Application of mathematical tools to improve the design and operation of activated sludge plants.  
Case study: the new WWTP of Galindo-Bilbao

Part I: Optimum design

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Abstract  This paper presents a mathematical formulation for the optimum design of a new activated sludge WWTP. The WWTP optimum design problem has been formulated as a Mathematical Programming problem, which is solved through a nonlinear optimisation method. The plant model has been based on the ASM1. The minimum volume of the biological reactors and the minimum total cost (including construction and exploitation costs) have been considered as optimisation criteria. Some practical results are also included, using as a case study the design of the second stage of the Galindo-Bilbao WWTP.

Keywords  Activated sludge; cost; mathematical models; optimum design; optimisation algorithms; simulators

Introduction: the problem of optimum WWTP design

In the design of an activated sludge wastewater treatment plant (WWTP), once the treatment configuration has been selected according to the required purification level, (elimination of carbonaceous matter or carbonaceous matter and nutrients), the designer will select, from among all the possible values of its design parameters, those that optimise its performance. The optimum design is the one which satisfies certain constraints and is the best with respect to a given criterion. The most common design parameters are the dimensions of the units that make up the plant (biological reactors and settlers) and its operational variables:
- Total volume of the biological reactors or Hydraulic Retention Time (HRT);
- Sludge waste flow or Solid Retention Time (SRT);
- Areas, volumes and sludge recycle flow of the settlers;
- Volume fractions of the biological reactors;
- Internal recycle flow between biological reactors;
- Influent flow fractioning.

The basic criterion commonly adopted by the designer is to minimise the total volume of the biological reactors, avoiding overloads in the solid flux through the settlers. However in some cases a minimum global cost, combining construction and exploitation costs, could be investigated.

For the selection of the optimum design parameters, many constraints must be taken into account. Several design constraints are related to effluent water quality, expressed as permitted maximum values of nutrient concentration. Others are related to the physical limitations on the design (e.g. maximum total volume or maximum aerated volume) or plant operation (e.g. a maximum concentration of mixed liquor suspended solid (MLSS) in the biological reactor before going to the settler).

The selection of an optimum solution for a WWTP design problem requires the evaluation of the objective and constraints for each of the possible sets of design parameters. This
evaluation is not simple, since usually the objective and the constraints are not direct functions of the design parameters. Therefore the numerical solution of the global mathematical model of the plant is required.

In this sense, the WWTP Simulators (WWTPS) are efficient computational tools able to help the designer to seek the optimum design parameters by solving the complex mathematical models. But generally the plant designer carries out a large part of the optimum design task in which the WWTPS plays the role of a specialised calculator. However the formulation of a WWTP optimum design problem as one of Mathematical Programming and its implementation in the simulators increases the role of WWTPS in optimum design. Moreover, this new field of application of WWTPS lets us consider new and more complex design criteria and constraints.

Model based optimisation of WWTP design

Theoretical basis

The global mathematical model of an activated sludge WWTP can be expressed as a differential equations system:

\[ \dot{e} = \Phi(e, \delta) \]  

where \( \delta \) is the vector of design parameters and \( e \) is the state vector in the biological reactors and settlers.

The steady-state mathematical model of the plant (2) is commonly used for design purposes. The steady-state is defined as the point reached when input variables (operational variables and influent load) are maintained constant. Mathematically, the steady-state solution corresponds to the values of the states that annul its time derivatives. From (1) the WWTP steady-state mathematical model can be obtained as:

\[ \Phi(e, \delta) = 0. \]  

If it were possible from Eq. (2) to obtain an analytical expression from the states

\[ e = \Psi(\delta) \]  

the objective and the constraints could be written easily as a function of the design parameters:

\[ \eta[\Psi(\delta), \delta] \leq 0 \]  

where \( \eta \) is the objective function and \( R \) is the vector of design constraints.

Nowadays, the most common mathematical models for WWTP are so complex that it is not possible to obtain an analytical expression like Eq. (3) and it is necessary to resort to computational tools to solve numerically Eq. (2). These computational tools are generally implemented in WWTP Simulators.

The WWTPS are very useful for the designer to solve numerically WWTP mathematical models. However to find the optimum design parameters, the designer generally uses a trial and error procedure where only his experience and knowledge of the processes can aid him. Sometimes the selection of the optimum design parameters can be extremely complicated using conventional trial and error procedures. In the cases of configurations with a large number of design parameters and/or complex criterion (e.g., to minimise the construction and exploitation cost of the plant), the number of possible combinations and factors that the designer has to take into account increase dramatically. This difficulty can be overcome by outlining mathematically the design problem as one of Mathematical Programming, where the independent variables are the design parameters (Rivas and Ayesa, 1997). In this
formulation each evaluation of the objective and the constraints involves solving the mathematical model (2) (Ayesa et al., 1998) and the final solution can be expressed as the following minimisation problem (5):

$$\min_{\delta} \left\{ \eta [\psi(\delta), \delta] : R[\psi(\delta), \delta] \leq 0 \right\}. \quad (5)$$

However, in order to eliminate the need to solve the mathematical model each time that an evaluation of the objective and constraints is done, the problem can be outlined in a slightly different way to (5). By adding states as additional independent variables and the steady-state model equations as constraints of the optimisation problem, it can be written as:

$$\min_{\delta, e} \left\{ \eta(e, \delta) : R(e, \delta) \leq 0, \Phi(e, \delta) = 0 \right\}. \quad (6)$$

then the minimisation of the objective function $e$ (Eq. (6)) is obtained simultaneously to the steady-state of the process (Eq. (2)), increasing the performance of the algorithm significantly.

**Practical implementation**

During the last few years the Centre of Studies and Technical Researches of Guipuzcoa (CEIT) has been developing the simulator DAISY (Suescun et al., 1994) based on Activated Sludge Model No. 1 (ASM1) of the IAWQ (Henze et al., 1987). In DAISY 2.0 the solution to problem (5) was implemented using the Penalty method (Fiacco and McCormick, 1968).

In DAISY 3.0, which is currently in the development stage for the Spanish Engineering firm CADAGUA S.A., problem (6) will be solved through the GRG2 method (Lansdon et al., 1978), an improvement on the Generalised Reduced Gradient Method (Abadie and Carpentier, 1969). A test has been carried out to prove the validity of both the formulation and the resolution method before implementing it in a new version of DAISY. The mathematical model of the case study has been written in *Microsoft Excel*© and *Premium Solver Plus*© (Frontline System Inc., 1998) has been used to solve this spreadsheet model.

**Design optimisation of the new WWTP of Galindo-Bilbao**

As an example of the proposed methodology, an optimum design of a WWTP for CN removal has been made. The selected case study corresponds to the second stage of Galindo (Bilbao-Spain) WWTP, which handles a daily average wastewater flow of 345,600 m³/day. The influent wastewater characterisation and model coefficients were previously obtained through pilot plant studies (De la Sota, 1995). Two configurations were considered for the new WWTP, Regeneration-Denitrification-Nitrification (RDN-contact-stabilisation for biological nitrogen removal) and Denitrification-Regeneration-Denitrification-Nitrification (DRDN) processes (Figures 1 and 2).

The RDN process was selected as an appropriate configuration for preventing filamentous bulking, but the possibility of operating the plant as a DRDN process was included in order to improve nitrogen removal during high load or low temperature periods. Therefore, the final implementation includes both configurations (Concha and Henze, 1996).

The design parameters taken into account in the RDN configuration are, the Hydraulic Retention Time (HRT), the Solid Retention Time (SRT), the fractions of the volume corresponding to each biological reactor ($f^R_V$, $f^D_V$ and $f^N_V$) and the internal recycle flow ($q^{N\rightarrow D}$). The sludge recycle flow is fixed at 50% of the influent flow and the oxygen concentration in the aerated reactors is maintained at 2.0 mgDO/l. In this configuration, a fraction of the total volume in the biological reactors must be fixed as a regeneration reactor ($R$) in order to maintain the contact-stabilisation properties required to avoid bulking (Wanner, 1994). For this reason, 10.7% of the total volume is fixed as $R$ volume and not considered in the
mathematical optimisation problem. Finally the design parameters for this configuration are:

$$\delta^{R}_{RDN} = \{HRT, SRT, f^D_V, f^N_V, q^{N \rightarrow D}\}.$$  \hspace{1cm} (7)

In the case of the DRDN configuration, the volume fractions corresponding to the $R$ and the $D_1$ reactors ($f^D_{V1}$ and $f^D_{V2}$) and the influent flow fractioning ($f^D_{Q1}$ and $f^D_{Q2}$) are added to the parameters of the RDN:

$$\delta^{T}_{DRDN} = \{HRT, SRT, f^D_{V1}, f^R_{V1}, f^D_{V2}, f^N_{V}, f^D_{Q1}, f^D_{Q2}, q^{N \rightarrow D}\}.$$  \hspace{1cm} (8)

The water effluent quality constraints imposed in both configurations, at a higher temperature than $13^\circ C$, are that the daily average concentration of ammonia must be smaller than $2.0 \text{ mgN/l}$ and that the sum of the daily average nitrate and ammonia effluent concentration must be smaller than $13.0 \text{ mgN/l}$. For optimum settler performance a maximum value of $3500 \text{ mg/l}$ of MLSS concentration to the settler is imposed:

Ammonia $\leq 2.0$

Ammonia + Nitrates $\leq 13.0$

MLSS$^N - 3500 \leq 0$.  \hspace{1cm} (9)

Finally, the volume fractions and influent flow fractioning are constrained by:

\begin{align*}
\text{(RDN)} & \\
(f^R_V + f^D_V + f^N_V) - 1 &= 0 \\
& \\
\text{(DRDN)} & \\
(f^D_{V1} + f^R_V + f^D_{V2} + f^N_V) - 1 &= 0 \\
(f^D_{Q1} + f^D_{Q2}) - 1 &= 0.
\end{align*}  \hspace{1cm} (10)

An additional constraint is considered in the DRDN, corresponding to the $R$ volume in the RDN process:

$$f^R_V + f^D_V = 0.107.$$  \hspace{1cm} (11)

Two criteria, minimum volume (minimum HRT) and minimum total cost of the plant have been taken into account. This second criterion is more complex since it requires knowledge of the construction and exploitation costs of the plant as a function of the design parameters, and/or of the magnitudes which are obtained once the mathematical model has been solved (e.g. the sludge production and air consumption). These cost functions were unknown but, for this exploratory case study, they have been estimated roughly from several cost data provided by the engineering firm that executed the Galindo WWTP project (CADAGUA S.A.). In order to estimate the variation in construction and exploitation costs in the neighbourhood of the design values proposed by the engineering firm, the following linear approximation for the cost curves (construction or exploitation) as a function of the...
variables (volume, sludge production, internal recycle flow pumping) was adopted:

\[
C_\lambda = \tilde{C}_\lambda + K_\lambda (\lambda - \tilde{\lambda})
\]  

(12)

where \(\tilde{C}_\lambda\) is the cost supplied by the engineering firm for a \(\tilde{\lambda}\) variable value. The \(K_\lambda\) coefficient was estimated for each type of cost.

Each annual exploitation cost, \(c_\lambda\), has been updated to its value in the year of the design according to the formula:

\[
c_\lambda = c_\lambda \frac{(1 + r)^N - (1 + a)^N}{(r - a)(1 + r)^N}
\]

(13)

where \(r\) is the money annual interest, \(a\) is the cost inflation and \(N\) is the working period of WWTP. Therefore, the total cost is the following:

\[
C_{\text{WWTP}} = C_{\text{CONSTRUCTION}} + C_{\text{EXPLOITATION}}
\]

\[
= C_V + C_S + C_R + C_A + c_i \frac{(1 + r)^N - (1 + a)^N}{(r - a)(1 + r)^N}
\]

\[
+ c_s \frac{(1 + r)^N - (1 + a)^N}{(r - a)(1 + r)^N} + c_R \frac{(1 + r)^N - (1 + a)^N}{(r - a)(1 + r)^N}
\]

(14)

where:

- \(C_V\): Construction cost of reactor volume
- \(C_S\): Construction cost of sludge treatment
- \(C_A\): Construction cost of aeration
- \(C_R\): Construction cost of internal recycle pumping
- \(c_i\): Exploitation cost of sludge production
- \(c_A\): Exploitation cost of aeration
- \(c_R\): Exploitation cost of internal recycle pumping

**Results**

Optimum design minimising the reactor volume

The optimum plant design minimising the total volume of the biological reactors at 13°C has been analysed for different effluent quality requirements. For this purpose, a collection of optimum design parameters have been calculated by the optimisation algorithm within the following range of effluent quality:

- \(7.0 \text{ mgN/l} < \text{effluent (ammonia + nitrate) concentration} < 12.0 \text{ mgN/l}\)
- \(0.4 \text{ mgN/l} < \text{effluent (ammonia) concentration} < 2.0 \text{ mgN/l}\)

All the optimum points have been generated by the solution of the optimisation problem (6), using the influent wastewater characterisation and the ASM1 coefficients previously obtained in pilot plant studies. The optimisation results have been condensed in the design charts presented in Figure 3 for the RDN configuration and Figure 4 for the DRDN configuration. These design charts have been created to facilitate the selection of the optimum design parameters, once the required effluent quality (inputs) has been fixed. From a required effluent (ammonia + nitrate) on the horizontal axis (for example 9.4 mgN/l) the designer can trace a vertical line in Figure 3 up to the intersection with the line corresponding to the desired ammonia concentration value (for example 0.6 mgN/l). Every value in the vertical axes indicates one of the optimum design parameters of the WWTP for the
required effluent quality (in this particular example HRT=11.0 hours, SRT=15.5 days, $f_{V}^{D}=26\%$ of total volume and $q^{N\rightarrow D}=108\%$ of influent flow).

The HRT plot in Figure 3 shows quantitatively how the volume of the plant increases when the required effluent (ammonia + nitrates) concentration is being reduced. Additionally, it is important to note that for a fixed value of effluent (ammonia + nitrates) concentration, the plant volume needed increases significantly when the required ammonia concentration is lower. Figure 3 also shows the SRT values of the optimum design that exhibit similar trends as the HRT values because of the MLSS concentration constraint. The optimum anoxic volume fraction ($f_{V}^{D}$) and the optimum internal recycle flow ($q^{N\rightarrow D}$) mainly depend on the required value of effluent (ammonia + nitrates) concentration.

The trends of the DRDN configuration results are similar to the RDN ones. Figure 4 shows that the volume required for the DRDN process are smaller than that required for the

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Figure 3 RDN optimum design charts

Figure 4 DRDN optimum design charts
RDN. However, because the plant must work with both configurations, the final design is established by the higher volume configuration (RDN). Because of constraint (11) the optimum volume fraction of reactor \( D_1 \) \( (f_{D_1}^V) \) remains constant at 4% of total volume for all the possible values of effluent nitrogen concentration constraints. The optimum influent flow fractioning \( (f_{Q_1}^{D_1}) \) is not very sensitive to effluent (ammonia + nitrates) concentration and varies from 15% to 25% of total influent flow depending on the required effluent ammonia.

Once the sensitivity of the optimum design parameters to the effluent requirements were deeply analysed, the engineering firm adopted the optimum design corresponding to an effluent concentration of 7.0 mgN/l of (ammonia + nitrates) and 1.0 mgN/l of ammonia. The design parameter values of this solution are HRT=13.3 hours, SRT=19.5 days, \( f_{D_1}^V=35.3\% \) of total volume and \( q_{N \to D_1}=232.2\% \) of influent flow. This design can be considered conservative considering the steady-state solutions, but guarantees a good dynamic behavior of the WWTP under the influent load experimentally measured at the former full-scale WWTP of Galindo.

Every optimum design for the DRDN configuration has 81 decision variables (9 design parameters and 72 states) and 78 constraints (Eqs. (9), (10), (11) and global mathematical model (2)). The computer time to find each solution is about 7 minutes, executing Microsoft Excel 97© on a 200 MHz Pentium© processor. Therefore the time required for building the design charts is 5 hours approximately.

Optimum design combining construction and exploitation costs

Once the optimum design parameters that minimises the total volume of the WWTP biological reactors have been selected, a complementary cost analysis has been carried out. This analysis has been focused to study the possible influence of considering the exploitation costs in the optimum design parameters.

For this purpose an approximate evaluation of the different construction and exploitation costs of the new plant has been estimated. Some results of this basic study are summarised in Table 1.

From these approximate data and the expected variation of the different costs near the design parameters previously obtained, some linear approximations of the costs functions (12) have been built. Then, the optimum design parameters for a global cost, including construction and exploitation, has been calculated, using a working period of 20 years, a money rate of 6% and a cost inflation of 6%.

For this particular WWTP the optimum design with combined costs is practically similar to that obtained minimising the volume of the biological reactors. This result is logical because the production of electric power from sludge incineration provokes a net exploitation cost negative for the sludge treatment. Then, the plant with minimum volume is also the plant with maximum sludge production and minimum exploitation cost.
Some fictitious cost functions have been evaluated in order to analyse the possibility to reduce the global combined cost of a new WWTP, selecting a biological volume bigger than that strictly needed and operating it with a high SRT and, consequently, with low sludge production. However, the possible variation of the optimum total volume of the biological reactors is not significant (2% of total volume for extremely high costs of sludge treatment and reasonable values of the other costs). This fact is motivated by the large increase in the reactor volume necessary to obtain a small diminution in sludge production, in this kind of WWTP.

Despite these results, our conclusion is that the combination of construction and exploitation costs could modify and improve significantly the optimum design of new WWTP. However, the active collaboration of the engineering firms in the development of realistic cost functions associated to the most important design parameters is completely unavoidable.

Conclusions
The formulation of optimum design as a Mathematical Program problem and its solution by optimisation methods implemented in a WWTP Simulator gives us a methodology and a very powerful tool to solve the problem of WWTP optimum design.

The introduction of the steady-state mathematical model in the optimisation problem and its numerical resolution with the GRG2 algorithm has worked adequately. For this reason this formulation and this algorithm is being incorporated in the new version of DAISY simulator. Since the method is very fast in solving the optimum design problem for any configuration, it is possible to build design charts that facilitate the sensitivity study of the optimum design depending on the values of any other parameters (e.g., effluent requirements, temperature, wastewater composition, etc.) (Uber et al., 1991).

The minimum combined cost is an applicable criterion to WWTP optimum design and should be a future optimisation criterion. However a lot of work has to be done in the development of reliable cost functions. In this sense, it is important that the engineering firms become aware of the practical possibilities of this methodology and they store, sort and rationalise the data from their projects.

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