Nordic Hydrology, 21, 1990, 153-164

The Speed of Two Moving Rainfall Events in Lund

M. H. Diskin

Technion, Haifa 32000, Israel

The speed and direction of movement of rainfall patterns, assumed constant during any given storm, can be derived from data sets comprised of the coordinates (x, y) of a number of rainfall recording stations and the times of arrival (T) of some prominent feature of the recorded hyetographs. The result depends on the nature of the feature adopted. A measure of the significance of the result can also be derived by computing the reduction in the value of the *RMS* deviation of arrival times due to the assumption of a moving storm in contrast to the alternative assumption of random fluctuations about an equal arrival time. The paper also outlines a procedure for estimating the accuracy of the results based on repeated computations of the speed and direction using subsets of the data obtained by removing records of one rainfall measuring station. The procedure is demonstrated with data recorded in the city of Lund, Sweden, by a network of 12 stations for two different storms.

Introduction

Moving storms are characterized by the fact that the rainfall hyetographs recorded at different stations are displaced along the time axis with respect to one another. The time shift between two such hyetographs is related to the speed and direction of storm movement. However, the definition of the time shift is not unique because the shapes of the recorded hyetographs at the various stations are, in general, not similar. It is thus necessary to specify a procedure for determining the time shift. Two approaches are available for defining the time shift. One is to adopt some recognizable feature of the hyetographs as the indicator of the time shift (Foster 1948, Niemczynowicz and Dahlblom 1984, Hindi and Kelway 1977). The feature may be the beginning of rainfall, the time of highest intensity, the centroid of the hyetograph, etc. The time shift is then defined simply as the difference between the arrival times of the specified feature at the two stations. The alternative approach (Shearman 1977) is to compute the correlation coefficients between the two hyetographs for various trial values of the time shift and adopt the value producing the highest correlation as the true time shift between the hyetographs recorded at the two stations.

In addition to the fact that the various methods produce different numerical values for any given record, there is also a significant difference in the nature of their outcome. This can be noticed if the time shifts for a group of three (or more) stations are computed. Using a method based on a recognizable feature for three recording rainfall stations, A, B and C, gives values that are additive, *i.e.,* the time shift between stations A and C is equal to the sum of the time shift between A and B and that between B and C. Values of the time shift computed by a method based on correlation coefficients are, on the other hand, not additive. This can lead to some difficulties if the data are used for the computation of the speed and direction of the movement of the rainfall pattern.

In a previous article (Diskin 1987) a procedure was proposed for the computation of the speed and direction of the movement of rainfall patterns from rainfall data recorded at a number of stations. The method uses the recognizable feature concept as a basis for the evaluation of the speed and direction of the movement from a set of data comprising the coordinates of the rainfall stations and the times of arrival of the selected feature. The latter is read directly, or computed, from the recorded hyetographs at the various stations. The article also proposes a measure of the significance of the results in the form of the ratio of two values of RMS deviations of times of arrival. One with respect to the arrival times predicted by the computed speed and direction and the second with respect to the alternative assumption of equal times of arrival for all stations.

Using available sets of data of two storm events recorded in the city of Lund, Sweden, the present paper examines the range of values of speed and direction of movement that can be produced under given conditions. The results are taken as an indication of the accuracy obtainable with the proposed method for given sets of data. This, in turn, has some implications for hydrologic studies related to moving storms. The effect of the choice of the recognizable feature is also considered.

Method of Computation

The procedure developed for the computation of the speed and direction of movement is described in detail elsewhere (Diskin 1987). Briefly it consists of the evaluation of the parameters, *a*, *b* and *c*, of the equation predicting the time of arrival (*t*) of the selected feature at a station of known coordinates, *x* and *y*. Assuming constant values of the speed and direction throughout the storm event, the prediction equation is linear and has the following form

$$t \equiv ax + by + c \tag{1}$$

The speed of movement is related to the parameters *a* and *b* by

$$V \equiv \frac{1}{\sqrt{a^2+b^2}} \tag{2}$$

and the direction of storm movement, relative to the *x*-axis, is given by

$$\theta \equiv \arctan\left(\frac{b}{a}\right) \tag{3}$$

The values of the parameters are obtained from data sets by minimizing the sum of the squared deviations between the observed arrival times (T_i) and the predicted values (t_i) computed by Eq. (1) for all stations. The equations derived for the three parameters can be expressed in the following form

$$a = \frac{AE-BC}{DE-C^2} \tag{4}$$

$$b \equiv \frac{BD-AC}{DE-C^2} \tag{5}$$

$$c = \frac{\Sigma(T_i)-a\Sigma(x_i)-b\Sigma(y_i)}{N} \tag{6}$$

where the summation (Σ) is over the *N* values and the constants *A*,*B*,*C*,*D*, and *E* are computed from the known set of data by the following equations

$$A = N\Sigma(T_i\,x_i)-\Sigma(T_i)\Sigma(x_i) \tag{7}$$

$$B = N\Sigma(T_i\,y_i)-\Sigma(T_i)\Sigma(y_i) \tag{8}$$

$$C = N\Sigma(x_i\,y_i)-\Sigma(x_i)\Sigma(y_i) \tag{9}$$

$$D \equiv N\Sigma(x_i^2)-\Sigma(x_i)\Sigma(x_i) \tag{10}$$

$$E = N\Sigma(y_i^2)-\Sigma(y_i)\Sigma(y_i) \tag{11}$$

The method adopted herein appears to be very similar to that described by Niemczynowicz (1987), and, in fact, Eqs. (1), (2) and (3) are the same in the two methods. There is, however, a very significant difference in the procedure of evaluating the three parameters, *a*,*b* and *c*, and the speed and direction of movement. Niemczynowicz (1987) writes Eq. (1) for all groups of three stations (triads) in the network and solves the sets of three simultaneous equations for each triad for the parameters and for the speed and direction. The mean values of speed and direction for all triads is then adopted as values describing the storm movement. In the method used herein, the values of the parameters are computed directly in such a way that the sum of squared deviations, between measured and computed times of arrival, for all stations in the network will be minimal. The speed and direction of movement computed by these parameters are in fact optimal for the data available.

The above procedure will yield some numerical results for any given set of recorded data. There remains, however, the question of the accuracy and significance of the results.

The Significance Ratio

Considering a given set of data, the prediction equation given above is not the only possible model to represent the data. A reasonable alternative assumption that could be adopted is that the observed different times of arrival are random fluctuations from equal arrival times for all stations. This equal arrival time is the mean of the observed arrival times. The choice between the two models can be based on a comparison of the sums of squared deviations obtained by adopting the two models.

Except for synthetic noiseless data, the agreement between observed times of arrival (T) and predicted values (t) computed by Eq. (1) is not perfect. At each station some deviation will remain between the two values. A measure of the degree of agreement is the root mean square (*RMS*) deviation between the two values

$$u = \sqrt{\frac{\Sigma(T_i - t_i)^2}{N}} \qquad (12)$$

The value of the *RMS* deviation computed by Eq. (12) will always be smaller than the *RMS* deviation between the observed arrival times and the mean arrival time, which is the predicted value according to the alternative model. The ratio of the two values of *RMS* deviation can be adopted as a measure of the significance of the results. The ratio

$$W \equiv \sqrt{\frac{\Sigma(T_i - t_i)^2}{\Sigma(T_i - \bar{T})^2}} \qquad (13)$$

is always in the range of (0...1). Values close to zero indicate high significance and values near 1 point to poor performance of the proposed model. Some high value of the significance ratio, for example 0.85 or 0.90, may be adopted as the limiting value, above which the hypothesis of equal arrival times can not be rejected.

Accuracy Considerations

The results produced by the proposed method, including the value of the significance ratio, are subject to errors. These are due to inaccuracies in the observed data as well as disagreement between the real behavior of the storm and the assumption of constant speed and direction during the storm, on which Eq. (1) is founded. An important part of any study of moving storms is the estimation of the magnitude of the errors in the results obtained. If the number of rainfall measuring stations is relatively large, the following procedure can be proposed for estimating the accuracy of the results.

Consider a case where rainfall data is available for a number of stations (N). Naturally, to get the best estimate of the speed and direction of movement, data from all stations should be used in the computations. However, if the number of stations is large, a reasonable estimate could also be obtained by omitting one station and basing the computations on data from only N-1 stations. The results, based on data for N-1 stations, will obviously depend on the location of the station omitted and, in general, be different from the corresponding results for all N stations.

Computing the speed and direction of storm movement for all possible groups of N-1 stations, obtained by omitting one station at the time, will result in a set of results containing N values of computed speeds and N values of computed directions. The means of these results are expected to be very close to values of speed and direction derived with all data for the N stations. The standard deviations of the results give an indication of their accuracy. Strictly speaking, this is the accuracy of values derived from (N-1) stations. However, if N is large, the value obtained can also be taken as a measure of the accuracy of the results obtained with all the data.

A Numerical Example

Rainfall data available for a network of 12 recording rainfall measuring stations in the city of Lund, Sweden, were used for the demonstration of the results obtainable with the proposed procedures. The data were derived from detailed information contained in Appendix 2 of a report by Niemczynowicz (1984). The data is in the form of lists of rainfall intensities at time intervals of one minute for ten extreme rainfall events during the two years period of 1979/1980. Only two of the events listed have entries for all 12 stations and these two storms were selected for the present study. The locations of the raingaging stations included in the network, covering an area of about 15 sq.km, is shown in Fig. 1. Detailed descriptions of the network and data are available in the above report (Niemczynowicz 1984).

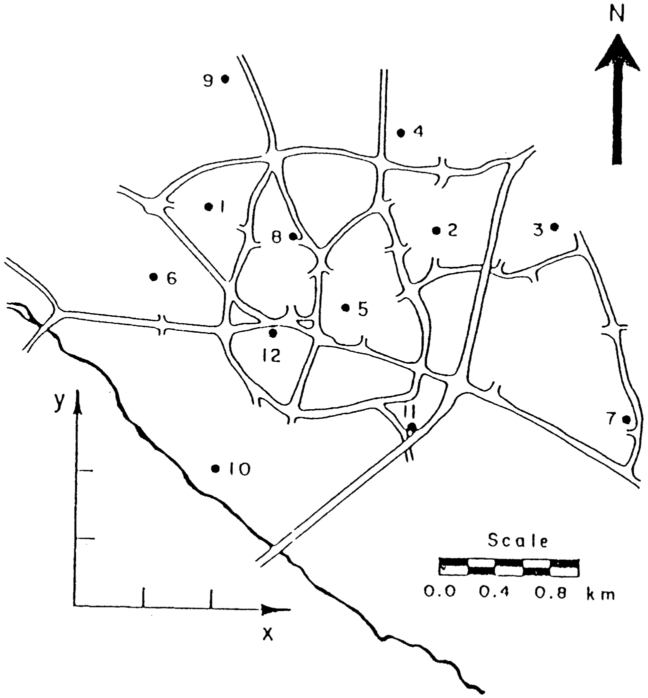

Fig. 1. Locations of the rainfall stations in the city of Lund.

Table 1 – Rainfall data for 2 storm events in the city of Lund

Gage No.	Coordinates		Storm No. 6 Date 79/09/21 zero time 14:00				Storm No. 9 Date 79/06/25 zero time 01:00			
	x km	*y* km	*d* mm	*tc* min	*tm* min	*tp* min	*d* mm	*tc* min	*tm* min	*tp* min
1	0.94	2.94	2.166	29.18	28.70	25.5	3.367	6.48	6.81	8.0
2	2.60	2.80	2.226	30.17	28.74	28.5	1.300	6.86	5.97	5.5
3	3.43	2.86	1.021	33.40	33.45	30.5	0.926	8.80	8.24	7.5
4	2.31	3.51	3.650	32.82	33.03	27.5	2.247	7.30	6.77	5.5
5	1.94	2.20	1.705	29.76	28.40	27.5	3.032	6.06	5.44	6.0
6	0.54	2.43	1.828	27.57	26.74	24.5	3.339	5.95	5.97	6.5
7	3.97	1.43	0.298	36.14	35.81	35.5	0.514	8.02	7.07	6.5
8	1.54	2.74	1.742	29.94	28.95	26.5	5.240	7.15	6.89	6.5
9	1.03	3.91	5.859	32.71	32.68	33.5	4.823	7.21	7.40	7.5
10	1.00	1.00	0.315	30.87	30.49	29.5	3.157	4.79	4.56	4.5
11	2.43	1.34	0.356	33.49	33.24	32.5	0.863	5.96	4.98	5.0
12	1.40	2.00	1.157	30.41	31.25	31.5	3.979	5.98	5.74	6.0

Data sets used in the present study are listed in Table 1 for the two storms studied. The values shown were either taken directly from the above report or derived from data listed in the report. The data set includes the *x* and *y* coordinates of the rainfall stations, the depth of total rainfall (*d*) for each station and storm event, and three values of times of arrival (*tc, tm, tp*) for each station and for each storm. The three features used for the definition of arrival times are: *tc* – the time to the centroid of the hyetograph, *tm* – the time to the median (50%) of the hyetograph, and *tp* – the time of highest rainfall intensity. The values of *tp* were taken at the center of the interval in which the peak intensity occurred. The origin of all time values is arbitrary, but it is, of course, the same for all stations in each storm.

Values of the speed and direction of movement were computed, using the complete sets of data in Table 1, for each of the three definitions of arrival time. The significance ratio was also computed for each case. The results are listed in Table 2 for the two storms studied. As expected, the values obtained for each storm using the various definitions of arrival time are not equal. The different values are, however, due to the different definitions of the arrival times and cannot be taken as the indication of the accuracy of the results.

The next step was the computation of the speed and direction of movement for partial data sets obtained from the complete set by omitting records of one of the stations. Doing so for all stations in turn resulted in 12 sets of data and 12 different values of speed and direction for each combination of storm date and arrival time definition. The results are listed in Tables 3 and 4 for speeds and directions, respectively. Table 3 contains the speed values computed for each of the three definitions of arrival times and for the two storms, and Table 4 lists the corresponding values of directions. Also listed in these tables are the range of values and their mean, standard deviation (S.D.) and coefficient of variation (C.V.). As expected, values of mean speed and mean direction in Tables 3 and 4 are very close to the corresponding values based on the complete set and listed in Table 2. The standard deviation and range, obtained for each combination of storm number and arrival time, represent the accuracy of the results.

Table 2 – Results obtained with the full set of data

	Storm No. 6			Storm No. 9		
Arrival time definition:	Centroid	Median	Peak	Centroid	Median	Peak
Speed (km/hr)	34.6	34.0	33.7	54.4	58.4	82.9
Direction (deg. from N)	91.0	89.9	113.7	46.8	28.1	4.2
Significance ratio	0.63	0.73	0.82	0.40	0.57	0.80

Table 3 – Computed speeds (km/hr) for two storms in Lund using N-1 stations

Omitted Gage	Storm No. 6			Storm No. 9		
No.	Cent.	Med.	Peak	Cent.	Med.	Peak
1	35.45	34.90	38.08	54.20	58.52	89.84
2	31.39	29.60	32.31	51.37	53.55	74.91
3	32.52	33.39	31.10	61.17	68.28	94.43
4	35.51	35.48	33.50	50.17	51.54	59.03
5	34.65	34.11	33.33	55.01	59.54	83.22
6	39.42	40.15	41.17	52.22	56.63	81.24
7	43.28	42.67	55.47	56.20	56.76	67.24
8	35.05	34.77	35.43	54.47	58.86	83.10
9	28.23	27.21	18.42	55.04	60.37	90.13
10	30.63	29.68	33.13	56.30	58.79	120.24
11	35.19	34.55	36.29	55.87	62.94	92.77
12	34.65	32.60	32.21	54.38	58.10	82.18
Min	28.23	27.21	18.42	50.17	51.54	59.03
Max	43.28	42.67	55.47	61.17	68.28	120.24
Mean	34.66	34.09	35.04	54.70	58.66	84.86
S.D.	3.78	4.12	8.07	2.68	4.08	14.61
C.V.	0.11	0.12	0.23	0.05	0.07	0.17

Table 4 – Computed directions for two storms in Lund using N-1 stations (Degrees from North)

Omitted Gage	Storm No. 6			Storm No. 9		
No.	Cent.	Med.	Peak	Cent.	Med.	Peak
1	90.10	88.94	112.66	47.70	31.13	15.82
2	86.23	83.59	108.85	46.85	29.87	9.29
3	88.91	89.32	107.97	45.11	21.92	−10.43
4	96.03	98.54	102.63	44.53	27.18	8.80
5	92.59	92.44	115.61	47.29	28.58	4.14
6	92.68	91.93	123.07	48.23	30.25	7.94
7	82.80	81.21	109.95	39.32	15.59	−13.44
8	90.09	88.37	112.31	48.01	29.29	4.27
9	111.96	112.90	138.43	48.07	30.09	6.11
10	80.18	78.50	106.67	47.43	28.14	− 8.09
11	84.41	82.47	106.74	49.24	31.83	6.00
12	91.08	87.21	106.50	46.77	28.21	4.52
Min	80.18	78.50	102.63	39.32	15.59	−13.44
Max	111.96	112.90	138.43	49.24	31.83	15.82
Mean	90.59	89.62	112.62	46.55	27.67	2.91
S.D.	7.77	8.78	9.29	2.51	4.38	8.46

Table 5 – Significance ratios for two storms in Lund using N-1 stations

Omitted Gage No.	Storm No. 6 Cent.	Storm No. 6 Med.	Storm No. 6 Peak	Storm No. 9 Cent.	Storm No. 9 Med.	Storm No. 9 Peak
1	0.66	0.75	0.86	0.39	0.55	0.78
2	0.55	0.62	0.81	0.34	0.48	0.75
3	0.65	0.76	0.82	0.44	0.61	0.80
4	0.63	0.72	0.81	0.36	0.53	0.64
5	0.61	0.69	0.81	0.38	0.55	0.81
6	0.69	0.79	0.87	0.39	0.56	0.80
7	0.77	0.85	0.95	0.39	0.52	0.73
8	0.63	0.73	0.83	0.37	0.56	0.81
9	0.43	0.57	0.37	0.40	0.60	0.86
10	0.61	0.71	0.82	0.48	0.67	0.89
11	0.63	0.73	0.84	0.39	0.59	0.85
12	0.64	0.71	0.79	0.41	0.58	0.81
Min	0.43	0.57	0.37	0.34	0.48	0.64
Max	0.77	0.85	0.95	0.48	0.67	0.89
Mean	0.62	0.72	0.80	0.40	0.57	0.79
S.D.	0.08	0.07	0.13	0.03	0.05	0.06
C.V.	0.13	0.10	0.16	0.08	0.09	0.08

Conclusions

As expected, the speed and direction of storm movement were different for the various definitions of the recognizable feature used with data for the same storm (Table 2). There is a tendency for results based on the centroid and those based on the median to be approximately the same, but this conclusion must be checked with data for a larger number of storms and for different networks of rainfall measuring stations.

Considering the results listed in Tables 3 and 4, it appears that speeds and directions computed from the arrival times of the centroid (*tc*) have the smallest errors, and values based on the arrival times of the peak (*tp*) have the largest errors. The values of the coefficient of variation for the computed speeds based on *tc* and *tm* are of the order of 10% while the corresponding values based on *tp* are about 20%. The above observations were found to be similar for the two storms studied. Comparing values of the standard deviations of the speeds and directions for the two storms, it appears that values computed for storm No. 9 have smaller errors than the corresponding values for storm No. 6. The ranges of values obtained for the speed and direction are of the order of 4 times the standard deviation.

Storm No. 9 was found to have also smaller and hence better values of the significance ratio than Storm No. 6. The values of this ratio, listed in Table 5, indicate that using either the time to the centroid or the time to the median as the recognizable feature leads to significant results, whereas values derived from data sets based on the time of the peak are near the limiting value mentioned above as the limit for random fluctuation about equal arrival time.

Notation

a – parameter in prediction equation
b – parameter in prediction equation
c – parameter in prediction equation
A – expression defined by Eq. (7)
B – expression defined by Eq. (8)
C – expression defined by Eq. (9)
D – expression defined by Eq. (10)
E – expression defined by Eq. (11)
N – number of rainfall stations
t – predicted time of arrival
T – observed time of arrival
$\bar{T}$ – mean arrival time
u – *RMS* deviation between observed and predicted arrival times
V – speed of movement of a rainfall pattern
W – significance ratio
x,y – coordinates of rainfall measuring station
θ – angle between direction of movement and x-axis

References

Diskin, M.H. (1987) On the Determination of the Speed of Moving Rainfall Patterns, *Hydrological Sciences Journal, Vol 32, No. 1,* pp. 1-14.

Foster, E.E. (1948) *Rainfall and Runoff*, The Macmillan Company, New York, N.Y.

Hindi, W.N.A., and Kelway, P.S. (1977) Determination of Storm Velocities as an Aid to the Quality Control of Recording Raingauge Data, *Journal of Hydrology, Vol. 32,* pp. 115-137.

Niemczynowicz, J. (1984) An Investigation of the Areal and Dynamic Properties of Rainfall and its Influence on Runoff Generating Processes. Report No. 1005, Department of Water Resources Engineering, Lund Institute of Technology, University of Lund, Sweden.

Niemczynowicz, J. (1987) Storm Tracking Using Rain Gauge Data, *Journal of Hydrology, Vol. 93,* pp. 135-152.

Niemczynowicz, J., and Dahlblom, P. (1984) Dynamic Properties of Rainfall in Lund, *Nordic Hydrology, Vol. 15, No. 1,* pp. 9-24.

Shearman, R.J. (1977) The Speed and Direction of Movement of Storm Rainfall Patterns with Reference to Urban Storm Sewer Design, Hydrological Sciences Bulletin, vol. 22, No. 3, pp. 421-431.

Received: 24 January, 1990
Accepted: 9 March, 1990

Address:
Faculty of Civil Engineering,
Technion,
Israel Institute of Technology,
Technion City,
Haifa 32000,
Israel.