Lateral Boundary Conditions in Regional Climate Models: A Detailed Study of the Relaxation Procedure

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ABSTRACT

In gridpoint regional climate models (RCMs), the lateral boundary conditions are usually provided by a procedure called relaxation. The technique was originally studied in the context of numerical weather forecasting. This paper complements the preceding theoretical studies in order to assess the practical choices of model relaxation coefficients. Several profiles of coefficients used in RCMs are then evaluated. The complexity of actual model numerics makes any definite choice of the coefficients out of reach of simple theoretical considerations, but these provide practical guidelines. The latter are confirmed by pragmatic considerations such as minimizing discontinuities and keeping relaxation rates in the range of the represented physical processes. The last part of the paper presents a sensitivity study with the Modèle Atmosphérique Régional (MAR).

1. Introduction

The technique of “nesting” a high-resolution limited area model within a general circulation model is widely used, specifically for the purpose of climate simulation (Houghton et al. 2001, chapter 10). This implies that large-scale (LS) atmospheric conditions must be provided to a regional climate model (RCM), and that spurious interactions between the model solution and the lateral boundaries must be avoided. Solving the problem of boundary conditions is not straightforward for fundamental mathematical reasons. Additional difficulties come from the principle of one-way nesting, because it seeks a unidirectional coupling between the external data and the flow of the regional model, which involves solutions of partly different equation sets on different grids.

The boundary data are provided either by a climate general circulation model (GCM) or by model-based analyses of observations, the latter being particularly useful for RCM validation. Since the primary purpose of RCMs is dynamical downscaling, it is evident that the RCM output should contain mesoscale details that are absent from the external data. The differences between the RCM and the provided large-scale data will derive mainly from the better representation of topography and other surface characteristics in the regional model, and often from the differences in the representation of subgrid processes. The acceptable amount of deviation of the RCM from the external data is a matter of debate. Since the external data are usually provided near the lateral boundaries only, large domains will result in significant RCM – forcing differences at the synoptic scale, in particular over the polar regions (Rinke and Dethloff 2000). It has been suggested that in most cases, RCM – forcing differences should provide only high-resolution details, and that it is possible to optimize the domain size for this purpose (Jones et al. 1995). The spectral nudging method (von Storch et al. 2000) is another way to mitigate the large-scale differ-
ences between the regional model and the driving fields. The lateral boundary issue may give less cause for concern when the RCM deviates less from the external forcing, but it still exists. A possible exception is the case of spectral models (Hong and Leetmaa 1999).

One-way nesting has been used in numerical weather prediction (NWP) for a long time. The case of RCMs is partly different, due to the longer timescales, but the experience gained from NWP can provide useful guidance. In both cases strong local forcing mechanisms will provide an added value to the regional simulation, which will be partly independent of the deficiencies in the large-scale fields and lateral boundary conditions (LBCs). In contrast, strong local forcing is not desirable near the lateral boundaries, because this increases the RCM–external flow inconsistencies and thus complicates the LBC problem. A general summary of recommendations related to lateral boundaries in regional NWP can be found in Warner et al. (1997).

In the context of NWP, some studies have focused on solving the initial-boundary-condition problem in a mathematically accurate way (e.g., Elvius and Sundström 1973; Davies 1973). The aim is to define boundary conditions so that the partial differential equations yield a unique solution that depends continuously on the provided data. This approach is called well posed. To fulfill this objective, it is not permitted to arbitrarily specify all model variables at the boundary: only an appropriate combination of these can be specified—namely the inward pointing characteristics. Adding more constraints at the boundary would make the analytical problem impossible. For the discretized system, the corresponding situation is called “overspecified.” This triggers numerical noise and may seriously degrade the model solution. However, the well-posed approach is difficult to implement, and research is on going (MacDonald 2000). In addition, as pointed out by Davies (1976), the complications associated with the well-posed approach accrue from seeking an analytically justifiable coupling between the boundary data and the interior flow. At the timescale of “climate” simulation, the situation is similar or worse than that for weather prediction because the regional model is not constrained by initial conditions, so that the model values may thus be less consistent with the external fields.

The above-mentioned problems of lateral boundary treatment are usually bypassed using “pragmatic” techniques. Rather than attempt to meet strict mathematical criteria, the aim is only to provide the large-scale data to the regional model without deleterious effects. Although other pragmatic techniques were tested in early limited area models (Davies 1983), the “relaxation” method is almost universally used in RCMs (see, e.g., Christensen et al. 1997). In this method, a relaxation component is continuously added to the model-generated fields to drive them to the large-scale forcing. Relaxation is applied specifically or exclusively in a “buffer zone” near the boundary. This enables an increase in the number of variables that can be specified at the boundary (typically all the main prognostic variables) while limiting the “noise” problems that normally result from the overspecification. In addition, it has long been accepted that relaxation can be used as a simple data assimilation scheme (e.g., Davies and Turner 1977), so that it has some ability to cope with inaccurate, unbalanced forcing data.

The use of relaxation in RCMs is widely accepted, but it has been suggested that this technique may lead to problems. A number of criticisms are listed in MacDonald (2000) in support of the development of well-posed NWP systems. It is thus interesting to have a look at these in the context of regional climate models. First, at points at which the characteristic velocity is pointing out of the domain, the externally imposed solution may introduce unnecessary errors. This mainly applies to NWP, for which the key input is the initial conditions while LBCs are often seen as a potential error source rather than a forcing. The case of outgoing characteristics may nevertheless be a problem in RCMs when the model differs from its external forcing. This may cause anomalies that might spread backward in the domain, but simple theoretical investigations suggest that it can be avoided (Davies 1983, discussed here). Second, the relaxation scheme may cause loss or gain of mass. This is suspected because there are slight differences between the total mass in an RCM and its equivalent in the forcing data. A possible origin of this issue can be found in the differences between the flow in the RCM and in the model providing the large-scale data (which are acceptable, at least at the mesoscale). Such flow differences are likely to cause small differences in the mass flux at the location of the boundaries. In RCMs, it is more relevant to check that the large-scale conditions are correctly provided (e.g., Giorgi et al. 1993). It is indeed usual to find that low pressure systems enter the domain accurately, causing about the same total pressure change as in the forcing data. Also important and commonly obtained is the absence of spurious long-term trends (e.g., mass loss or gain). Third, when both the externally provided fields and the model are in geostrophic equilibrium, the relaxation scheme may cause imbalance. This argument is based on the fact that the linear combination of two sets of balanced fields can result in fields that are not balanced (MacDonald 2000; Staniforth 1997). However, the analytical form of the relaxation scheme does not involve a simple replacement of the model fields by such linear combination (see section 1). Even in the discretized form, the changes introduced by relaxation are usually progressive. Except close to the boundary, where relaxation should be very fast, any imbalance introduced by the scheme is mitigated by the slowness of the relaxation process.

Relaxation is thus a widely used technique that, in spite of its simplicity and imperfections, does not have annoying drawbacks. The purpose of this paper is to make a detailed investigation of the relaxation parameters with
a view toward optimizing common models. We first review how the relaxation method works (section 2); then we present a theoretical assessment of its parameters (section 3). This work is based on Davies (1983, hereafter referred to as D83). The theory involves large simplifications in comparison to RCMs, so that its results can be mainly regarded as indications of possible model behavior. It is then possible to complement the existing developments in order to study the diffusion type of relaxation and compare theoretically optimum parameters with actual model values. Section 4 presents sensitivity tests with the Modèle Atmosphérique Régional (MAR) (Gallée and Schayes 1994), which globally confirm the theoretical results and lead to an improved assessment for the relaxation parameters.

2. The relaxation schemes

a. General formulation

The relaxation boundary scheme consists of progressively constraining the main prognostic variables of the regional model to match the corresponding “large scale” values in a buffer zone next to the lateral boundaries. Two terms are commonly added to the internal model tendency for each variable:

\[
\frac{\partial a}{\partial t} = \ldots - N(a - a_{LS}) + D\nabla^2(a - a_{LS}),
\]

where \( a = a(x, y, z, t) \) is one of the model variables for which information is incorporated from the large-scale, while \( a_{LS} \) is the equivalent variable from the large-scale (LS) data, with \( x = \text{distance (perpendicular)} \) to the boundary; \( N = N(x) \) is the Newtonian relaxation factor; and \( D = D(x) \) is the diffusive relaxation factor. A specific term is sometimes added to the tendency of the tangential velocity \( (u) \):

\[
+ \int_{x'}^{x} \frac{\partial N}{\partial x} (a - a_{LS}) \, dx,
\]

where the integral \( x' \) is computed from the inner domain (where the relaxation factors vanish) to the current \( x \) position, named \( x' \). The aim of this term is to damp the vorticity component of the fields. This was proposed by Davies and Turner (1977), but it was later suggested that it is unnecessary when the two other terms are used (D83), and it will not be studied here.

In addition to relaxation, the regional model variables are forced to be equal to the large-scale data along each lateral boundary, as summarized in Fig. 1.

The first term, called Newtonian relaxation (following Davies 1976), continuously removes a fraction of the difference between the model and the large-scale forcing. Mass and waves enter the domain correctly because the modeled field is equal to the forcing at the boundary, and departures from that situation are only damped in the buffer zone. On the other hand, for outgoing perturbations the relaxation provides a progressive damping toward the large-scale condition. Intuitively, this damping explains that the noise problem that may result from overspecification is mitigated.

The diffusive relaxation term is also based on the difference between the regional model and the forcing data, but its effect is to diffuse that difference horizontally. Note that this is not the same as the “diffusive boundary zone” in which the diffusion term is applied to the model solution. In the diffusive relaxation scheme, the diffusion is applied to the difference between modeled and large-scale fields. This is important because the purely diffusive scheme may degrade the incoming flow and is very sensitive to numerical shortcomings (D83). In contrast, relaxation diffusion should have the advantages of the Newtonian relaxation explained above. To our knowledge, the Newtonian term is included in all models using a relaxation formulation, and most models also include the diffusive relaxation term.

b. Relaxation profiles

The implementation of relaxation in various RCMs differs by the large-scale condition. In Fig. 1. Schematic view of the boundary treatment and notation.

large-scale condition. Intuitively, this damping explains the noise problem that may result from overspecification is mitigated.

The principle of relaxation is that the flow is “relaxed” to the large-scale values in a buffer zone near the boundaries, with a view to avoiding overspecification problems. Intuitively, this means that the relaxation factors are such that the model — forcing differences are canceled out for any flow and waves reaching the boundary, while no adverse effects such as wave reflection are generated in the domain. This principle was the basis of an example of optimization of the relaxation coefficients in D83. We follow the same approach, but we make a comparative evaluation of the common relaxation profiles described above rather than an optimization, and include the diffusive relaxation term.
TABLE 1. Investigated relaxation profiles (relaxation magnitude as a function of the distance to the boundary). The acronym summarizes the name of the profiles (e.g., LIN) and the number of points, s.

<table>
<thead>
<tr>
<th>Type of profile</th>
<th>No. of points</th>
<th>Acronym</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>(N_s = \frac{s + 1 - j}{s - 1})</td>
<td>5</td>
<td>LIN-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>LIN-9</td>
</tr>
<tr>
<td>Parabolic</td>
<td>( \tilde{N}_s = \left( \frac{s + 1 - j}{s - 1} \right)^2 )</td>
<td>5</td>
<td>PAR-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>PAR-9</td>
</tr>
<tr>
<td>Exponential</td>
<td>(\tilde{N}_s = \exp \left( \frac{2 - j}{M} \right))</td>
<td>5</td>
<td>EXP7-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>EXP1-9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>EXP3-11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>EXP3-30</td>
</tr>
<tr>
<td>Optimization from Davies (1983)</td>
<td>(for (c_{\text{max}}/c_{\text{min}} = 100))</td>
<td>5</td>
<td>D83-5</td>
</tr>
<tr>
<td>Optimization from Lehmann (1993)</td>
<td>(for (c_{\text{max}}/c_{\text{min}} = 1000))</td>
<td>9</td>
<td>L93-9</td>
</tr>
</tbody>
</table>

a. Relaxation in a simple system

We first restrict our attention to the Newtonian relaxation term in (1). As in D83, the relaxation scheme is applied on a simple system representing small, hydrostatic perturbations of an isothermal atmosphere in uniform flow, with neither rotation nor viscosity effects, restricted to variations in a vertical plane. This system can be described by linearized equations, which allows a separation of the variables and equations for the horizontal and vertical dependency (see, e.g., D83 or Phillips 1973). The horizontal equations are then equivalent to a linearized shallow-water system:

\[ u_t + \overline{u} u_x = -gh_u \quad \text{and} \]
\[ h_t + \overline{u} h_x = -Hu_u, \quad (3) \]

where \(u\) is the horizontal wind perturbation, \(\overline{u}\) is the mean horizontal wind, \(g\) is the gravity constant, \(h\) is related to the perturbation component of the geopotential, and \(H\) is a constant whose value depends on the vertical mode (the subscripts \(t\) and \(x\) stand for the time and horizontal-space derivatives).

We introduce the relaxation term:

\[ u_t + \overline{u} u_x = -gh_u + \alpha' \]
\[ h_t + \overline{u} h_x = -Hu_u - Nh', \quad (4) \]

where \(u' = u - u_{\text{LS}}\) and \(h' = h - h_{\text{LS}}\) (\(N\) is the relaxation factor).

With a simple linear combination, one obtains the characteristic form

\[ a_t + ca_x = -Na', \quad (5) \]

This is identical to the basis equation in D83 [Eq. (21a)]. Note that we introduced the relaxation term as usual, that is, in the basis equation (4), and then obtained the characteristic form (5). This last expression will show that in the analytical form, the relaxation scheme does not cause wave reflection at the boundaries. Indeed, the solutions of (5) without relaxation (i.e., \(N = 0\)) represent perturbations moving at the speed \(c_u\) or \(c_\cdot\). The important point is that the relaxation term is only adding a damping to each of the equations: even if there is a discontinuity in the values of \(N\), there is no coupling between the perturbations traveling with speeds \(c_u\) and \(c_\cdot\). (An imaginary example of the opposite behavior is to impose an abrupt change in the constant \(H\) at some location: this would result in reflected waves similar to those produced by the radiation phenomenon). In contrast, some other boundary schemes may produce wave reflection at the boundaries (e.g., the tendency modification scheme in D83).

We assume that the large-scale forcing is consistent, that is, that \(a_{\text{LS}}\) is a solution of the unmodified system \(\{a_{\text{LS}}, c(a_{\text{LS}})\} = 0\). The equation for the “error” \(a' = a - a_{\text{LS}}\) is then

\[ a' + ca' = -Na', \quad (7) \]

which emphasizes the fact that the error field is “advected” like the physical fields and progressively damped when it reaches a boundary region. Since the equations for characteristic speeds \(c_u\) and \(c_\cdot\) are identical, we shall now refer to either of these two values as a scalar, \(c\).

b. Diffusive relaxation

The diffusive relaxation term is mentioned but not studied explicitly in D83. The application of diffusion to the model variables is evaluated therein, but as discussed above, it is a different boundary scheme. When the diffusive relaxation term is included, the error equation (7) becomes

\[ a' + ca' = -Na' + D\nabla^2(a'). \quad (8) \]

There is considerable mathematical similarity between the diffusion and diffusive relaxation schemes. The error equation with relaxation diffusion has indeed the same form as the equation for the basis system with diffusion. The difference lies in the involved variable, which is based only on model values \((a)\) in the diffusion case while it is based on the difference between model and forcing \((a')\) in the relaxation–diffusion case. Consequently, the two cases have some similar properties, particularly with regard to the magnitude of numerical
reflection for outgoing characteristics (discussed below). However, the diffusion will modify the model values at all buffer zone points while relaxation diffusion will act only where forcing - model differences exist. The relaxation–diffusion scheme will thus likely impose the incoming characteristics to the model without alteration, while the diffusion scheme will add an unnecessary smoothing.

c. Boundary reflection

Despite the satisfying behavior of the analytical system, wave reflection (or propagation of perturbation from the boundary in the domain) is possible when the equations are discretized. Therefore, the minimization of this numerical problem may be taken as a criterion to propose an “optimal” choice of the relaxation coefficients (D83).

Following D83, we write a simple finite-difference scheme for the error equation (8):  
\[ 1 \frac{1}{2 \Delta t} (a^{n+1}_j - a^{n-1}_j) + c \frac{1}{2 \Delta x} (a^{n+1}_{j+1} - a^{n+1}_{j-1}) \]
\[ = -N_j a^{n+1}_j + \frac{D_j}{(\Delta x)^2} (a^{n+1}_{j-1} - a^{n+1}_j - a^{n+1}_{j+1} + a^{n+1}_{j+2}) \]  
(9)

where \( \Delta x \) and \( \Delta t \) are the model grid size and time step; \( n \) and \( j \) are the time and horizontal indexes, respectively (\( x_j = j \Delta x \)), and we have dropped the prime (′) symbol for simplicity; and \( N_j \) is the relaxation coefficient. As shown in Fig. 1, \( N_j \) is undefined because the model value is forced to be equal to the large-scale value at the

\(^1\)The relaxation term is taken at time \( r + 1 \) as recommended by D83 (implicit scheme). Note that a similar discretization of the basis equation (4) would give the same form for the error equation.
boundary (i.e., \( a_1 = 0 \)), and the relevant coefficients are \( N_2 \ldots \ N_s \), while \( N_j = 0 \) for \( j > s \) (note that this definition of \( s \) follows D83).

To examine the reflection problem, we consider a perturbation coming from the inner domain to the boundary, that is, \( c < 0 \). In the inner part of the domain, the solutions are of the form (D83)

\[
a_j^* = 1 \exp[-i(k_jx + \omega t)] + r(-1)^j \exp[-i(k_jx - \omega t)]
\]  

(10)

where \( k \) is the wavenumber, the frequency is \( \omega/2\pi \), \( t \), \( \omega \), \( n \), \( k \), \( b \), and \( r \) is the reflection coefficient. The first term is the incident wave, which was set to a unit amplitude. The second term is the reflected wave. If the “physical wave” is spatially well resolved \( (\Delta x/k \ll 1) \), the \((-1)^j\) factor changes the sign of the reflected perturbation from mesh to mesh: this is a 2\( \Delta x \) wave, which is modulated by the physical signal.

The minimization of the reflection coefficient, \(|r|\), in (10) was taken as the optimization criterion for the relaxation coefficients in D83, and extensively studied in Lehmann (1993, hereafter referred to as L93). However, these works are based on a simple numerical scheme that was presented as an example in D83. It is thus important to remember that the results may be the consequence of the discretization used for the advection term, which enables 2\( \Delta x \) noise in the solution (see below). Things may be different in practical applications since 2\( \Delta x \) noise is easily filtered out and since the numerical schemes may be more sophisticated. On the other hand, our analytical equations do not enable reflection, although we were starting from a simple but physically based system: this suggests that the most important problem is indeed that of numerical origin. It would be very difficult to study this problem for sophisticated numerical schemes. Consequently, we try to gain some indications about optimal relaxation coefficients from the theory presented in D83 even though we are aware of its limitations.

d. Reflection coefficient

We shall now compute the reflection coefficient, using a method from L93. This method is simplified and restrictive. However, as pointed out in L93, it gives the same result as the more general development in D83. We consider the limit case of an incoming (initial) solution of infinite wavelength. The system will therefore approach a steady state, so that (9) becomes

\[
\frac{c}{2\Delta x}(a_{j+1} - a_{j-1}) = -N_ja_j + \frac{D_j}{(\Delta x)^2}(a_{j-1} - 2a_j + a_{j+1}).
\]  

(11)

In the inner model domain where \( (N_j = D_j = 0) \),

\[
a_{j+1} = a_{j-1} \quad \text{for} \quad j > s.
\]  

(12)

In this limiting case, it is evident that the centered-differences scheme enables 2\( \Delta x \) noise. In agreement with the definition of the reflection coefficient in (10), we have

\[
a_t = a_{s+2} = \ldots = 1 + r,
\]

\[
a_{s+1} = a_{s+3} = \ldots = 1 - r.
\]  

(13)

A simple algebraic manipulation gives an expression for the reflection coefficient \(|r|\):

\[
|r| = \frac{1 - \mu}{1 + \mu}.
\]

(14)

with

\[
\mu = \frac{a_{s+1}}{a_s}.
\]  

(15)

This is obtained from the properties of the numerical system in the inner domain. To connect \(|r|\) with the relaxation coefficients, we must now use the properties of the system in the boundary region [from Eq. (11)]:

\[
a_s = N_s a_s - D_s (-2a_s + a_1)
\]

\[
a_4 - a_2 = N_s a_3 - D_s (a_2 - 2a_1 + a_1)
\]

\[
a_s - a_i = N_s a_4 - D_s (a_4 - 2a_3 + a_3)
\]

\[
\quad \vdots
\]

\[
a_{s+1} - a_{s-1} = N_s a_s - D_s (a_{s-1} - 2a_s + a_{s+1}).
\]  

(16)

with

\[
N_s = -\frac{2N_\Delta x}{c} \quad \text{and}
\]

\[
D_s = -\frac{2D_\Delta x}{c}\Delta x.
\]  

(17)

(18)

The system (16) involves \( s - 1 \) equations with \( s \) unknowns \( (a_1) \) so that it is possible to obtain the ratio \( \mu = a_{s+1}/a_s \) as a function of the coefficients \( N_s \) by successively eliminating all the other \( a_j \). The resulting expression for the reflection coefficient is given in D83 for the Newtonian relaxation case. When diffusive relaxation is introduced, the analytical form becomes cumbersome. Since we seek numerical values of the reflection coefficient, the substitution can be done while computing numerical values. We divide (16) by \( a_2 \), compute all the \( a_i/a_2 \), and in turn obtain \(|r|\). We checked that this gives the same results for the specific cases presented in D83, that is, the Newtonian relaxation alone and the diffusion with a constant coefficient (the result of D83 was presented in the context of diffusion rather than relaxation diffusion, but the reflection curves are the same, as explained in section 3b).

In summary, the formulas (14)–(18) relate the reflection coefficient \(|r|\) to a choice of the relaxation coefficients \( N_s \), \( D_s \), \( \Delta x \), and a given characteristic speed \( c \).
e. Assessment basis

In the context of this study, the optimum relaxation coefficients are those that minimize the reflection coefficient $|r|$ for waves reaching the boundary. We shall now detail the application of this criterion.

As explained above, $|r|$ is a function of the nondimensional numbers $N_j^T, D_j^s (j = 2, 3, \cdots s)$. To facilitate the discussion of the optimization process, we recast Eq. (17) as

$$
N_j^T = N_j^T \bar{N}_j, \quad \text{with} \quad N_j^T = -\frac{2N_j \Delta x}{c}, \quad (19)
$$

so that we separate $\bar{N}_j$, the normalized horizontal profile of the coefficients, and $N_j^T$, which will be referred to as the leading coefficient, and which defines the overall amplitude of relaxation (the index is $j = 2$ because this is the first point that involves relaxation, while at $j = 1$ the model is fixed to the large-scale values). Similarly,

$$
D_j^s = D_j^s \bar{D}_j, \quad \text{with} \quad D_j^s = -\frac{2D_j}{c \Delta x}. \quad (20)
$$

The primary need for our criterion is to evaluate the range of characteristic speed $c$. The maximum of this variable is about the same as the maximum phase speed for gravity waves, as shown by Eq. (6) in which $(gH)^{1/2} \gg \bar{U}$. In complex models, the maximum speed of gravity waves (and sound) will also be close to the maximum phase speed for any kind of perturbation. The choice of the maximum speed is thus not directly connected with the simplicity of the present theoretical framework. We set this upper bound to $c_{\text{max}} = 350 \text{ m s}^{-1}$. The minimal speed is probably more difficult to estimate. In D83, the principle is that “slowly moving waves will not penetrate sufficiently into the boundary zone during the period of integration.” Our problem is in finding a criterion applicable to climate simulation (i.e., without reference to the timescale of meteorological prediction). We suggest estimating $c_{\text{min}}$ as “a few $\Delta x$” divided by “a maximal time between changes in the large-scale flow.” The length scale, a few mesh sizes, is assumed to be the minimum distance that a perturbation must travel so that spurious interaction with the boundary could happen. Note that this length is usually shorter than the buffer zone and is not linked to it: the underlying motive is that reflection can start anywhere in this zone. The retained time is assumed to be the maximum time that a perturbation can interact with the boundary. After that time period, the large-scale conditions will usually have changed significantly, thus making a persistent model — forcing difference unlikely on longer timescales. For $\Delta x = 50 \text{ km}$ in midlatitude regions, our estimation is $c_{\text{min}} = 150 \text{ km per 6 days} = 0.3 \text{ m s}^{-1}$. At finer scales or in other regions, it may possibly be useful to consider lower speeds.

The ratio of the maximum to minimum phase speed is therefore $c_{\text{max}}/c_{\text{min}} = 1000$. In comparison, D83 uses $c_{\text{max}}/c_{\text{min}} = 100$ as an example, while L93 presents results for 10, 100, and 1000. It seems thus reasonable to seek a ratio of 1000 in the climate application context. Consequently, the ratio of the minimum and maximum values of $N_j^T$ and $D_j^s$ of interest is

$$
\frac{N_j^T_{\text{max}}}{N_j^T_{\text{min}}} = \frac{D_j^s_{\text{max}}}{D_j^s_{\text{min}}} = \frac{c_{\text{max}}}{c_{\text{min}}} = 1000. \quad (21)
$$

We may now evaluate each profile $\bar{N}_j$ with regard of the reflection problem. As an example, let us consider Newtonian relaxation alone, with the “optimized” profile from D83. We compute the reflection coefficient as a function of $N_j^T$, using Eqs. (14)–(16), (19), and (20). The result is given in Fig. 3. In this example, the requested range of minimal reflection is $N_j^T_{\text{2min}}/N_j^T_{\text{2max}} = 100$, because the coefficients are computed to match D83. If the profile is optimized for a larger $N_j^T_{\text{3max}}/N_j^T_{\text{3min}}$ ratio, the mean and the peak reflection increase in this $N_j^T$ interval. Therefore, the profiles are a matter of compromise between two needs: to account for a large band of speeds and to obtain a low peak reflection.

To complete our Newtonian relaxation example, it is necessary to decide which part of the $N_j^T$ domain will be used. A way to do this is to select the best value for $N_j^T_{\text{2min}}$, the minimal $N_j^T$ that will be used and therefore corresponds to $c_{\text{min}}$ through Eq. (19). From this value and the previously selected profile, all the coefficients ($N_j$) are known.

In summary, our study of relaxation coefficient profiles will consist of (i) defining the profile, that is, $\bar{N}_j (j = 2, 3, \cdots s)$; (ii) assessing for the reflection properties of this profile, that is, find out the size of the $N_j^T$ interval over which the reflection is low and the amount of unavoidable reflection; and (iii) assessing for the choice of the overall relaxation magnitude, that is, decide the value of $N_j^T_{\text{2min}}$ with $c_{\text{max}} = 350 \text{ m s}^{-1}$. The same prin-
pilciples apply to diffusive relaxation. The two terms will first be investigated separately, then an assessment will be done for their association.

f. Assessment for the profile

1) NEWTONIAN RELAXATION

The reflection profiles are plotted for Newtonian relaxation alone in Fig. 2, for the relaxation profiles presented in Table 1. According to the above theory, our "quality" assessment for the profiles is based on their ability to provide only weak reflection over a suitably large range of speed, which we seek to be equivalent to a maximum-to-minimum speed ratio equal to at least 10. The results are summarized in the first column of Table 2, which gives the range of speed (or \(N_s\)) over which it is possible to obtain less than 10% of reflection. It is clear that some profiles reach the requested speed ratio very well, while others do not.

The linear profile was used in the early climate simulations with the Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model, version 4 (MM4) (Anthes et al. 1989), with relaxation on three points only (\(s = 4\)). With regard to our minimal reflection criterion, this profile was clearly not appropriate, even with a larger boundary zone. The Second-Generation Regional Climate Model (RegCM2) team came to the same conclusion after experimenting with large domains, and turned to the exponential profile (Giorgi et al. 1993).

The parabolic profile was used in Davies (1976), and the \(s = 5\) version is the standard choice in MAR, which will be described in the next section (however, diffusive relaxation is dominant in this model, as explained below). Although the results are better than for the linear profile, these are far from matching our requirement of \(c_{\text{max}}/c_{\text{min}} = 1000\).

The exponential profiles reported here are based on experiments with RegCM2 from Giorgi et al. (1993). A peculiar feature of these profiles is that the relaxation is activated over the whole domain. We try to evaluate this choice by including two boundary region sizes in our study. The case \(s = 11\) (EXP3-11) corresponds to a last nonzero coefficient equal to 5% of the first coefficient (i.e., \(N_s = 0.05\)): the relaxation zone is "abruptly" cut after the point. In the framework of the present theory, this causes much more reflection than allowing relaxation over all or a large part of the model domain, as in the case EXP3-30. To achieve low reflection while applying relaxation to a small buffer zone, it proves useful to choose a fast exponential decay, as in EXP1-9. To compare results, we also test a five-point exponential profile, which proves to be better with an even higher exponential decay (EXP7-5). In summary, it seems necessary and logical to avoid "cutting" the exponential profile at a point that should still have a significant relaxation coefficient.

### Table 2. Performances of the various profiles. The \(c_{\text{max}}/c_{\text{min}}\) columns give the speed ratio that can be achieved with no more than 10% reflection (ratios higher than \(4 \times 10^4\) exceed our computation range and the expected needs). These results are given for Newtonian relaxation (\(N\)) and for diffusion relaxation (\(D\)). The last column shows the reflection that must be tolerated to obtain the required speed ratio (\(c_{\text{max}}/c_{\text{min}} = 10^3\); see text).

| Abbreviation | \(c_{\text{max}}/c_{\text{min}}(10\%)\) | For \(D\): max \(\{|r|\ (\%), c_{\text{max}}/c_{\text{min}} = 10^3\) |
|--------------|-----------------|-----------------|
| LIN-5 | 3 | 286 | 11 |
| LIN-9 | 11 | >4.10^4 | 5 |
| PAR-5 | 17 | 714 | 10 |
| PAR-9 | 100 | >4.10^4 | 4 |
| EXP7-5 | 76 | 2460 | 9 |
| EXP1-9 | 920 | >4.10^4 | 1 |
| EXP3-11 | 16 | >4.10^4 | 4 |
| EXP3-30 | >3600 | >4.10^4 | <0.1 |
| D83-5 | 123 | 3465 | 8 |
| L93-9 | 2070 | >4.10^4 | 0.7 |

The "optimized" profiles, obtained in D83 for \(s = 5\) and in L93 for \(s = 9\) (as an example), perform as expected; that is, they provide a low reflectivity up to the speed ratio for which they are designed (100 for D83 and 1000 for L93). Because of this optimization, the profile L93-9 performs better than all the others, but the corresponding exponential profile (EXP1-9) is quite close to the optimum (Table 2).

2) DIFFUSIVE RELAXATION

The reflection curves obtained for the various relaxation profiles in the context of the diffusive relaxation are shown in Fig. 4. The obtained reflection coefficients are much lower than for Newtonian relaxation. All profiles provide a weak or very weak reflection over a large range of speeds as reported in the right column of Table 2. The linear and parabolic profiles perform in a relatively similar fashion. The parabolic is still somewhat better, and the maximum ratio of speed that enables, at most, 10% reflectivity is nearly a match to our request of \(c_{\text{max}}/c_{\text{min}} = 1000\). All exponential profiles give excellent results. We note that the extension of relaxation to the whole domain (EXP3-30) results in almost no reflection at the boundaries for any speed.

3) COMBINATION OF NEWTONIAN AND DIFFUSIVE RELAXATION

Several models use both the Newtonian and diffusive relaxation terms. To assess this combination, we selected a satisfying profile from the previous tests, the exponential profile over nine points (EXP1-9). We have applied this profile to both terms, but with several different weights. To achieve this, we define a new overall relaxation parameter, \(K\), and let \(N_{\text{new}} = xK, D_{\text{new}} = (1 - x)K\). Therefore, \(K\) is also proportional to \(1/c\) and can be used in the same way as \(N_{\text{new}}^\frac{s}{2}\) and \(D_{\text{new}}^\frac{s}{2}\). The resulting reflection curves are plotted in Fig. 5.
The reflection curve for the combined cases always lies between the “pure” Newtonian and diffusive curves. The results for pure diffusive relaxation are the best. To keep a similarly low reflection while introducing Newtonian relaxation, it is necessary to maintain the relative weight of the Newtonian term at a few percent or less (in terms of $N_0^*/K$).

**g. Assessment for the leading coefficient**

We will now focus on the overall amplitude (or rate) of relaxation. Our analysis uses the “leading coefficients” introduced above ($N_0^*, D_0^*$), which provide reflection curves that are not explicitly dependent on the time step and space resolution of the model, nor on a wave speed, $|r| = fct(N_0^*, D_0^*)$, for each profile. According to the theory, the relaxation coefficients are connected to the parameters ($N_0^*, D_0^*$) by Eqs. (19) and (20). This introduces the time step and horizontal resolution, and may be expressed differently in models (see the appendix). In the following discussion, we use the parameters $N_0^*_{(2150)}$ and $D_0^*_{(2150)}$ to state the overall amplitude of relaxation in the model: this notation emphasizes the fact that it corresponds to $c = 350 \text{ m s}^{-1}$, a speed reasonably faster than any wave. Therefore, it should be remembered that the reflection obtained for $N_0^*_{(2150)}$ is not always low, as is normally required for $N_0^*_{\min}$; this is simply because some of the profiles do not provide low reflection over the required range of speeds.

Several choices of relaxation coefficients are described in Table 3, either on the basis of model values or theoretical options. The model values where selected independently of D83 and subsequent theoretical works and are assumed to be mainly based on empirical testing. The models use both relaxation terms simultaneously, but it is important to note that one term dominates: for the MM4 and RegCM cases, the dominant term is Newtonian relaxation, while it is diffusive relaxation for MAR (this estimation is based on the ratio $N_0^*_{(2150)} / D_0^*_{(2150)}$ and the above discussion.

For both model and theoretical parameter choices, the lines $c_{\min105}$ and $c_{\max105}$ give, respectively, the maximum and minimum wave speeds that do not cause more than 10% of the reflection. This is an approximate range of phase speeds for which the scheme is theoretically efficient.

For some of the profiles, we have shown that it is not possible to cover the requested range of speed, that is, $c_{\max} / c_{\min} \approx 1000$; there is no theoretical optimum for the relaxation amplitude. To make a minimal estimation,
Table 3. Theoretical properties (shown in rows) of different relaxation configurations (in columns) $N^2$ and $D_t$ are the leading relaxation coefficients used to define the overall relaxation rate; taken together with the profile, these define the relaxation entirely. Model rows show relaxation rates from modeling practice (available only for the configurations that come from models). Theory rows show theoretically optimal relaxation rates. Here, $c_{\text{min}}$ and $c_{\text{max}}$ give the range of phase speeds that cause no more than 10% of reflection at the boundaries (according to theory, when each term is used alone). Here, $1/N_2$ is the e-folding time for Newtonian relaxation (in min) and $\tau_{[4\Delta x, 50\text{km}]}$ is the e-folding time for diffusion relaxation (in min), for 4\Delta x wavelength perturbations (see text).

<table>
<thead>
<tr>
<th>Model reference</th>
<th>Profile</th>
<th>MM4 (Anthes et al. 1989)</th>
<th>This study</th>
<th>MAR standard</th>
<th>RegCM2 (Giorgi et al. 1993)</th>
<th>This study</th>
<th>Theory (D83,L93)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile Abbreviation</td>
<td>Linear</td>
<td>s = 4</td>
<td>PAR-9</td>
<td>PAR-5</td>
<td>s = dom.</td>
<td>EXP3-10</td>
<td>L93-9</td>
</tr>
<tr>
<td>No. of points</td>
<td>s = 9</td>
<td>s = 9</td>
<td>EXP1-9</td>
<td>EXP7-10</td>
<td>s = dom.</td>
<td>EXP3-10</td>
<td>L93-9</td>
</tr>
<tr>
<td>$N^2$ $[m^2 s^{-2}]$</td>
<td>0.2</td>
<td>0.06</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
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<td>$c_{\text{max}}$ [10%]</td>
<td>79</td>
<td>2.1</td>
<td>123</td>
<td>123</td>
<td>123</td>
<td>123</td>
<td>123</td>
</tr>
<tr>
<td>$c_{\text{min}}$ [10%]</td>
<td>28</td>
<td>0.13</td>
<td>0.03</td>
<td>0.03</td>
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<tr>
<td>$1/N_2$ [50 km]</td>
<td>24</td>
<td>800</td>
<td>24</td>
<td>24</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Theory $N^2$ $[m^2 s^{-2}]$</td>
<td>0.1-0.6</td>
<td>0.02-0.9</td>
<td>(0.6-0.9)</td>
<td>0.95</td>
<td>0.1-0.9</td>
<td>0.1-0.9</td>
<td>0.1-0.9</td>
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<tr>
<td>$c_{\text{max}}$ [10%]</td>
<td>60-350</td>
<td>8-350</td>
<td>$\geq 350$</td>
<td>350</td>
<td>40-350</td>
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<td>350</td>
</tr>
<tr>
<td>$c_{\text{min}}$ [10%]</td>
<td>0.6-3.5</td>
<td>0.5-21</td>
<td>$\leq 0.15$</td>
<td>0.4</td>
<td>0.6-5</td>
<td>0.18</td>
<td>0.18</td>
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<tr>
<td>$1/N_2$ [50 km]</td>
<td>48-8</td>
<td>240-5</td>
<td>8-5</td>
<td>5</td>
<td>48-5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Model $D^2_t$ $[s^{-2}]$</td>
<td>0.04</td>
<td>0.6</td>
<td>0.04</td>
<td>0.6</td>
<td>0.04</td>
<td>0.6</td>
<td>0.04</td>
</tr>
<tr>
<td>$c_{\text{max}}$ [10%]</td>
<td>35</td>
<td>410</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>$c_{\text{min}}$ [10%]</td>
<td>$&lt;10^{-4}$</td>
<td>0.58</td>
<td>$&lt;10^{-4}$</td>
<td>0.58</td>
<td>$&lt;10^{-4}$</td>
<td>0.58</td>
<td>$&lt;10^{-4}$</td>
</tr>
<tr>
<td>$\tau_{[4\Delta x, 50\text{km}]}$</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Theory $D^2_t$ $[s^{-2}]$</td>
<td>0.03-0.45</td>
<td>0.6</td>
<td>0.3-0.5</td>
<td>(0.3-0.8)</td>
<td>0.5-0.8</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>$c_{\text{max}}$ [10%]</td>
<td>23-350</td>
<td>350</td>
<td>210-350</td>
<td>$&gt;350$</td>
<td>$&gt;350$</td>
<td>$&gt;350$</td>
<td>415</td>
</tr>
<tr>
<td>$c_{\text{min}}$ [10%]</td>
<td>0.5-7</td>
<td>$&lt;10^{-2}$</td>
<td>0.3-5</td>
<td>$&lt;10^{-2}$</td>
<td>$&lt;10^{-2}$</td>
<td>$&lt;10^{-2}$</td>
<td>$&lt;10^{-2}$</td>
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<tr>
<td>$\tau_{[4\Delta x, 50\text{km}]}$</td>
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<td>7-4</td>
<td>(7-2)</td>
<td>(4-2)</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

It must be further decided whether “slow” or “high” speeds should be favored, that is, given the priority to correspond to the low reflection part of the $N^2$ curve. In this case, a range of values is given for the relaxation parameter, with corresponding values for $c_{\text{max}}$ and $c_{\text{min}}$. For the EXP1-9 and EXP3-30 cases, we also give a range (set in parentheses in Table 3), but the reason for this is different: these values are not all “optimal” but they still provide less than 10% reflection because of the very large low reflection band ($c_{\text{max}}/c_{\text{min}} \gg 1000$).

### h. Discussion

We will now try to identify pragmatic explanations of the theoretical findings, which will help indicate that many results are not the consequences of the simplifications involved in the theory.

**Newtonian compared to diffusive.** The theoretical developments suggests that the diffusive relaxation provides lower reflection at the boundaries than the straightforward Newtonian relaxation. An intuitive explanation for this is connected with the “likely location” of the potential RCM–LS forcing conflict at outflow points. It is probable that such a conflict will often occur closer to the lateral boundary in the case of diffusive relaxation. Indeed, the diffusion form allows differences between RCM and LS fields, provided that these do not involve short waves. In addition, Tatsumi (1980) attributes the better behavior of diffusive relaxation found in testing to the fact that the short waves are damped more rapidly in that case. Putting the two ideas together, it seems that the advantage of diffusive relaxation is to let long-wave RCM–LS differences penetrate the buffer zone while damping the short-wave differences very efficiently. In the context of data assimilation (over an entire domain), it was shown theoretically for a 3D model (Davies and Turner 1977) that the Newtonian form is not efficient for short wavelengths, while there is no problem for the diffusive relaxation. In D83, it was also suggested that diffusive relaxation has the extra advantage of providing a better damping of outgoing vorticity perturbations, so that it would compensate for the absence of the specific vorticity term [Eq. (2)]. When diffusion and Newtonian relaxation are combined, the reflection curve lies between the two pure cases, thus giving no advantage to the combination.

**Profiles.** According to theory, the best performing profile is the exponential one, followed by the par-
able one, while the linear profile causes much stronger reflection. Following D83, it is possible to reduce boundary reflection somewhat more than with exponential profiles by using optimized relaxation coefficients designed for that purpose. An interesting result obtained here is that the same values of the coefficients provide excellent results for the diffusive relaxation case (for which the coefficients were not optimized). As pointed out by H. C. Davies (1999, personal communication), the optimal choice of coefficients will almost certainly depend on the numerical schemes of each model. Furthermore, finding optimal coefficients is relatively difficult, even in the simplified case considered here (see L93). In this context, the exponential profiles may have an additional advantage: they can cover a large range of relaxation magnitudes, so that we suggest that there should always be a good part in the curve: in the absence of certainty, this may be a wise choice.

**Overall relaxation rate.** In summary, the theoretical best value for the leading coefficient is around 0.9 for $N_{*}$ and 0.8 for $D_{*}$ for all profiles that can satisfy our low reflection requirement. If lower relaxation coefficients are used, typically with a leading coefficient under 0.5 (for both terms), the low reflection band extends to lower speeds, but then large reflection may occur at high speeds. In the case of a narrow (five point) boundary buffer, most configurations cannot provide low reflection for both low and high speeds (with the speed ratio equal to 1000). In such a case, the theory does not provide reasons for favoring high or low speeds.

To better understand these results, characteristic relaxation times are reported in Table 3 for a mesh size of 50 km. For the Newtonian case, the reported value is $1/N_{*}$, which is the e-folding time for the decay of the model forcing difference if the relaxation can be regarded as constant in space [see, e.g., Eq. (7)]. Thus $1/N_{*}$ is the time required to reduce the error to less than 40% of its previous value. For diffusive relaxation, the reported value is again the e-folding time of the damping process (denoted $\tau_{D}[4\Delta x, 50km]$), but specifically for $4\Delta x$ wavelength perturbations (see the appendix). Note that $2\Delta x$ perturbations will be damped about four times faster. By reporting the e-folding time for $4\Delta x$ waves, we try to account for the fact that model results suggest the existence of boundary noise at wavelengths larger than $2\Delta x$ (see section 4). For the profiles that can fully satisfy our low reflection requirement, the typical relaxation time is 5 min for the Newtonian form and about 4 min or less for the diffusive form. We can now understand that these theoretically optimal relaxation rates are reasonable values. Longer relaxation times would mean little relaxation effect on fast waves: for example, traveling at 330 m s$^{-1}$ on a 50-km mesh takes about 2.5 min, so that the above relaxation time corresponds to a damping that is still significant but not very large. Reducing relaxation coefficients under the above optimal values is clearly not desirable except for the case that requires a compromise to allow for the damping of slow waves, as detailed above. On the other hand, using higher relaxation coefficients is normally useless. A side effect for the diffusive term is that such very high relaxation rates can cause an obvious numerical instability. With excessive relaxation rates, the first relaxed point near the lateral boundary ($j = 2$), and then other points in the boundary buffer, behave as the points located on the last boundary row: the model value is forced equal to the large scale. This essentially reduces the size of the buffer zone since it brings the “boundary wall” toward the interior of the model domain. Increasing all relaxation coefficients may also be a problem at the last relaxed point next to the free model domain ($j = s$), where the relaxation coefficient can be too large. The result may be a wall effect on the free model side of the buffer zone, which is expected to cause noise mainly related to slow incoming waves (because on these points, the relaxation should still be too weak to act excessively on fast waves). Another possible mechanism is that a strong forcing area may inherently generate noise that should normally be damped in a low relaxation area before reaching the free integration area, which is missing if the relaxation is strong everywhere. In summary, a kind of smooth transition of the relaxation rates is necessary between the free domain and the boundary, while inappropriate tuning of the relaxation should cause adverse effects either near the lateral boundary (lack of relaxation) or next to the buffer zone, in the free model domain (excessive relaxation).

4. Sensitivity experiments

a. Description of the model and the experiments

A general description of MAR is given in Gallée and Schayes (1994) and Gallée (1995). The present model setup is close to that of Brasseur (2001). MAR is a hydrostatic primitive equation model in which the vertical coordinate is the normalized pressure ($P$). The advection of temperature, humidity, and wind fields is represented by a semi-Lagrangian scheme. The pressure terms are included using centered second-order-in-time and fourth-order-in-space schemes. To avoid the generation of short-scale spurious horizontal waves, the low-pass filter proposed by Raymond and Gardner (1988) is used. The weighting parameter of this filter is set to a moderate value (0.0075); this is expected to completely remove $2\Delta x$ waves and to have a weak damping effect at slightly longer wavelengths. The solar radiation computations are based on the SUNRAY scheme of Fouquart and Bonnel (1980). The longwave radiation scheme follows a wide-band formulation of the radiative transfer equation (Morcrette 1984). The vertical subgrid-scale fluxes are treated using the 1.5-order turbulent
The domain size is 3000 km for the representation of deep convection. The model also includes a convective adjustment scheme derived from Fritsch and Chappell (1980). The model also includes a convective adjustment scheme derived from Fritsch and Chappell (1980). The kinetic energy that is exclusively due to high-resolution circulations. It is computed as $1/2(U_m^2 + V_m^2)$, where $(U_m, V_m)$ are the mesoscale component of the velocity, obtained by subtracting the average velocity field over $5 \times 5$ gridpoint squares from the total velocity field (Giorgi et al. 1993). We have maintained the limit size at five meshes, so that the physical length is smaller in the present study (250 km) than in the work of Giorgi et al. (approximately 350 km). This maintains the principle that the MKE approximately relates to scales that can be represented in the RCM but not in the driving analysis, since both have a somewhat higher resolution in the present study. As shown in Giorgi et al. (1993), the MKE associated with the physically sound motions is increasing from the edge of the grid toward the interior of the domain, because high-resolution features are absent in forcing data and are progressively generated in the RCM. Therefore, high MKE values found near the boundary are expected to be related to noise. The selection of the separation length between MKE and the lower resolution is thus connected to the expected length scale of nonphysical signals that might be generated or reflected at the boundaries. While two-grid-length noise was found in the theoretical investigation, this finding cannot be directly extrapolated to models, which are much more complex and which include in particular a filtering of such waves. However, including larger scales in the definition of MKE does not seem to reveal more noise and is, thus, uninteresting because some large-scale signal becomes mixed with the investigated noise. Note the MKE results reported here relate only to the outflow points (outgoing velocity points, not outgoing characteristics). When all points are included in the computation of the mean MKE, the resulting curves are very similar although the peaks associated with boundary errors are sometimes a little lower.

The rms difference between LS and RCM wind fields is also used to evaluate boundary anomalies in Giorgi et al. (1993). However, it does not appear to reveal more than the MKE. This suggests that all significant boundary errors involve small-scale motions, since these are the only ones shown in the MKE field.

b. Assessment tools

1) MESOSCALE KINETIC ENERGY

The aim of the sensitivity experiments is to investigate the quality and potential flaws of the various boundary configurations in a “real” RCM, that is, to find out if the LBC scheme actually does provide the large-scale information to the model with no adverse effects. A practical method for detecting errors coming from the lateral boundaries in different LBC configurations has been presented in Giorgi et al. (1993). They compute average variables over all grid points at a given distance from the grid edges, and show the results as a function of that distance. From their results, it appears that plots of mesoscale kinetic energy (MKE) are clearly showing faulty boundary configurations. The MKE is the part of the kinetic energy that is exclusively due to high-resolution circulations. It is computed as $1/2(U_m^2 + V_m^2)$, where $(U_m, V_m)$ are the mesoscale component of the velocity, obtained by subtracting the average velocity field over $5 \times 5$ gridpoint squares from the total velocity field (Giorgi et al. 1993). We have maintained the limit size at five meshes, so that the physical length is smaller in the present study (250 km) than in the work of Giorgi et al. (approximately 350 km). This maintains the principle that the MKE approximately relates to scales that can be represented in the RCM but not in the driving analysis, since both have a somewhat higher resolution in the present study. As shown in Giorgi et al. (1993), the MKE associated with the physically sound motions is increasing from the edge of the grid toward the interior of the domain, because high-resolution features are absent in forcing data and are progressively generated in the RCM. Therefore, high MKE values found near the boundary are expected to be related to noise. The selection of the separation length between MKE and the lower resolution is thus connected to the expected length scale of nonphysical signals that might be generated or reflected at the boundaries. While two-grid-length noise was found in the theoretical investigation, this finding cannot be directly extrapolated to models, which are much more complex and which include in particular a filtering of such waves. However, including larger scales in the definition of MKE does not seem to reveal more noise and is, thus, uninteresting because some large-scale signal becomes mixed with the investigated noise. Note the MKE results reported here relate only to the outflow points (outgoing velocity points, not outgoing characteristics). When all points are included in the computation of the mean MKE, the resulting curves are very similar although the peaks associated with boundary errors are sometimes a little lower.

The rms difference between LS and RCM wind fields is also used to evaluate boundary anomalies in Giorgi et al. (1993). However, it does not appear to reveal more than the MKE. This suggests that all significant boundary errors involve small-scale motions, since these are the only ones shown in the MKE field.

2) SEA LEVEL PRESSURE NOISE

The sea level pressure noise (SLPN) field represents short-wave perturbations in the sea level pressure field. The technique is similar to that used for the computation of MKE. The five-gridpoint running mean is first subtracted from instantaneous mean sea level pressure fields, then the rms of these values is computed for the
time period. This technique is used because we found noise in the surface pressure field of some simulations in circumstances that are not always the same as those that cause significant MKE peaks.

c. Results

The relaxation configurations tested are listed in Table 4, along with a summary of the results. These configurations involve buffer zone sizes of \( s = 5 \) points (Fig. 7) and 9 points (Fig. 8). The simulation named P5/D1 is used as a reference; it corresponds to the standard MAR configuration, in which relaxation is dominated by the diffusive term. While Figs. 7–8 give actual values of MKE and SLPN, Table 4 reports differences between each experiment and P5/D1 to highlight the best configurations. The results are given at grid point \( i = 6 \), which is the first nonrelaxed point for the five-gridpoint buffer zones. This position proved to be satisfactory for obtaining a summary of the relaxation configurations. In addition we report the position of the MKE peak, if there is one.

While reviewing the various cases, we will first focus on two important issues: assessing the choice of the overall relaxation rate parameters \( (N_{\text{MAR}}^{5(350)}, D_{\text{MAR}}^{5(350)}) \) and checking that the performance of the profiles ranks as in the above theoretical considerations.

1) Five-point Newtonian relaxation

For the parabolic profile (simulations P5/N1–P5/N4 in Table 4), \( N_{\text{MAR}}^{5(350)} = 0.5 \) or above gives very significant MKE peaks next to the buffer zone, in the inner model domain. Lower relaxation parameters result in lower MKE peaks, and furthermore these are located closer to the boundary. This suggests that the lower relaxation magnitude is to be preferred. However, \( N_{\text{MAR}}^{5(350)} \leq 0.2 \) results in higher values of our surface pressure noise.

### Table 4. Summary of sensitivity experiments. (top) Five- and (bottom) nine-point relaxation buffer sizes are considered. In both cases, experiments with Newtonian relaxation alone or dominant are presented \((N \gg D)\), as well as experiments with dominant diffusive relaxation \((D \gg N)\). Reported results: difference in MKE \((\text{m}^2 \text{s}^{-2})\) between the experiment and the control run \((P5/D1)\), gridpoint index of the MKE peak (when occurring), and amplitude of short waves in MSLP field (hPa) at grid point \( 6 \) (difference with control). Bold values emphasize potential weaknesses of the relaxation configuration. Here "(up)" indicates that the peak was only or mainly found in the upper-tropospheric levels.

<table>
<thead>
<tr>
<th>Profile</th>
<th>( N_{\text{MAR}}^{5(350)} )</th>
<th>( D_{\text{MAR}}^{5(350)} )</th>
<th>Expt name</th>
<th>MKE difference at ( i = 6 )</th>
<th>SLPN difference at ( i = 6 )</th>
<th>Comments</th>
</tr>
</thead>
</table>
| **Five-point buffer zone**
| \( N \gg D \) | | | | | | |
| PAR-5 | 0.05 | 0 | P5/N1 | +0.11 (up) | 4 | +0.12 |
| | 0.2 | 0 | P5/N2 | +0.45 | 5 | | +0.09 |
| | 0.5 | 0 | P5/N3 | +1.02 | 6 | | +0.06 |
| | 0.8 | 0 | P5/N4 | +1.46 | 6 | | +0.06 |
| EXP7-5 | 0.2 | 0 | E5/N1 | +0.12 | 4 | | +0.08 |
| | 0.6 | 0 | E5/N2 | +0.28 | 5 | | +0.05 |
| | 0.6 | 0 | O5/N1 | +0.02 (up) | 4 | | +0.00 |
| **Nine-point buffer zone**
| \( N \gg D \) | | | | | | |
| EXP1-9 | 0.2 | 0.4 | E9/N1 | −0.01 (4)(up) | | +0.06 |
| | 0.2 | 0 | E9/N2 | +0.01 (4)(up) | | +0.08 |
| | 0.5 | 0 | E9/N3 | +0.03 | 6 | | +0.03 |
| | 0.9 | 0 | E9/N4 | −0.01 (6) | | +0.01 |
| | 0.9 | 0 | O9/N1 | +0.05 | 6 | | +0.01 |
| **D \gg N** | | | | | | |
| PAR-9 | 0 | 0.7 | P9/D1 | −0.51 | | −0.01 |
| | 0 | 0.5 | E9/D1 | +0.02 | 3 | | +0.08 |
| | 0 | 0.7 | E9/D2 | −0.13 | 4 | | 0.00 |
| | 0 | 0.9 | E9/D3 | −0.17 | 4 | | 0.00 |
| | 0 | 0.9 | E9/D4 | −0.19 | 4 | | 0.00 |
| | 0 | 0.7 | O9/D1 | −0.15 | 4 | | 0.00 |
index. This can be a concern because high SLPN values extend in the model domain, outside the boundary buffer (Fig. 7, bottom panel, simulation P5/N1). The best choice for the relaxation rate parameter is thus about 0.2 (or a little higher). The case of large MKE peaks in the inner model domain close to the buffer zone is likely explained by the wall effect of excessive relaxation rates over the entire buffer zone. On the other hand, a likely consequence of low relaxation rates is that some internal model features, not present in the forcing data, can reach the boundary, which now plays the role of the wall and generates numerical noise. It is clear that SLPN is highest near the boundary (and so is the MKE peak), and it seems reasonable to conclude that noise generated close to the boundary spreads back in the model domain (where it is progressively damped, at least, by the filtering over all model grid points). However, it would be necessary to elaborate more on
this to understand why the noise is seen in SLPN fields and not in wind (MKE) fields in this situation. It might be related to the use of a fourth-order centered-difference scheme for the advection of the mass of air columns in the MAR model. It is interesting to note that the highest relaxation rates causes MKE peaks that are larger near the surface, while low relaxation rates are associated with (lower) MKE peaks that are larger in the upper troposphere. This suggests that, in the case of low relaxation, the problems are caused by fast perturbations in the upper levels (which cannot be damped due to the combination of high wind speed and low relaxation rate).

According to the theoretical assessment, the five-grid point parabolic profile is not very efficient when Newtonian relaxation is used alone. Indeed, the range of wave speed for which low reflection can be obtained is small, and the consequence is that the theoretical considerations provide a range of equally acceptable relaxation rates rather than a single value (Table 3). The best value found with the empirical testing ($\approx 0.2$) falls between the middle and the higher part of that theoretical
range \( (N^*_2|_{3500} = 0.02\text{–}0.9) \). Although we must remember the limitations of the theoretical considerations, these suggest that the empirically selected relaxation rates give priority to an efficient damping in the phase speed range of approximately \( 6\text{–}100 \text{ m s}^{-1} \).

For the exponential profile with \( N^*_2|_{3500} = 0.2 \) [experiment (E5/N1)], the SLPN result is about the same as for the parabolic profile (P5/N2). However, the peak of MKE is lower and it is located in the buffer zone, near the boundary rather than in the meaningful part of the domain. With \( N^*_2|_{3500} = 0.6 \), the result is better for SLPN while more noise is seen in MKE. Thus values between 0.2 and 0.6 should be a good compromise, probably closer to 0.6. It is indeed reasonable to expect that outside this range the results would degrade in either the SLPN or MKE indexes as it does for the parabolic profile. This is in good agreement with the theoretical conclusions.

The result obtained with the optimized profile (O5/N1) is excellent, rather better than expected from the simple optimization procedure, which does not account for the actual model numerical schemes. The evaluation based on MKE and SLPN suggests that there is almost no noise generation at the boundaries (Fig. 7), in contrast to the cases of parabolic and exponential profiles. Although changes in the overall relaxation rate were not tested, \( N^*_2|_{3500} = 0.9 \) is a reasonable candidate on the basis of the theoretical assessment (theoretical range: \( 0.2\text{–}0.9 \), not shown in the summary table). Note that this profile is the only one that provides results in the Newtonian relaxation case that are as good as for diffusive relaxation. The quality of this result is somewhat surprising because the five-gridpoint profile is optimized for a speed ratio \( c_{\text{max}}/c_{\text{min}} = 100 \) as in D83, while we propose a ratio of 1000 for climate simulations (section 3e). Since the minimum speed is difficult to estimate, we may have selected an excessively low value \( (c_{\text{min}} = 0.3 \text{ m s}^{-1}) \). However, the minimum speed may depend on the meteorological conditions, and our choice may be merely careful.

**2) Five-point diffusive relaxation**

As a general rule, diffusive relaxation performs better than the Newtonian form. The linear profile is somewhat less efficient than the other tested profiles, but the result is still good in comparison to all Newtonian relaxation cases.

For the parabolic profile with \( D^*_2|_{3500} = 0.5 \), the result is very good: no peak of MKE and no detectable SLPN are found (except in the buffer zone but close to the boundary). With \( D^*_2|_{3500} = 1.0 \), the performance is not as good but the change is small. There is a very small MKE peak at point 6 (as expected, due to the wall effect of excessive overall relaxation). In contrast, using \( D^*_2|_{3500} = 0.05 \) proves much less efficient, showing a significant MKE peak as well as SLPN over a relatively large area. The empirical estimation of the best overall relaxation rate is therefore in satisfactory agreement with the theoretical values \( (0.3\text{–}0.5) \).

The exponential and optimized profiles (with \( D^*_2|_{3500} = 0.7 \)) perform very similarly to the parabolic one. A test with the exponential profile shows a very significant degradation of the results when the relaxation rate is decreased to \( D^*_2|_{3500} = 0.2 \) (not shown); the result is rather worse than for the linear profile in this case. Although this is in agreement with the theoretical ranges (see Table 3), it is likely that the origin of the poor behavior of the exponential profile (with low \( D^*_2|_{3500} \)) is simply that it does not provide a significant relaxation on a minimum of points, specifically for fast waves.

In conclusion, diffusive relaxation is efficient with all profiles. This supports the idea that a very large part of the noise generated near boundaries has a very short wavelength, most probably 2 \( \Delta x \). Indeed, the process only involves diffusion and works only on four points \((s - 1, \text{ or even less if a given level of relaxation is required})\), so it cannot be expected to damp longer-wave-length perturbations.

The configuration P5/D1 involves both Newtonian and diffusive terms, but the Newtonian relaxation is very weak. This configuration was selected for standard use in MAR much before the present study. The differences between this simulation and P5/D3 are extremely small, and can be regarded as insignificant—at least on the basis of the present analysis methods and simulation framework. This result is in agreement with the theoretical result for \( N^*_2/D^*_2 = 0.01 \) (section 3f).

**3) Nine-point Newtonian relaxation**

For the exponential profile with \( N^*_2 = 0.2 \) (E9/N2), an increase of both MKE and SLPN is seen near the boundary (Fig. 8). This was identified above as the typical consequence of a relatively low overall relaxation rate. Higher relaxation coefficients provide results similar to those of the best five-point experiments. The best value is found to be close to 0.9 (E9/N4).

For the exponential profile, the nine-point version performs better than the five-point one (even the relatively poor configuration E9/N2 performs better than its five-point equivalent, E5/N1; the MKE peak in E5/N2 clearly suggests that further increasing the overall relaxation rate as in E9/N4 would degrade the results in the case of the five-point buffer zone). The optimized profile does not perform better than the exponential one.

Making the comparison with the theoretical considerations, it is satisfying to see that the nine-point case requires somewhat higher relaxation rates. Indeed, the theoretical assessment proposes a range of relaxation rates including lower values in the case of five-point and/or less appropriate profiles. In the nine-point cases, the theory suggests that the high relaxation rates are the best choice, in agreement with the practical findings.

Experiment E9/N1 investigates a combination of N and D relaxation. This configuration is very similar to
one of those that were tested with RegCM2: the overall relaxation rates are exactly the same and the exponential decay is one of those used. The main difference in our case is that the buffer zone is cut at the 10th point, while it is not in RegCM2 (Giorgi et al. 1993). However, we expect this to be a minor change in our simulations, because no MKE peak was found near the cutting point (such a peak is indeed the likely consequence of the abrupt introduction of relaxation at the edge of the buffer zone). The results of this experiment are similar to the results of the corresponding experiment without diffusive relaxation, E9/N2, although the SLPN increase in the buffer zone is a little lower when diffusion is added (Fig. 8). This is in agreement with the theoretical conclusions.

4) Nine-point diffusive relaxation

For the exponential profile, overall relaxation coefficients under \( D^*_{\text{E9/N2}} = 0.5 \) are clearly too low, while all values from about 0.5 and above give satisfying and similar results; \( D^*_{\text{E9/D3}} = 0.9 \) results in a slight increase of SLPN close to the boundary. This is in agreement with the theoretical results, which suggest \( D^*_{\text{E9/N2}} = 0.7 \). The result obtained with the optimized profile is very similar, although marginally less good than that for the exponential experiment with the same overall relaxation rate, E9/D3.

With regard to the absence of noise production near the boundary, the parabolic profile gives an excellent result: there is no peak, and there is a smooth decrease of MKE from the interior to the boundary. However, this experiment shows a large reduction of SLPN in the buffer zone, and it has a tendency to spread in the domain: in fact, the parabolic profile over nine points provides a much stronger forcing (larger relaxation) than any other configuration investigated here. The production of mesoscale features by the model seems hampered over a larger zone. While the progressive buildup of mesoscale features with distance to the boundary is normal in RCM modeling, it is probable that the forcing is excessive in the present case (at least in the low levels). However, profiles–buffer zone sizes combinations that provide strong forcing over a larger area may be more useful when the model domain is much larger.

5. Conclusions

We used a simple theoretical framework to assess the quality of usual configurations of the lateral boundary relaxation in regional climate models. Further evaluation of these relaxation configurations is then obtained from sensitivity experiments with the MAR model. While the theory has limitations, there are indications that the key ideas regarding the choice of the profile and the amplitude of relaxation may come from general principles. Broadly speaking, this results in a smooth transition in the relaxation rates everywhere from the inner free model area to the fixed boundary row, and specifically at the edges of the buffer zone.

The sensitivity tests are based on an evaluation of nonphysical effects (noise) near the boundaries. This is mainly seen in the wind field (MKE), but we found that looking at small-scale changes in the surface pressure field (SLPN) may sometimes emphasize boundary-related noise that is not seen in the MKE field. Both theory and sensitivity tests show that the best performing of the standard relaxation profiles is the exponential decrease, followed by the parabolic one. For the most difficult type of configuration, Newtonian relaxation on a five-point buffer zone, the best simulation results are obtained with the “theoretically optimized coefficients.” Most sensitivity tests clearly agree with the theoretical guidelines. Both the theory and the sensitivity study suggest that diffusive relaxation causes much less noise in the model domain than the Newtonian form. While poorly performing configurations have been investigated to assess the differences between the theoretical and empirical results, it should be clear that these configurations should not be used. In particular, this means that the combination of Newtonian relaxation and small buffer zones should be avoided or, at least, implies the use of an exponential or optimized profile. If the theoretically optimized profile is used, it should be verified that it works well with the involved model, as it does here.

To define the overall magnitude of the relaxation, we use parameters that should theoretically not depend on the grid and time step of the model, \( N^*_{\text{E9/D3}} \) and \( D^*_{\text{E9/D3}} \) (called leading coefficients). For all profiles that are expected to give good performance on a theoretical basis (i.e., low reflection at boundaries), both theoretical and empirical approaches suggest values ranging between 0.5 and 0.9, for both Newtonian and diffusive leading coefficients. For the poorly performing profiles, the theory gives a large range of possible values for the leading coefficients, which emphasizes the need for a compromise between the damping of slow or fast waves. The empirical evaluation gives more precise results, suggesting that the leading coefficients should be moderately lower than for the profiles that perform well.

As a complement to the present study, it may be interesting to test buffer zone sizes (area with significant relaxation), which increase with altitude, in a way similar to Giorgi et al. (1993). We note that in their case, the exponential rate of decrease is changed to obtain the desired increase of the forcing in the free atmosphere, but the relaxation coefficient is kept constant near the boundary: this is in agreement with the present work. As explained in the introduction, we think that the idea of a stronger forcing of the model in the free atmosphere is a matter of debate, specifically for large domain sizes.

The numerical results from this study should not be applied to other models without checking on a case by case basis, because this involves at least differences in the numerical schemes. However, the convergence of
the theoretical and empirical assessment, and the pragmatic considerations that have been found to explain many aspects, strongly supports the conclusion that this work can provide good guidelines for selecting or improving relaxation coefficients in other models. Previous work with other models produced results that are in agreement with the present findings. For example, the advantages of replacing the linear profile by an exponential profile was initially found with the RegCM2 model (Giorgi et al. 1993). In addition, we did not find advantages to the combination of the Newtonian and diffusive relaxation terms, so it would be interesting to know if this plays a significant role in models that use both terms, such as RegCM2. In summary, there are easily applicable guidelines that may at least be helpful as a starting point or when LBCs problems are suspected: (i) rely mainly or solely on relaxation diffusion, (ii) use exponential or optimized profiles, and (iii) set the highest relaxation coefficient so that $D_{2\times3\zeta} = 0.7$.

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APPENDIX

Relation between the Relaxation Parameters Computed from the Theory and Those Used in Some RCMs

The theoretical developments presented here provide relaxation parameters that can be directly used in models. However, time step and space resolution choices are introduced in models such as MAR and RegCM2 (Giorgi et al. 1993) in a way that is not exactly equal to the theoretical expression. These models use the following equations [obtained in D83 and the present paper, Eq. (19)]:

\[
N_x = N_x^* - \frac{c_{\text{max}}}{2\Delta x} \quad \text{(A1a)}
\]

\[
= \frac{1}{\Delta t} \quad \text{(A1b)}
\]

where $N_x$ is a model constant. We assume that these two expressions can be simultaneously accepted because this simply means that a change in the horizontal resolution will be accompanied by a proportional alteration of the time step. In particular, this is valid when the time step approaches the Courant–Friedrichs–Lewy (CFL) criterion, that is, $c_{\text{max}} \Delta t/\Delta x \leq 1$ (this is necessary for the stability of explicit schemes). The corresponding relations for the diffusive relaxation term are

\[
D_x = D_x^* \frac{c_{\text{max}} \Delta x}{2} \quad \text{(A2a)}
\]

\[
= D_M (\Delta x)^2 \quad \text{(A2b)}
\]

where $D_M$ is a model constant.

Damping of Waves by Diffusive Relaxation

The diffusive relaxation term provides a damping of outgoing waves that depends on the wavelength ($\lambda$). A characteristic relaxation time can be obtained from the analytical equation of the system (8), which has solutions of the type

\[
a^t = a^* \exp(-t/\tau) \exp[i2\pi(t/c\lambda - x/\lambda)] \quad \text{(A3)}
\]

Introducing this expression in (8) with $N = 0, D \neq 0$ gives the following e-folding time:

\[
\tau_e = \lambda^2/(4\pi^2 D).
\]

To illustrate the relaxation time corresponding to the proposed values of the relaxation parameter, we let $\lambda = 4\Delta x$. This choice is somewhat arbitrary but it is based on the observation that boundary noise takes the form of short waves and that if $4\Delta x$ waves are appropriately damped, $2\Delta x$ waves will be even better canceled (the actual damping in the model will be somewhat different because such waves are not resolved, but the analytical evaluation still provides a good estimate). For these waves, $\tau_e \approx 0.41\Delta x^2/D$. At the first relaxed point near the boundary, the approximate e-folding time is then obtained from Eq. (20): $\tau_e \approx 0.82\Delta x/(c_{\text{max}} D_x^*)$.

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