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Effects of Excess Rainfall Time Distribution on Catchment Area Hydrograph

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The effect of triangular excess hyetographs on catchment area runoff process is investigated, thereby fixing the precipitation time. The approach accounts for the kinematic wave theory and allows a discussion of arbitrary hyetograph shapes. The results are compared with these originating from a uniform excess rainfall and underline the importance of proper account for the hyetograph shape.

Introduction

The classical approach to the rainfall-runoff process is to consider a *uniform* precipitation distribution over a time lapse t_p. Consequently, neither *local* nor *temporal* effects of the rainfall load on the resulting runoff are accounted for. For relatively *small* catchment areas the local distribution of the rainfall intensity may be overlooked. However, as has first been outlined by Huff (1967), its time distribution is of more or less triangular shape in most of the cases (single-peaked hyetographs), of which the peak intensity is located at the first or second quartile of the complete rainfall duration, t_p, provided t_r has the order of 2 to 4 hours. Evidently, these rainfalls may provoke *heavy storms,* while longer events have lower peak intensities (Raudkivi 1979).

Although various design hyetographs have been proposed in the past (Raudkivi 1979), their effect on the resulting hydrograph is not yet systematically investigated. In order to study the significant features of a time-dependent excess hyetograph, a simple rectangular catchment area of length x will be considered. According to Hager (1984b), its topography and the roughness characteristics may be approximated using appropriate averages. To this end, let S_o be the average bottom slope and $K=1/n$ the corresponding average roughness coefficient according to the Manning-Strickler formula. Furthermore, the following considerations include only *excess rainfall* hyetographs, $p=p(t)$ of which the precipitation time t_p is fixed. The simplest model hyetograph allowing a rather arbitrary *shape variation* accounts for a triangular time distribution (see Fig. 1 later). If time to peak is denoted by t^*, the ratio $\tau=t^*/t_p$ may alter between the limits $0<\tau<1$. It is simple to demonstrate that the corresponding uniform hyetograph with equal precipitation time t_p has an intensity $p_u=p^*/2$. This curve is shown as dotted in line in Fig. 1

The first problem to be considered examines hydrographs at various locations $x>0$ ($x=0$ corresponds to the highest point of the catchment area), thereby varying the hyetograph shape parameter τ for fixed properties of the catchment area (K, S_o) and the excess hyetograph (p^*,t_p). Results allow a discussion of the effects of excess rainfall time-distribution on the runoff characteristics with respect to the considered reach. These results can be of particular interest when applied on more complex hydrological systems.

The second problem considers the total rainfall height (in m) as independent of space and time and analyses the hydrographs at various locations of the catchment area in terms of variable precipitation time and hyetograph shape. The results are again compared with the usual approach according to Henderson and Wooding (1964). The differences of the solutions with respect to the hydrograph peak quantities and the hydrograph shapes are discussed in detail and allow general recommendations regarding the choice of the excess rainfall hyetograph.

Hydrographs Resulting from Constant Time of Precipitation

Analysis

Given a catchment area of average bottom slope S_o, average roughness coefficient K and length L. The longitudinal coordinate is denoted by x, and $x=0$ corresponds to the higest point of the reach. The continuity equation balances the mass transfer and reads according to Raudkivi (1979)

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = p \qquad (1)$$

in which h is flow depth, q discharge per unit width, p excess precipitation and t time. In overland flows the governing conditions for the kinematic wave theory as given by Hager (1984a) and Raudkivi (1979) are usually fulfilled, such that the dynamical flow equation reduces to the pseudo-uniform flow condition

$$S_0 = S_f \tag{2}$$

in which S_f is the frictional slope. For fully turbulent flow as may appear under heavy excess rainfall, this latter quantity can be expressed using the Manning-Strickler flow formula

$$S_f \equiv \frac{q^2}{K^2\, h^{10/3}} \tag{3}$$

$K=1/n$ being the roughness coefficient.

The usual procedure for overland flow computations is to assume excess precipitation independent of time and space, whence by prescribing the function $p=p_u$, in which index 'u' indicates uniform precipitation during the precipitation time t_p. However, as has been outlined by Huff (1967) among others, the effective precipitation and thus also the excess rainfall depend significantly on time t. Therefore, it is interesting to explore in more detail the effect of time dependent excess rainfall on the resulting hydrograph at the catchment area outlet, thereby fixing the catchment area geometry (L, S_0), the roughness coefficient K, and the precipitation characteristics (p^*, t_p) in which p^* denotes the precipitation peak. The simplest approach accounts for a *linear* variation of the function $p(t)$. For relatively *small* catchment areas, spatial effects of the excess rainfall may furthermore be overlooked, so that $p=p(t)$ only. The model hydrographs, plotted in Fig. 1, can be represented as

$$\begin{aligned} p &= p^*\left(\frac{t}{t^*}\right) \quad , \quad 0 \le t \le t^* \\ p &= p^*\left(\frac{t-t_p}{t^*-t_p}\right) \quad , \quad t^* \le t \le t_p \\ p &\equiv 0 \quad , \quad t < 0 \quad , \quad t > t_p \end{aligned} \tag{4}$$

It will be advantageous to introduce the following scalings

$$T \equiv \frac{t}{t_p} \quad , \quad P = \frac{p}{p^*} \tag{5}$$

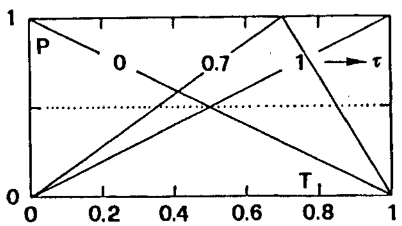


Fig. 1. Model excess hyetographs, $P(T)$ for fixed time of precipitation, constant maximum peak precipitation but varied time to peak. (...) uniform excess hyetograph.

in which quantities marked by (*) are peak quantities. As is seen from Fig. 1, p^* remains constant for all shapes of hyetographs, provided the precipitation time is fixed. It is further interesting to note that the precipitation intensity of the uniform excess rainfall is given by $p_u = p^*/2$, thus $\bar{P} = 2P$ if $P = p/p_u$.
Let

$$X = \frac{x}{KS_0^{1/2}\, p_u^{2/3}\, t_p^{5/3}} \quad , \quad Y = \frac{h}{p_u\, t_p} \quad , \quad Q = \frac{q}{KS_0^{1/2}\, p_u^{5/3}\, t_p^{5/3}} \qquad (6)$$

then Eqs. (1) to (3) transform into (Hager 1984a)

$$\frac{3}{5}\frac{1}{Q^{2/5}}\frac{\partial Q}{\partial T} + \frac{\partial Q}{\partial X} = P(T) \qquad (7)$$

in which

$$\begin{aligned} P(T) &= 2\frac{T}{\tau} \quad , \quad 0 \le T \le \tau \\ P(T) &= 2\left(\frac{T-1}{\tau-1}\right) \quad , \quad \tau \le T \le 1 \qquad (8) \\ P(T) &\equiv 0 \quad , \quad T < 0,\ T > 1 \end{aligned}$$

$\tau = t^*/t_p$ will be the parameter to be varied in the present approach. It accounts for the time distribution of excess rainfall and has the domain $0<\tau<1$. For *uniform* precipitation, Eq. (8) must be modified to read

$$\begin{aligned} P(T) &= 1 \quad , \quad 0 \le T \le 1 \\ P(T) &\equiv 0 \quad , \quad T < 0,\ T > 1 \end{aligned} \qquad (9)$$

Eq. (7) is a first-order, non-linear partial diffential equation for the sought discharge Q as a function of space and time, $Q=Q(X,T)$. It must be solved by prescribing an initial and a boundary condition. Evidently, discharge (and flow depth according to Eq. (3)) vanish at the highest point of the reach, whence $Q(X=0,\ T)=0$. For an initially dry catchment area (or if maximum discharge is much higher than the baseflow), discharge and flow depth initially also vanish, thus $Q(X,T=0)=0$. These conditions are usually imposed in problems encountered in overland flow, see Woolhiser (1975).

Let us now consider the *rising hydrograph,* for which excess precipitation is given by Eq. (8a). According to the method of characteristics (Abbott 1966), Eq. (7) may equally be expressed as the system

$$\frac{5Q^{2/5}}{3}\,dT = dX = \frac{\tau}{T}\frac{dQ}{2} \qquad (10)$$

Equating the left and the right hand sides yields upon integration

$$Q \equiv \left[\frac{T^2 - T_0^2}{\tau} \right]^{5/3} \quad , \quad 0 \leq T_0 < \tau \tag{11}$$

in which T_o is the constant of integration.

The characteristics curves on which the above solution is valid can be evaluated by the left and center part of Eq. (10). However, except for the case $T_o=0$, the integral

$$X = \frac{5}{3} \int \left(\frac{T^2 - T_0^2}{\tau} \right)^{2/3} dT \quad , \quad 0 \leq T_0 < \tau \tag{12}$$

subject to the condition $X(T=T_o)=0$, can only be solved numerically. The result for $T_o=0$ is

$$X = \frac{5}{7} \left(\frac{T^7}{\tau^2} \right)^{1/3} \quad , \quad 0 \leq T < \tau \tag{13}$$

Once time $T=\tau$ has passed, the excess rainfall intensity must be modified according to Eq. (8b). Inserting the respective term into the right hand-side of Eq. (7) then yields for the second portion of the rising hydrograph

$$\begin{aligned} Q(T) &= \left[\frac{T^2 - 2T - \tau^2 + 2\tau}{\tau - 1} + \frac{\tau^2 - T_0^2}{\tau} \right]^{5/3} \quad , \quad 0 \leq T_0 < \tau \\ Q(T) &\equiv \left[\frac{T^2 - 2T - T_0^2 + 2T_0}{\tau - 1} \right]^{5/3} \quad , \quad \tau \leq T_0 < 1 \end{aligned} \tag{14}$$

in which the conditions $Q(T=\tau)=\{(\tau^2-T_o^2)/\tau\}^{5/3}$ and $Q(T=T_o)=0$, respectively are inserted.

The locii on which Eqs. (14) hold are again evaluated by the left and center portions of the characteristic equation, namely

$$X = \frac{5}{3} \int Q^{2/5} (T, T_0, \tau)\, dT \tag{15}$$

thereby considering the conditions $X(T=\tau)=X_1$ and $X(T=T_o)=0$, respectively, in which $X_1=X(T=\tau,T_o)$ according to Eq. (12).

Finally, for $T>1$, the above relations must be modified by accounting for the excess rainfall according to Eq. (8c). The *receding hydrograph* portion is given by

$$\begin{aligned} Q &= \left[\frac{\tau - T_0^2}{\tau} \right]^{5/3} && 0 \leq T_0 \leq \tau \\ Q &= \left[\frac{(T_0 - 1)^2}{1 - \tau} \right]^{5/3} \{\text{along } X \equiv X_2 + \tfrac{5}{3} Q^{2/5}(T-1) \text{ for}\} && \tau \leq T_0 \leq 1 \\ Q &= 0 && T_0 \geq 1 \end{aligned} \tag{16}$$

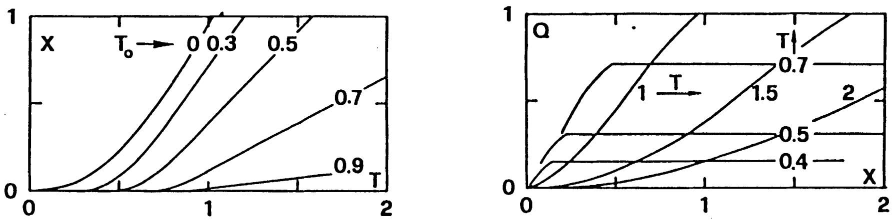


Fig. 2. Intermediate results of the method of characteristics with respect to X-T relation for various parameters T_o (right) and $Q(X)$ relation for various times T ($\tau=1/2$).

in which $X_2=X(T=1,T_o,\tau)$. Note that the complete solution $Q(X,T)$ is matched of particular solutions valid in five different domains as specified above, provided $\tau\neq0$ or $\tau\neq1$.

Fig. 2 shows a typical evaluation of the above relations for the particular case $\tau=1/2$. The final result, the hydrograph $Q(T)$ at various locations X is plotted in Fig. 3 for $\tau=0$, $\tau=1/2$ and $\tau=1$.

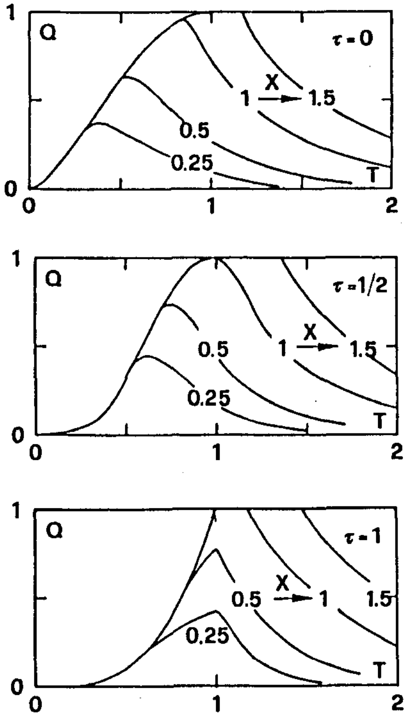
These curves ressemble strongly to the ones presented by Hager (1984a) for a continuous hyetograph relation of Maxwellian time-distribution. In the latter case, the procedure then becomes much simpler with respect to the division of the solution in various domains of validity.

Discussion of Results

The hydrographs $Q(T)$ at various locations X according to Fig. 3 reflect partly the hyetograph shape. For $\tau=0$ time to peak, $T_{max}=t_{max}/t^*$, is shorter than for $\tau>0$. Moreover, the hydrographs for the first case are more asymmetric to the left, while hydrographs are nearly symmetric for $\tau=1$. However, for equal non-dimensional catchment area length, $X<1$, peak discharge $Q_{max}=q_{max}/(KS_o^{1/2}p_u^{5/3}t_p^{5/3})$ is lowest for $\tau=0$ and highest for $\tau=1$. For $X>1$, peak discharge is always $Q_{max}=1$ independent of the location X. Fig. 4 represents the non-dimensional peak discharge Q_{max} as a function of non-dimensional catchment area length, X. Also included in the plot is the result for uniform excess precipitation, see Appendix I. The same figure shows also non-dimensional time to peak, T_{max}, as a function of X for various, significant τ.

It is now important to note that, at the same location $X<1$, maximum discharge Q_{max} is significantly lower for a uniform hyetograph than when accounting for the hyetograph shape (triangular hyetograph).

Differences are smallest for $\tau=0$, while these with respect to $\tau=1$ can be as high as 35 %. Note too, that these deviations are highest for $X\simeq0.5$. Consequently, the effective hyetograph shape should always be accounted for, provided $X<1$. For $X>1$, however, $Q_{max}=1$ independent of X and τ (thereby excluding the case $\tau=0$, for which Q_{max} (1)=0.96).



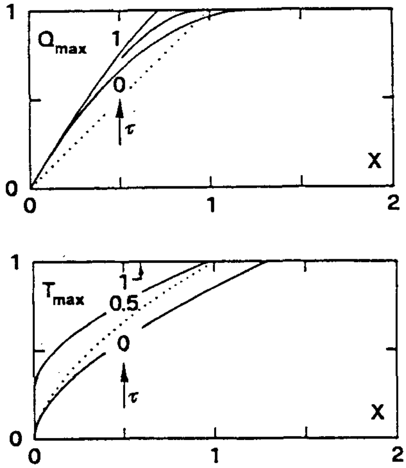


Fig. 4.
Hydrograph peak quantities, Q_{max} (X) (top) and $T_{max}(X)$ (bottom) as functions of hyetograph shape parameter τ. Also included as (...) are the results originating from a uniform hyetograph.

Fig. 3. Non-dimensional hydrographs $Q(T)$ for various locations X and $\tau=0$ (top), $\tau=1/2$ (center), $\tau=1$ (bottom).

Differences of $T_{max}(X)$ for $\tau=0$ and the result regarding uniform excess precipitation are smallest. However, the deviation becomes significant for $\tau=1$, for which $T_{max}=1$ independent of X. Finally, it should be noted that the overall agreement between the hydrographs according to Fig. 3 and Fig. 6 (see Appendix I) is best for $\tau=0.5$. To the lowest approximation, a nearly symmetrical excess hyetograph may therefore be replaced by a uniform excess hyetograph, of which the precipitation times are equal but $p^*=2p_u$.

Hydrographs under Constant Excess Rainfall Depth

The above analysis clearly indicates the effect of the temporal distribution of excess rainfall on the resulting hydrograph. However, these results are only partly of practical importance, since one will set the precipitation time equal to the time to peak of the hydrograph (time of concentration equal to precipitation time). For a location $X<X_c$, in which X_c corresponds to the location at which $T_p=T_{max}$, the

precipitation time T_p will be chosen shorter such that $T_p=T_{max}$. This, in turn, rises the governing uniform and maximum precipitation p^*, since $p_u \propto t_p^{-1}$. Consequently, it will be more realistic to consider the total excess precipitation, H (m), as independent of precipitation time and maximum precipitation intensity. The simplest relation for the precipitation depth states that, for a fixed rainfall frequency at a particular location, the product of precipitation duration times the uniform excess precipitation remains constant, e.g.

$$H = p_u t_p \tag{17}$$

Although not verified for arbitrary rainfall events, this simple relation retains the most significant features (the longer t_p is, the better Eq. (17) becomes). It was also adopted by Henderson and Wooding (1964) in their study regarding the uniform precipitation. The present investigation considers in addition the excess rainfall time distribution, and a comparison with the first solution indicates the effects of variable excess rainfall intensity on the hydrograph. The following analysis will also be performed for the identical characteristics of the catchment area, thus considering both K and S_o as independent of space and time.

Analysis

The runoff phenomenon on a uniform catchment area characterised by the averages of the bottom slope, S_o, and the roughness coefficient, K, is described by Eqs. (1) to (3). If the total excess rainfall height, H, remains constant, maximum precipitation is $p_u=H/t_p$ according to Eq. (17). The problem to be investigated considers K,S_o,H as fixed quantities, and studies the solutions $q(x,t)$ for variable t^*,t_p. Consequently, the scalings according to Eq. (16) must be modified except for the non-dimensional discharge $Q=q/(KS_o^{1/2}H^{5/3})$. As will be shown below it is convenient to let

$$X = \frac{S_0^{5/2}\,x}{H} \quad , \quad T = KS_0^3\left(\frac{t}{H^{1/3}}\right) \quad , \quad Q = \frac{q}{KS_0^{1/2}\,H^{5/3}} \tag{18}$$

for which Eq. (7) is the governing runoff relation, provided

$$P(T) = \frac{p(t)}{KS_0^3 H^{2/3}} \tag{19}$$

Note that the scalings according to Eqs. (7) and (18) are not identified by an index, and that the latter apply only to section B. Non-dimensional excess precipitation $P(T)$ according to Eq. (19) may further be developed by accounting for Eqs. (4), namely

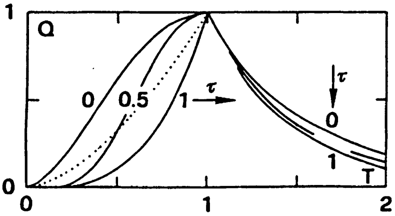


Fig. 5. Hydrographs $Q(\bar{T})$ for various hyetograph shapes, τ, for which the time of concentration is equal to the precipitation time; (...) uniform precipitation.

$$P(T) = \frac{2T}{\tau\gamma^2} \quad , \quad 0 \le T \le \tau\gamma$$

$$P(T) = \frac{2(T-\gamma)}{\gamma^2(\tau-1)} \quad , \quad \tau\gamma \le T \le \gamma \qquad (20)$$

$$P(T) \equiv 0 \quad , \quad T < 0 \; , \; T > \gamma$$

in which

$$\gamma \equiv \frac{KS_0^3 t_p}{H^{1/3}} \qquad (21)$$

The governing Eq. (7) must be solved using the initial condition $Q(X,T=0)=0$, and the boundary condition is $Q(X=0,T)=0$, in accordance with the problem in section A.

In contrast to the first problem treated in section A, the present investigation depends on two parameters, namely τ and γ. However, the transformations

$$\bar{X} = \frac{X}{\gamma} \; , \quad \bar{T} = \frac{T}{\gamma} \qquad (22)$$

and $\bar{Q}=Q$ yield not only Eq. (7), but Eqs. (20) are also transformed into the original Eqs. (8). Consequently, the problem of section B can be solved with the solutions according to section A by accounting for the transformations Eq. (22).

Of particular interest are the solutions which yield Q_{max} at the shortest distance possible. Fig. 5 has been drawn as a result of Fig. 3 and indicates the hydrographs at locations $\bar{X}$, for which the precipitation time is equal to the time of concentration. It is noted that the usual approach (uniform excess precipitation) and the case τ=0.5, ressemble most in terms of distance and in the hydrograph shape. However, deviations become significant with respect to the hydrograph shape for the two remaining cases τ=0 and τ=1.

Regarding the non-dimensional length of the catchment area, the values $\bar{X}(\tau=0)=1.23$, $\bar{X}(\tau=0.5)=0.96$ and $X(\tau=1)=0.71$ are found.

Example

Consider a catchment area with an average roughness coefficient $K=10\ m^{1/3}s^{-1}$, an average bottom slope $S_o=0.05$. The precipitation height producing direct runoff amounts to H=0.0114 m (for a fixed frequency and a particular location). Find the hydrographs of maximum peak discharge q_{max} per unit width for a hyetograph of symmetrical shape (τ=0.5).

Consider first a precipitation time t_p=1,800 s, for which γ=10 according to Eq. (21). With $\bar{X}_{max}$=0.96 (see above) the length of the catchment area becomes $x=\bar{X}_{max}(KS_o^{1/2}t_pH^{2/3})$=196 m, and the peak discharge is Q_{max}=1, thus q_{max}=1.3 1/(sm). The hydrograph then can be obtained using Fig. 5.

Consider now t_p=90 s, for which γ=0.5. With $\bar{X}_{max}$=0.96, the length of the catchment area that yields maximum discharge is only x=9.8 m. However, peak discharge is also q_{max}=1.3 1/(sm), since Q according to Eq. (18c) is independent of γ.

Finally, it is simple to demonstrate that time to peak, $\bar{T}_{max}$ in both cases is equal to the precipitation time. Both catchment areas thus produce the identical peak discharge, although the peak excess precipitations are different.

Conclusions

The present investigation analyses the effect of time-dependent excess rainfall on the resulting hydrograph of a plane catchment area of constant bottom slope and roughness coefficient. Distinction is made between the cases in which the precipitation time is fixed, and the excess rainfall height is regarded as a constant. The results have been obtained using the kinematic wave approach, and the following conclusions are immediate:

1. The effect of the excess hyetograph shape on the *peak discharge* is only subordinated if the precipitation time is set equal to the time of concentration. However, deviations between the usual approach (assuming a uniform precipitation) and the approach accounting for the excess hyetograph shape become significant, if the catchment area length is smaller than the aforementioned.
2. Since the rational formula accounts only for the *peak discharge,* effects of the hyetograph shape are insignificant for the final result, provided the condition time of concentration equal to the precipitation time is accounted for.
3. Significant loss of information yields the conventional approach with respect to the *hydrograph.* For equal catchment area length, hyetographs with maximum precipitation at the begin of the event (τ=0) yield a higher peak discharge and time of concentration is shorter than when computed with a uniform excess hyetograph. For excess hyetographs of which the peak intensity is at the end of the precipitation time (τ=1), peak discharge and time of concentration are longer than when computed with a uniform excess hyetograph. Consequently, it is important to account for the excess hyetograph shape whenever the resulting *hydrograph* is sought.

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Notation

The following symbols may appear with index 'max' and then refer to peak quantities. Note that the notation of sections A and B is sometimes identical, but the meaning may be different.

h	(m)	– flow depth
H	(m)	– excess precipitation height
K	($m^{1/3}s^{-1}$)	– roughness coefficient
p	(ms^{-1})	– excess precipitation
p^*	(ms^{-1})	– peak excess precipitation
p_u	(ms^{-1})	– uniform (average) excess precipitation
P	(–)	– non-dimensional hyetograph
q	(m^2s^{-1})	– discharge per unit width
Q	(–)	– non-dimensional discharge per unit width
S_o	(–)	– bottom slope
S_f	(–)	– frictional slope
t	(s)	– time
t^*	(s)	– time to peak of hyetograph
t_p	(s)	– precipitation time
T	(–)	– non-dimensional time
T_o	(–)	– constant of integration
x	(m)	– length of catchment area
X	(–)	– non-dimensional space coordinate
Y	(–)	– non-dimensional flow depth
τ	(–)	– excess hyetograph shape parameter
γ	(–)	– relative precipitation time

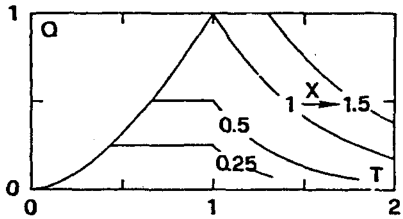

Fig. 6. Non-dimensional hydrographs $Q(T)$ for various locations X resulting from uniform excess precipitation.

Appendix I: Uniform Excess Precipitation

The hydrograph $Q(X,T)$ resulting from uniform excess precipitation according to Eqs. (9) under the conditions as assumed herein is given by Woolhiser (1975)

$$\begin{aligned} Q &= T^{5/3} &&, \quad 0 \leq T \leq T_c = X^{3/5} \\ Q &= X &&, \quad T_c < T < 1 \\ Q - X + \frac{5}{3} Q^{2/5} (T-1) &= 0 &&, \quad T > 1 \end{aligned} \tag{23}$$

for equilibrium conditions (for which the time of concentration t_c is shorter than the precipitation time t_p), and

$$\begin{aligned} Q &= T^{5/3} &&, \quad 0 \leq T \leq 1 \\ Q &= 1 &&, \quad 1 < T < 1 + \frac{3}{5}(X-1) = \frac{2+3X}{5} \\ Q - X + \frac{5}{3} Q^{2/5} (T-1) &= 0 &&, \quad T \geq \frac{2+3X}{5} \end{aligned} \tag{24}$$

for partial equilibrium conditions ($t_p < t_c$). A typical solution is plotted in Fig. 6. This allows the completion of Fig. 4.

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