The Friction of Recording Pens on Paper

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1. INTRODUCTION

One of the factors which determine the accuracy of a recording instrument is the friction of the pen on the chart paper. For our present purpose, the mechanism that controls the pen may be considered as a lever with the pen on one end and a controlling force on the other (Fig. 1). For a given value of the quantity being measured, and in the absence of friction, there will be a unique position assumed by the lever, and therefore by the pen. If now a couple \( M_t \) is applied to the lever, it will rotate until an equal and opposite couple \( M \) is developed by the deformation of the various members of the mechanism of the instrument. The magnitude of this couple for rotation through a specified angle has been called the control \([1]\). The control differs greatly for different instruments, being much greater in a thermograph, for example, than in a hair hygrograph.

Now if we perform the opposite experiment, holding the pen in a fixed position while the measured quantity is allowed to change, a similar couple \( M \) will be developed by the measuring element. This must be resisted by a couple \( M_t \) applied to hold the pen, this couple being equal to \( M \) at each instant. If \( M \) exceeds \( M_t \), the pen will move.

It is clear that the couple \( M_t \) may be produced by the friction of the pen on the paper, and that an error in the indication of the instrument will result from this. Whether or not this is of practical importance in a given installation depends on the required precision of the record and the magnitude of the frictional forces. In any well-constructed instrument, the pen friction will be much greater than that due to the various pivots and bearings in the mechanism, mainly because of the relatively much greater lever arm on which the pen friction is applied.

The experiments to be described are an attempt to find out something about the behaviour of pens on paper, and to estimate the magnitude of the frictional forces.
2. Apparatus

The apparatus used for the investigation is shown in Fig. 2. A plate A is rotated at a rate of one turn in several hours by a synchronous motor B. The plate is supported on a tubular shaft running in ball bearings. Inside the tubular shaft, but not touching it, is a second tube C which forms the lower portion of a frame carrying the torsion wire D and its associated parts. The entire frame can be pulled out and replaced without disturbing the plate A. The torsion wire, which can be stretched by a nut E, passes through a fitting F on which is mounted a plane galvanometer mirror G. This fitting is provided with two miniature ball bearings H, in which rotate conical pivots forming an axis for the beam J which carries the pen K under investigation. A damping vane L, moving in Dow-
Corning fluid (viscosity 350 cs at 25°C), and a sliding balance weight M, are also arranged on the beam. N is a removable rider which can be placed in a notch as shown so as to press the pen down with the desired force. Riders of from 18 mg to 150 mg have been used. The paper, cut in a 10-inch circle, is laid on the upper surface of A and held down by the large washer P and the nut Q.

Light from a lamp R, which has a straight vertical filament, is focussed by the achromatic lens S, so as to form a sharp image of the filament on the drum T after reflection from G. The slit U confines the image on T to a spot, producing clear traces on the bromide paper carried by the drum. Once every hour the motor B closes a pair of contacts, V, for a brief period, lighting a small lamp (not shown) in the same horizontal plane as the slit. This produces time marks in the form of transverse lines on the bromide paper. The drum revolves once in about 26 hours.

3. CALIBRATION

The torsion wire was calibrated by using the wire and beam assembly as a torsion pendulum. A pair of similar weights in the form of thin discs were arranged to slide along the beam, and the time of a number of oscillations was taken with these in various positions, the weights being balanced on the beam and the radius r to the centre determined by measuring the distance between them.

The moment of inertia \( I \) (gm cm\(^2\)) of a torsion pendulum, the torsion constant \( L \) (dyne-cm per radian) of the wire, and the time \( T \) (seconds) of one complete oscillation, are connected by the well-known relation

\[
T^2 = 4 \pi^2 \frac{I}{L} \tag{1}
\]

If the moment of inertia of the bare beam and other moving parts, without the weights, is \( I_s \) gm cm\(^2\) and the mass of each weight \( M \) gm, then the moment of inertia of the complete pendulum is

\[
I = I_s + 2 Mr^2 \tag{2}
\]

For any value of \( r \),

\[
T^2 = 4 \pi^2 \left( I_s + 2 Mr^2 \right) / L \tag{3}
\]

A number of values of \( r \) were taken, and \( L \) determined from the resulting values of \( T \) by the method of least squares. The precision of this measurement is very much greater than that of any other part of these experiments.

If the point of the pen under test is at a radial distance \( R \) from the centre of the wire, tangential force \( L/Rg \) grams applied at the pen point will be necessary to deflect the beam through one radian. If \( l \) is the distance from the mirror to the recording drum, the force necessary to move the spot of light through 1 cm will therefore be \( 1000L/(2L Rg) \) milligrams.

4. PROCEDURE

In the actual experiment, the beam is balanced so that the pen point just clears the paper, and a sheet of bromide paper placed around the drum. After leaving the apparatus undisturbed for a few seconds in order to record a zero, a rider is placed in the notch at N (Fig. 2), pressing the pen down on the paper with a force \( p = \left( R_{\infty}/R \right) Wg \), where \( W \) is the weight of the rider and \( R_{\infty} \) the distance from the centre of the wire to the notch. The pen is then dragged by the moving chart paper, deflecting the beam at a constant rate until it slips part of the way back. The appearance of a typical record is shown in Fig. 3, and it will be seen that it takes the form of a series of steeply-sloping lines, the light being adjusted so that it will not record the return motion of the pen. The bold vertical lines are time marks, produced hourly as explained in Section 2 above. The zero line has been marked in ink on the record, joining the spots produced when the rider was changed. The length of the sloping lines is a measure of the difference between static and kinetic friction. The perpendicular distance from the zero line to the significant features of the record can be measured, and the results converted into milligrams tangential force as explained in Section 3.
There is an uncertainty in the value of \( p \), produced by the unavoidable friction in the horizontal (beam) bearings and by the change in the immersion of the damping vane as the pen is lowered on to the paper. From an examination of the records it may be deduced that the error in the frictional force resulting from these causes is of the order of \( 100/R \) mg. It is, however, substantially constant during any particular experiment and therefore does not seriously affect the measurements of the coefficient of friction.

Considered in detail, paper is far from uniform, a fact brought out by such runs as No. 33 (Fig. 4) in which the apparatus was left to record overnight with \( p = 120 \) mg. Small irregularities and also more extensive ones will be observed. The arrow at about 0130h denotes the instant when the pen began to mark over the trace produced some 15 hours before, the plate having made one complete turn. The increase in friction was always noticeable under these circumstances.

It is this irregularity of the paper which places the most serious limitation on the numerical significance of the measurements. The following discussion will therefore stress their general qualitative and comparative aspects.

5. SOME RESULTS

There is practically no limit to the number of combinations of elements that could be investigated with this apparatus. Some of the variables are (1) paper, (2) ink, (3) pens, (4) relative humidity, (5) pressure of pen on paper, (6) chart speed, (7) effect of printing on chart paper, and (8) vibration. Each experiment takes several hours and up to this time not all the above variables have been investigated fully. In particular, the relative humidity has been held constant at \( 60 \pm 5\% \).

One of the most surprising facts is the
narrow range of values of the coefficient of kinetic friction, as determined from the slope of a straight line drawn through the graph of $f_k$ against $p$, where $f_k$ is the mean kinetic frictional force. Practically all the values lie between 0.40 and 0.60, whether the ink is thick or thin or absent, for two widely different types of pen, four kinds of paper, and a 5 to 1 range of chart speeds. The average coefficient of static friction varies rather more, especially between different papers, say from 0.6 to 1.2. The maximum static friction, corresponding to the catching of the pen in small roughnesses of the paper, varies widely, and its expression as a coefficient is probably of no significance. It varies more widely with the type of chart paper, but for the chart papers studied, a maximum tangential force equal to twice the pen pressure was frequently observed.

The effect of the ink is interesting. The slope of the graph of $f_k$ against $p$ is about the same with or without ink in the pen, but there is an added, nearly constant force due to the ink. Because of the uncertainty in $p$ mentioned above, quantitative estimation of this effect is difficult, but the extra force is of the same order of magnitude as that required to overcome the surface tension across the width of the line.

Moderate vibration has little effect on the kinetic friction, though naturally a great deal on the static.

To investigate the effect of the printing on a chart, alternate 45° sectors of some charts were printed solid by offset lithography. There was an unimportant diminution of the kinetic friction on the printed sectors, but the main effect was to reduce the spread between the average static and kinetic friction. **Figure 5** will illustrate this.

6. **Application**

The application of these results may be illustrated by reference to the barograph. The Canadian barograph has a pen arm hung on a hinge which is tilted inwards towards the drum at an angle of $71\frac{1}{2}$° so that the pen rests against the drum by gravity. The normal force is approximately 75 mg. The tangential force due to kinetic friction is about half this under most conditions, say 40 mg. We might suppose that the maximum static friction to be expected would be more important and we have suggested above that this might be taken as twice the normal force, say 150 mg. Direct experiment shows that one gram at the pen deflects it 3.0 mb; one might therefore expect frictional errors of as much as 0.45 mb. It is probable, however, that the constant movement of the drum, combined with any vibration that may be present, renders the conditions more nearly those corresponding to the kinetic state, in which the maximum error would be about 0.1 mb. This interpretation of the experiment is supported by the complete absence of steps in the record having any such amplitude as 0.45 mb.

7. **Conclusion**

There is practically no end to the possible combinations on which experiments might be performed. It would be of particular interest to extend the present experiments to determine the effect of high relative humidity.

**Reference**