Axial Angular Momentum: Vertical Fluxes and Response to Torques

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ABSTRACT
The horizontally averaged global angular momentum at a certain height reacts only to the vertical divergence of the angular momentum flux at least above the crest height of the earth’s orography. The flux is tied to the torques at the surface. Data are used to evaluate the flux and the response of the angular momentum to the torques. It is shown that the accuracy of the data is sufficient for an investigation of this interaction. It is found that the horizontally averaged angular momentum in the upper troposphere and lower stratosphere tends to be negative before an event of positive friction torque. Downward transports of negative angular momentum from these layers allow the angular momentum to further decrease near the ground, even shortly before the event although the friction torque is positive at that time. The impact of the mountains during this process is demonstrated. The ensuing positive response to the friction torque is felt throughout the troposphere. The final decay of this reaction involves downward transports of angular momentum with typical velocities of 1–2 km day−1.

The angular momentum in the lower troposphere tends to be negative before an event of positive mountain torque. There is a short burst of rapid upward transport of positive angular momentum during the event itself, which reaches the stratosphere within 1–2 days. A phase of decay follows with slow downward transport of positive angular momentum.

1. Introduction
Atmospheric angular momentum is transferred at the surface of the earth by pressure drag and frictional processes. For example, Rossby waves induced by a large-scale mountain massif are able to transport angular momentum toward this obstacle, where it is taken up by the earth. Turbulent motion in the boundary layer transfers momentum toward the ground, where it is removed from the atmosphere by surface friction. The global budget of these processes is captured by the axial momentum equation

$$\frac{dM}{dt} = T_0 - T_f,$$

(1.1)

where

$$M = \rho \int \left( u + \Omega a \cos \varphi \right) a \cos \varphi \, dV,$$

(1.2)

is the global angular momentum ($T_0$ is mountain torque; $T_f$ frictional torque; $\rho$ density; $u$ zonal wind; $\Omega = 2\pi$ day−1; $a$ earth’s radius; $\varphi$ latitude; and $V$ volume of the atmosphere). The total torque $T = T_0 + T_f$ describes the transfer of angular momentum from the earth to the atmosphere.

Of course, (1.1) does not tell us how the angular momentum transferred at the ground is transported and distributed in the vertical. In the past, specific cases of vertical and horizontal momentum transports and absorption have attracted considerable interest. For example, Rossby waves excited by the Tibetan Plateau may propagate up- and southward to be absorbed near the critical surface where the velocity of the zonal mean flow changes sign. Westerly momentum is removed from the Tropics by this process and is transported to the plateau, where it is transferred to the earth (e.g., Held et al. 2002). Easterly angular momentum is transported by Rossby waves to Antarctica, where it is redistributed by the deep vertical circulation of Antarctica linked to the slope winds to be transferred, finally, to the Antarctic ice dome via surface friction (Egger 1992; Juckes et al. 1994). Downward propagation of zonal-mean wind anomalies from the stratosphere to the troposphere as described by Kodera et al. (1990) and others appears to imply a corresponding downward transport of angular momentum. Anderson and Rosen (1983) found vertical and meridional transports of angular mo-
momen t related to the Madden–Julian oscillation. However, links between vertical transports and torques have not been discussed in these last two papers. Here, we wish to analyze the relation of vertical transports of angular momentum by atmospheric motions to torques on a climatological basis. To be more specific, let us look at the conservation equation for angular momentum

$$\frac{\partial pm}{\partial t} + \nabla \cdot (\rho vm) = -\frac{\partial p}{\partial \lambda} + a \cos \varphi \frac{\partial \tau_e}{\partial z} - \Omega a^2 \cos^2 \varphi \frac{\partial}{\partial z} H, \quad (1.3)$$

where

$$m = (u + \Omega a \cos \varphi) a \cos \varphi$$

is the specific axial angular momentum resolved by the analysis (v is velocity; p pressure; \(\lambda\) longitude; \(z\) height; \(\tau_e\) vertical component of zonal stress; and \(H = \frac{\rho w}{F}\) is vertical mass flux due to unresolved subgrid motions). We choose height coordinates in (1.3) instead of the more customary pressure coordinates (e.g., Peixoto and Oort 1992) because the height of pressure surfaces varies in time. As usual we apply some averaging to facilitate the analysis. Peixoto and Oort (1992) averaged (1.3) over time and longitude and presented a time-mean budget of the axial angular momentum and its vertical and horizontal fluxes. They showed inter alia that the angular momentum gained via surface friction of the trade winds is first transported upward in the Tropics to flow then toward the midlatitudes near the tropopause. Weickmann et al. (2000) and, quite recently, Weickmann (2003) provided information on the meridional transport of the vertically integrated angular momentum during torque events. The vertical structure of circulation anomalies related to fluctuations of \(M\) has been investigated by several authors (e.g., von Storch 1999; Kang and Lau 1994). However, the related vertical transports have not been discussed as yet. To simplify the analysis we suppress meridional variability by averaging (1.3) horizontally to obtain

$$\frac{\partial (\rho m)}{\partial t} + \frac{\partial (\rho wm)}{\partial z} = - \int (p_e - p_w) a^2 \cos \varphi \, d\varphi$$

$$+ \frac{\partial}{\partial z} (a \cos \varphi \tau_e)$$

$$- \Omega a^2 \frac{\partial}{\partial z} (\cos^2 \varphi H), \quad (1.4)$$

where the global integral

$$q = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} rq \cos \varphi a^2 \, d\varphi \, d\lambda \quad (1.5)$$

is a function of height and time only. We prescribe \(r = 0\) in (1.5) if the coordinate surface \(z = \) constant is located within a mountain and \(r = 1\) elsewhere. The pressure integral in (1.4) must be evaluated along the eastern and western part (subscripts e, w) of the intersection of the surface \(z = \) constant with the orography. Removing meridional averaging is planned for future work but, as will be seen, quite meaningful results can be obtained by investigating (1.4).

Above crest height, (1.4) reduces to

$$\frac{\partial \mu}{\partial t} = -\frac{\partial F}{\partial z}, \quad (1.6)$$

where

$$\mu(z, t) = (\rho m) \quad (1.7)$$

is the global angular momentum per unit layer thickness and

$$F = (\rho wm) - a (\cos \varphi \tau_e) + H \Omega a^2 \cos^2 \varphi \quad (1.8)$$

is the vertical flux of angular momentum due to resolved and unresolved motions. Vertical divergences of the flux induce changes of the mean angular momentum \(\mu\). With (1.6) the central issues of this paper can be stated more clearly. Given, say, an event of a positive torque so that \(M\) increases, we would like to know from which levels the negative angular momentum is transported downward before this event, which is then handed over to the earth by the torque. We likewise wish to find out how fast and up to which height the positive angular momentum thus transferred is redistributed in the atmosphere. This analysis can be performed for mountain and friction torques separately.

To resolve these issues, data from the European Centre for Medium-Range Weather Forecasts Reanalysis Project (ERA; Gibson et al. 1997) are used to calculate \(\mu\) as a function of height and time. The resolved part of the fluxes can be evaluated directly while the total flux \(F\) is accessible only as a residuum. The torques are evaluated as in Egger and Hoinka (2002). Note that we do not include the torque due to gravity waves in (1.1) nor in (1.4). This term is relatively small and difficult to estimate (e.g., Weickmann et al. 2000). Temporal relations between these variables are investigated by looking at covariance functions \(C(a, b | \tau) = E[a(t)b(t + \tau)]\) of pairs of variables \(a, b\), where \(E\) is the expectation and \(\tau\) the lag.

The analysis will be based on the covariance equations related to (1.6). After multiplying (1.6) by \(T(t - \tau)\), switching to lag \(\tau\) and taking expectations follows

$$\frac{\partial}{\partial \tau} C(T, \mu | \tau) = -\frac{\partial}{\partial \tau} C(T, F | \tau). \quad (1.9)$$

The change of the covariance function of the torque \(T\) at the ground and \(\mu\) at height \(z\) with lag is due to the vertical divergence of the covariance of torque and flux \(F\). We can go one step beyond (1.9) and relate (1.6) to the mountain and friction torques separately. We may look, for example, at
\[
\frac{\partial}{\partial \tau} C(T_0, \mu | \tau) = - \frac{\partial}{\partial z} C(T_0, F | \tau) \tag{1.10}
\]
or replace \(T_0\) by \(T_j\). Altogether, (1.9) and (1.10) provide the framework for our investigation.

2. Data

The data cover the years 1979–92. All three components of the wind, temperature, and density are available at the 31 \(\sigma\) levels of the analysis scheme four times a day. All fields are represented in a spherical grid with mesh size 1.125\(^\circ\) in zonal and meridional direction. The dataset contains also the surface pressure and the surface stress. Mountain and friction torques are evaluated on this basis. The equations [(1.9) and (1.10)] to be analyzed are formulated in \(z\) coordinates in the foregoing section. Obviously it would have been more convenient to use the hybrid \(\sigma-p\) coordinates of the ERA analysis scheme. However, the height of \(\sigma\) surfaces varies in time, so the interpretation of fluxes and vertical velocities in \(\sigma\) systems is rather difficult. To evaluate \(\mu\) and \(F\) as functions of height we have to interpolate every 6 h all data from the \(\sigma\) surfaces to \(z\) surfaces where we choose equidistant surfaces at heights \(z_j\) where the layer thickness \(\Delta z = z_{j+1} - z_j\) is 1000 m and \(1 \leq j \leq 28\) (see Fig. 1). The interpolation formula is linear. Fluxes are defined at the \(z\) surfaces as well. They are evaluated both directly and diagnostically. Details also on Fig. 1 are given in the appendix.

The annual and semiannual cycle is removed by subtracting the corresponding Fourier mode. This way we exclude the impact of the annual cycle on the extremely slow decay of the autocovariance function of \(M\) with increasing lag (Egger and Hoinka 2002). The daily cycle is suppressed by applying a running mean over four consecutive data points. All filtering procedures are applied to \(\mu\), \(F\), and \(T\) and not to the original data.

The basic equation (1.6) is adopted to the vertical grid by integration over the depth of the layers. With

\[
\bar{\mu}_j = \int_{z_j}^{z_{j+1}} \mu \, dz \tag{2.1}
\]

follows

\[
\frac{\partial}{\partial t} \bar{\mu}_j + F_{j+1} - F_j = 0 \tag{2.2}
\]

above the crest level. The resolved fluxes \(F_{j+1} = [p\omega w m]\) are evaluated directly on the basis of the available fields of \(u, w,\) and \(\rho\). The ERA data do not contain information on the stress profiles needed to determine the fluxes \(F\) [see (1.8)]. That is, however, not a serious shortcoming above the lowest level because turbulent fluxes may be assumed to be small there. It is more serious that there is also no information available on the subgrid mass fluxes \(H\). Therefore, it is necessary to determine the total fluxes as residuals. Given \(F\) and the tendency of \(\bar{\mu}\), as available from the observations, the flux \(F_{j+1}\) can be calculated from (2.2). It remains to determine the fluxes below crest level. Figure 1 shows that 28\% of the volume of the lowest layer is occupied by orography compared to 7\% of the second layer. In principle, one could partition the torques on the basis of such information. Such attempts to distribute the torques over several layers have been made but finally rejected as involving too many assumptions. Instead, we simply assume

\[
F_i = T \tag{2.3}
\]

and apply (2.2) in all layers. The fluxes in the lowest two layers are overestimated this way, but we expect on the basis of Fig. 1 that the effect of this assumption is small for \(j \geq 4\).

Summation of (2.2) over all layers gives

\[
F(z_{29}, t) = T - \sum_{j=1}^{28} \frac{\partial \bar{\mu}_j}{\partial t}, \tag{2.4}
\]

where \(z_{29}\) is the top surface of the data domain (see Fig. 1). If, in particular, (1.1) is satisfied exactly by the data, we have

\[
F(z_{29}, t) = 0. \tag{2.5}
\]

Although the ERA data have been shown to satisfy (1.1) quite well, time-mean (Egger and Hoinka 2002) deviations remain and (2.5) is not fulfilled exactly. Such deviations may be due to the exchange of momentum with the layers above the top of our analysis scheme in the atmosphere. Then, (2.5) is not appropriate. It is, however, easy to demonstrate that momentum fluxes to the upper stratosphere and mesosphere are not the key problem. It follows from (1.1) that the variance of the tendency of \(M\) must equal the variance of the torque. Thus the data have to satisfy

\[
C \left( \frac{dM}{dt}, \frac{dM}{dt} \right) \big| 0 = C(T, T | 0). \tag{2.6}
\]

In practice, however, the left-hand term is always larger than that on the right-hand side. We obtain a ratio 1.3 of both terms for the data used here. The ratio climbs up to a value of 2.7 if there is no averaging of the data over 1 day. Obviously there is a pronounced imbalance of torques and angular momentum tendencies for short time scales that is certainly not caused by transports to the mesosphere.

Two approaches to this problem are tested. The first one (subscript \(A\)) is to accept (2.2)–(2.4) directly. All errors sum up to produce a flux \(F_{z_{29}}\) at the top height \(z_{29}\) which may deviate considerably from (2.5). This procedure is simple but the resulting fluxes near the top level reflect essentially the inaccuracies of the data and not stratospheric conditions. The other possibility is to adapt the torques such that
Fig. 1. Layering of the $\sigma$ levels of the ERA data above (left) the point with the lowest height $h$-minimum = -300 m and the (right) top point with $h$-maximum = 5700 m for intercomparison with (center) the $z$ system chosen here ($z = 100$ m, $\Delta z = 1000$ m). The percentages give the relative volume of a layer not occupied by orography. The numbers in the left and right row are those of the $\sigma$ levels; those in the center count the layers of the $z$ system.

\[ T_{\mu} = \sum_{j=1}^{28} \frac{\partial \bar{\mu}}{\partial t} J_j \]  

replaces $T$. This procedure redefines the torques and satisfies (2.5) exactly. It is, however, not possible to split $T_{\mu}$ into a mountain and a friction torque.

3. Results

a. Fluxes

Figure 2a shows the variance of the flux $F$ as a function of height for both options as well as for the resolved flux $F_{\mu}$. As for $F_{\mu}$, we observe a decrease of the variance...
with height close to the ground, but the variance is almost constant above the lowest 10 km. This high level of stratospheric variance follows from the mismatch of torques and observed variability of $dM/dt$ noted above. The autocovariance of $F_A$ decays within a few days (not shown). The variance of $F_B$ at the ground is much larger than that of $F_A$. This reflects the fact that the observed variability of $\bar{\mu}$ is larger than that implied by the variance of the observed torques [see (2.6)]. Therefore, the adjustment of the torques as in procedure B leads to an enhancement of the variance of the fluxes near the ground. The variance of $F_B$ decays with height and is quite small above 15 km. This decrease of the variance with height is what one would expect. If most of the angular momentum transferred to the earth is taken from the troposphere, the vertical derivative of $F$ must be small in the stratosphere and thus with (2.5) also $F$. This indicates that $F_B$ is more realistic than $F_A$. Attempts to distribute the torques over several layers in order to incorporate topographic effects lead to a reduction of the variance of the fluxes $F_A$ and $F_B$ near the surface, but there is no visible deviation from the profiles in Fig. 2a for $z > 2$ km. The variance of the resolved fluxes $F_r$ vanishes, of course, at the ground and comes relatively close to that of $F_B$ near $z \sim 5$ km but is always less. The covariance $C(T_o, F_z|0)$ decreases rapidly with height and is quite small above the height $z \sim 15$ km (Fig. 2b). This result agrees with expectations and suggests that the fluxes $F_A$ can be used to study at least the response of the atmosphere to the torques. Note that the large stratospheric variances of $F_A$ in Fig. 2a are not reflected here. They are due to inconsistencies of the observed tendencies of angular momentum and the torques. The response of the resolved fluxes is weaker than that of $F_A$.

### b. Response to torques

We turn now to the main topic of this paper, that is, to the analysis of the reaction of $\mu$ to the torques. Let us introduce the vector

$$
\mathbf{c} = [C(T_o, \mu|\tau), C(T_o, F|\tau)]
$$

in the $(\tau, z)$ plane, where $T_o$ may be chosen for $T_a$ as well as $T_f$ or $T$. This vector is nondivergent

$$
\nabla \cdot \mathbf{c} = 0
$$

in this plane according to (1.9) and (1.10), so that we can introduce a function $\psi$ with

$$
C(T_o, \mu|\tau) = -\frac{\partial \psi}{\partial z},
$$

$$
C(T_o, F|\tau) = \frac{\partial \psi}{\partial \tau}.
$$

As can be seen from (3.3) and (3.4), $\psi$ satisfies the same rules as the streamfunction of two-dimensional nondiagonal flow, but $\mathbf{c}$ is not a velocity in the $(\tau, z)$ plane and there is no “flow” along lines of constant $\psi$ to be called $\psi$ lines in the remainder. It will be seen, however, that there exist subdomains in the $(\tau, z)$ plane where the $\psi$ lines are straight and nearly parallel. In that case, $C(T_o, F|\tau) = \lambda C(T_o, \mu|\tau)$, where the slope $\lambda$ is constant and the $\psi$ lines are even characteristics of (1.9) in that subdomain (e.g., Whitham 1974). Since $C(T_o, \mu|\tau)$ is constant on a $\psi$ line in that case, this subdomain is an area of “nonacceleration” and the slope $\lambda$ reveals the speed of ascent or descent. Normally, however, the $\psi$ lines bend so that angular momentum is stored or removed.

Let us briefly recapitulate what is known about the vertically integrated forms

$$
\frac{d}{d\tau} C(T_f, M|\tau) = C(T_f, T_f|\tau) + C(T_f, T_o|\tau),
$$

$$
\frac{d}{d\tau} C(T_o, M|\tau) = C(T_o, T_f|\tau) + C(T_o, T_o|\tau).
$$
of (3.2), which follow also immediately from (1.1), before turning to a discussion of the vertical structure of the field $\mathbf{c}$. The various covariance functions in (3.5) and (3.6) are displayed in Fig. 3. The covariance function $C(T_f, M | \tau)$ is seen to decrease for negative, increasing lags to reach a minimum near $\tau \sim -2$ days. The covariance of mountain torque and global angular momentum is also negative for $\tau < -1$ day, but there is a rapid increase with lag near $\tau = 0$ that results in a positive maximum at $\tau \sim 2$ days (see also Weickmann et al. 2000; Egger and Hoinka 2003; and further references therein). The autocovariance of the friction torque on the right of (3.5) is positive for $|\tau| < 15$ days (Fig. 3b), while that of the mountain torque is negative for $5 \leq \tau \leq 15$ days. In principle, the covariances $C(T_f, M | \tau)$ and $C(T_o, M | \tau)$ can be obtained by integrating (3.5) and (3.6) over the lag. If $C(T_f, T_o | \tau)$ would vanish, the right-hand sides of (3.5) and (3.6) would be symmetric with respect to $\tau = 0$ and, correspondingly, $C(T_f, M | \tau)$ and $C(T_o, M | \tau)$ would have to be antisymmetric with negative values near $\tau = 0$ for $\tau < 0$. Figure 3a shows clearly that both covariances contain a strong symmetric part so that the cross-covariance $C(T_o, T_f | \tau)$ must be quite important in (3.5) and (3.6). By and large, $C(T_o, T_f | \tau)$ is antisymmetric with negative values for $\tau > 0$ (Fig. 3b), but the minimum for $\tau > 0$ is more pronounced than the maximum for $\tau < 0$. Positive mountain torques are preceded by positive friction torques. Midlatitude wave trains are important in these processes (Weickmann 2003). In turn, $C(T_f, T_o | \tau)$ is negative for $\tau < 0$ and positive for $\tau > 0$. Hence, the incorporation of $C(T_f, T_o | \tau)$ on the right-hand side of (3.5) leads to the continuous decrease of $C(T_f, M | \tau)$ with increasing $\tau$ for negative lags. This cross covariance is more important than the autocovariance of friction torque, which is positive for $|\tau| < 15$ days (see Fig. 3b). It is the opposite with (3.6). The positive value of $C(T_o, M | \tau)$ at lag $\tau = 0$ is due to the action of the friction torques for $\tau < 0$, which are positively correlated with the mountain torque at $\tau = 0$.

The vector field $\mathbf{c}$ for $T_o = T_f$ is shown in Fig. 4 for $|\tau| < 15$ days. The evaluation of the vertical component of $\mathbf{c}$ is based on the fluxes $F_x$. The field for $F = F_x$ is similar to that for $F = F_y$ and is, therefore, not shown. The orientation of the vectors in Fig. 4 is difficult to estimate if the vectors are small. It can, however, be derived easily from Fig. 5 where the corresponding $\psi$ lines are presented. By and large, $\mathbf{c}$ is oriented toward the left throughout the figure with an upward component. In other words, the covariance of the angular momentum $\mu$ with the friction torque at lag $\tau$ is nearly always negative except for the upper troposphere and the stratosphere for, say, $\tau \simeq 3–5$ days. There is mainly upward (downward) flux of positive (negative) angular momentum if $T_o$ is positive at $\tau = 0$. The sign and the length change of the horizontal component of $\mathbf{c}$ with lag correspond well with Fig. 3a. This component is negative at all levels for $\tau < 0$ and decreases further with $\tau$ to reach a minimum near $\tau \sim 0$ at least in the troposphere. As has been mentioned above, this strengthening is due to the influence of the mountains. The mountain torque removes positive angular momentum before an event of positive friction torque. For $\tau > 0$, $C(T_o, \mu | \tau)$ is increasing, but positive values are found in the stratosphere only. Note that the vertical component of $\mathbf{c}$ at the ground represents the covariance function $C(T_f, T | \tau)$ of the friction torque with the total torque, that is, the right-hand side of (3.5). As can be
seen from Fig. 3, this component is negative for $\tau \leq -2$ days but is positive and large for $0 \leq \tau \leq 10$ days. It would be strictly antisymmetric with respect to $\tau = 0$ without the mountain effect.

Figure 4 reveals distinct differences between the lower troposphere ($0 \leq z \leq 5$ km) and the layers above. For $\tau < -10$ days, say, $|\mathbf{c}|$ is quite small in the lower troposphere, but the horizontal component $C(T_f, \mu | \tau)$ of $\mathbf{c}$ is negative and clearly visible above that layer. Hence, the global angular momentum tends to be neg-
ative in the layer $6 \leq z \leq 15$ km long before an event of positive friction torque. The angular momentum in the lower troposphere decreases then rapidly with increasing $\tau$ while $\textbf{c}$ is turning clockwise at upper levels. The covariance $C(T_f, \mu|\tau)$ reaches its minimum for $\tau \sim -1$ day close to the ground. Strong easterlies near the surface are required to provide a positive friction torque. For positive $\tau$, the response to the torque fades first in the upper troposphere and is quite small throughout the atmospheric column for $\tau \sim 12$ days.

The corresponding $\psi$ field is displayed in Fig. 5a. The vector field $\textbf{c}$ is oriented along the $\psi$ lines and with high values of $\psi$ to its right as is conventional. We see that $\partial\psi/\partial\tau > 0$ almost everywhere except for $\tau < -5$ days in the lower troposphere. Moreover, $\partial\psi/\partial z > 0$ except for the stratosphere and $\tau > 5$ days. The $\psi$ lines are almost straight and parallel for, say, $0 \leq \tau \leq 7$ days, $0 \leq z \leq 7$ km. We are able to determine flux directions in such areas of “nonacceleration”. The atmospheric motions transport negative angular momentum anomalies downward with a speed of about 2 km day$^{-1}$ ($|w| \sim 0.02$ m s$^{-1}$) in this domain. At the bottom, these fluxes are transferred to the earth as a positive friction torque. As shown by Weickmann (2003), this transfer occurs mainly in the belt $10^\circ \leq \varphi \leq 40^\circ$N (at least in the boreal winter season).

Vertical momentum transfers can be studied quantitatively by considering the momentum balance of boxes. For example, box I in Fig. 5a covers part of the domain in the lower troposphere where $C(T_f, \mu|\tau)$ decreases with increasing lag despite the fact that the friction torque is positive (see also Fig. 3). Integration of (3.2) over this box gives

$$\int_{z_1}^{z_2} [C(T_f, \mu|\tau) - C(T_f, \mu|\tau_1)] dz$$

$$= \int_{\tau_1}^{\tau_2} [C(T_f, T|\tau) - C(T_f, F, \tau)]_{z=0} d\tau. \quad (3.7)$$

The integral on the left-hand side describes the decrease of the angular momentum in the interval $\tau_1 \leq \tau \leq \tau_2$ and is obviously negative. The first term on the right-hand side equals the mean torque and is positive. Therefore, the decrease of the angular momentum with increasing $\tau$ in box I must be caused by the flux through the upper boundary of the box. Although the sign of the flux does not allow us to determine a direction, we learned above that the transports are directed downward near box I. Thus it is presumably negative angular momentum that is transported downward from the upper troposphere so that $C(T_f, \mu|\tau)$ decreases in the box. In turn, $C(T_f, \mu|\tau)$ increases above the box, as can be seen also in Fig. 4. In box II, the surface torque is positive and dominant with respect to the inflow of negative angular momentum from above so that $C(T_f, \mu|\tau)$ increases with respect to the box. This statement appears to contradict the notion of “nonacceleration” for straight flow lines as in box II. However, it is only along a straight flow line that $\textbf{c}$ does not change. The spacing of parallel flow lines may change with $\tau$ and does so indeed near box II so that the total inflow on top is less than the outflow at the bottom.

The $\psi$ field for $F = F_B$ as displayed in Fig. 5b is quite similar to that for $F = F_A$ in the troposphere. But while $\psi$ lines ascend to the stratosphere in Fig. 5a they turn here downward for decreasing negative lags. In particular, the downward flow of negative angular momentum for $\tau < -2$ days, say, as induced by the mountain torque is much more visible than in Fig. 5a. It should be kept in mind, however, that the total torque $T_B$ in Fig. 5b is not the sum of the observed torques $T_f$ and $T_o$ but has been adjusted. On the other hand, the interpretation of Fig. 5a applies to Fig. 5b with only slight modifications.

Altogether, the following picture emerges. The covariances $C(T_f, \mu|\tau)$ tend to be negative for $\tau \leq -5$ days in the upper troposphere and lower stratosphere. Downward transports of negative angular momentum into the lower troposphere lead to a rapid decrease of angular momentum there and, presumably, to an increase of the friction torque. This downward transport is partly caused by mountains. The loss of negative angular momentum in the stratosphere results in positive values of $C(T_f, \mu|\tau)$ there, for $\tau > 0$. The downward transport continues for $\tau > 0$. Nevertheless, the angular momentum increases in the troposphere in the interval, say, $0 \leq \tau \leq 10$ days. This growth is due to the dominance of the torque, which overrules the impact of the downward transports. It is again the mountains that are important in this process because $C(T_f, T_o|\tau) > 0$ for $\tau > 0$.

The orographic torques excite a “response” (Fig. 6) where the vectors $\textbf{c}$ are oriented to the left for $\tau < 0$ and to the right for $\tau > 0$, the horizontal components being substantially larger for $\tau > 0$ than for negative lags. These features correspond, of course, with Fig. 3 but the vertical structure of the response is again of interest. In particular, $C(T_o, \mu|\tau)$ vanishes almost for $\tau < 0$, $z > 5$ km, while appreciable covariances are found up to heights of 15 km for $\tau > 0$. The clockwise turning is restricted to a strip near $\tau = 0$. Thus there are strong upward (downward) fluxes of positive (negative) angular momentum for $|\tau| \lesssim 2$ days while positive (negative) momentum is transported downward (upward) for larger positive $\tau$. The $\psi$ field (Fig. 7a) tells us which choice to make. The contours of $\psi$ “descend” steeply for $\tau < -1$ day from the stratosphere to the ground for $F = F_B$. They are densely packed near $\tau = 0$, $z = 0$ where there is the maximum of the covariance $C(T_o, T|\tau)$. For $\tau > 0$, the $\psi$ lines “ascend” steeply for $\tau < 3$ days to turn downward for larger lags with $C(T_o, T|\tau)$ being negative. There is at least one subdomain for $\tau > 0$ where the $\psi$ lines are reasonably straight and parallel, that is, for $2 \leq \tau \leq 8$ days, $0 \leq z \leq 12$ km. This time it is positive angular momentum that is transported.
downward to the surface if the mountain torque is positive at $\tau = 0$. The related velocities are quite small ($w \sim -5 \times 10^{-3}$ m s$^{-1}$). The situation is somewhat different for $F = F_B$ (Fig. 7b) where a minimum of $\psi$ is seen near $\tau \sim -7$ days at $z = 0$. For $\tau < -7$ days, the covariance of the total torque and $T_0$ is negative, and the decrease of $C(T_0, \bar{\mu} | \tau)$ with increasing $\tau$ in the lower troposphere as seen in Fig. 6 is supported by the torques at least for $\tau < -7$ days. Note that the horizontal component of $c$ in Fig. 6 remains the same when we switch to the fluxes $F_{\bar{\mu}}$ (not shown) while the vertical component would point downward for $\tau < -7$ days, $z \leq 4$ km. The torques are positive for $\tau > -7$ days but $C(T_0, \bar{\mu} | \tau)$ decreases further until $\tau \sim -2$ days. In that

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{Vector field $[C(T_0, \bar{\mu} | \tau), C(T_0, F_x | \tau)]$ in units of $10^{14}$ (kg$^2$ m$^4$ s$^{-3}$) as a function of height and lag $\tau$. Maximum vector length is 1.0.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{Function $\psi$ according to (3.3) and (3.4) for the mountain torque in unit of $10^{14}$ (kg$^2$ m$^4$ s$^{-3}$) for $|\tau| \leq 15$ days: (a) $T_0 = T_0$; $F = F_0$, and (b) $T_0 = T_0$; $F = F_x$. Otherwise as in Fig. 5.}
\end{figure}
case, clearly, it is the downward transport of negative angular momentum throughout the troposphere that overrides the effect of the torques. There is no descent from the stratosphere for negative lags in Fig. 7b, but the response to the torque near $\tau = 0$ and for $\tau > 0$ is fairly similar to that in Fig. 7a. An intercomparison of Figs. 4 and 6 demonstrates that the interaction of friction torque and $\mu$ involves longer time scales than that of the mountain torque (see also Weickmann 2003).

To sum up, we find a weak interaction of the troposphere with the torques for $\tau \leq -2$ days. The fluxes at the ground become quite strong and positive for $|\tau| < 2$ days. The rapid clockwise turning of $c$ up to the lower stratosphere must be seen as a response to the fluxes at the surfaces. The corresponding velocities of transfer are at least 10 km day$^{-1}$. For $\tau > 3$ days, say, a slow downward transport sets in that removes angular momentum from all layers so that the decay becomes first visible in the stratospheric layers.

An inspection of the vector field $c$ for $T_a = T, F = F_a$ does not reveal any new features beyond a superposition of Figs. 4 and 6. However, the $\psi$ field for $F = F_B$ and for $T = T_g$ is presented in Fig. 8 because $T_g$ is not the sum of $T_o$ and $T_g$. Although $\psi(\tau, 0)$ is symmetric in this case with respect to $\tau = 0$, the $\psi$ field is not symmetric for $z > 0$. The $\psi$ lines reach larger heights for $\tau > 0$ than for $\tau < 0$. This asymmetry reflects mainly the impact of the mountains. Note that the variability of $\mu$ represented in Fig. 8 need not cover the full variance of $\mu$ as displayed in Fig. 2 because the torques are not the only source of variability of the angular momentum $\bar{\mu}$ at a certain level.

4. Discussion

Although the ERA data satisfy the covariance Eqs. (3.5) and (3.6) reasonably well (Egger and Hoinka 2002), substantial uncertainties remain. Figures 5 and 7 suggest that it is mainly the torques that cause the problems. The results obtained with the modified total torque $T_B$ are more plausible than those based on the original data where the flux is $F = F_a$.

The dynamic interpretation of our results is hampered by the fact that we are looking at global data only. It is hardly possible this way to separate tropical processes from those at midlatitudes. Let us nevertheless make an attempt to compare our results to current dynamical ideas about angular momentum transfer, where it must be said that the effect of mountains on the atmospheric circulation has attracted much more interest than that of friction. It has been found (Figs. 4, 5) that negative angular momentum is transferred from the upper troposphere to the ground before events of positive friction torque. Midlatitude mountain torque events play a role in this process. One may speculate that Rossby waves excited by the mountains propagate southward to remove westerly angular momentum from tropical/subtropical regions where the positive friction torque is exerted. Such processes have been studied mainly within the framework of barotropic dynamics (e.g., Brunet and Haynes 1996), but the downward transport of negative angular momentum seen so clearly in Figs. 4 and 5 has as yet not been recognized. Of course, there do not exist any related models. The ensuing transfer of positive angular momentum from the ground to the troposphere is understood reasonably well. The transfer from the ground to the planetary boundary layer is mainly achieved by turbulent motions (e.g., Stull 1997). Simple concepts like Ekman layer suction explain the further transfer to the large-scale mass circulation. Altogether the impact of surface friction generates a meridional circulation that transfers angular momentum to upper levels. These processes have been described quite early
by Eliassen (1952). On the other hand, the following phase of downward transport and decay (see Figs. 4, 5) has as yet not been described theoretically.

As for the mountain torque there exists as yet no explanation why there is a layer of negative angular momentum in the lower troposphere before an event of positive torque (see Fig. 6). The main event itself is due to the interaction of large-scale waves with topography (e.g., Weickmann 2003). Theories of a form drag instability describe the interaction of baroclinic waves with mountains including the response of the mean angular momentum (e.g., Buzzi et al. 1984). These theories are, however, not designed to explain the interplay of mountain and friction torques. The rapid vertical redistribution of mountain torques seen in Figs. 6 and 7 can be understood in terms of the vertical propagation of Rossby waves (e.g., Andrews et al. 1987). The ensuing phase of decay has not attracted any interest as yet. The slowness of the downward transports suggests that Rossby waves are unimportant during that stage.

Further progress requires analyzing the meridional transports of angular momentum as well.

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APPENDIX

Interpolation Problems

Although the distribution of the layers in Fig. 1 is simple, there are problems partly because the surface may intersect the ground and partly because the transformation from the surface to the surface depends on the height of the orography. This latter problem is illustrated in Fig. 1, in which the central part depicts the location of the layers \( z_j \leq z \leq z_{j+1} \) (layer index \( j = 1 \ldots 28 \)) in the vertical. The lines on the left-hand side give the position of the 31 \( \sigma \) surfaces at the deepest point of the global orography (orography \( h = -300 \) m) as prescribed in the ERA set. The location of the \( \sigma \) surfaces above the highest point (\( h = 5700 \) m) is presented on the right-hand side of Fig. 1. The \( \sigma \) layers 1–5 are inside the mountain in this case. The information contained in the 31 \( \sigma \) layers is distributed over 23 \( z \) layers at this point while 27 \( \sigma \) levels provide the information for 28 layers in the first row. Moreover, the \( z \) layers 25–28 extract their data from two levels only (\( \sigma = 30, 31 \)) so that details of the vertical profiles above, say, \( z = 24 \) km are hardly reliable. All these considerations show clearly that there is a loss of accuracy when switching to the \( z \) coordinates. However, Table A1 suggests that these losses are not very large. In this table, the mean values and the standard deviation of the so-called wind and mass terms are given, that is, the contribution of the relative velocity to \( \mu \) and that of the earth’s velocity. This separation is introduced at this point because the mean value of the mass term dwarfs that of the wind term. Mean values and standard deviations of mass and wind term deviate by a few percent only from the “correct” value obtained from the \( \sigma \)-level representation if interpolated to the \( z \) system with \( \Delta z = 1000 \) m and \( z_1 = 100 \) m, as used in this paper. The height \( z_1 = 100 \) m of the lowest surface has been chosen on the basis of trial and error. Table A1 demonstrates that in most cases the closer results come to “reality,” the better the resolution. Note, however, that the improvement is not dramatic if we switch from the low-resolution case with \( \Delta z = 2000 \) m to a high resolution with \( \Delta z = 500 \). The problems caused by the intersection of \( z \) surfaces with orography [see (1.5)] are more serious. The ERA orography is available at the same grid points as all other variables. The integrals must be calculated on the basis of these data. One may think of introducing a fine grid with small horizontal and vertical mesh sizes so as to approximate the intersection lines to great detail. There is, however, little point in establishing such a rather costly scheme because the orography is available with crude horizontal resolution only. Moreover, this orography is smoothed as well so that the accuracy achieved this way is partly spurious. Instead, orography is taken into account by imposing the simple rule that a grid point of the \( z \) system does not contribute to integrals (1.5) if its height is less than that of the ERA orography at this location. It is seen from the percentages in the central row of Fig. 1 that 27.8% of the space of the lowest layer is covered by orography if this rule is applied. This orographic part is quite small above the second layer.

To obtain a feeling for the uncertainties involved we

| \( \sigma \) system: 30 layers | 1.725/0.557 | 1.015/0.264 |
| \( z \) system: \( \Delta z = 500 \) m 56 layers | 1.767/0.502 | 0.972/0.243 |
| \( z \) system: \( \Delta z = 1000 \) m 28 layers | 1.770/0.498 | 0.962/0.237 |
| \( z \) system: \( \Delta z = 2000 \) m 14 layers | 1.779/0.492 | 0.945/0.227 |
| \( z \) system: high resolution, near ground | 1.765/0.503 | 0.983/0.247 |
have calculated for a few months some fields on the basis of a rather dense layering ($\Delta z = 100$ m for $z < 2000$ m, $\Delta z = 500$ m for $2000 \leq z \leq 4000$ m; see also last row of Table A1). The standard deviations of $\mu_w$ and $\mu_m$ are presented in Fig. 9 for January 1979 for the lowest 3 km as given by the standard and by the high-resolution scheme. The agreement for the wind terms is quite good while the differences between the estimates of the mass term are relatively large in the lowest layer. Note, however, that the deviations are small in the second layer. Given the fairly large numerical expense needed to impose the high-resolution scheme with $\Delta z = 100$ m and the fact that uncertainties appear to be relatively large in the lowest 1000 m only, it has been decided to accept the inaccuracies of the evaluation of the mass term in the lowest layers and to stick to the scheme with $\Delta z = 1000$ m throughout the domain.

REFERENCES


