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Overland Flow on Pervious, Converging Surface

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A desk-top method based on kinematic overland flow and Green-and-Ampt infiltration models is developed to determine the time of concentration of and the peak runoff rate from a converging basin. The governing equations of the rainfall-infiltration-overland flow process are written in terms of various dimensionless parameters. Common values of these parameters indicate hydrologic similarity of an infinite number of different converging basins, and therefore the results obtained in terms of these physically-based parameters can be generalized. Using this similarity concept, two charts have been developed to determine the time of concentration of a basin and the peak runoff rate. As illustrated by a sample application, these charts can be used for quick estimates of the design discharge for storm drainage facilities.

Introduction

Overland flow has caught considerable attention in the past because of its hydrologic significance. Most of the earlier research efforts were devoted to plane overland flow resulting from constant rainfall excess (Henderson and Wooding 1965, Wooding 1965). Variable rates of losses due to infiltration have been taken into account in more recent overland flow studies (Akan 1985b, Akan and Yen 1981, Hjelmfelt 1978, Smith and Woolhiser 1971).

As pointed out by Woolhiser (1969), an actual watershed hydrograph has a steeply rising portion caused by concentration of runoff, and this portion may not be reproduced by plane overland flow models. A converging surface, however, accounts for runoff concentration, and it provides a better representation of watershed runoff. Veal (1966) derived and attempted to obtain numerical solutions to the equations of flow on converging surfaces. However, he experienced stability problems. Woolhiser (1969) solved the characteristic equations of kinematic flow on a converging surface numerically. Singh and Woolhiser (1976) reported a kinematic converging flow model for watershed runoff and verified the model with field data. Campbell *et al.* (1984) proposed a similarity solution for the kinematic equations of converging flow. Among these studies only Singh and Woolhiser (1976) considered variable infiltration, and they employed a simple arithmetical formula to calculate the losses due to infiltration.

The physically-based mathematical model presented herein is based on the kinematic overland flow and Green-and-Ampt infiltration equations. These equations are combined and rewritten in nondimensional form, and they are obtained in terms of various dimensionless parameters representing the basin geometry, the subsurface soil characteristics, and the rain conditions. Since common values of these parameters indicate hydrologic similarity of an infinite number of different converging basins, the results of the model can be generalized. The time of concentration, and the peak discharge charts presented in this paper have been developed using the similarity concept.

Infiltration Model

The Green-and Ampt model is adopted to evaluate losses due to infiltration. It has been shown by Mein and Larson (1971) and Morel-Seytox and Khanji (1974) that the Green-and-Ampt model has a precise physical basis. Also its results agree well with those of Richards equation (Mein and Larson 1971).

An initially unsaturated soil subjected to a constant rainfall rate i is considered. The whole rainfall is assumed to infiltrate until ponding begins at time t_s, which is expressed as (Mein and Larson 1971)

$$t_s = \frac{P_f \phi (1-S_i)}{i\left(\frac{i}{K_s} - 1\right)} \qquad (1)$$

where ϕ, is the soil porosity, S_i is the initial degree of saturation, K_s is the hydraulic conductivity, and P_f is the characteristic suction head that can be obtained from the suction pressure-relative permeability relationship of the soil. As the ponding time is reached the potential rate of infiltration drops below the rate of rainfall, and that part of rainfall which cannot infiltrate runs off over the ground. After the overland flow commences, that is for $t > t_s$ where t is time, the rate of infiltration, f, is expressed as

$$f = K_s\left(1 + \frac{P_f\,\phi\,(1-S_i)}{i\,t_s + \int_{t_s}^{t} f\,dt}\right) \qquad (2)$$

The depth of flow is not included in Eq. (2), since it has negligible effect on infiltration (Akan and Yen 1977).

Overland Flow Model

The kinematic-wave approximation is adopted to represent the overland flow component of the model since the kinematic solutions were shown to give very reliable results for most hydrologically significant cases (Woolhiser and Liggett 1967, Akan 1985a).

Overland flow on a converging surface that has uniform properties is described using the expression

$$\frac{\partial y}{\partial t} + \alpha\, m y^{m-1}\,\frac{\partial y}{\partial t} = i - f + \frac{\alpha\, y^m}{L_0 - r} \qquad (3)$$

where y is the flow depth, t is time, r is the space coordinate, i is the rate of rainfall, f is the rate of infiltration, L_o is the radius of the flow region as shown in Fig. 1, $L_o(1\text{-}c)$ is the flow length, c is the ratio of the radius of the bottom segment to L_o, and α and m are constant parameters relating the flow depth, y, to local velocity, v, as

$$v = \alpha\, y^{m-1} \qquad (4)$$

The values of α and m depend upon the friction formula used. In this paper the Manning formula is adopted because of its popularity. Provided that the roughness factor, n, is treated as a fitting parameter the Manning formula can be used satisfactorily for overland flow (Woolhiser 1975, Engman 1983). This "effective n" is assumed to include the effects of various factors like channelization and obstacles on flow resistance. For the Manning formula $m = 5/3$.

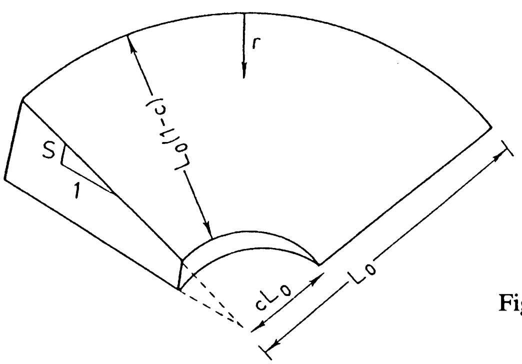

Fig. 1. Geometry of a Converging Surface.

$$\alpha = \frac{k}{n} S^{\frac{1}{2}} \tag{5}$$

where k = 1.0 $m^{1/3}$ /sec = 1.49 $ft^{1/3}$ /sec.

Since the overland flow will commence at $t = t_s$, the initial condition for Eq. (3) is $y = 0$ at $t = t_s$ for all r. Likewise, the upstream boundary condition is $y = 0$ at $r = 0$ for all t. No downstream boundary condition is needed.

Nondimensional Equations

It is possible to rewrite the governing equations of the overland flow-infiltration process in nondimensional form in terms of

$$P \equiv \frac{P_f \phi (1-S_i)}{i t_e} \tag{6}$$

$$K = \frac{K_s}{i} \tag{7}$$

$$F = \frac{f}{i} \tag{8}$$

$$T = \frac{t}{t_e} \tag{9}$$

$$T_s = \frac{t_s}{t_e} \tag{10}$$

$$Y = \frac{y}{i t_e} \tag{11}$$

$$R = \frac{r}{L_o} \tag{12}$$

where t_e is the equilibrium time for an impervious, rectangular basin which has a length of L_o, slope of S, and an effective roughness factor of n. Under a constant rate of rainfall i the equilibrium time is expressed as

$$t_e \equiv \left(\frac{L_o}{\alpha i^{m-1}}\right)^{\frac{1}{m}} \tag{13}$$

Substituting Eqs. (6) to (12) into Eqs. (1), (2), and (3), one obtains

$$T_s \equiv \frac{PK}{1-K} \tag{14}$$

$$F = K\left(1 + \frac{P}{T_s + \int_{T_s}^{T} F\,dt}\right) \tag{15}$$

and

$$\frac{\partial Y}{\partial T} + m\,Y^{m-1} \frac{\partial Y}{\partial R} \equiv 1 - F + \frac{Y^m}{1-R} \tag{16}$$

Characteristic Equations

Using the method of characteristics Eq. (16) can be transformed into a set of two ordinary differential equations

$$\frac{dY}{dT} = 1 - F + \frac{Y^m}{1-R} \tag{17}$$

and

$$\frac{dR}{dT} = mY^{m-1} \tag{18}$$

Eq. (17) represents the growth of local flow depth, and it is valid along a characteristic curve described by Eq. (18). The characteristic curves trace out space-time paths of disturbances in surface flow as explained below.

In terms of the dimensionless parameters the rain starts at $T = 0$. However, the overland flow will commence at $T = T_s$, at which time the infiltration capacity of the surface soil drops just below the rate of rainfall. As the overland flow commences, the local depth will increase everywhere at a rate given by Eq. (17) except at the upstream boundary, $R = 0$, where the dimensionless depth, Y, remains zero. This surface flow deficit will create a disturbance at the point defined by $T = T_s$ and $R = 0$ in the R-T plane. The disturbance will travel along the "limiting characteristic" described by the expression

$$R = m \int_{T_s}^{T} \left[\int_{T_s}^{T} \left(1 - F + \frac{Y^m}{1-R}\right) dT \right]^{m-1} dT \tag{19}$$

Eq. (19) is obtained from Eqs. (17) and (18) with $R = 0$ and $T = T_s$. A typical path of an initial disturbance along the limiting characteristic in the R-T plane is shown in Fig. 2.

Let T_t denote the time needed for the disturbance to reach the downstream boundary of the surface, which is defined by $R = L_0\,(1\text{-}c)\,/\,L_0 = (1\text{-}c)$, after it has been created at the upstream end at time $T = T_s$. Then with respect to the starting time of the rainfall, $T = 0$, the initial disturbance will reach the basin outlet at $T = T_c$ where

$$T_c = T_s + T_t \tag{20}$$

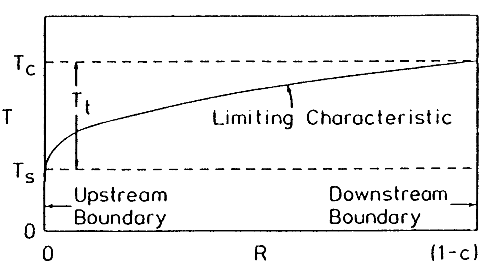

Fig. 2. Limiting Characteristic and Time of Concentration.

Time of Concentration

The arrival of the initial disturbance at the basin outlet marks the time the whole basin starts contributing to basin discharge, and it represents the time of concentration of the basin (Henderson and Wooding 1964, Hjelmfelt 1978). Thus the dimensionless parameter T_c may be called the relative time of concentration. With reference to Eq. (9), T_c becomes

$$T_c = \frac{t_c}{t_e} \tag{21}$$

where t_c is the time of concentration of the basin.

As shown in Fig. 2, the limiting characteristic passes through the point that has the coordinates $T = T_c$ and $R = L_o\,(1\text{-}c)/L_o = (1\text{-}c)$ in the R-T plane. Hence substituting T_c and (1-c) into Eq. (19) for T and R, respectively, one obtains

$$1 - c = m \int_{T_s}^{T_c} \left[\int_{T_s}^{T_c} \left(1 - F + \frac{Y^m}{1-R} \right) dT \right]^{m-1} dT \tag{22}$$

Note that R is maintained on the right hand side of Eq. (22) as a variable since this variable takes the value of (1-c) not before $T = T_c$.

The relative time of concentration, T_c, can be calculated by approximating Eq. (22) in finite difference form as

$$1-c = m' \sum_{j=1}^{N} \left[\sum_{j=1}^{N} \left(1-F_j + \frac{Y_j^m}{1-R_j} \right) \Delta T \right]^{m-1} \Delta T \tag{23}$$

where j is the time step of computation and $\Delta T = \Delta t/t_e$ dimensionless computational time increment. In this study $\Delta T = 0.0001$ is used. The time step $j = 1$ marks the commencement of the surface runoff, and $j = N$ is the time step at which the right hand side of Eq. (23) becomes equal to (1-c). Therefore, $T_c = (N)\,(\Delta T)$. If at the N-th time step the right hand side exceeds (1-c), the time increment ΔT is reduced for the last time step so that Eq. (23) is satisfied.

The dimensionless quantities F_j, Y_j, and R_j are calculated using the finite difference approximations to Eqs. (15), (17) and (18), respectively, as

$$F_j = K \left(1 + \frac{P}{T_s + \sum_{k=0}^{j-1} F_k \, \Delta T} \right) \tag{24}$$

$$Y_j = Y_{j-1} + \left(1-F_j + \frac{Y_{j-1}^m}{R_{j-1}} \right) \Delta T \tag{25}$$

and

$$R_j = R_{j-1} + m \Delta T \, Y_j^{m-1} \tag{26}$$

where k represents a time step with $k = 0$ marking the commencement of the overland flow.

Peak Discharge

The peak discharge from the basin will occur at $t = t_c$ if the duration of the rain is equal to the time of concentration. Defining the dimensionless peak discharge, Q_c, as

$$Q_c = \frac{Q_p}{iA} \tag{27}$$

where Q_p is the peak discharge, and A is the basin area. It can be shown that for a converging surface

$$Q_c = \frac{2c}{1-c^2} \, Y_n^m \tag{28}$$

In Eq. (28), y_n is the dimensionless flow depth at the basin outlet which is determined using Eq. (25) at $T = T_c$, that is at $t = t_c$.

Time of Concentration and Peak Discharge Charts

The Manning formula is adopted herein to represent the flow resistance, and therefore $m = 5/3$. For a constant value of m, a close examination of Eqs. (14), (20) and (23) to (28) reveals that T_s is a function of P and K, while T_t, T_c, and Q_c are functions of P, K, and c. Therefore an infinite number of different converging surfaces can be said to be hydrologically similar if they have common values for P, K, and c. Using the similarity concept the results of the mathematical model presented herein can be generalized.

For systematically chosen combinations of the dimensionless parameters P, K, and c, the mathematical model was employed to calculate $T_t = T_c$-T_s and Q_c. The results were then plotted to develop the family of curves displayed in Figs. 3 and 4. The chart given in Fig. 3 can be used to determine the time of concentration of any converging, infiltrating surface subjected to a constant rainfall rate of any magnitude. Then the chart given in Fig. 4 can be used to estimate the peak discharge produced by a rainfall that has a duration equal to the time of concentration of the basin.

In order to apply these charts one needs to estimate the soil characteristics ϕ, K_s, and P_f, and the Manning roughness factor n. The soil characteristics can be obtained from the soil texture (Rawls and Brakensiek 1983). The "effective n" values are also available for a variety of natural and agricultural surfaces (Woolhiser 1975, and Engman 1983).

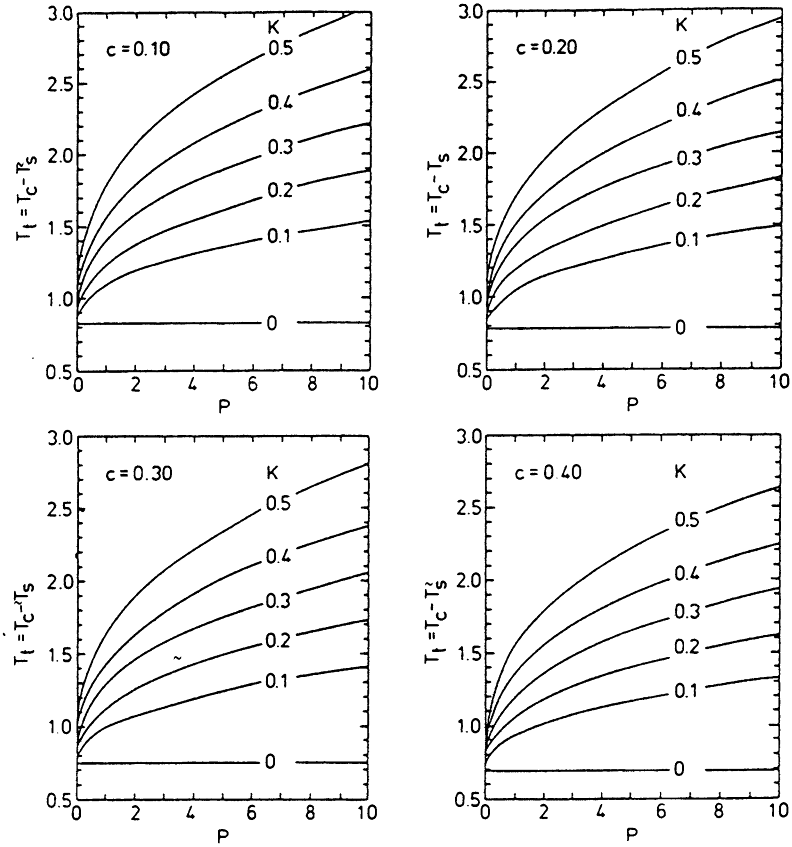

Fig. 3. Relative Time of Concentration Charts.

Practical Application

The use of the charts presented can be illustrated by a hypothetical example. A culvert to be placed under a roadway in the Knoxvile, Tennessee area will be designed for a 25-year storm. The intensity-duration curve for 25-year return period for this area has been constructed using the information from Overton and Meadows (1976), and it is shown in the form of a solid line in Fig. 5. Let the catchment be approximated by a converging surface with A = 18,600 m^2, L_o = 305 m, c = 0.20, S = 0.04, and n = 0.10. The antecedent degree of soil saturation for the design condition is chosen as S_i = 0.40. Let the Green and Ampt infiltration parameters be ϕ = 0.50, K_s = 12.7 mm/hr, and P_f = 0.305 m.

To determine the design discharge for the culvert it will be assumed that, within the context of the design return period, the maximum peak discharge is produced

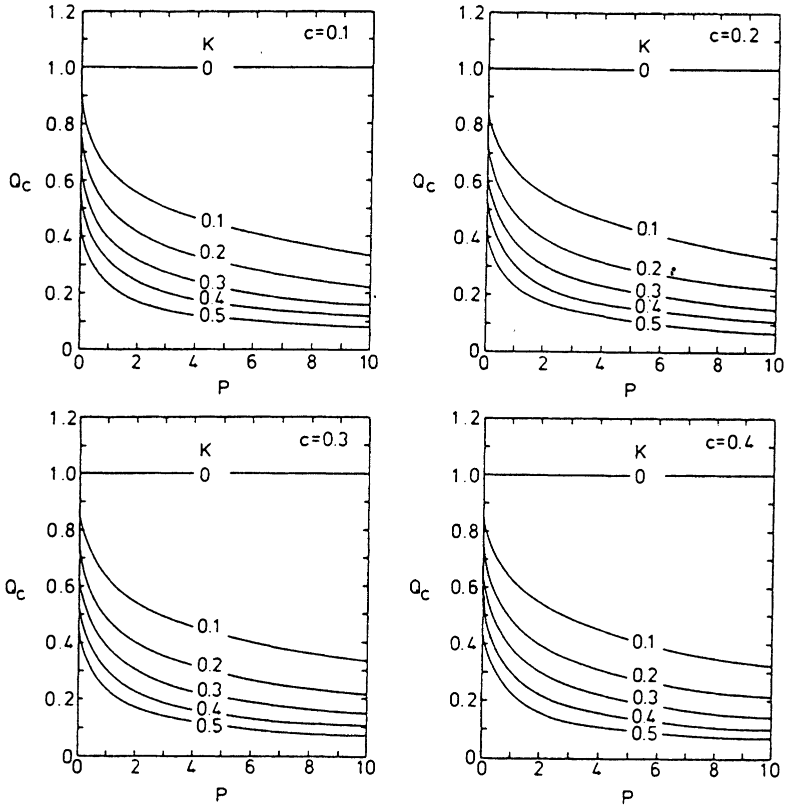

Fig. 4. Peak Discharge Charts.

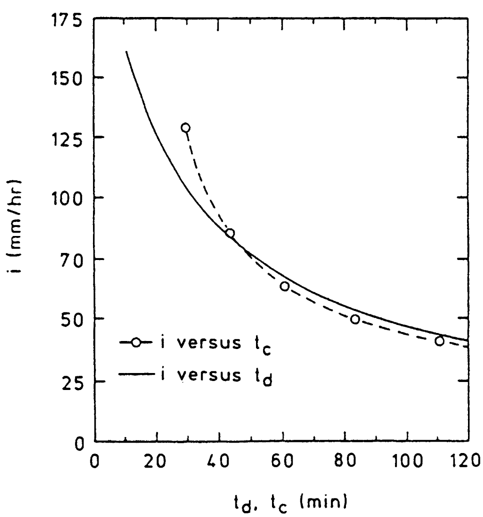

Fig. 5. Example Application.

Table 1 –Practical Application

i (mm/hr) (1)	t_e (sec) (2)	K (3)	P (4)	T_s (5)	T_t (6)	T_c (7)	t_c (sec) (8)	t_c (min) (9)
42.3	1,908	0.30	4.07	1.74	1.76	3.50	6,678	111.3
50.8	1,775	0.25	3.65	1.22	1.60	2.82	5,006	83.4
63.5	1,624	0.20	3.19	0.80	1.45	2.25	3,654	60.9
84.7	1,448	0.15	2.69	0.47	1.33	1.80	2,606	43.4
127.0	1,230	0.10	2.11	0.23	1.19	1.42	1,747	29.1

by a rainfall whose duration is the same as the time of concentration of the catchment. This assumption concurs with the rational method and is broadly used in engineering practice. The first step in the solution procedure is to determine the rate and the duration of the design rainfall. Since the duration is equal to the time of concentration of the basin, both the intensity-duration curve shown in Fig. 5 and the time of concentration relationships given in Fig. 3 should be satisfied by the design rainfall intensity and duration. Then if a plot of i versus t_c is superposed over the intensity-duration curve the point of intersection of the two curves will give the design rainfall intensity and duration.

To develop an intensity-time of concentration plot the following procedure is repeated for various intensities chosen. For any rainfall intensity, i, the equilibrium time, t_e, is calculated from Eq. (13), and P and K are evaluated using Eqs. (6) and (7), respectively. Then T_t is obtained from Fig. 3, T_c is calculated using Eq. (20), and t_c is found from Eq. (21). Listed in Table 1 are P, K, the T_s, T_c, and t_c calculated for various rainfall rates, i. It should be noted that as determined from Eq. (5) $\alpha = 2\text{m}^{1/3}$ /sec for this basin.

Next column 1 of Table 1 is plotted against column 9 and superposed over the intensity-duration curve as shown in Fig. 5. The point of intersection gives i = 80.6 mm/hr and $t_d = t_c = 46$ min. Then for the design rainfall t_d, P, and K are obtained as being 1,480 sec, 2.78, and 0.16 using Eqs. (13), (6), and (7), respectively. From Fig. 4, Q_c is determined as being 0.43, and finally the design discharge, Q_p, is calculated from Eq. (27) as being 0.18 m^3/sec.

Concluding Remarks

Previously reported kinematic overland flow and Green and Ampt inflitration models have been utilized to develop a desk-top method to determine the time of concentration and the peak runoff rate from converging surfaces. This method can be used for quick estimates of the design discharge for drainage facilities. Despite its simplicity the method can account for the surface geometry, subsurface soil characteristics, antecedent soil moisture content, and the rainfall characteristics of a basin. The major assumptions of the method in addition to those of the kinematic-wave and Green and Ampt models are that the rainfall rate is constant and the basin properties are uniform.

To apply the proposed method one needs to approximate a real catchment by an idealized converging surface. If the dominating type of surface runoff in the real basin is overland flow, one-to-one correspondence between the elements of the real catchment and the idealized system may be possible. In that event the method presented may be viewed as being physically-based since the parameters used in the method, such as n, ϕ, P_f, and K_s can be obtained from available sources (Engman 1983, Woolhiser 1975, and Rawls and Brakensiek 1983). However, if the idealized converging surface is meant to represent a complex watershed with various overland flow and channel flow elements the model parameters should be treated as fitting parameters, and the model calibration is unavoidable. In any event, verification of the proposed method with field data is most desired.

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