streamwise velocity component $S_u$ were computed in 60-s intervals, with a Hamming window applied as described above. Following the procedure described in Oncley et al. (1996), $f^{5/3} S_u(f)$ was computed over the frequency band $0.5 < f < 2$ Hz. These bounds were chosen empirically by examining several hours of data; they also are close to the range 0.5–5 Hz used in Wyngaard and Coté (1971) and Oncley et al. (1996). The upper bound at 2 Hz was chosen below the Nyquist frequency of the sonic anemometers, thereby limiting the influence of line averaging on the calculation. The frequency band
where \( f^{3/2}S_u(f) \) achieved a maximum, as well as the two adjacent frequency bands, were used to compute a mean value of \( f^{3/2}S_u(f) \), which was used in (17). The mean wind speed \( U \) was interpolated from the 900-s average value, and a Kolmogorov constant of \( \alpha_i = 0.53 \) was used.

c. Kolmogorov’s four-fifths law

Another method for calculating dissipation rate is Kolmogorov’s four-fifths law, an exact relation derived directly from the Navier–Stokes equations using the assumptions of homogeneity and isotropy. The four-fifths law is given by

\[
\Delta u^3 = [u(x+r) - u(x)]^3 = -\frac{4}{5} \epsilon r \tag{18}
\]

(Kolmogorov 1941b), where the left-hand side is the third-order streamwise velocity structure function, with the condition that the separation distance \( r \) must lie in the inertial subrange.

The four-fifths law gives an appealing method for estimating dissipation rate because there are no undetermined coefficients, unlike (17). The four-fifths law has been studied recently by Praskovsky (1992) and Dhruva et al. (1997), and used in Katul et al. (1994) and Albertson et al. (1997). Sreenivasan and Antonia (1997) suggest that the four-fifths law may provide a less ambiguous estimate of \( \epsilon \) than the direct method (15), and that it can also be used to fix the extent of the inertial subrange. Dhruva et al. (1997) assert that the four-fifths law appears to hold even in inhomogeneous turbulence.

Taylor’s hypothesis is used to convert (18) to the time domain. If \( r = U\tau \), where \( U \) is mean wind speed and \( \tau \) is a displacement in time, then (18) can be rearranged to provide a formula for \( \epsilon \):

\[
\epsilon = -\frac{5}{4} \left( \frac{1}{U\tau} \right) \Delta u^3, \tag{19}
\]

with the stipulation now that the separation distance \( U\tau \) must fall within the inertial subrange of scales. There is no high-intensity turbulence correction to Taylor’s hypothesis for the third-order structure function (Hill 1996). Figure 1 suggests that the sonic anemometers used in the MICROFRONTS experiment do not have the frequency response necessary to use Kolmogorov’s four-fifths law to calculate dissipation rate, since the four-fifths law technique requires measurements in the inertial subrange, and evidence of an inertial subrange in the 3-m sonic data is tenuous. The hot-wire anemometer can, however, measure velocities in the inertial range, so data from each are used with (19). Estimates of \( \epsilon \) using (19) were made in 60-s intervals with the hot-wire anemometer data. The value of \( r \) was chosen to be 0.03 m. For the range of \( U \) values during the fronts, this choice of \( r \) gives values of \( f = \tau^{-1} \) that fall within

![Figure 5](http://journals.ametsoc.org/doi/abs/10.1175/1520-0469(2004)061<1768:SLTMDA>2.0.CO;2?journalCode=josd)
Fig. 6. Dissipation rate calculations from the 3-m hot-wire and sonic anemometers for the 20 Mar 1995 front. Values of $\epsilon$ are calculated in 60-s intervals. Error bars denote 95% confidence intervals on the means. (a) Direct dissipation technique from the hot wire (HD). Note the expanded ordinate. The error bars are smaller than the plot symbols. The hatched areas indicate when $U > 7$ m s$^{-1}$. (b) Inertial dissipation technique from the sonic (SI).

Fig. 7. Scatterplots of the 60-s $\epsilon$ calculations for the 20 Mar 1995 front. Symbols refer to calculations made within the frontal zone ($\times$) and before and after the front ($\bullet$). The solid lines show a 1:1 correspondence in the $\epsilon$ values. (a) Comparison between inertial dissipation (HI) and direct dissipation (HD) calculations from the hot-wire anemometer. The comparison is only made for wind speeds $U < 7$ m s$^{-1}$. (b) Comparison between inertial dissipation (HI) and Kolmogorov’s four-fifths law (HK) calculations from the hot-wire anemometer. (c) Comparison between inertial dissipation calculations from the hot-wire (HI) and sonic (SI) anemometers.

The confidence intervals for the HI and HK calculations displayed in Fig. 5 are small, even during the frontal passage, indicating random errors are controlled well. The confidence intervals on the HD calculation are smaller than the plot symbols because of the large number of points used in the calculation in comparison to the tiny integral scales (on the order of the Kolmogorov time scale $\tau$) of the velocity derivative. The confidence intervals on the SI calculation are uncomfortably large. The confidence in these $\epsilon$ values could be increased by further averaging in time or by including more points in the average through more spectral bands. Another means by which the confidence could be increased is by including independent calculations of $\epsilon$ from the transverse velocity component of the sonic anemometer. However, Kaimal et al. (1972) show that the transverse component does not display $-5/3$ power-law behavior for frequencies as low as those in the streamwise spectrum. Because the SI calculation is made under tenuous evidence of an inertial subrange within the bandwidth capabilities of the 3-m sonic anemometer, the chances of obtaining inertial subrange estimates of $\epsilon$ from the transverse velocity component are not good.

Ideally, the four calculations of $\epsilon$ (HD, HI, HK, and SI) should yield the same value for the dissipation rate in each 60-s time interval. However, due to systematic errors—for example, the hot-wire frequency response limit in the HD calculation or the lack of true inertial subrange measurements in SI—and random errors, the four calculations do not give the same value in each interval. Figure 7 shows scatterplots comparing the values of $\epsilon$ calculated with the four techniques over the 6-h period. In each panel, the 1:1 line is used instead of a least-squares line of regression because none of the $\epsilon$ calculations can be considered independent for the purposes of regression. The 1:1 line gives the ideal limit of where the individual data points should lie on each scatterplot. The idea of neutral regression, when neither variable is independent, is discussed in Emery and Thomson (1997). Because the HD method cannot be used in the front, HI is used instead for a basis of comparison with the other calculations. Figure 7 shows that the values of HK and SI are visibly well correlated with the values of HI, as evidenced by the agreement with the 1:1 line in each plot. The agreement is favorable even during the frontal passage. The scatter between the calculations tends to increase with increasing $\epsilon$. The scatter between the methods is mostly due to random error, since increasing the averaging interval reduces the amount of scatter.

Table 5 shows the cross-correlation coefficient $\rho$ and
neutral regression error $e_r$ between each $e$ calculation. Values are displayed for the 6-h analysis period ($N = 360$ points) and for the 35-min frontal zone ($N = 35$ points), with the exception of the HI–HD comparison, which is only shown when $U < 7 \text{ m s}^{-1}$. The cross-correlation coefficients achieve their maximum at a lag of 0 s. The neutral regression errors represent the mean value of the set of distances of each point in Fig. 7 from the 1:1 line, with each value normalized by the mean value of the two $e$ values being compared. This corresponds to

$$e_r = e_{AB} = \frac{2}{N} \sum_{i=0}^{N-1} \left| \frac{e_i - e_{Bi}}{e_{Ai} + e_{Bi}} \right|,$$  \hspace{1cm} (20)

where $A$ and $B$ represent any two of the methods (HD, HI, HK, or SI) used to calculate $e$.

The values of the parameters shown in Table 5 suggest that there is a high degree of correlation between the three calculations. The highest degree of correlation is between the calculations made with the hot-wire anemometer, presumably because the buoyancy of inertial range measurements from the hot wire tend to reduce random errors. The inertial dissipation method from the sonic anemometer also compares fairly well with the hot-wire anemometer calculations. This result is important because it shows that $e$ calculations can be made with a sonic anemometer that compare favorably with those made from a hot-wire anemometer, and sonic anemometers are much sturdier and easier to use and maintain in the field (Onclcy et al. 1996).

A larger question is how well do the $e$ calculations compare within the frontal zone? This frontal zone took 35 min to pass the MICROFRONTS site. The three calculations that use hot-wire anemometer data are still well correlated with each other, although their maximum correlation values have decreased due to scatter, since only $N = 35$ calculations are used in the computation of the cross-correlation coefficients. The correlation between the hot-wire calculations and the sonic inertial dissipation calculation has dropped, although the value of the correlation is still reasonably high. This suggests that $e$ calculations from the sonic anemometer can be used when data from the hot-wire anemometer are not available, and that the $e$ values from the sonic will be fairly reliable, even in frontal situations.

### Table 5. A matrix of correlation coefficients $\rho$ and neutral regression error $e_r$ for the four dissipation rate calculations.

<table>
<thead>
<tr>
<th>Techniques</th>
<th>$N$</th>
<th>$\rho$</th>
<th>$e_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HI–HD</td>
<td>242</td>
<td>0.95</td>
<td>0.13</td>
</tr>
<tr>
<td>HI–HK</td>
<td>360</td>
<td>0.94</td>
<td>0.20</td>
</tr>
<tr>
<td>HI–SI</td>
<td>35</td>
<td>0.85</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>360</td>
<td>0.89</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.73</td>
<td>0.23</td>
</tr>
</tbody>
</table>

**Fig. 8.** Dimensionless dissipation rate $\phi_e$ plotted against the Monin–Obukhov scaling parameter $\zeta$ for the dissipation rate calculations made in the 6-h period surrounding the 20 Mar 1995 front. Plot symbols correspond to HD (○), HI (●), HK (×), SI 3 m (△), and SI 10 m (+). The curve is suggested by Wyngaard and Coté (1971).

### e. Normalized dissipation rate

Dissipation rate can be normalized with Monin–Obukhov surface layer similarity scales. The normalized dissipation rate $\phi_e$ is given by

$$\phi_e = \frac{kz\epsilon}{u_b^3},$$  \hspace{1cm} (21)

(Wyngaard and Coté 1971), where $k$ is the von Kármán constant, $z$ is the height above the surface, and $u_b$ is the friction velocity. The dimensionless dissipation rate $\phi_e$ is hypothesized to be a universal function of $\zeta = z/L$, where $L$ is the Obukhov length. Because $\phi_e$ is expected to be a universal function, it provides a good way to compare the $\epsilon$ values calculated here with other $\epsilon$ values presented in the literature.

Dimensionless dissipation rate calculations are presented in Fig. 8. Data from the 3-m hot-wire and the 3- and 10-m south tower sonic anemometers are displayed, using all available methods. The dissipation rate calculations are averaged into the 900-s intervals used to calculate $u_b$ and $L$. Each point therefore corresponds to an average of 15 of the 60-s $\epsilon$ calculations displayed in Figs. 5 and 6. A value of $k = 0.365$ was used for the von Kármán constant (Onclcy et al. 1996). The line represents the curve fit suggested by Wyngaard and Coté (1971):

$$\phi_e = (1 + 2.5\zeta^{0.5})^{\sqrt{2}}.$$  \hspace{1cm} (22)
In the previous section, the methods used to calculate $\epsilon$ were shown to agree well with one another, indicating a fair degree of precision in the calculations. Figure 8 shows that the $\epsilon$ calculations were also fairly accurate, given that they collapse well when normalized with surface layer scales and the normalized values compare favorably with an accepted $\phi_e$ curve fit. The dimensionless $\epsilon$ data are slightly lower in magnitude than the curve fit predicts over the range of $\zeta$ displayed. Oncley et al. (1996) also found that their $\epsilon$ data near $\zeta = 0$ in stable conditions were smaller than the values predicted by the Wyngaard and Coté curve fit. Similar scatter is found by other researchers (see, e.g., discussion in Oncley et al. 1996).

5. Discussion

Ground truth has been established for turbulence levels within a surface frontal transition zone, including measurements of Kolmogorov spectra and turbulent kinetic energy dissipation rate. These results can be used in assessing the effects of friction in the surface layer in traditional semigeostrophic models of frontal collapse (Hoskins and Bretherton 1972; Williams 1974), or in models with an ageostrophic feedback mechanism (Orlanski and Ross 1984).

Examination of energy spectra shows that the fine scales of turbulence obey Kolmogorov scaling, even when the large scales are subjected to the substantial anisotropic and nonstationary forcing of a cold front. The data quality is high enough to permit testing of the first and second moments of the normalized spectrum. They are found to agree with field data from other experiments. The normalized spectrum is also compared with a recent theoretical model of the equilibrium range proposed by Pope (2000). The theoretical model does not agree particularly well with the current data or the other experimental data through the dissipative range of scales, on the range $0.1 < \eta_k < 1.0$.

Direct and indirect methods for calculating dissipation rate are used on data collected before, during, and after the passage of a cold front. Both sonic and hot-wire anemometers are utilized. The calculations from the direct and indirect methods are found to compare well, even though information from different scales of turbulence are used in the calculations, and despite the fact that the calculations are obtained using wind field measurements from different instruments. The agreement in the calculations suggests that the indirect methods can be used safely to calculate $\epsilon$ at other height levels where only one turbulence sensor is located, and with data from other fronts, where no direct calculations of $\epsilon$ are available. The dissipation rate in the surface layer is found to increase by an order of magnitude in the 20 March 1995 frontal passage to a maximum value of $\sim 1.2 \text{ m}^2 \text{s}^{-3}$, compared to prefrontal values of $\sim 0.05 \text{ m}^2 \text{s}^{-3}$. Dissipation rate levels remain high even after the passage of the frontal zone. When normalized by Monin–Obukhov surface layer scales, the dissipation rate values agree well with a curve fit proposed by Wyngaard and Coté (1971).

The results of this study may have some immediate utility. For example, information on how the surface layer interacts with fronts and density currents can be useful for pollutant dispersion prediction (Yee et al. 1998; Reitebuch et al. 2000). Additionally, accurate observations of dissipation rate will assist in the modeling of the stably stratified boundary layer (Kosović and Curry 2000).

Acknowledgments. We are indebted to our advisor Bill Blumen, for his guidance and support in science and in life.

Support for MP has been provided by the Air Force Office of Scientific Research under Grant F49620-95-1-0141 and under AASERT Grant F49620-97-1-0448. Support for MP and JKL at the University of Colorado has been provided by the Atmospheric Sciences Division, Mesoscale Dynamics Program of the National Science Foundation under Grant ATM-9903645. MP also thanks Research Systems, Inc., for allowing a leave of absence.

This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.


