The Determination of Vertical Velocities in Thunderstorms (II)*

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THE FOLLOWING DISCUSSION shows the derivation of the equations and gives a more detailed presentation of the assumption involved in the three cases mentioned in the first section of this paper.* It will be noted that the single expression given there for the vertical velocity is in reality only the principal term of the equations derived here for those three separate cases.

The treatment following is based in part on that used by J. Levine.† For the sake of brevity we start with the relation

$$\frac{\delta p_s}{p_s} = \frac{\delta p}{p} + \frac{\mu}{R} \int_0^z \frac{\delta g}{T} \, dz \qquad (1),$$

where z is the altitude, p the normal pressure, δp the change from normal pressure at the altitude z, T the temperature at the altitude z, and δg the change in acceleration from the normal for static conditions.

Now we assume (following Levine) that

$$\delta g = A z (1 - z/h), \qquad (2).$$

where h is the altitude at which the vertical velocity is at a maximum. In terms of the vertical velocity, we have

$$w^2 = 2 \int_0^z \delta g \, dz = A z^2 \left(1 - 2z/3h \right),$$

whence

$$A = \frac{w^2}{z^2 (1 - 2z/3h)} \qquad (3).$$

Now the vertical velocity will be a maximum at h, so putting

$$w_{MAX} = V$$

we have

$$A = 3V^2/h^2 \qquad (4).$$

If we let H represent the altitude at which the vertical velocity becomes zero again, we have $H = 3h/2$. Substituting (2) and (4) in (1), and letting $T = T_s - \lambda z$, we have

$$\frac{\delta p_s}{p_s} = \frac{\delta p}{p} + \frac{\mu}{R} \frac{3V^2}{h^2} \int_0^z \frac{z(1 - z/h)}{T_s - \lambda z} \, dz \qquad (5).$$

Carrying out the integration and using the approximation

$$\log \left(1 - \frac{\lambda z}{T_s} \right) = - \left[\frac{\lambda z}{T_s} + \frac{1}{2} \left(\frac{\lambda z}{T_s} \right)^2 + \frac{1}{3} \left(\frac{\lambda z}{T_s} \right)^3 \right],$$

we obtain

$$\frac{\delta p_s}{p_s} = \frac{\delta p}{p} + \frac{\mu}{R} \frac{3V^2}{h^2} \frac{z^2}{T_s} \left[\frac{1}{2} - \frac{z}{3h} + \frac{2z}{3T_s} \right] \qquad (6).$$

For the cases mentioned before, we now introduce the assumptions:

Case A: $\delta p = 0$ for $z \geqq H = 3/2\,h$
Case B: $\delta p = 0$ for $z \geqq h$
Case C: $\delta p = 0$ for $z \geqq h^*$ ($h^* < h$) .

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*See pp. 94-95 of the March 1943, BULLETIN.

†Levine, J., The Effect of Vertical Accelerations on Pressure During Thunderstorms. *Bull. of the Amer. Met. Soc.*, v. 23, no. 2, Feb. 1942, pp. 52-61.

(Note that in case C the maximum vertical velocity is reached at the altitude h^* where $h^* < h$, so that in this case we must use (3) instead of (4), so that w is replaced by V, and z by h^*.)

Making the proper substitutions in (6) for each of the three cases, we obtain the three expressions:

$$\text{Case A: } \frac{\delta p_s}{p_s} = \frac{1}{2}\frac{\mu}{R}\frac{V^2}{T_s} \times \frac{27}{4}\frac{\lambda h}{T_s} \qquad (7),$$

$$\text{Case B: } \frac{\delta p_s}{p_s} = \frac{1}{2}\frac{\mu}{R}\frac{V^2}{T_s}\left(1 + \frac{2\lambda h}{T_s}\right) \qquad (8),$$

$$\text{Case C: } \frac{\delta p_s}{p_s} = \frac{1}{2}\frac{\mu}{R}\frac{V^2}{T_s}\left(1 + \frac{2\lambda h}{T_s}\frac{h^*}{3h - 2h^*}\right) \qquad (9).$$

In these three expressions, we note that each is approximately the equivalent of

$$\frac{\delta p_s}{p_s} = \frac{1}{2}\frac{\mu}{R}\frac{V^2}{T_s} \qquad (10)$$

from which follows the equation that was given in the first section of the paper,

$$V^2 = 2\frac{R}{\mu}T\,\frac{\delta p_s}{p_s} \qquad (11).$$

Jan Mayen, "A Birthplace of Weather"

The island is 34 miles long and 9 miles wide at its broadest section in the north. It lies at almost equal distances from Iceland and Greenland (290 and 255 nautical miles respectively) and 555 miles from Spitzbergen and the Scandinavian coast. Situated just where the south bound Polar Stream hugging the coast of Greenland skirts the Gulf Stream that is heading the opposite direction. A zone of storms, whirlwinds and fogs results, that is perhaps without equal in the world. Directly in the path of most cyclones that cross Iceland on their way to northern Normay, there is therefore no more ideal place than Jan Mayen from which to observe them long before they reach Europe. Norwegians erected a Meteorological post on the island in 1929 to establish full sovereignty. Winds reach as high a velocity as 190 mph. The number of fine days per year averaged over the ten-year period is 2.4. For 238 days of the year the sky is completely overcast. During 33 days hurricanes or storms occur. The above mentioned 2 forces are constantly at work brewing a dense clinging fog. There is a mountain (extinct crater) 8094 ft. high. Driftwood on the island provides much enlightening data on the currents of the Polar basins. Named after Captain Jan Mayen who maintained a whaling station there from 1611 to 1635.—*Courtesy of F. A. Baughman.*
