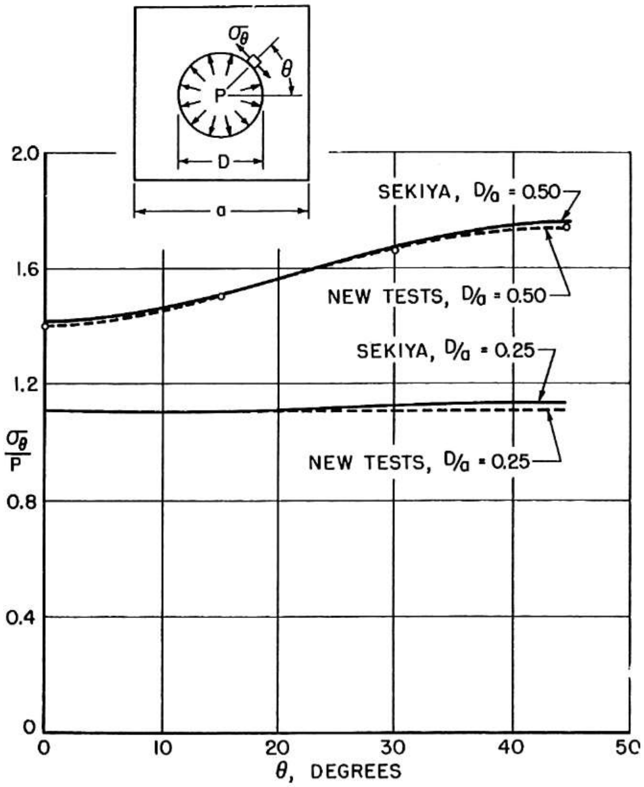

Fig. 5 Hydrostatically loaded central circular hole in a square plate. (Comparison of results obtained theoretically by Sekiya and results obtained from a new series of photoelastic tests.)

Kawaguchi and one of the authors has clarified the following points: (*a*) Kawaguchi's series is a convergent series; (*b*) the apparent divergence of the last term in Kawaguchi's approximate solution was due to the fact that all terms larger than the seventh order were neglected and the actual sixth term plus the remainder was set equal to an approximate sixth term.

Reference was made on a different type of loading jig for applying hydrostatic pressure to a circular hole. Dr. Flynn's idea of using lucite covers to provide boundary visibility is indeed good and may improve the author's loading jig. However, the overlapping of the lucite cover by as much as $^1/_2$ in. may impose a restriction on the free movement of the model due to friction between the model and lucite cover surfaces. Such friction could affect the accuracy of the experiment.

The authors do not believe that the method of determining the effective pressure at the hole boundary by integrating σ_x along a reference line using the shear difference method, as described by Dr. Flynn, is easy or accurate. The authors prefer to use the calibration jig described or a calibration jig which utilizes Lamé's formula for a thick-walled cylinder under internal pressure.

Under the light of the new tests, the extrapolation of the curves representing photoelastic results, Fig. 8 of the paper, should not go below ordinate 1 and should approach this value from above.

The authors would like to assure Dr. Flynn that repeated checks on the authors' experimental techniques were made from time to time not only by the authors but also by other investigators[7] and that the results published in this paper or in another paper by one of the authors[8] are free of the error which was pointed out.

[7] D. D. Ordahl and M. L. Williams, "Preliminary Photoelastic Design Data for Stresses in Rocket Grains," *Jet Propulsion*, vol. 27, June, 1957, pp. 657–662.

[8] A. J. Durelli and W. F. Riley, "Stress Distribution in Strips With Hydrostatically Loaded Central Circular Holes," Proceedings of the Second Mid-Western Conference on Solid Mechanics, Sept., 1955, pp. 81–93.

A New Method to "Lock-In" Elastic Effects for Experimental Stress Analysis[1]

M. M. LEVEN.[2] The authors are to be congratulated on devising an ingenious method for "locking-in" elastic strains in a photoelastic material without the application of heat.

However, some of the inherent difficulties in making practical application of such a method should, perhaps, be pointed out. The highly exothermic nature of epoxy-resin polymerization when using amine curing agents is well known. It is not unusual for the temperature at the center of a 2-in-diam cylindrical casting to rise as much as 100 deg C above ambient shortly after the start of gellation. Under these conditions, with a three-dimensional model of any appreciable size, there would exist an inhomogeneity in physical and optical properties at the time of initial loading. This would seriously affect the resulting stress pattern. Where there is no built-in calibration in the model, a separate calibration model must be employed, with no guarantee that the properties of the test model and calibration piece would correspond at the time of loading.

The difficulties of mixing and pouring viscous materials in any appreciable quantities without obtaining air bubbles have already been pointed out by D'Agostino, et al.[3] The impossibility of performing any machining operations after casting would be a serious detriment to practical tests rather than an advantage.

A possible explanation of the phenomena which the authors have discovered may be obtained by considering the critical temperature of the resin. In the semicured state the resin has a critical temperature (or second-order transition point) in the vicinity of room temperature. Then, as the load is applied and additional polymerization occurs, the critical temperature of the resin rises to some value probably higher than 100 C. As a result, elastic strains are fixed into the model not in the usual manner of lowering the temperature from the critical value but by raising the critical temperature of the model material.

The proposed method of "locking-in" elastic stresses has great appeal and simplicity for the case of gravitational forces and would be a very valid tool for such cases if the polymerization process could be controlled to give uniform curing rates throughout complicated models and calibration pieces.

Authors' Closure

The authors wish to thank Mr. Leven for his constructive comments. The explanation of the "freezing" phenomena based on an initial critical temperature in the vicinity of room temperature seems probable. This theory could be checked by heating a model with a locked-in fringe pattern to its new critical temperature and noting if the distortions are relieved.

Mr. Leven's cautioning comments on the exothermic nature of the epoxy resin are well founded. Most of the studies conducted thus far using this method were either two-dimensional or nearly two-dimensional and hence the temperature of the plastic could be controlled easily by slowing the rate of polymerization. The addition of an appreciable amount of plasticizer to the resin mix as well as a water-cooled mold aided in controlling the exothermic reaction.

A vacuum system is not necessary to avoid appreciable bubble formations. Recent work with liquid rubbers which are more

[1] By J. W. Dally, A. J. Durelli, and W. F. Riley, published in the June, 1958, issue of the Journal of Applied Mechanics, vol. 25, *Trans. ASME*, vol. 80, pp. 189–195.

[2] Westinghouse Research Laboratories, Pittsburgh, Pa.

[3] J. D' Agostino, D. C. Drucker, C. K. Liu, and C. Mylonas, "An Analysis of Plastic Behavior of Metals With Bonded Birefringent Plastic," *Proc. SESA*, vol. 12, 1954, pp. 115–122.

difficult to handle has shown that a good vacuum system practically eliminates the presence of bubbles.

The material can be machined in its semicured state by using a high-speed routing tool. However, this machining process may be applied only to relatively simple models. For more complicated geometries it is necessary to cast to final shape.

The Calculation of Optimum Concentrated Damping for Continuous Systems[1]

F. M. LEWIS.[2] The author has made an interesting contribution to the general theory of the optimum damping in vibrating systems, and the results apply to any system, distributed, concentrated, or mixed.

The formula (3*c*) which he derives for the optimum C is of delightful simplicity, but its application to actual cases may be of considerable difficulty.

I will note that this formula can be derived in an elementary manner by taking

$$\frac{d}{d\omega}|M_{31}|^2 = 0$$

and then substituting $b_2 = -b_3$.

The paper covers only one aspect of the problem. In the mechanical applications we are generally interested in minimizing displacements, or relative displacements, rather than velocities, and the damping may be either external or on the relative motion of two points.

The commonest torsional application involves minimizing the relative motion of two points with the relative motion of two other points damped, and exciting forces applied at a set of other points.

The writer has a prejudice against the "mobility method" in favor of a "compliance" which is the ratio of displacement force. Using this one does not so readily become lost in a forest of imaginaries.

To minimize a displacement, write:

$$x_1 = a_1F_1 + a_{12}F_2$$
$$x_2 = a_{21}F_1 + a_{22}F_2$$
$$x_3 = a_{31}F_1 + a_{32}F_2$$

with $F_2 = -jC\omega x_2$. The a are all real.

Solving for $R^2 = |x_3|^2/|F_1|^2$ and equating $dR^2/d\omega$ to zero at the fixed point there is obtained

$$C^2 = \frac{d/d\omega R_0}{a^2_{22}\omega^2\, d/d\omega} R_\infty$$

where R_0 is the value of R for $C = 0$ and R_∞ for $C = \infty$.

Problems of internal damping can be handled in the same manner, and the author must have made use of such a solution in his check of the damped dynamic absorber, although the procedure is not given in the paper.

Author's Closure

The author would like to thank Professor Lewis for his comments which extend the application of the suggested method. Following up his suggestion by differentiating $|M_{31}|^2$, equation [3] of the paper, it turns out that optimum damping is given by

$$|a_{22}|C = \left(\frac{M_{\rm I}'}{M_{\rm II}'}\right)^{1/2} \text{for both } b_2 = -b_3 \text{ and } b_2 = +b_3.$$

The paper should have stated more explicitly that M_{31} is a generalized mobility; v_3 may be the time derivative of any displacement dependent quantity: displacement, angle, strain, or stress, whether absolute or relative. Likewise F_1 may be any forcelike quantity; force, torque, unbalance, or fluid pressure, also absolute or relative. This means that we may treat the torsional problem cited by Professor Lewis by the methods demonstrated.

I am also pleased that Professor Lewis has pointed out that the same results apply to displacement and acceleration as were derived for velocity. The problem of mobility versus compliance is the subject of study by ASA committee S2-W38, which I trust will come up with a universally acceptable set of nomenclature.

[1] By R. Plunkett, published in the June, 1958, issue of the JOURNAL OF APPLIED MECHANICS, vol. 25, TRANS. ASME, vol. 80, p. 219.

[2] Professor of Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

Ring Damping of Free Surface Oscillations in a Circular Tank[1]

GARRETT BIRKHOFF.[2] It seems to the writer that in this interesting paper one unstated assumption is made; namely, (*d*) that the absorption of energy by a baffle from the dominant mode of sloshing is equal to the work required to move a baffle against a static fluid with the same relative motion.

Author's Closure

It appears to the author that the assumption suggested by Professor Birkhoff would be implicit only if the drag coefficient C_D were determined by moving a baffle against a static fluid, whereas the drag coefficient actually used—namely, that determined by Keulegan and Carpenter—was determined for a fixed baffle in a sinusoidally oscillating current. The author is further inclined to the opinion that, although not necessary for the application in question, the suggested assumption would be reasonable for the absorbtion of energy (but not, of course, for the inertial drag coefficient).

[1] By J. W. Miles, published in the June, 1958, issue of the JOURNAL OF APPLIED MECHANICS, vol. 25, TRANS. ASME, vol. 80, pp. 274–276.

[2] Department of Mathematics, Harvard University, Cambridge, Mass.

Creep Deflections and Stresses of Beam-Columns[1]

L. W. HU.[2] The author is to be commended for having considered the total creep deformation in his excellent analysis of creep of beam-columns.

In recent investigations, many analyses of the creep of structural and machine members have been made by taking into consideration only the secondary creep (or linear creep). Although this approach does render considerable mathematical simplicity,

[1] By T. H. Lin, published in the March, 1958, issue of the JOURNAL OF APPLIED MECHANICS, vol. 25, TRANS. ASME, vol. 80, pp. 75–78.

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