

**DISCUSSION**

difficult to handle has shown that a good vacuum system practically eliminates the presence of bubbles.

The material can be machined in its semicured state by using a high-speed routing tool. However, this machining process may be applied only to relatively simple models. For more complicated geometries it is necessary to cast to final shape.

## The Calculation of Optimum Concentrated Damping for Continuous Systems<sup>1</sup>

**F. M. LEWIS.**<sup>2</sup> The author has made an interesting contribution to the general theory of the optimum damping in vibrating systems, and the results apply to any system, distributed, concentrated, or mixed.

The formula (3c) which he derives for the optimum  $C$  is of delightful simplicity, but its application to actual cases may be of considerable difficulty.

I will note that this formula can be derived in an elementary manner by taking

$$\frac{d}{d\omega} |M_{31}|^2 = 0$$

and then substituting  $b_2 = -b_3$ .

The paper covers only one aspect of the problem. In the mechanical applications we are generally interested in minimizing displacements, or relative displacements, rather than velocities, and the damping may be either external or on the relative motion of two points.

The commonest torsional application involves minimizing the relative motion of two points with the relative motion of two other points damped, and exciting forces applied at a set of other points.

The writer has a prejudice against the "mobility method" in favor of a "compliance" which is the ratio of displacement force. Using this one does not so readily become lost in a forest of imaginaries.

To minimize a displacement, write:

$$\begin{aligned} x_1 &= a_{11}F_1 + a_{12}F_2 \\ x_2 &= a_{21}F_1 + a_{22}F_2 \\ x_3 &= a_{31}F_1 + a_{32}F_2 \end{aligned}$$

with  $F_2 = -jC\omega x_2$ . The  $a$  are all real.

Solving for  $R^2 = |x_3|^2/|F_1|^2$  and equating  $dR^2/d\omega$  to zero at the fixed point there is obtained

$$C^2 = \frac{d/d\omega R_0}{a^2_{22}\omega^2 d/d\omega} R_\infty$$

where  $R_0$  is the value of  $R$  for  $C = 0$  and  $R_\infty$  for  $C = \infty$ .

Problems of internal damping can be handled in the same manner, and the author must have made use of such a solution in his check of the damped dynamic absorber, although the procedure is not given in the paper.

### Author's Closure

The author would like to thank Professor Lewis for his comments which extend the application of the suggested method.

<sup>1</sup> By R. Plunkett, published in the June, 1958, issue of the JOURNAL OF APPLIED MECHANICS, vol. 25, TRANS. ASME, vol. 80, p. 219.

<sup>2</sup> Professor of Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

Following up his suggestion by differentiating  $|M_{31}|^2$ , equation [3] of the paper, it turns out that optimum damping is given by

$$|a_{22}|C = \left(\frac{M_{11}'}{M_{11}''}\right)^{1/2} \text{ for both } b_2 = -b_3 \text{ and } b_2 = +b_3.$$

The paper should have stated more explicitly that  $M_{31}$  is a generalized mobility;  $v_3$  may be the time derivative of any displacement dependent quantity: displacement, angle, strain, or stress, whether absolute or relative. Likewise  $F_1$  may be any forcelike quantity; force, torque, unbalance, or fluid pressure, also absolute or relative. This means that we may treat the torsional problem cited by Professor Lewis by the methods demonstrated.

I am also pleased that Professor Lewis has pointed out that the same results apply to displacement and acceleration as were derived for velocity. The problem of mobility versus compliance is the subject of study by ASA committee S2-W38, which I trust will come up with a universally acceptable set of nomenclature.

## Ring Damping of Free Surface Oscillations in a Circular Tank<sup>1</sup>

**GARRETT BIRKHOFF.**<sup>2</sup> It seems to the writer that in this interesting paper one unstated assumption is made; namely, (d) that the absorption of energy by a baffle from the dominant mode of sloshing is equal to the work required to move a baffle against a static fluid with the same relative motion.

### Author's Closure

It appears to the author that the assumption suggested by Professor Birkhoff would be implicit only if the drag coefficient  $C_D$  were determined by moving a baffle against a static fluid, whereas the drag coefficient actually used—namely, that determined by Keulegan and Carpenter—was determined for a fixed baffle in a sinusoidally oscillating current. The author is further inclined to the opinion that, although not necessary for the application in question, the suggested assumption would be reasonable for the absorption of energy (but not, of course, for the inertial drag coefficient).

## Creep Deflections and Stresses of Beam-Columns<sup>1</sup>

**L. W. HU.**<sup>2</sup> The author is to be commended for having considered the total creep deformation in his excellent analysis of creep of beam-columns.

In recent investigations, many analyses of the creep of structural and machine members have been made by taking into consideration only the secondary creep (or linear creep). Although this approach does render considerable mathematical simplicity,

<sup>1</sup> By J. W. Miles, published in the June, 1958, issue of the JOURNAL OF APPLIED MECHANICS, vol. 25, TRANS. ASME, vol. 80, pp. 274–276.

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<sup>1</sup> By T. H. Lin, published in the March, 1958, issue of the JOURNAL OF APPLIED MECHANICS, vol. 25, TRANS. ASME, vol. 80, pp. 75–78.

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