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MICRO-SCALE CFD MODELING OF OSCILLATING FLOW IN A REGENERATOR

M. J. Cheadle, G. F. Nellis, and S. A. Klein

University of Wisconsin
Madison, WI, 53706, USA

ABSTRACT

Regenerator models used by designers are macro-scale models that do not explicitly consider interactions between the fluid and the solid matrix. Rather, the heat transfer coefficient and pressure drop are calculated using correlations for Nusselt number and friction factor. These correlations are typically based on steady flow data. The error associated with using steady flow correlations to characterize the oscillatory flow that is actually present in the regenerator is not well understood. Oscillating flow correlations based on experimental data do exist in the literature; however, these results are often conflicting. This paper uses a micro-scale computational fluid dynamic (CFD) model of a unit-cell of a regenerator matrix to determine the conditions for which oscillating flow affects friction factor. These conditions are compared to those found in typical pulse tube regenerators to determine whether oscillatory flow is of practical importance. CFD results clearly show a transition Valensi number beyond which oscillating flow significantly increases the friction factor. This transition Valensi number increases with Reynolds number. Most practical pulse tube regenerators will operate below this Valensi transition number and therefore this study suggests that the effect of flow oscillation on pressure drop can be neglected in macro-scale regenerator models.

KEYWORDS: Regenerator, Oscillating Flow, Valensi number, CFD, Friction Factor, Pulse Tube Cryocoolers

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INTRODUCTION

There has been much debate in the literature about the effect of the oscillating flow on the friction factor and the associated adequacy of the steady-flow correlations that are typically used to predict the performance of pulse tube regenerators [1-6]. This paper examines this issue by defining the operation of a pulse tube regenerator in terms of logical independent similarity parameters, establishing the range of these parameters where typical pulse tube regenerators operate, and developing a CFD model to predict friction factor as a function of these similarity parameters within the established operating range.

The CFD modeling is for a two-dimensional (2-D) micro-scale computational domain consisting of a staggered array of cylinders. That is, the model examines flow through a small number of unit cells in order to explicitly model and examine the fluid-to-surface interaction. Clearly, actual regenerator matrices are three-dimensional (3-D), consisting of screens or spheres. However, the model does represent the tortuous fluid path associated with typical regenerator matrices and, while the results may not be quantitatively accurate, the qualitative trends are useful and are the focus of this study. Similar research focusing on using unit-cell CFD models to obtain information about heat transfer and hydrodynamic parameters can be found in the literature [7-9].

The CFD results show that the friction factor is indeed sensitive to the oscillating nature of the flow above a threshold Valensi number and that the Valensi number where this transition occurs (referred to as the Valensi transition number) increases with increasing Reynolds number. However, most pulse tube cryocooler regenerators will operate at a Valensi number that is below the Valensi transition number. At very low regenerator operational temperatures, the Valensi number becomes very large due to the decrease in the kinematic viscosity of the working fluid. However, there is an accompanying increase in the Reynolds number at low temperature (for the same reason) and therefore the transition Valensi number also increases. As a result, this study suggests that the effect the oscillatory nature of the flow has on pressure drop is not likely to be important even at very low operating temperature.

TYPICAL REGENERATOR OPERATING CONDITIONS

The similarity parameters used to characterize the regenerator operating condition are the Reynolds number and the Valensi number. The Reynolds number is defined as:

$$Re = \frac{\rho D_h u}{\mu} \tag{1}$$

where ρ is the density, D_h is the hydraulic diameter of the regenerator passage, u is the pore velocity, and μ is the viscosity. The Valensi number is defined as:

$$Va = \frac{\rho \omega D_h^2}{\mu} \tag{2}$$

where ω is the angular frequency of operation. The Reynolds number is the ratio of inertial to viscous effects. The Valensi number accounts for the effects of oscillating flow and can be thought of as the ratio of the hydraulic diameter of the passage to the momentum diffusion distance during one cycle. A low Valensi number indicates that momentum has sufficient time to penetrate from one side of the channel to the other during a cycle whereas a large Valensi number suggests that the diffusion of momentum is confined to the near wall region.

Pfotenhauer et al. [10] carried out an extensive parametric study using the simulation software REGEN 3.2 in order to identify the characteristics of an optimally designed regenerator for use in cryocoolers with a cold end temperature in the 60 K to 80 K range. The results of this analysis showed that the optimal ratio of regenerator cross-sectional area (A_c) to acoustic power ($\dot{W}_{ac}$) is very nearly a constant, insensitive to frequency, cold end temperature, acoustic power, and other operating parameters. The value of this constant was found to be approximately:

$$\frac{A_c}{\dot{W}_{ac}} = K \approx 7.7\text{x}10^{-5}\ \text{m}^2/\text{W} \tag{3}$$

We can use this observation to identify, at least approximately, the range of Reynolds numbers and Valensi numbers that will characterize cryogenic regenerators for pulse tube refrigerators. The acoustic power associated with an ideal gas experiencing sinusoidal pressure and mass flow variations is given by:

$$\dot{W}_{ac} = \frac{R\,T_C\,\dot{m}\,\tilde{P}}{2\,\bar{P}}\cos(\theta) \tag{4}$$

where R is the ideal gas constant, $\dot{m}$ is the amplitude of the mass flow rate variation, $\tilde{P}$ is the amplitude of the pressure variation, T_C is the cold end temperature, $\bar{P}$ is the charge pressure, and θ is the phase angle between the pressure and mass flow rate. The optimal phase angle is approximately $\theta = 45°$, although this value may vary somewhat depending on the capability of the phase shifter that is employed. Solving Equation (4) for the mass flow rate amplitude leads to:

$$\dot{m} = \frac{2\,\bar{P}\,\dot{W}_{ac}}{R\,T_C\,\tilde{P}\cos(\theta)} \tag{5}$$

The pore velocity is given by:

$$u = \frac{\dot{m}}{A_c\,\rho\,\phi} \tag{6}$$

where ϕ is the porosity of the matrix. Substituting Equation (5) into Equation (6) leads to:

$$u = \frac{2\,\bar{P}\,\dot{W}_{ac}}{A_c\,\rho\,\phi\,R\,T_C\,\tilde{P}\cos(\theta)} \tag{7}$$

Note that the ratio of cross-sectional area to acoustic power in Equation (7) must be approximately constant for a well-designed regenerator, according to Equation (3):

$$u = \frac{2\,\bar{P}}{K\,\rho\,\phi\,R\,T_C\,\tilde{P}\cos(\theta)} \tag{8}$$

Substituting Equation (8) into Equation (1) leads to:

$$Re = \frac{D_h \, 2\,\bar{P}}{\mu \, K \, \phi \, R \, T_C \, \tilde{P} \cos(\theta)} \tag{9}$$

Substituting the definition of the pressure ratio, *PR*, into Equation (9) leads to:

$$Re = \frac{D_h \, 2(PR+1)}{\mu \, K \, \phi \, R \, T_C \, (PR-1)\cos(\theta)} \tag{10}$$

Equation (10) is interesting as it shows clearly the dependence of the Reynolds number on the operating parameters associated with the regenerator. Notice that Reynolds number associated with a regenerator is a maximum at the cold end (where viscosity is lowest) and is minimum at the warm end. The maximum Reynolds number will occur in regenerators that operate at lower temperatures (both μ at the cold end and T_C in Equation (10) will decrease). The Reynolds number does depend on D_h, PR, K, ϕ, and θ ; however, the dependence on these parameters is linear and these parameters are not likely to change by orders of magnitude from cryocooler to cryocooler. The maximum Reynolds number in the regenerator is most strongly related to T_C - approximately as $T_C^{-1.7}$ (assuming that viscosity is proportional to temperature to the 0.7 power) and T_C is likely to change substantially depending on the application of the cryocooler.

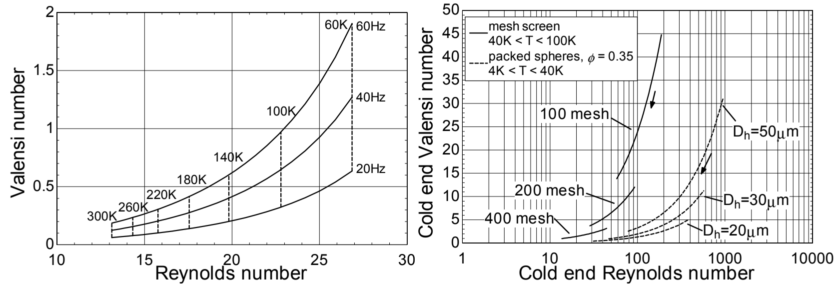
Substituting the ideal gas law into Equation (2) leads to:

$$Va = \frac{\bar{P} \, \omega D_h^2}{R \, T \, \mu} \tag{11}$$

Equation (11) is also interesting as it shows that the Valensi number, and therefore the importance of the oscillating nature of the flow, increases as the temperature decreases. The maximum Valensi number in the regenerator will occur at the cold end and will scale approximately as $T_C^{-1.7}$.

FIGURE 1(a) illustrates the local Reynolds number and Valensi number within a regenerator with a pressure ratio of 1.2, charge pressure of 20 bar, cold end phase angle of 45°, and cold end temperature of 60 K. The working fluid is helium and the regenerator is made of 400 mesh screen (D_h = 55.4 μm, ϕ = 0.686). Contours of temperature and frequency are shown in the parameter space of *Re* and *Va*. The Reynolds number is unaffected by frequency but varies with temperature while the Valensi number is affected by both frequency and temperature. At the cold end of the device operating at 60 Hz, the Valensi number is approaching 2 - indicating that oscillating flow effects may be important locally.

FIGURE 1(b) illustrates the maximum Reynolds number and Valensi number (i.e., the values of these parameters at the cold end of the regenerator) for a regenerator composed of different sized mesh screen as the cold end temperature is changed from 40 K to 100 K and different sized packed sphere beds as the temperature is changed from 40 K to 4 K. Clearly the Valensi number in these regenerators will exceed unity over much of the regenerator and therefore the potential for oscillating flow to affect their behavior exists. Further, the Reynolds number range of primary interest for pulse-tube regenerators is from approximately 10 to a few hundred.



(a) (b)

FIGURE 1. (a) Contours of constant temperature and frequency in the Valensi number and Reynolds number space. (b) Maximum Valensi and Reynolds numbers (at the cold end of the regenerator) for mesh screens as the cold end goes from 40 K to 100 K and packed spheres (ϕ = 0.35) as the cold end goes from 4K to 40K. Arrows indicate the direction of increasing temperature.

CFD MODEL DETAILS

Geometry and Boundary Conditions

The geometry chosen for CFD analysis is a staggered cylinder array, FIGURE 2. Typical regenerator matrices are packed spheres or wire screens. Complete models of these configurations would require a complex 3-D mesh; the associated simulation time is prohibitive given the type of parametric studies required by this project. However, the 2-D staggered cylinder array provides a useful means to investigate trends in the behavior of the fluid-to-solid interaction in a geometry that has most of the important characteristics (i.e., a tortuous path with developing boundary layers and wake regions) at a small fraction of the computational time. Geometric quantities and the associated nomenclature and definitions for a staggered cylinder array are shown in FIGURE 2.

The CFD model used to simulate steady and oscillating flow in a cylindrical array is shown in FIGURE 3. In order to avoid end effects, a total of nine unit-cells were modeled. The unit cell forming the basis of the model, outlined in FIGURE 2, is composed of a fluid domain bounded by an inlet, an outlet, two walls, and four fluid edges. The steady state model, used to compare the model against established experimental correlations for staggered cylinder arrays, utilizes the following boundary conditions: uniform inlet velocity, constant outlet pressure, no-slip walls, and symmetry at unit-cell fluid edges. The oscillating flow model differs only in that a uniform mass-flux that varies in time is applied at the inlet. All models were run at a charge pressure of $\overline{P}$ = 1.0 MPa.

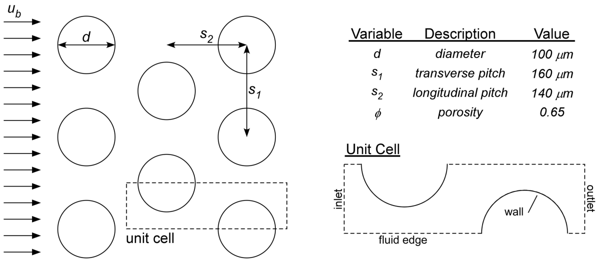


FIGURE 2. Staggered cylinder array geometry, nomenclature, and values used in the CFD model. The dashed box defines a unit-cell of the cylinder array.

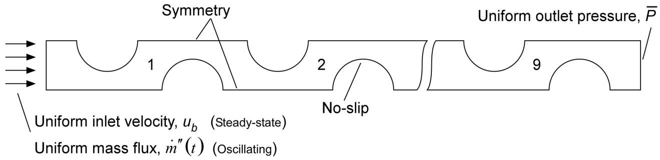


FIGURE 3. Boundary conditions on a micro-scale CFD model of a regenerator matrix. Note that for steady flow a uniform inlet velocity was applied as a boundary condition, whereas for oscillating flow a time-varying mass flux was applied.

RESULTS AND DISCUSSION

Model Validation and Steady Flow Results

Under steady flow conditions, assuming constant properties for helium at 300 K, the mesh was refined until the model results were independent of mesh spacing. It was also verified that the flow was fully developed beyond the third unit-cell in the model; that is, the velocity distribution and unit-cell pressure drop for all subsequent cells were the same. Steady flow model results, generated across a range of Reynolds numbers by varying the mass flow rate and consistent with typical regenerator operation, were used to calculate the friction factor. The CFD predictions were compared to previously published experimental data found in the Heat Exchanger Design Handbook (HEDH) [11]. The friction factor was calculated according to

$$f = \frac{\Delta P\, D_h / L}{1/2\, \rho u_{\max}^2} \tag{13}$$

where ΔP is the pressure drop across a single unit cell, L is the length of a unit-cell, and u_{max} is the maximum velocity of the fluid in the cylinder array and is defined as

$$u_{\max} = \frac{a}{a-1} u_b \tag{14}$$

where a is ratio of the transverse pitch to the diameter, s_1/d, and u_b is the uniform velocity of the fluid at the inlet. Model results and HEDH experimental correlations are compared in FIGURE 4. The discrepancy between the two ranges from 10.7% at low Reynolds number to 26.4% at high Reynolds number.

Oscillating Flow Results

In order to determine when oscillating flow affects the regenerator friction factor, it was necessary to model the flow under oscillating conditions. The oscillating flow results are compared to the steady flow results shown above in order to show that they limit to these results at low frequency and also identify the deviation that occurs at high frequency. Oscillating flow was imposed on the model by applying a sinusoidal time-varying mass-flux at the inlet. The results of the oscillating flow models are presented in FIGURE 5. For each run, the model was allowed to reach an oscillatory steady state and the maximum pressure drop across unit-cell number five (i.e., the middle unit cell) over the course of a cycle was used to calculate the friction factor according to Equation 13. The model was considered to be at steady state when the pressure drop amplitude across unit-cell five and

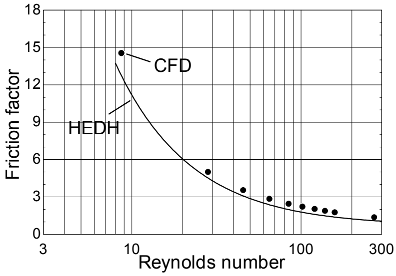


FIGURE 4. CFD model predictions and HEDH correlations of friction factor as a function of Reynolds number for steady flow.

the average velocity amplitude at the inlet to unit-cell five did not change in consecutive iterations.

In FIGURE 5(a) the friction factor was calculated for various values of the Valensi numbers assuming constant properties at a temperature of 300 K and a mass-flux amplitude of 2.11 kg/m^{2}s, which corresponds to a Reynolds number of 23.0. The plot clearly shows that oscillating flow affects the friction factor, but only beyond a transition Valensi number of approximately 10.0. The Valensi number for these runs was varied by varying the frequency. However, in order to obtain Valensi numbers that are high enough at these temperatures the frequency was increased beyond 60 Hz, which is beyond the range of most cryocoolers. Therefore, for the runs shown in FIGURE 5(b) the Valensi number was varied by reducing the operating temperature (and hence the properties) and the Reynolds number was held constant by varying the mass flux amplitude. As the plot shows, there is no difference in the calculated results indicating that the similarity properties Reynolds number and Valensi number are adequate to characterize the flow conditions.

The model was also run in order to determine potential effect of compressibility. Results for these runs are shown in FIGURE 5(c). As one might expect, density changes

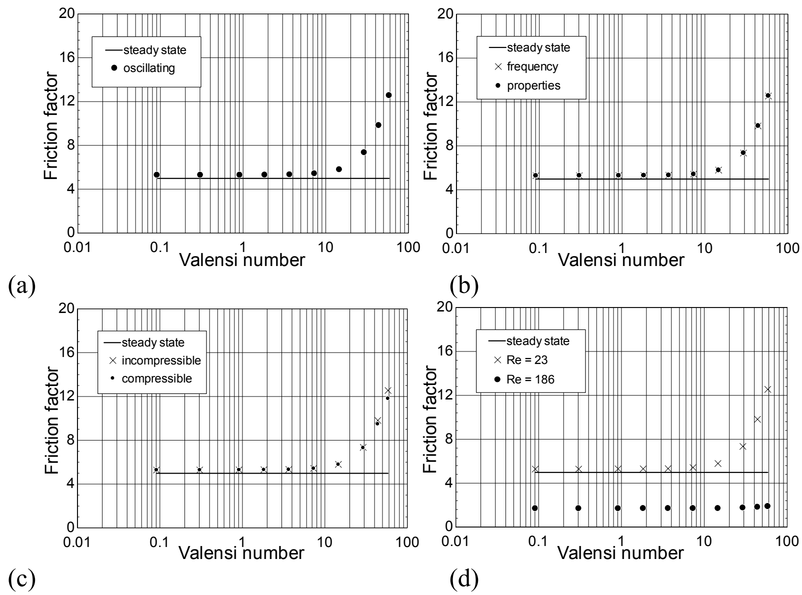


FIGURE 5. Friction factor as a function of Valensi number for (a) steady (b) properties (c) compressible (d) Reynolds number.

across the unit cell are small and do not have any noticeable effect on friction factor.

Finally, FIGURE 5(d) illustrates the effect of increased Reynolds number on the Valensi transition number; the results in FIGURE 5(d) are calculated for Reynolds numbers of 23.0 and 186.0. The increase in the Reynolds number shifts the transition Valensi number from a value of 10 to approximately 50. Comparing this to FIGURE 1(b) shows that even as the cold-end temperature of a regenerator decreases causing the Valensi number to increase, the effect of flow oscillations are still not significant due to the increased Valensi transition number at higher Reynolds numbers.

CONCLUSION

The behavior of pulse tube regenerators has been characterized by two similarity parameters, the Reynolds number and the Valensi number. Typical pulse tube regenerators experience Reynolds numbers that are in the range from 10 to a few hundred and Valensi numbers from less than one to near 50. CFD simulations were used to calculate the friction factor over this parameter space. Initial results show that there is a Valensi transition number beyond which oscillating flow affects friction factor. However, the parameter space that defines typical pulse tube cryocooler operation appears to always lie below the transition Valensi number even at low temperatures where Valensi number is at its highest.

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