Simple Adjoint Retrievals of Microburst Winds from Single-Doppler Radar Data

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ABSTRACT

Although the simple adjoint (SA) method for retrieving the low-altitude winds from single-Doppler scans was previously examined with the Phoenix II data collected for rather uniform wind fields on nonstorm days with aluminum chaff dispensed from an aircraft (to enhance the reflectivity), the method has not been tested with storm data for complex flow fields. To examine this problem, the SA method is further developed and tested with the Denver airport microburst data through a series of numerical experiments. In addition to the earlier upgrading of the SA method, three new objectives are fulfilled to improve the retrievals. In particular, it is found that by imposing a weak vorticity constraint, by using the previous time-level retrieval as the initial guess of the retrieval at the current time level, and finally by incorporating the surface anemometer data into the method, the averaged rms difference between the retrieved (from FL-2 radar) and dual-Doppler observed winds reduces from 3.87 to 2.89 m s$^{-1}$, then to 2.66 m s$^{-1}$, and finally to 2.55 m s$^{-1}$, while the averaged correlation coefficient increases from 88% to 94%, then to 95%, and finally to 98%. The surface anemometer data can make a significant improvement, especially when the retrieval purely based on radar data is relatively poor. Factors responsible for the retrieval error are also examined and discussed with physical interpretations.

1. Introduction

It is well known that Doppler radar can scan large volumes of the atmosphere at high spatial and temporal resolutions, but the direct measurements are limited to reflectivity and the velocity component in the direction of the radar beam. The velocities perpendicular to the axis of the radar beam are often critical in some hazardous weather situations (especially low-level wind shear associated with convective downbursts). The complete wind and thermodynamic fields are necessary to allow initialization of a numerical model that could predict hazardous weather conditions in advance. Because of this and the fact that the NEXRAD (Next Generation Weather Radar) network will provide only single-Doppler scanning over most areas in the United States, single-Doppler velocity retrievals and assimilation techniques have been subjects of intensive research. In this regard, many methods have been developed (Tuttle and Foote 1990; Sun et al. 1991; Kapitza 1991; Liou et al. 1991; Qiu and Xu 1992) and substantial progress has been made not only in proof of concept by using simulated data but also in demonstration of feasibility with real Doppler data (Xu et al. 1993, 1994a,b; Gal-Chen and Zhang 1993; Shapiro 1993; Sun and Crook 1994; Crook and Tuttle 1994; Laroche and Wawrzynski 1994). A review of these previously developed methods can be found in Qiu and Xu (1992), and this paper reports the recent refinements of the simple adjoint (SA) method (Qiu and Xu 1992; Xu et al. 1993, 1994a,b) tested with the Denver airport microburst data.

The potential merits of the SA method were explored and tested first with simulated data by Qiu and Xu (1992, henceforth referred to as QX92), and the most important features for the method can be summarized as follows: (i) It uses the reflectivity conservation equation or radial-component momentum equation to retrieve the time-mean (or running-mean) wind field averaged over several time levels of radar scans, and using data over multiple time levels can reduce or eliminate the problem of "underdetermined condition." (ii) It uses the variational formulation or, say, the adjoint technique in particular (Lewis and Derber 1985; LeDimet and Talagrand 1986; Talagrand and Courtier 1987). In this sense, the method is similar to the full adjoint method (Sun et al. 1991; Kapitza 1991), except that the control equations contain only those prognostic equations in which the predicted variables are directly observable (such as reflectivity and radial wind). In this way, the boundary conditions can be well determined from observations. [The full adjoint method uses a complete set of equations and the boundary conditions involve nonobserved variables (cross-beam winds, temperature, etc.), so large errors could be introduced by poorly estimated boundary values for those nonobserved variables.]

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The SA method was also recently upgraded and tested with the Phoenix II data (Xu et al. 1994a,b, henceforth referred to as XQY94a,b). The major results can be briefly reviewed as follows: (i) Using multiple time-level data with the adjoint formulation makes the retrieval more accurate and less sensitive to the observational error. (ii) Imposing a weak divergence constraint (i.e., a weak form of nondivergence constraint) can suppress the spurious divergence caused by the data noise and improve the retrieval. (iii) Retrieving the eddy coefficients improves the wind retrieval. (iv) Retrieving the time-mean residual term improves the wind retrieval.

Although the results in XQY94a,b were encouraging, the SA method was tested only with the Phoenix II data for rather uniform wind fields, and the Phoenix II data were collected on nonstorm days with aluminum chaff dispensed from an aircraft (to enhance the reflectivity). The real challenge is to retrieve complex flow fields from storm data, and this problem was not previously examined. To examine this problem, the Denver airport microburst data are selected in this paper to test the SA method. In this case, as we will see in Tables 1 and 2, directly applying the previously developed SA method to the microburst data cannot produce the same satisfactory results as those in XQY94a,b. To improve the retrievals, we have tried and tested many different modifications and refinements of the SA method with 50 datasets [25 from FL-2 radar and 25 from UND (University of North Dakota) radar], and the following three refinements are found to be significant: (i) refinement of the SA method in XQY94b by imposing a weak vorticity constraint or, say, a weak form of zero vorticity constraint (Sasaki 1970); (ii) refinement by continuous retrievals (i.e., using the previous time-level retrieval as the initial guess of the retrieval at the current time level); and (iii) refinement by using the surface wind data. With these three refinements, the SA method produces quite satisfactory retrievals, for the first time, from real storm data for complex wind fields. The results are reported in this paper. In particular, the weather condition and data preparation are described in the next section. The SA method is reviewed in section 3 with a weak vorticity constraint added to the formulation. Experiment designs and results are presented in section 4, showing that the retrievals can be significantly improved by the above three refinements. Error analyses are performed in section 5 to identify the major factors that affect the accuracy of the retrievals. Conclusions follow in section 6.

2. 11 July 1988 microburst case

Dual-Doppler data were collected by MIT Lincoln Laboratory during the operational test and evaluation of the Terminal Doppler Weather Radar (TDWR) in Denver, Colorado, during the summer of 1988. On 11 July, a very strong microburst (greater than 35 m s\(^{-1}\) differential velocity) occurred at the approach end of one of the runways (Schlickemeyer 1989). The low reflectivity microburst occurred downstream from the main precipitation shaft and was initiated by moisture blowing off the main cell around 5 km AGL (Elmore et al. 1990; Proctor and Bowles 1992). Several commercial flights received TDWR microburst alerts as they approached the runway, and those pilots felt that the TDWR warnings were extremely valuable in enabling them to successfully execute missed approaches in the midst of very strong wind shear.

The operational scan strategy executed by the TDWR testbed radar (FL-2, operated by MIT Lincoln Laboratory) included a surface sector scan over the airport every minute, expressly for detecting low-altitude wind shear. This surface scan was matched nearly simultaneously (average within 3.5 s) by a second Doppler radar operated by University of North Dakota. The radar locations and the location of the Stapleton International Airport runways are shown in Fig. 1. In the Lincoln Laboratory dual-Doppler analysis, the polar data from each radar were thresholded at 5 dB signal-to-noise ratio, median smoothed with a 5 gate by 3° filter (at least 8 good values out of 15 required), and dealiased according to the NSSL (National Severe Storm Laboratory) algorithm (Eilts and Smith 1990). The data were then sampled to a 250-m resolution Cartesian grid at the level of 190 m above the FL-2 radar site. The grid was 89 × 89 points (not shown) with the origin 23 km west and 1 km south of the FL-2 radar site. The portion (37 × 30) of this grid used in this study is shown in Fig. 2a, which covers the inner rectangular domain in Fig. 1.

![Fig. 1. Locations of airport runways, radars, and LLWAS stations. The inner rectangular domain indicates the region where the winds are retrieved in Fig. 2. The dots are the LLWAS anemometers and the lines are runways.](http://journals.ametsoc.org/mwr/article-pdf/123/6/1822/4173306/1520-0493(1995)123_1822_saromw_2_0_co_2.pdf)
Surface anemometer data from the 12 station Low Level Wind Shear Alert System (LLWAS) were also collected during the experiment. Several of the stations in 1988 suffered from wind sheltering problems (Lepins et al. 1990) that have since been remedied by raising the sensor height. The location of the 1988 Stapleton LLWAS sensors are also shown in Fig. 1.

3. Method description

As in XQY94b; the radial-component wind \( v_r \) is used as a "tracer" field, but the radial momentum equation is expressed in the Cartesian coordinates \((x, y, z)\). In this case, the equation has the following form:

\[
\partial_t v_r + u \partial_x v_r + v \partial_y v_r - \frac{v_r^2}{r} + w \partial_z v_r + \frac{\partial p}{\rho} = \kappa \nabla_r^2 v_r + \nu \partial_r^2 v_r, \tag{3.1}
\]

where \( \mathbf{v} = (u, v) \) is the horizontal wind vector, \( v_r = (xu + yv) r^{-1} \) is the radial-component wind, \( v_a = (xu - yv) r^{-1} \) is the tangential-component wind, \( r^2 = x^2 + y^2 \), \( w \) is the vertical velocity, \( p \) is the pressure, \( \rho \) is the density. \( \nabla_r^2 \) is the horizontal Laplacian, \( \kappa \) is the coefficient of horizontal eddy viscosity, and \( \nu \) is the coefficient of vertical eddy viscosity.

The time-mean (or running-mean) operator is denoted by \( (\quad)_m = \tau^{-1} \int_0^\tau (\quad) dt \), where \( \tau = N \Delta \tau \) is the averaging period covering \( N + 1 \) sequential radar scans and \( \Delta \tau \) is the time elapsed for one scan. The horizontal wind can be partitioned into a time-mean part \( v_m \equiv (u_m, v_m) \) and a temporal fluctuation part \( v' \equiv (u', v') \). Substituting \( \mathbf{v} = v_m + v' \) into (3.1) (but with \( v_r \) retained as a "tracer" field) gives:

\[
\partial_t v_r + u_m \partial_x v_r + v_m \partial_y v_r - \frac{v_m^2}{r} - \kappa \nabla_r^2 v_r,
\]

\[
= \nu \partial_r^2 v_r - \frac{w}{\tau} \partial_x v_r - u' \partial_y v_r - v' \partial_z v_r,
\]

\[
+ \frac{1}{r} (2 v_{am} v'_{am} + v'_{am}^2) - \frac{\partial_p}{\rho} \equiv F = F_m + F', \tag{3.2}
\]

where \( v_{am} = (xu_m - yu_m) r^{-1}, \ v'_{am} = (xu' - yu') r^{-1}, \) and \( F_m \) and \( F' \) are the time-mean part and temporal fluctuation part of the residual forcing \( F \), respectively. As in XQY94b, the vertical eddy viscous term is combined into the residual forcing \( F \), so the method can apply to data on a single level.

As explained in XQY94b, it is impossible to retrieve the residual forcing field (including its temporal fluctuation) because the total number of unknowns will be greater than the number of the discretized model equations plus the number of data points. It is possible, however, to retrieve the time-mean part of the unknown residual forcing. This feasibility has been tested with both simulated data (Xu et al. 1992) and real data (XQY94b). Here, neglecting \( F' \) from (3.2) can be a valid approximation as long as the retrieving period is relatively short (\( \tau = 4 \) min is used in this paper). In this case, (3.2) reduces to:

\[
\partial_t v_r + u_m \partial_x v_r + v_m \partial_y v_r - \frac{v_m^2}{r} - \kappa \nabla_r^2 v_r = F_m. \tag{3.3a}
\]

The boundary and initial values are given by

\[
v_r(t, x, y) = v_{\text{ob}}(t, x, y) \quad \text{at the domain boundary},
\]

\[
v_r(0, x, y) = v_{\text{ob}}(0, x, y), \tag{3.3b}
\]

where the subscript ob denotes observed value. The coefficient of horizontal eddy viscosity \( \kappa \) is assumed as an unknown constant and will be retrieved.
The objective is to find the best estimate of \((v_m, \kappa, F_m)\) in (3.3a) that gives the best "prediction" of the radial wind \(v_r\) in terms of minimizing the following cost function:

\[
J = \{ P_1 \Delta^2 + P_2 \Delta^2 + P_3 \Delta^2 + P_4 \Delta^2 \} \).
\]

Here, \(\{ ( ) \} = \Omega^{-1} \int \int ( ) d\Omega\) is the area-mean operator over the retrieval domain \(\Omega\); \(P_1\) and \(P_2\) are nondimensional weights, \(\Delta = v_{vobs} - v_{vobs}\), and \(\Delta_m = v_{vobs} - v_{vobs}\); and \(P_1\) and \(P_2\) are dimensional weights (in units of square meters), \(\Delta = \nabla_{\mu} \cdot v_m\) the divergence, and \(\zeta_m = k \cdot \nabla_{\mu} \times v_m\) the vorticity. The minimum of \(J\) can be approached by numerical iteration with the conjugate gradient method. The gradient of \(J\) with respect to \((v_m, \kappa, F_m)\) is computed at each step of iteration by an explicit expression derived from the adjoint formulation.

Following the methods of QX92 and QXY94a,b, one can show that the gradient components of \(\nabla J\) with respect to \((u_m, v_m, \kappa, F_m)\) can be explicitly expressed by

\[
\frac{\partial J}{\partial u_m} = \left( v^* \partial_{v_m} \right) + \frac{2y v^* v_{u_m}}{r^2} + \frac{2x P_3 \Delta_m}{r} - 2P_3 \partial_{\mu} d_m + 2P_4 \partial_{\nu} \zeta_m,
\]

\[
\frac{\partial J}{\partial v_m} = \left( v^* \partial_{v_m} \right) - \frac{2x v^* v_{u_m}}{r^2} + \frac{2y P_3 \Delta_m}{r} - 2P_3 \partial_{\mu} d_m - 2P_4 \partial_{\nu} \zeta_m,
\]

\[
\frac{\partial J}{\partial F_m} = -\{ v^* \}_{m}, \quad \frac{\partial J}{\partial \kappa} = -\{ v^* \nabla^2 v \}_{m},
\]

where \(v^*\) is the solution of the following associated adjoint problem:

\[
\partial_t v^* + \partial_t (u_m v^*) + \partial_t (v_m v^*) + \kappa \nabla^2 v^* = 2P_1 \Delta \equiv 2P_1 (v_r - v_{vobs}),
\]

\[
v^*(t, x, y) = 0 \quad \text{at the boundary of domain } \Omega,
\]

\[
v^*(t, x, y) = 0.
\]

The standard conjugate gradient algorithm (UMCGG in the IMSL Math Library) is used together with the cost function \(J\) in (3.4) and its gradients in (3.5) to search the minimum of \(J\) in the function-parameter space of \((u_m, v_m, \kappa, F_m)\). The iterative procedures are similar to that in QX92. The general mathematical theory of the adjoint method can be found in Cacuci (1981) and Talagrand and Courtier (1987). The standard leapfrog scheme, with the Euler scheme for the initial time step, is used to integrate the control equation and adjoint equation. The numerical code is designed carefully to ensure the adjoint property between the discretized control and adjoint operators. The detailed process of finding the discrete adjoint from a discrete model can be found in Thacker and Long (1988).

Before proceeding to the next section, it is worth mentioning the problem with scaling, ellipticity of the supersurface of the cost function, and convergence rate of iteration. This problem is not new and has been previously experienced by other authors (e.g., Moore 1991). In our case, if the standard units of meters per second squared are used for \(F_m\), then \(\partial J/\partial F_m\) will be much larger than the remaining three gradient components in (3.5). This means that the supersurface of the cost function is highly elliptical and the above descent algorithm may hardly converge unless \(F_m\) is properly rescaled. The proper scale for \(F_m\) can be estimated from the observed mean absolute value of the time-mean of the first term in (3.3a); that is, \(\{ v^* \}_{m}\). With this scale, \(\partial J/\partial F_m\) has about the same nondimensional magnitude as the remaining gradient components in (3.5), so the supersurface of the cost function becomes locally close to spherical or, at least, less elliptical and the descent algorithm converges rapidly.

### 4. Experiments and results

#### a. Experiment design and selection of weights

As mentioned in the introduction, the SA method was previously tested with both simulated data (QX92) and real data (QXY94a,b). Several specific objectives were previously examined in order to improve the retrievals. These include (i) recovering the eddy diffusion terms and retrieving the eddy coefficients, (ii) interpolating the data field to the dense computational time levels, (iii) choosing an optimal retrieving time length, (iv) using proper function forms and values for the weights in the cost function, and (v) retrieving the time-mean part of the unknown residual forcing (or source) term. These five objectives have also been re-examined with the Denver airport microburst data. Since the results confirm the earlier findings—that is, the retrieval can be improved significantly as these objectives are fulfilled—the details are omitted here. As mentioned in the introduction, the new objectives examined in this paper concern how to further improve the retrievals (i) by imposing a weak vorticity constraint, (ii) by upgrading the initial guess with the previous time-level retrieval, and (iii) by using the surface wind data. Three experiments (I–III) are designed to fulfill the above three objectives and the results are presented in the following three subsections.

Before we proceed to the next subsection, we need to describe the selection of the weights. Varieties of weight specifications have been examined, and the results show that the optimal or nearly optimal selections of the weights are

\[
P_1 = \left( \frac{\tau}{t + \Delta t} \right)^{1/2},
\]

\[
P_2 = \mu P_{1m} \quad \text{with } \mu = 0.02 \quad \text{and} \quad P_{1m} = (P_1)_m,
\]

\[
P_3 = k_3 \sigma_r^2 P_{1m} \quad \text{with } k_3 = 80 \, \text{s}^2
\]

(or between 30 and 200 s²),
Table 1. Statistics of the retrievals from FL-2 radar data.

<table>
<thead>
<tr>
<th></th>
<th>$V_m$ (m s$^{-1}$)</th>
<th>$d_m$ (10$^{-3}$ s$^{-1}$)</th>
<th>$\zeta_m$ (10$^{-3}$ s$^{-1}$)</th>
<th>$F_m$ (10$^{-2}$ m s$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_4 = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms ( )</td>
<td>3.87</td>
<td>4.11</td>
<td>4.97</td>
<td>1.45</td>
</tr>
<tr>
<td>RRD ( )</td>
<td>0.45</td>
<td>0.73</td>
<td>1.64</td>
<td>0.75</td>
</tr>
<tr>
<td>SCC ( )</td>
<td>0.88</td>
<td>0.69</td>
<td>0.39</td>
<td>0.68</td>
</tr>
<tr>
<td>$k_4 = 400$ s$^{-2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms ( )</td>
<td>2.89</td>
<td>3.49</td>
<td>2.82</td>
<td>1.21</td>
</tr>
<tr>
<td>RRD ( )</td>
<td>0.33</td>
<td>0.62</td>
<td>0.91</td>
<td>0.63</td>
</tr>
<tr>
<td>SCC ( )</td>
<td>0.94</td>
<td>0.79</td>
<td>0.53</td>
<td>0.78</td>
</tr>
</tbody>
</table>

$$P_d = k_4 \sigma_{\nu_r}^2 P_{1\nu} \quad \text{with} \quad k_4 = 400 \text{ s}^2$$ (4.1)

(4.1)

where $\sigma_{\nu_r}$ is the rms amplitude of $\nu_r$. The choice of the time-dependent form for $P_1$ was explained in QX92. With the above specified value for $P_2$, the weak form of the constraint $\Delta_m = 0$ can reduce the error in the estimated cross-beam wind. This weak constraint is better than the strong constraint that requires $v_m = v_{\text{obs}}$, because allowing a small difference $\Delta_m$ can reduce the error in the tangential wind. Besides, it is not necessary to require $v_m = v_{\text{obs}}$, as $v_{\text{obs}}$ contains observational error. The relative strength of the weak divergence (or vorticity) constraint is controlled by $k_3$ (or $k_4$). As long as $k_3$ (or $k_4$) is in the optimal range shown in (4.1), the retrieval is not very sensitive to $k_3$ (or $k_4$). The weights in (4.1) are consistent with those in QXY94a,b, but $k_4$ and the last term in (3.4) are new here. The retrieving time period $\tau$ is selected to cover 5 sequential scans; that is $\tau = 4\Delta \tau \approx 4$ min and this selection is found to give the most accurate retrievals in general.

b. Experiment I—The effect of weak vorticity constraint

Experiment I is designed to check whether and how much the retrievals can be improved by imposing a weak vorticity constraint, as proposed in the above objective (i). To this end, 25 retrievals are performed with the microburst data from FL-2 radar for the period of 2204–2233 UTC (which contains 29 consecutive low-elevation scans each about one minute apart). Since a zero initial guess is used in this experiment, all the retrievals are done by 185 iterations. This large number of iterations are found to be generally necessary for optimally convergent retrievals. The statistics (averaged results over 25 retrieved fields) with $k_4 = 0$ and 400 s$^{-2}$ are listed in Table 1, where the rms difference, relative rms difference (RRD), and spatial correlation coefficient (SCC) between the retrieved and dual-Doppler observed variables are computed as follows:

$$\text{rms}(\ ) = \frac{\{\{\ast(\ ) - (\ )_{\text{obs}}\}_{1}\}^{1/2}}{\{\{\ast\}_{1}\}^{1/2}}, \quad (4.2a)$$

$$\text{RRD}(\ ) = \frac{\text{rms}(\ )}{\{\{\ast\}_{1}\}^{1/2}}, \quad (4.2b)$$

$$\text{SCC}(\ ) = \frac{\{\{\ast(\ )\}_{1}\}^{1/2}}{\{\{\ast\}_{1}\}^{1/2}}, \quad (4.2c)$$

where $\ast(\ ) = (\ ) - \{\{\ )\}$. The observed time-mean forcing, $F_{\text{obs}}$, is estimated by substituting the dual-Doppler winds and $\kappa$ into (3.2). Clearly, as shown in Table 1, the weak vorticity constraint improves not only the velocity and vorticity retrievals but also the retrieved divergence and time-mean forcing term. This remains true for the continuous retrievals performed in the next subsection.

The effect of the weak vorticity constraint is also tested with the Phoenix II data. It is found that the retrievals are improved significantly for all the four cases in Table 2 of QXY94b and the averaged rms difference between the retrieved and dual-Doppler observed winds reduces from 1.19 to 0.87 m s$^{-1}$. Like the weak divergence constraint, the weak vorticity constraint can suppress the spurious vorticity anomalies (caused by the data noise) and improve the retrieval. Although these two constraints tend to smooth the retrieved wind field, the strong divergent flow in the downburst region and strong shear flow along the gustfront are well retrieved (see Figs. 2–5). In general, these two constraints are found very effective in improving the retrievals, and the results are not sensitive to the weight $k_4$ as long as the weight is between 100 and 600 s$^{-2}$.

c. Experiment II—Continuous retrieval

Experiment II is designed to examine the above proposed objective (ii), that is, to see whether and to what extent the retrievals can be improved if the wind field retrieved at the previous time level is used as the initial guess for the retrieval at the current time level. Note that the velocity is hardly retrieved but mainly determined by the initial guess in a region where the gradient of the tracer field remains nearly zero during the re-

Table 2. Statistics of the continuous retrievals.

<table>
<thead>
<tr>
<th></th>
<th>$V_m$ (m s$^{-1}$)</th>
<th>$d_m$ (10$^{-3}$ s$^{-1}$)</th>
<th>$\zeta_m$ (10$^{-3}$ s$^{-1}$)</th>
<th>$F_m$ (10$^{-2}$ m s$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL-2 radar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms ( )</td>
<td>2.66</td>
<td>3.38</td>
<td>2.74</td>
<td>1.20</td>
</tr>
<tr>
<td>RRD ( )</td>
<td>0.31</td>
<td>0.60</td>
<td>0.89</td>
<td>0.63</td>
</tr>
<tr>
<td>SCC ( )</td>
<td>0.95</td>
<td>0.90</td>
<td>0.55</td>
<td>0.78</td>
</tr>
<tr>
<td>UND radar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms ( )</td>
<td>4.06</td>
<td>4.60</td>
<td>3.20</td>
<td>1.35</td>
</tr>
<tr>
<td>RRD ( )</td>
<td>0.48</td>
<td>0.82</td>
<td>1.01</td>
<td>0.68</td>
</tr>
<tr>
<td>SCC ( )</td>
<td>0.88</td>
<td>0.61</td>
<td>0.41</td>
<td>0.72</td>
</tr>
</tbody>
</table>
Fig. 3. As in Fig. 2 but for the worst case from FL-2 radar data in experiment II. The retrieval is performed for the period of 2223–2227 UTC 11 July 1988. In this case, \( \text{rms} = 3.39 \, \text{m s}^{-1} \) and SCC = 0.926.

trieval period (because the tracer movement can hardly be “detected” in this region). In this case, the initial guess provided by the retrieval at the previous time level (which is obviously better than the zero initial guess) will not only reduce the CPU cost but also improve the retrieval.

All the retrievals in this experiment are done by 120 steps of iteration, so the CPU cost is reduced in comparison with 185 steps of iteration used in experiment I. The results are summarized in Table 2. In comparison with the results in Table 1, we can see that the continuous retrievals are more accurate than the retrievals with a zero initial guess. The improvements are seen not only in the retrieved velocity field but also in the retrieved divergence, vorticity, and time-mean forcing term. The best and worst cases among the 25 retrievals (2206–2231 UTC) from FL-2 radar are shown in Figs. 2a,b and Figs. 3a,b, respectively. The best and worst cases for the 25 retrievals from UND radar are shown in Figs. 4a,b and Figs. 5a,b, respectively. The correlation diagrams between the retrieved winds and dual-Doppler observed winds are shown in Figs. 6a and 6b for the retrievals (at every fifth time level) from FL-2 radar and UND radar, respectively.

d. Experiment III—Use of LLWAS data

To use the LLWAS data, we need to extrapolate the wind vectors measured by surface anemometers to the grid level of \( z = 190 \, \text{m} \). The correlation between the anemometer winds and Doppler winds for the Denver microburst case has been examined by Liepins et al. (1990). They found that the wind speeds (but not the wind directions) measured by the anemometers (at the

Fig. 4. As in Fig. 2 but for the best case from UND radar data in experiment II. The retrieval is performed for the period of 2227–2231 UTC 11 July 1988. In this case, \( \text{rms} = 3.09 \, \text{m s}^{-1} \) and SCC = 0.927.
Fig. 5. As in Fig. 2 but for the worst case from UND radar data in experiment II. The retrieval is performed for the period of 2210–2214 UTC 11 July 1988. In this case, rms = 5.11 m s⁻¹ and SCC = 0.822.

surface level) and by dual-Doppler radar (at the levels from 200 to 300 m) were quite comparable. For our purpose, the problem is difficult due to the following two reasons: (i) Not only the wind speed but also the wind direction needs to be extrapolated to the grid level (z = 190 m). (ii) Only single-Doppler radial winds are operationally available at the grid level to determine the relation between the wind vectors at the two different levels or, say, to find a formulation for the extrapolation. According to Liepins et al. (1990, see their

Fig. 6), the wind directions at the surface level and at the level of the Doppler radar scans (about 200 m) were poorly correlated during the microburst period (2206–2226 UTC). It seems even more difficult to use single-Doppler radial winds to find a good formulation for the extrapolation.

Many different methods have been tried to find a useful extrapolation formulation. The best method so far obtained is described as follows. The method assumes that the radial wind \( v_r \) and tangential wind \( v_\theta \) at

\[
\begin{align*}
V(\text{RT}) & \quad (a) \\
V(\text{RT}) & \quad (b)
\end{align*}
\]

Fig. 6. The correlation diagrams between the retrieved winds and dual-Doppler observed winds for the retrievals (total 5 among the 25 retrievals with 1 out of every 5) in experiment II from (a) FL-2 radar with rms = 2.59 m s⁻¹ and SCC = 0.952, and (b) UND radar with rms = 3.96 m s⁻¹ and SCC = 0.885.
the grid level (190 m) are approximately related to the radial wind \( u_a \) and tangential wind \( u_{aa} \) at the anemometer level, respectively, by

\[
v_r = a + b v_{ra}, \quad (4.3a)
\]

\[
v_{aa} = a + b v_{aa}, \quad (4.3b)
\]

where \( a \) and \( b \) are constants for each retrieving period, but can change from one retrieving period to another. For each retrieving period \( \tau \), constants \( a \) and \( b \) are determined by least-squares fitting of (4.3a) with the radial winds measured by the Doppler radar and anemometers over the 12 stations and at the seven time levels that cover the retrieving period (five time levels) with one more time level before and one more time level after the retrieving period. Once \( a \) and \( b \) are computed, (4.3b) is used to compute the tangential wind at the grid level from the tangential wind at the anemometer level. Thus, the time-mean wind vectors are obtained at the grid level over the 12 LLWAS stations. The rms difference and relative rms difference (RSD) between the extrapolated winds from the 12 LLWAS stations and the corresponding dual-Doppler winds are listed in Table 3. Clearly, the rms difference is smaller for the radial winds than for the tangential winds, because constants \( a \) and \( b \) are determined from the radial winds. The rms differences for the extrapolated tangential winds are not small (2.72 m s\(^{-1}\) for FL-2 radar and 4.06 m s\(^{-1}\) for UND radar) but hardly to be further improved, even though various power functions, logarithm functions, as well as the classic Ekman boundary flow solution have been tried to replace the linear function in (4.3a) and (4.3b), and many ways have been tried in least-squares fitting of (4.3a) and (4.3b) with differently grouped data. Since all these extrapolation formulations are tested only with the data selected in this paper and, as mentioned in section 2, several of the LLWAS stations in 1988 suffered from wind-sheltering problems (Lippins et al. 1990), the above extrapolation method will need to be further improved when it is tested with future high-quality LLWAS data.

There are two basic ways to use the above extrapolated wind vectors as a weak constraint to improve the retrievals. The first way directly uses the 12 extrapolated wind vectors to constrain the retrieved wind vectors at the grid points in the vicinity of each station (within a proper radius), whereas the weight function can be the Cressman type [see (4.5b)]. The second way interpolates the 12 wind vectors onto the grid points in the vicinity of the LLWAS stations and then the interpolated (or grided) wind vectors are used to constrain the retrieved wind vectors in the vicinity of each station. Tested with the real data, the second way is found better than the first way, so only the second way and its results are presented here.

The Cressman scheme used to interpolate the 12 wind vectors onto the grid points in the vicinity of the

<table>
<thead>
<tr>
<th>TABLE 3. Statistics of the extrapolated winds from LLWAS data.</th>
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<tbody>
<tr>
<td>( V_{uw} ) (m s(^{-1}))</td>
</tr>
<tr>
<td>FL-2 radar</td>
</tr>
<tr>
<td>rms (  )</td>
</tr>
<tr>
<td>RRD (  )</td>
</tr>
<tr>
<td>UND radar</td>
</tr>
<tr>
<td>rms (  )</td>
</tr>
<tr>
<td>RRD (  )</td>
</tr>
</tbody>
</table>

The LLWAS stations. In particular, the time-mean wind vector \( v_{mi} \) at the \( i \)th grid point is obtained by

\[
v_{mi} = \frac{\sum_j \eta_{ij} v_{mj}}{\sum_j \eta_{ij}}, \quad (4.4a)
\]

where \( v_{mj} \) is the time-mean wind vector at the \( j \)th station, \( \eta_{ij} \) is the Cressman weight defined by

\[
\eta_{ij} = \max \left( 0, \frac{R^2 - r_{ij}^2}{R^2 + r_{ij}^2} \right) \quad \text{with}
\]

\[
R = 5 \quad \text{(or between 4 and 6),} \quad (4.4b)
\]

\( R \) is the dimensionless Cressman radius (scaled by the grid resolution \( \Delta x = \Delta y = 250 \) m), and \( r_{ij} \) is the dimensionless horizontal distance between the \( i \)th grid point and the \( j \)th station. The interpolation is performed only for the grid points within the Cressman radius with respect to, at least, one of the 12 stations.

With the above interpolated (or grided) wind vectors, a weak constraint can be imposed on the retrieved wind vectors at the same grid points within the Cressman radius of the LLWAS stations. In particular, the following weak form of constraint is added to the cost function in (3.4):

\[
J_5 = \{ \{ P_5 | v_m - v_{mi} |^2 \} \}, \quad (4.5a)
\]

where \( v_m \) is the retrieved wind, \( v_{mi} \) is the interpolated wind at the \( i \)th grid point, and \( P_5 = P_5(x,y) \) is the Cressman-type weight function defined by

\[
P_5 = k_5 \mu P_{1m} \max_j \eta_{ij} \quad \text{with}
\]

\[
k_5 = 0.2 \quad \text{(or between 0.1 and 0.4).} \quad (4.5b)
\]

Here, \( \mu \) and \( P_{1m} \) are as in (4.1). The gradient components of \( \nabla J_5 \) with respect to \( (u_m, v_m) \) can be directly derived from (4.5a); that is,

\[
\frac{\partial J_5}{\partial u_m} = 2 P_5 (u_m - u_{mi}),
\]

\[
\frac{\partial J_5}{\partial v_m} = 2 P_5 (v_m - v_{mi}). \quad (4.6)
\]

These two terms need to be added to the first two gradient components in (3.5), respectively.
The above method is tested with different settings of $R$ and $k_s$. It is found that using the LLWAS data can, indeed, improve the retrievals. The improvements are statistically significant and not very sensitive to the settings of $R$ and $k_s$ as long as $R$ is between 4 and 6 and $k_s$ is between 0.1 and 0.4. The results with $R = 5$ and $k_s = 0.2$ are shown in Table 4. Note that the improvements are less significant for the retrievals from FL-2 radar than from UND radar. The reason seems mainly due to the fact that the retrievals from UND radar are relatively poor and, thus, there is more room for improvement when the LLWAS data are used. This is generally true and the differences in their improvements for the retrievals from FL-2 radar and UND radar are notable from the correlation diagrams in Figs. 7a,b in comparison with those in Figs. 6a,b.

With the LLWAS data, the four cases of retrievals in Figs. 2–5 (experiment II) are all improved and the detailed results (not shown) indicate that the improvement is more significant for a relatively poor retrieval. Thus, the LLWAS data can be more useful when the retrievals purely from radar data are less accurate. As an example, the improved wind field (for the worst FL-2 case) is shown in Fig. 8. In comparison with the worst FL-2 case in Figs. 3a,b (without the LLWAS data), improvements in Fig. 8 are seen mainly in the middle and northern central portion of the domain, that is, in the area covered by the LLWAS stations (see Fig. 1). Clearly, the localized improvements are associated with the optimal radius of influence used in (4.4b); that is, $R = 5$ or $5\Delta x$ from each station. Without using the LLWAS data, the winds are relatively poorly retrieved in the area covered by the LLWAS stations, and the rms differences computed over these 12 stations are not only larger than those computed over the entire domain in Table 2 but also larger than the rms differences for the extrapolated winds in Table 3. This explains why the extrapolated winds can improve the retrievals even though the rms differences for the extrapolated tangential winds are not smaller than the rms differences for the retrieved winds in Table 2.

5. Error analysis

Factors responsible for retrieval errors are examined in relation to the terms in (3.2). The area-averaged running mean absolute values of the terms in (3.2) are

![Figure 7](http://journals.ametsoc.org/mwr/article-pdf/123/6/1822/4173306/1520-0493(1995)123_1822_saromw_2_0_co_2.pdf)

Fig. 7. As in Fig. 6 but for the retrievals in experiment III from (a) FL-2 radar with rms = 2.45 m s$^{-1}$ and SCC = 0.959, and (b) UND radar with rms = 3.78 m s$^{-1}$ and SCC = 0.899.
computed with the radial winds obtained from FL-2 radar and UND radar, respectively. The diffusion term is estimated by using the averaged value of the retrieved \( \kappa \) (=200 m² s⁻¹), which is in the commonly accepted range for the PBL flow. The time-mean forcing \( F_m \) and transient perturbation forcing \( F' \) are estimated by substituting the dual-Doppler winds and \( \kappa \) into (3.2). Corresponding to the 25 time intervals (or 25 retrieving periods), 25 values are computed for each term. The mean, maximum, and minimum of the 25 values are listed for each term in Table 5. As shown, the local tendency and mean advection terms are the largest, the time-mean and transient perturbation forcing terms are relatively small, the diffusion term is even smaller, and the curvature term is the smallest. As explained and illustrated in XQY94ab, although the diffusion term is small, including this term and retrieving the eddy coefficient \( \kappa \) can further suppress short-wave noises and, thus, improve the wind retrieval.

If the above computed forcing terms \( F_m \) and \( F' \) are considered as known terms in (3.2), then the SA method can produce almost "perfect retrievals" (with respect to the dual-Doppler analysis) and the relative rms difference between the "retrieved" winds and the corresponding dual Doppler winds is smaller than 1% for all the cases. The detailed results are omitted here. Clearly, the retrieval error is caused by the difference between the above computed total forcing, \( F_m + F' \), and the retrieved part of the time-mean forcing, which is a combination of the equation error and data error. Since the forcing terms are relatively small (as shown in Table 5) and the time-mean forcing is partially retrieved (as shown in Tables 1, 2, and 4), the equation error and data error are relatively small. This explains why the method works well with the FL-2 radar data. Note also from Table 5 that the mean advection term computed from the UND radar data is smaller than that from the FL-2 radar data, whereas the source terms are slightly larger than those from the FL-2 radar data. The relatively small mean advection term for the UND radial winds is related to the fact that the wind vectors are largely perpendicular to the UND radar beams, while the slightly large source terms for the UND radial winds could be due to the relatively poor data accuracy (since the UND radar has relatively low resolution and is farther distant from the domain). These may explain why the retrievals from UND radar are less accurate than those from FL-2 radar (see Table 2).

In the following part of this section, a detailed error analysis is presented for the 25 retrievals from FL-2 radar in experiment I (with \( k_s = 400 \) s²). The 25 retrievals in experiment II or III are statistically less independent than those in experiment I, because the retrieval at an earlier time level in experiment II or III affects the retrieval at the later time level. Because of this, the retrievals in experiments II and III are not used for the error analysis.

The statistic relations of the RRD and SCC to the terms in Table 5 are examined and the results are shown in Table 6, where the correlation coefficient (CC) is defined by

\[
CC[a, b] = \frac{\sum a'' b''}{\sqrt{\left(\sum a''^2\right)\left(\sum b''^2\right)}}^{1/2},
\]

with \( a'' = a - \Sigma a/25, b'' = b - \Sigma b/25 \). The summation is over the 25 cases, RRD is defined in (4.2b) and

### Table 5. Absolute values of the terms (10⁻² m s⁻¹) in Eq. (3.2)

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>\vec{V}_{V_mW} )</td>
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</tr>
<tr>
<td>(</td>
<td>\vec{V}_{V_mW} )</td>
</tr>
<tr>
<td>FL-2 radar</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.25</td>
</tr>
<tr>
<td>Max</td>
<td>2.94</td>
</tr>
<tr>
<td>Min</td>
<td>1.79</td>
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</table>

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<tr>
<th>Term</th>
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<td>(</td>
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<td>(</td>
<td>\vec{V}_{V_mW} )</td>
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<tr>
<td>UND radar</td>
<td></td>
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<tr>
<td>Mean</td>
<td>1.90</td>
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<tr>
<td>Max</td>
<td>2.32</td>
</tr>
<tr>
<td>Min</td>
<td>1.57</td>
</tr>
</tbody>
</table>
Table 6. Correlations between RRD(V_w) or SCC(V_w) and the terms in Table 5. Here, V_0 is the retrieval from FL-2 radar in experiment I (with k_0 = 400 s^2).

<table>
<thead>
<tr>
<th></th>
<th>Adv</th>
<th>Tend</th>
<th>Diff</th>
<th>F_0/Adv</th>
<th>F'/Adv</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC[RRD(V_0), ( )]</td>
<td>-0.73</td>
<td>0.01</td>
<td>-0.45</td>
<td>0.28</td>
<td>0.65</td>
</tr>
<tr>
<td>CC[SCC(V_0), ( )]</td>
<td>0.76</td>
<td>0.10</td>
<td>0.42</td>
<td>-0.23</td>
<td>-0.58</td>
</tr>
</tbody>
</table>

SCC( ) is in (4.2c). The results in Table 6 indicate that RRD(V_w) is small and SCC(V_w) is large when the mean advection and/or diffusion are strong, or the normalized (by Adv) transient forcing is weak. The mean forcing is only weakly correlated to the retrieval error, and this weak correlation may be explained by the fact that the mean forcing term is already partially retrieved. The RRD(V_0) and SCC(V_0) are also found well correlated to the mean wind speed and wind fluctuation. The correlation coefficients are listed in Table 7. Clearly, the retrieval is more accurate for strong wind and/or weak transient wind fluctuation.

Based on the results in Table 6, it is natural to speculate that the retrieval may be more accurate when the local tendency term \( \partial_t V \) and mean wind advection term \( v_0 \cdot \nabla V \) in (3.3a) are better correlated over the retrieving period \( \tau \). To verify this speculation, we need to compute the domain-averaged time correlation coefficient (denoted by TCC1) between \( \partial_t V \) and \( v_0 \cdot \nabla V \) for each retrieving period \( \tau \) and then, to compute the correlation coefficients between TCC1 and RRD(V_0) and between TCC1 and SCC(V_0) over the 25 cases. The results are CC[TCC1, RRD(V_0)] = -0.67 and CC[TCC1, SCC(V_0)] = 0.82, where CC[(), ()] is defined in (5.1). Thus, the above speculation is true.

It is also interesting to compute the domain-averaged time correlation coefficient (denoted by TCC2) between the local tendency term \( \partial_t V \) and transient forcing term \( F' \) in (3.2) over each retrieving period \( \tau \) and then, compute the correlation coefficients between TCC2 and RRD(V_0) and between TCC2 and SCC(V_0) over the 25 cases. The results are CC[TCC2, RRD(V_0)] = 0.71 and CC[TCC2, SCC(V_0)] = -0.71, where CC[(), ()] is defined in (5.1). This suggests that the retrieval is less accurate when the local tendency is caused more due to the transient forcing.

6. Conclusions

The simple adjoint (SA) method for retrieving the low-altitude winds from single-Doppler scans (XQY94a,b) is further developed and tested with the Denver airport microburst data. The new objectives concern how to further improve the retrievals (i) by imposing a weak vorticity constraint; (ii) by continuous retrievals or, say, by using the previous time-level retrieval as the initial guess of the retrieval at the current time level; and (iii) by incorporating the surface anemometer data into the method. Tests are performed over a continuous period (2206–2231 UTC 11 July 1988). During this period, 25 consecutive (running mean) wind fields are retrieved from single-Doppler wind data measured by either one (FL-2 or UND) of the two radars. The horizontal wind fields are retrieved on high-resolution grids (250 m) at the low altitude (190 m above FL-2 radar) every 60 s as the updates are received (from FL-2 or UND radar). By fulfilling the above objectives (i), then objective (ii), and finally objective (iii), the averaged (for the 25 retrievals from FL-2 radar) rms difference between the retrieved winds and the corresponding dual-Doppler winds reduces from 3.87 to 2.89 m s^{-1}, then to 2.66 m s^{-1}, and finally to 2.55 m s^{-1}, while the averaged correlation coefficient increases from 88% to 94%, then to 95%, and finally to 96% (see Tables 1, 2, and 4). It is also found that the LLWAS data can make a larger improvement when the retrieval is less accurate (cf. the results from FL-2 with those from UND in Tables 2 and 4). Thus, the LLWAS data can be more useful when the retrievals based purely on radar data are less accurate.

Factors responsible for retrieval errors are examined in relation to the terms in the radial wind momentum equation [see (3.2)], which are estimated by using the dual-Doppler winds. It is found that the terms neglected by the model equation (3.3a) are smaller than the retained terms (see Table 5). This explains why the method works well with the FL-2 radar data, similarly to those with the Phoenix II data in XQY94b. Based on the statistics of the 25 retrievals (from FL-2 radar), the following factors are found to have significant impact on the accuracy of the retrievals: (i) The retrieval error is small when the mean wind advection is strong, or the normalized transient forcing is weak (see Table 6). (ii) The retrieval is more accurate when the local tendency and mean advection terms are better correlated over the retrieving period \( \tau \). The retrieval is less accurate when the local tendency is caused more due to the transient forcing. (iii) The retrieval is more accurate for strong wind and/or weak transient wind fluctuation (see Table 7). These statistical results quantify and extend the early results of error analysis (performed for only four retrieved fields) in XQY94b.

Because the SA method uses the data over several sequential time levels and retrieves the time-mean (or running-mean) winds from the movements of the radial wind (or reflectivity) patterns measured by a single-
Doppler radar, it is easy to understand why the above factors (i) – (iii) have significant impact on the accuracy of the retrievals. We have actually tested the SA method with the Denver airport microburst data through a much greater number of numerical experiments than presented in this paper. We have not only tested the three new objectives, but also reexamined most of the old objectives in XQY94a,b (see the major results briefly reviewed at the beginning of the introduction section). The test results for the old objectives confirm those in XQY94a,b and the details are omitted in this paper.

Based on tests performed in this and previous studies (XQY94a,b), the SA method seems very promising for research and operational uses. Since the SA method has very low CPU cost (90s on VAX-6000), high spatial resolution, reasonably good accuracy, and flexibility in using the surface anemometer data and other additional data, this technique will permit reporting of the true along-runway component of wind shear and the cross-runway winds regardless of the TDWR viewing angle. The SA method is currently being further tested with a broader spectrum of real cases, and the results obtained so far remain positive. The important fact revealed by our extensive experiments (with a broad spectrum of different types of Doppler data) is that the effective time window for small-scale data assimilation is actually very short (5 – 30 min) and intrinsically limited by the timescale during which the small-scale features can be correctly predicted. With a short time window and high frequency of data supply, reasonably accurate adjoint single-Doppler retrievals can be produced with very low CPU cost on a workstation. This seems to suggest an optimal scenario for future operational applications of Doppler data assimilation.

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