A Comparison of Length Scales and Decay Times of Turbulence in Stably Stratified Flows

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(Manuscript 20 September 1985, in final form 14 April 1986)

ABSTRACT

An examination of average values of a buoyancy length scale, \( L_R = (\epsilon / N^2)^{1/2} \), and a Thorpe scale, \( L_T \), computed from vertical profiles of oceanic turbulence at 150°W in the tropical Pacific Ocean, shows reasonable agreement with the relation \( L_R = 0.8L_T \) found by Dillon. The present study uses direct measurements of velocity microstructure to compute \( L_R \). It is also shown that if \( L_R = 0.8L_T \), and the Brunt–Väisälä period \( 2\pi/N \) is constant, the decay–time constant of kinetic energy of turbulence in a stably stratified fluid is one-third to one-half of the Brunt–Väisälä period. Further investigation, using an exact formula for the energy, reveals that the assumption of constant \( N \) leads to an overestimate of the energy by a factor of 3. Correction for this factor reduces the decay time to between one-sixth and one-tenth of the Brunt-Väisälä period.

These results are compared with previous observations. Only one previous study investigates the covariation of \( N \) and \( \epsilon \) within a patch of turbulence, finding a decay time of about one-tenth of \( 2\pi/N \). Other studies, such as vertical profiles in decaying grid turbulence in a water tunnel and horizontal profiles in the ocean and the atmospheric stratosphere, assume \( N \) to be constant. Decay times in these studies are between 0.2 and 0.6 of \( 2\pi/N \) and are most likely high by a factor of 3 owing to the assumption of constant \( N \). Therefore, these experiments show the decay time to be a small fraction of the Brunt–Väisälä period, with no evidence of a dependence of this time upon features of the turbulence or the large-scale flow.

1. Introduction

A recent paper by Dillon (1982) examines the ratio \( L_R/L_T \) determined from turbulence data at three locations: in the wind mixing layer of the ocean, in the seasonal thermocline of the ocean, and near the surface of a lake. Values of \( L_R \) are computed from the ratio \( (\epsilon / N^2)^{1/2} \), where \( \epsilon \) is the rate of dissipation of mechanical energy by viscous forces, and \( N \) is the Brunt–Väisälä frequency over a patch of turbulence for which \( \epsilon \) is computed. This ratio was proposed by Dougherty (1961) and Ozhimov (1965) to be proportional to the height of the largest eddy to be unaffected by buoyancy in stratified, turbulent shear flow. Thorpe (1977) computed a vertical scale of eddies from high-resolution profiles of temperature in a lake and suggested that this scale \( (L_T, or the Thorpe scale) should be proportional to \( L_R \).

To test this hypothesis, Dillon (1982) computed \( L_R \) from values of \( \epsilon \) and \( N \) derived from temperature microstructure measurements (Dillon and Caldwell, 1980). Average values of \( \epsilon \) and \( N \) over turbulent patches were computed and combined to give the patch-averaged values of \( L_R \). This value was compared to \( L_T \), computed as the rms vertical displacement of fluid elements away from their stable position. A "bubble sort" routine was used to compute these vertical displacements. (This computer algorithm rearranges the elements in the digital record of temperatures to produce a profile of monotonically decreasing temperatures.) His results indicated that \( L_R = 0.8L_T \), and individual data points follow this relation (to a factor of 2) over a wide range of \( L_T \) of more than two decades.

There have been several papers written recently describing the implications of these measurements upon the relative merits of fossil turbulence (Gibson, 1980) and a "steady-state turbulence" in the ocean (Dillon, 1984; Caldwell, 1983). The intent of this paper is not to repeat these arguments, which require rather lengthy introductions, but to introduce three new pieces of evidence: first, that the ratio \( L_R/L_T \), observed in the tropical Pacific using shear measurements to compute \( \epsilon \), agrees with the observations of Dillon; second, that the ratio \( L_R/L_T \) itself implies a relatively small decay time for the kinetic energy of turbulence in the absence of a continuous source of energy; and third, that observations of \( L_R/L_T \) in the atmospheric stratosphere, in a water tunnel, and in a lee wave in a fjord predict similar decay times for turbulence to within a factor of 2.5. The decay time of turbulence is investigated in section 4. It is found that although the ratio \( L_R/L_T \) can be used to compute decay times, its use requires that \( N \) be constant through a patch of turbulence. It is more accurate to compute the energy in turbulence with the local value of \( N \).

2. Calculation of \( L_R \) and \( L_T \)

The values of \( \epsilon \) in this study are the 2.5 m vertical averages of the turbulent dissipation rate described by
Crawford and Osborn (1981) and Crawford (1982) from the cruise of the Canadian Survey Ship Parizeau to 150°W in the Pacific at the equator in 1979. Measurements of $\frac{du_i}{dx_j}$ ($i = 1, 2$) at microstructure scales using the profiler CAMEL during this cruise permit the direct computation of two components of the turbulent dissipation rate, and the remaining terms in the expression
\[ \varepsilon = \frac{\nu}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 \] (1)
are inferred from isotropic relations. This method offers a more direct route for computing $\varepsilon$ than chosen by Dillon (1982), who calculated $\varepsilon$ over 50 cm layers from Batchelor spectra of microstructure temperature data.

Turbulent patches were selected only if a minimum of three neighboring 2.5 m thick layers had a dissipation rate above the noise level for $\varepsilon$. Therefore, the minimum patch size is 7.5 m. For regions of extensive turbulence a patch size of 20 m was chosen because this is the interval selected by Crawford and Osborn (1981). Patches were excluded if salinity decreased with depth to avoid the possibility that the observed overturns at 10 m scales, were generated by doubly diffusive processes rather than shear flows. All profiles of the Thorpe displacement were plotted to identify and eliminate regions where intrusive flows rather than turbulence generated the overturns. These intrusive flows appear as smooth overturns, dominated by large displacements, with an absence of smaller overturns.

The turbulence profiler has no conductivity sensor to provide measurements of salinity and density, but from CTD casts taken within one hour of the turbulence profiles, the $\sigma_T - T$ relation was tabulated for each 2.5 m layer and used to convert the temperature record to a profile of $N_c$, the local Brunt-Väisälä frequency.

The value of $L_T$ is computed from the temperature record. The original record was digitized at 2.5 mm intervals, low-pass filtered to eliminate high frequency oscillations, and subsampled to 2 cm spacing. The sorting program moves the temperature data points the minimum distance to produce a monotonically decreasing temperature profile. Where turbulence produces overturns and inversions in the temperature record, the sorting routine will return the data points to a stable temperature profile and record the distance moved. The rms distance over each 2.5 m layer was computed, giving $L_T$. A comparison of 1.7, 2.5 and 10 cm low-pass filters reveals little difference for $L_T \leq 10$ cm. The 2.5 cm filter is used here.

3. Results

Comparisons are noted for three sets of data: 1) turbulence on the equator on a day of intense mixing, 2) all other measurements on the equator, and 3) patches found off the equator. Profiles 11–17 comprise set 1 and record a day of strong westward wind, a strong eastward undercurrent, weak stratification in the upper 110 m of water, and a strong shear in the average current (Crawford, 1982). Therefore, high dissipation rates in these profiles are to be expected and, because they are unique, these profiles are presented separately. Figure 1 shows the scattergram of $L_R$ and $L_T$ of these profiles (solid circles). All but one value of $L_T$ are over 60 cm in length, and the range in values is too small to show a strong linear relationship. However, results for other equatorial values (open circles) show a wider variation in length. Here $L_T$ varies from 8 to 250 cm. Turbulence profiles further removed from the equator are also summarized in Fig. 1 (solid triangles). Here the range in $L_T$ is greatest, from 8 to 660 cm, and the variation of $L_R$ with $L_T$ is most noticeable. Combining all three datasets gives $L_R/L_T = 0.66 \pm 0.27$ (±1 standard deviation). The line $L_R = 0.66L_T$ is plotted in Fig. 1.

4. Energetics of turbulence in a stably stratified fluid

The potential energy of a water column at a depth $z$ relative to a reference column is given by Holliday and McIntyre (1981) as
\[ APEF = \frac{E(z)}{\rho} = \frac{g}{\rho} \int_0^z \frac{\rho(z - \xi)}{\partial z} \partial z \] (2)
where $\partial \rho(z - \xi)/\partial z$ is the depth gradient of density at the point $(z - \xi)$ in the reference profile, $\xi$ a dummy variable representing displacement, and $\xi$ the vertical displacement of a fluid element away from its depth in a reference profile. If the sorting program is used to find this displacement, then $\langle \xi^2 \rangle = L_T^2$, since $\xi$ is the minimum vertical distance a fluid element must move to reach a depth at which it is statically stable. The angle brackets denote an average quantity.

Fig. 1. Comparison of $L_R$ and $L_T$ for turbulence observed during cruise of CSS Parizeau to the equator at 150°W in Jan–Feb 1979. ○ represents profiles 11–17, on equator; ○ represents other equatorial profiles; ▲ profiles off equator. Dashed line represents $L_R = 0.66L_T$. 
The choice of a reference profile is not trivial. Dillon (1982) notes that if the monotonic profile given by the sorting algorithm is selected for reference, the value of \( E_p \) computed will be the available potential energy of the turbulent fluctuations, which he defines as \( \text{APEF} = E_p/\rho \).

If one can assume that the ordered (and therefore monotonic in temperature and density) profile has \( \partial \rho/\partial z \) and \( \alpha \) constant over the interval \( \xi \), \( \alpha \) is defined by \((1/\rho) \partial \rho/\partial z = -\alpha (\partial T/\partial z)\), and \( \rho(z) \) and \( T(z) \) are the monotonic profiles of density and temperature, then the Brunt-Väisälä frequency can be written \( N^2 = -(g/\rho) \partial \rho/\partial z = g \alpha (\partial T/\partial z) \), and one can define \( T' = \xi (\partial T/\partial z) = -\xi/\rho \partial \rho/\partial z \). Here \( T' \) is the difference in temperature at a given depth between the original and the ordered, monotonic profile. Substituting \( \partial \rho/\partial z = -\rho N^2/g \) into (2) gives

\[
\text{APEF} = \frac{E_p}{\rho} = N^2 \int_0^T \frac{\xi d \xi}{\sqrt{\xi}} = N^2 \xi^2/2 \tag{3}
\]

while the substitutions \( \xi = \tilde{T}'(\partial T/\partial z) = -\tilde{T}'g \alpha N^{-2} \)

give

\[
\text{APEF} = \frac{E_p}{\rho} = g^2 \alpha^2 N^{-2} \int_0^T \tilde{T}' d \tilde{T}' = g^2 \alpha^2 T'^2/2N^2 \tag{4}
\]

where \( T' \) is a dummy variable of integration (with units of temperature, such that \( \partial T'/\partial z = \partial T/\partial z \)) ranging between zero and \( T \). In this paper vertical averages are computed over 2.5 m layers and these are denoted as follows:

\[\overline{\text{APEF}} = \overline{N^2 \xi^2}/2 \tag{5a}\]

\[= g^2 \alpha^2 \overline{T'^2}/2N^2. \tag{5b}\]

Although APEF given by the average of (2) is exact while (5a) and (5b) require \( N^2 \) to be constant, there are advantages to the simple equations given by the linear density gradient. Since \( \langle \xi^2 \rangle = L_T^2 \), APEF is readily computed from (5a) once the Thorpe scale is known. Equation (5b) allows calculation of APEF from measurements of \( T'^2 \) along a horizontal path. In this case \( T' \) is the deviation of \( T \) from the mean. One must assume that overturns are isotropic, and the value of \( N \) may have to be measured independently. Such observations are discussed in section 5.

For turbulence, the simplest balance of terms in the Reynolds equation for the rate of change of turbulent kinetic energy is

\[\frac{1}{2} \frac{\partial u_i^2}{\partial t} + u_i u_j \frac{\partial U_i}{\partial z} = -\xi - g \gamma \overline{\rho w}/\rho \tag{6}\]

if the divergence terms are neglected. In term P, \( i \) is summed over 1 and 2. Other authors have discussed the assumptions made here (Osborn, 1980; Dillon, 1984; Caldwell, 1983) and the reader is referred to these papers for a complete discussion. In steady state turbulence, the ratio of the buoyancy term B to the production term P is the flux Richardson number, and is commonly assigned a value of about 0.2 (Oakley, 1982; Osborn, 1980). Normally the temporal gradient term \( T \) is considered to be small, and a ratio of \( B/P = 0.2 \) leads to the ratio \( B/D = 0.25 \).

It is interesting to determine the lifetime of turbulence in the absence of any production. We can accomplish this by taking the ratio of the turbulent energy to the rate of dissipation. Since \( \epsilon \) is a measure of the rate of decay of kinetic energy of turbulence, it is appropriate to compare \( \epsilon \) to the total kinetic energy in the turbulence. The value of APEF represents potential energy, but it is reasonable to assume that each component of velocity contributes equally to the kinetic energy and that the APEF is equal to the kinetic energy in any one component, in which case the total kinetic energy is \( 3 \text{APEF} \) (Dillon, 1982). Experimental evidence for this ratio is available only for measurements in water-tunnel turbulence by Stüllinger et al. (1983), who observe a ratio of kinetic to potential energy of 3:1 in the region of the flow tunnel where the turbulence length scales are limited by buoyancy and not contaminated by internal waves.

The lifetime, or decay time, of kinetic energy of turbulence is approximately

\[\tau_d = 3 \text{APEF}/\epsilon \approx 3N^2 \xi^2/2\epsilon \tag{7}\]

where a constant \( N \) is assumed within a patch. This equation is admittedly simple, because the decay of kinetic energy cannot be separated from that of potential energy. However, the total energy is 4 APEF and the complete decay rate is about 1.25\( \epsilon \) \([B + D \in Eq. (6)]\), with the ratio of these not significantly different than for kinetic energy and \( \epsilon \) alone. Therefore, the simple expression (7) is chosen to represent the decay time of turbulence.

A natural time for internal waves in the ocean is \( 2\pi/N \). This period is also a natural one for turbulence whose largest overturns are limited by the stable stratification as represented by \( N \). We can express the decay time in units of \( 2\pi/N \) as

\[\tau_c = \frac{\tau_d N}{2\pi} = \frac{1.5N^3 \xi^2}{2\pi \epsilon} = \frac{1.5(L_T/L_R)^2}{2\pi} = 0.37, \tag{8}\]

if the substitution \( L_R = 0.8L_T \) is made, and \( \tau_c = 0.49 \) for \( L_R = 0.7L_T \). (The subscript on \( \tau_e \) denotes the use of a constant, layer-averaged Brunt-Väisälä frequency.)

Therefore, the decay-time constant of kinetic energy in the absence of any source of turbulence is about one-third to one-half of a Brunt-Väisälä period, a result which follows directly from the relation between \( L_R \) and \( L_T \).

At first glance this result appears to contradict Dillon (1984), who observes a much shorter decay time. In his notation, a \( \tau_e \) of 0.37 would give \( m = 2.2 \), much larger than his observed \( m \). There are two reasons for his shorter decay time. First, his temperature micro-
structure measurements permit a calculation of the term $N^2 D C_x$, where $D$ is the molecular diffusivity of heat and $C_x$ is the Cox number. This term is normally assumed to be equal to term B in (6). His decay term is $\epsilon + N^2 D C_x$, rather than $\epsilon$ used here, and should be larger than the decay $\epsilon$ by a fraction $N^2 D C_x/\epsilon \approx 0.25$ if we use the observations of Oakey (1982). However, Dillon uses $C_x = 3(\partial T/\partial z)^2/(\partial T/\partial z)^2$ while Oakey (1982) uses $C_x = 2(\partial T/\partial z)^2/(\partial T/\partial z)^2$. Therefore, if the observations of Oakey and Dillon are consistent, the term $\epsilon + N^2 D C_x$ used by Dillon should be 1.4$\epsilon$. Instead, he observes $\epsilon + N^2 D C_x = 1.93\epsilon$ and 2.58$\epsilon$ for his series A and B, respectively, much larger than Oakey's values and larger than normally observed (Osborn, 1980). Since his rate of decay is now 1.93 and 2.58 times $\epsilon$, rather than the nominal value of 1 assumed in my calculations, his decay time will be smaller.

The second source of discrepancy is the approximation $\textup{APEF} = N^2 \langle \psi^2 \rangle/2$ [Eq. (5a)] for the true APEF given by (2), that is, the assumption that $N$ is constant through a patch of turbulence. To test this assumption, two energylike functions were computed for all the data points in Fig. 1. They are

1. $\int_0^1 \frac{\partial T}{\partial z}(z - \tilde{z})d\tilde{z}$
2. $\frac{\xi^2 \Delta T}{2\Delta z}$

where the first represents true potential energy, while the second is proportional to (5a). As represented above, both functions give values at a point; when averaged over the profile the ratio of 2:1 will represent the relative accuracy of (5a) in computing APEF. The results, plotted in Fig. 2, show that (5a) always overestimates APEF. The average ratio between the two is 2.8:1 with a standard deviation of 2.0. One possible explanation for this bias of (5a) is that if $\epsilon$ tends to be large in a turbulent patch in a region where $N$ is locally small, then (5a) will overestimate APEF, since the local decrease in $N$ cannot be included in the calculation. Since turbulence mixes the water column and decreases $N$, this tendency is to be expected and is, in fact, observed in the individual 2.5 m patches. This overestimate reduces the decay times computed above to $\tau = 0.13$ for $L_T = 0.8L_T$ and $\tau = 0.17$ for $L_T = 0.7L_T$. Once these two factors are taken into account a nondimensional decay time of $\tau_c = 0.37$ is equivalent to a value of $m = 0.5$, which is a reasonable fit to the data plotted by Dillon (1984, Fig. 1) for his oceanographic data, but not for his lake measurements, which are too near the surface for this comparison to be valid.

5. Comparison with other flows

In an attempt to determine a decay time for turbulence, I have progressed through two distinct steps. In the first a nondimensional decay time ($\tau_c$) of kinetic energy of turbulence is computed, subject to the assumption that $N$ is a constant throughout each individual patch; in the second step corrections for these decay times are computed and found to be appreciable. It would be more straightforward to compute the correct decay time ($\tau$) immediately, but I have avoided this in order to compare the results here with additional observations described in the literature which do not take into account the simultaneous variation of $\epsilon$, $\xi$ and $N$ within a turbulent patch. A summary of measurements in several flows is presented in Table 1. Values of decay times in this table should be observed for turbulence in any stably stratified fluid where

(i) the vertical scale $L_T$ is much less than the distance to the boundary;
(ii) the heights of the largest overturns are limited by the stable stratification; and
(iii) the sources of energy are internal waves or slower oscillations of period $2\pi/N$ or longer, or there is no source and the turbulence is decaying.

a. Turbulence near the sill in Knight Inlet

A remarkable set of observations has been reported by Gargett et al. (1984; hereafter referred to as GON84). They traversed a horizontal path through a very active turbulent regime in a submersible, equipped with a
Table 1. Decay times for observations of turbulence in stratified fluids. The decay time $\tau_c$ is computed from the original relationships reported, all of which assume $N$ to be constant through the turbulence. Results in this paper show that the variation of $N$ through turbulent patches reduces the computed APEF by a factor of 2.8, and the resulting true decay time $\tau$ is also lower. Listed values of $\tau$ are $\tau_c/2.8$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Medium</th>
<th>Profile direction</th>
<th>Original relationship</th>
<th>Method of measurement</th>
<th>Nondimensional decay times $\tau_c$ $(\tau)$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dillon (1982)</td>
<td>Seasonal thermocline in lake and ocean</td>
<td>Vertical</td>
<td>$L = 0.8L_T$</td>
<td>$\varepsilon$ from Batchelor cutoff of temperature spectra. $L_T$ from vertical sorting.</td>
<td>0.37 (0.13)</td>
<td>—</td>
</tr>
<tr>
<td>Crawford (this paper)</td>
<td>Permanent thermocline in tropical Pacific Ocean</td>
<td>Vertical</td>
<td>$L_R = (0.66 \pm 0.27)L_T$</td>
<td>$\varepsilon$ from direct observations of velocity gradients. $L_T$ from vertical sorting.</td>
<td>0.49 (0.18)</td>
<td>Much of the scatter in data points is due to uncertainties in the $T - \sigma_T$ relation.</td>
</tr>
<tr>
<td>Gargett et al. (1984)</td>
<td>Lee wave behind a sill in Knight Inlet</td>
<td>Horizontal</td>
<td>$\bar{u}_T^2N/4\pi\varepsilon = 0.25 \pm 0.1$</td>
<td>$\varepsilon$ from direct measurements of velocity gradients. $u_T$ from direct observations of large-scale velocity.</td>
<td>0.24 (0.09)</td>
<td>—</td>
</tr>
<tr>
<td>Weinstock (1982)</td>
<td>Stratosphere</td>
<td>Horizontal</td>
<td>$\varepsilon = 0.4\bar{u}_T^2N$</td>
<td>$\varepsilon$ and $u_T$ from velocity measurements.</td>
<td>0.60 (0.21)</td>
<td>Survey of recent measurements.</td>
</tr>
<tr>
<td>Stillinger et al. (1983)</td>
<td>Grid turbulence in a water tunnel (salt stratified)</td>
<td>Horizontal</td>
<td>$L_R = 0.7L_T$ when buoyancy effects are first observed</td>
<td>$\varepsilon$ from direct observations of velocity gradients. $N$ from conductivity measurements.</td>
<td>0.49 (0.18)</td>
<td>$L_R = 0.7L_T$ was inferred from horizontal profile and was only valid when buoyancy effects were first observed.</td>
</tr>
<tr>
<td>Itsweire (1984)</td>
<td>Grid turbulence in a water tunnel (salt stratified)</td>
<td>Horizontal/vertical</td>
<td>$L_R = 0.65 \pm 0.1L_T$ for vertical profiles</td>
<td>Same as Stillinger et al.</td>
<td>0.57 (0.20)</td>
<td>$L_R = 0.65 \pm 0.1L_T$ was found downstream of point where buoyancy effects were first observed.</td>
</tr>
</tbody>
</table>
wide range of turbulence and large-scale flow sensors. For most turbulent flows encountered, except the most active, they were able to resolve the complete velocity spectra of downstream and two cross-stream components, from the energy-containing eddies through to the dissipation scales. They could then determine simultaneously the kinetic energy of turbulence and the rate of dissipation for classes of turbulence graded by the ratio $I = k_{0S}/k_{0b}$, where $k_{0S}$ is the Kolmogoroff wavenumber $[k_{0S} = (\varepsilon / \nu)^{1/4}]$ and $k_{0b} = L_{R}^{-1}$. In their results an average $\bar{N}$ is used; therefore, comparison with $\tau_{c}$ is appropriate.

Their results are summarized in Table 2, which is adapted from Table 3 in GON84. For classes 2 to 4 their average $\tau_{c}$ is 0.24. The estimated $\tau_{c}$ for class 1 turbulence of GON84 is 0.6, which is greater than given by the vertical profiles, but because they could not resolve the entire spectrum of velocity fluctuations for this class of turbulence, the uncertainty in this value is large and the decay time is not significantly different from the averages of classes 2 to 4.

b. Atmospheric stratosphere

In a review paper describing measurements in the stratosphere, Weinstock (1981) gives the relation $\varepsilon = C_{0} \langle u_{3}^{2} \rangle \bar{N}$, where $C_{0}$ is a constant observed to be equal to 0.4. This equation fits well into a wide range of observations in this layer, and gives a nondimensional decay time $\tau_{c} = 3 \langle u_{3}^{2} \rangle \bar{N} / 4\pi \varepsilon = 3(4\pi C_{0})$. Gargett and Holloway (1984) note that the method of computing $\langle u_{3}^{2} \rangle$ used in studies of stratospheric turbulence may overestimate the variance due to turbulence, since the summation of variance is over all spectral values, and contributions from all wavenumbers are included, which may add internal wave energy at low wavenumbers. This factor may explain the difference between results of Weinstock (1981) and GON84 for values of $\tau_{c}$.

Weinstock uses a standard $\bar{N} = 2.1 \times 10^{-1}$ s$^{-1}$ for the stratosphere. If $N$ is locally small where $\langle u_{3}^{2} \rangle$ is large, as is expected for the measurements in the ocean, then the value of $C_{0}$ that fits his observations may be too small. In a theoretical work (Weinstock, 1978), he shows that if vertical oscillations are damped for wavenumbers less than $k = \bar{N} / \langle u_{3}^{2} \rangle^{1/2}$, then the expected value of $C_{0}$ is 0.5, giving $\tau_{c} = 0.48$. If one uses $C_{0} = 0.5$ in the relation $\varepsilon = C_{0} \langle u_{3}^{2} \rangle \bar{N}$, the small wavenumber limit of Weinstock is $k = (0.5 \bar{N}^{3})^{1/2} = 0.7 k_{0b}$. This value is not significantly different from the low wavenumber limit of 0.5$k_{0}$ assumed by GON84. Precise determination of such a limit is difficult in existing datasets of geophysical flows, and the use of different values contributes to uncertainties in $\tau_{c}$ if the complete spectrum is not resolved.

c. Grid turbulence in a water tunnel

A set of experiments reported by Stillinger et al. (1983) reveals the decay of turbulence behind a grid in a salt-stratified water tunnel and the transition to internal waves. They maintain sensors at a fixed position in a horizontal flow and measure density fluctuations. Their length scale is $2\lambda = -\langle \rho' \theta' \rangle^{1/2} / (\partial \theta / \partial z)$. From the definitions of $\xi$ and $L_{T}$ in section 4 it can be seen that $L_{T}^{2} = \lambda^{2}$. Initially, behind the grid, the stable stratification has no influence upon the length scale $\lambda$, but further downstream the largest scales of turbulence are suppressed by the stable stratification. At this point they observe that $L_{R} = 0.7 \lambda$. This relation is equivalent to $\tau_{c} = 0.49$. Just downstream of this region they observe a kinetic to potential energy ratio of 3:1, as expected for the equipartition of energy and assumed in section 4.

In a follow-up study using the same water tunnel and grid, Itsweire (1984) repeated the observations of Stillinger et al. (1983), but also observed density variations in a series of vertical profiles. This permits the immediate comparison of $L_{T}$ from both vertical and horizontal profiles, the first experiment to do so.

To avoid confusion, the length scale determined from horizontal measurements will be denoted $\lambda = -\langle \rho' \theta' \rangle^{1/2} / (\partial \theta / \partial z)$, and $L_{T}$ will indicate the Thorpe scale given by the vertical profile. The first observation of Itsweire is that $\lambda$ and $L_{T}$ are equal only at the point where the effects of stable stratification are first observed, which is the point at which Stillinger et al. (1983) found $L_{R} = 0.7 \lambda$. Closer to the grid, $L_{T}$ is slightly greater than $\lambda$, and further downstream $L_{T}$ is much smaller. As noted earlier, buoyancy effects are small near the grid, and buoyancy scaling is not valid. Far from the grid, internal waves begin to influence the flow and they contribute significantly to the measure-

Table 2. Nondimensional decay times for measurements of Gargett et al. (1984, GON84). Values for Class 1 turbulence are estimated by using a 5/3 law to integrate individual components of variance out to a low wavenumber of $k_{0}/2$.

<table>
<thead>
<tr>
<th>Class</th>
<th>$I = k_{0}/k_{0b}$</th>
<th>$3u_{3}^{2}N / 4\pi \varepsilon$</th>
<th>$3u_{3}^{2}N / 4\pi \varepsilon$</th>
<th>$3u_{3}^{2}N / 4\pi \varepsilon$</th>
<th>$\tau_{c} = u_{3}^{2}N / 4\pi \varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3000</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>2</td>
<td>600-900</td>
<td>0.31 ± 0.08</td>
<td>0.31 ± 0.09</td>
<td>0.23 ± 0.08</td>
<td>0.28 ± 0.09</td>
</tr>
<tr>
<td>3</td>
<td>200-300</td>
<td>0.29 ± 0.06</td>
<td>0.18 ± 0.01</td>
<td>0.14 ± 0.03</td>
<td>0.20 ± 0.06</td>
</tr>
<tr>
<td>4</td>
<td>50-100</td>
<td>0.29 ± 0.05</td>
<td>0.26 ± 0.04</td>
<td>0.15 ± 0.02</td>
<td>0.23 ± 0.05</td>
</tr>
</tbody>
</table>
6. Discussion

The value of $L_T$ is more reliably computed from vertical profiles than from horizontal ones, since the former are not contaminated by internal waves. The reason is that turbulence mixes fluid elements but waves do not. In order to mix vertically, fluid elements must overturn, and these overturns contribute to $L_T$ in vertical profiles. Because mixing is a necessary condition for turbulence, the value of $L_T$ is a reliable parameter for gauging its strength. Horizontal profiles cannot distinguish between waves and turbulence since both can move fluid elements vertically. Both Itsweire (1984) and Caldwell (1983) make this point. The work of Itsweire (1984) illustrates this difference, showing that vertical profiles in water tunnels give a stable ratio of $L_R/L_T$, while the horizontal measurements do not.

Among the three sets of horizontal profiles, measurements by GON84 give the lowest values of $\tau_e = 0.24$, those summarized by Weinstock (1981) have $\tau_e = 0.60$, while the Stillinger et al. (1983) results show $\tau_e$ to increase with distance downstream from an initial value of 0.49. As noted earlier, the larger values for horizontal measurements may be due to the contribution of waves to the velocity variance. Gargett et al. classify their results with the intensity ratio $I = k_d/k_b = L_R/L_S$, where $L_S$ is the Kolmogoroff length scale. Stillinger et al. note that the relative contribution of internal waves to the velocity variance increases with decreasing $I$, and for values of $I$ near 10 the turbulence is completely suppressed. (They use $R_b = \epsilon/\nu N^2 = I^{4/3}$ and therefore find that turbulence is suppressed at $R_b = 25$.)

The strongest influence of waves is for the experiment of Stillinger et al. They report maximum values of $I = L_R/L_K$ in their notation of 25 to 40 for their turbulent water-tunnel flow. These intensities are close to the value $I = 10$ where turbulence ceases and the strong influence of waves in their experiment is to be expected. Gargett et al. report nearly uniform values of $\tau_e$ for classes 2 to 4, which lie in the range $50 < I < 900$ and are therefore active turbulent regions, less influenced by internal waves.

The Weinstock (1981) summary of turbulence in high altitude stable layers covers a wide range of values of $I$, from a low of 12 to 42 to a high of about 200. However, the reported values of $\epsilon$ for the low $I$ case are averages over all measurements and include regions where turbulence is too weak to observe. If only the turbulent regions are considered, the values of $\epsilon$ might increase by a factor of 10, raising $I$ to a level where internal wave effects should be small.

However, the waves referred to are those generated by the decaying turbulence itself. The waves not accounted for by the scaling factor $I$ are those at larger scales in a geophysical flow, if these waves are generated by some unusual feature. Lee waves behind mountains are one example. Velocity spectra in the stably stratified layers of the upper troposphere and stratosphere often display a $-3$ slope in the wavenumber range below the dissipation region (Vinnichenko et al., 1980). In such a case, these long wavelength fluctuations, which may be attributed to waves or even convection from below, add to the velocity variance $\langle u_s^2 \rangle$, and therefore increase the value of $\tau_e$.

7. Conclusions

An examination of the ratio of a buoyancy length scale $L_R = (\epsilon/N^3)^{1/2}$ to a Thorpe scale $L_T$ for oceanographic flows, using direct measurements of turbulent velocities, finds $L_T = (0.90 \pm 0.05) L_R$, in agreement with the results of Dillon (1982). The Thorpe scale is computed from the digital record of a temperature profile as measured by a fast response probe on a free-falling instrument. The elements of this digital record are rearranged by the minimal amount to produce a monotonic temperature profile, and the rms distance moved by the elements is the Thorpe scale.

It is also possible to compute the gravitational potential energy (APEF) of the original temperature profile relative to the rearranged, monotonic profile, if the local value of the Brunt-Väisälä period $(2 \pi / N)$ is known. The rate of turbulent dissipation $\epsilon$ can be computed from the output of turbulence probes on the profiler. The ratio of $\langle 3 \text{APEF} \cdot N \rangle$ to $(2 \pi \epsilon)$ is a non-dimensional decay time of kinetic energy of turbulence, $\tau$, where 3APEF is the kinetic energy of turbulence. This ratio is between 0.1:1 and 0.2:1 for the oceanographic data if the product APEF $\cdot N$ is computed point by point through a profile. Therefore, the decay-time constant of oceanographic turbulence is much less than a Brunt-Väisälä period.

If it is assumed that the Brunt-Väisälä period is constant through a patch of turbulence, then the decay time is $3\langle \text{APEF} \cdot N \rangle/2 \pi \epsilon = (3/4 \pi) (L_T / L_R)^2 = \tau_e$, which is 0.49 if $L_R = 0.7 L_T$ and 0.37 if $L_R = 0.8 L_T$; thus $\tau_e > \tau$. An investigation of these decay times finds $\tau_e = (2.8 \pm 0.2) \tau$ owing to a tendency for turbulent overturns in a water column to be large when the local value of $N$ is small. Therefore, the assumption of a constant Brunt-Väisälä period is not valid and $\tau_e$ is too high.

These results are compared with observations reported in the literature. The value $\tau$ of 0.1 to 0.2 is consistent with observations of Dillon (1984), although
he does not compute a numerical value for $\tau$ from his plotted data. Other investigators assume a constant Brunt-Väisälä period in their various comparisons of the energy in turbulence and the dissipation rates. Although none actually compute a nondimensional decay time, $\tau_c$ can be computed from their observations. Values of $\tau_c$ range from 0.25 to 0.6 for various types of turbulence. These results agree with the values determined in this paper.

One rather surprising result of this comparison is that the decay time $\tau_c$ is similar for both oceanic and grid turbulence. The oceanic observations at the equator are of intense, relatively steady turbulence above the core of the equatorial undercurrent. Since a series of observations (profiles 11–17) over a period of seven hours show turbulence levels to be slowly increasing at the equator, there is no way this can be considered decaying turbulence. Yet it has a similar ratio $L_R/L_T$ to that observed in decaying grid turbulence by Itsweire (1984). The period of the shears that provide the source for equatorial turbulence must be longer than the Brunt-Väisälä period $2\pi/N$, since this is the shortest period of internal waves. When the decay time is nondimensionalized by $2\pi/N$, the resulting nondimensional decay time is between 0.1 and 0.2. There is then a distinct separation between the period of shears providing the energy for turbulence and the decay time of turbulence. The decay is so rapid that it can quickly adjust to changes in the energy of turbulence, and therefore it is reasonable for the ratio of kinetic energy to dissipation rate to approach a constant fraction of the Brunt-Väisälä period.

Experiments described here show $\tau_c$ to vary between 0.25 and 0.6. Additional measurements combining dissipation, potential energy and all components of kinetic energy are necessary to determine if there is any variation in $\tau_c$ and $\tau$ with some of the large-scale features of the turbulence.

Acknowledgments. Tom Dillon provided the sorting algorithm and the inspiration to begin this work, and Greg Holloway drew my attention to the paper by Holliday and McIntyre. Ann Gargett contributed an interesting discussion on the comparison of horizontal and vertical measurements and valuable comments on an early manuscript.

REFERENCES


