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Drawdowns at the Free Water Table During Pumping Tests in Artesian and Semiartesian Aquifers

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In an earlier paper I have shown an example of how long term drawdowns can be used for the computation of long term storage in artesian and semiartesian areas. In most cases the long term storage is more or less equivalent to the specific yield at the water table; the storage mechanisms of consolidation playing a minor role in long term situations. The specific yield in artesian areas is a very important parameter in the prediction of long term effects of ground water withdrawal. Especially the stream depletion will often mainly be governed by draw-downs in upper non-pumped aquifers near the water table, and these drawdowns depend to a great extent on the specific yield at the water table.

A determination of long term storage will often necessitate long term drawdown data, however, under certain circumstances a determination can be made on the basis of a pumping test of limited duration (3-5 weeks) provided drawdown observations at the water table can be made. In this paper some formulas dealing with water table drawdowns in different geohydrologic systems are reviewed, and two cases in which these formulas have been used in practice are presented.

Introduction

In a paper of 1978 I showed an example of computation of long term storage in a semiartesian aquifer system on the basis of long term drawdown data caused by heavy ground water extraction from the system. The analysis was carried out on the basis of a model published by Cooley and Case (1973), using the type curve system of Boulton (1954), describing the drawdowns in a pumped aquifer overlain by a water table aquitard; in the following called the aquifer/aquitard case (Fig. 1). This model proved to be effective for the analysis of long term data in a semiartesian system, provided no constant head boundaries (streams, lakes, shores) have been significantly reached by the drawdowns. The example also showed, that knowledge of the long term storage in a geohydrologic system is extremely important in the forecasting of long term drawdowns, as the use of simpler models, not allowing for long term storage, often will lead to very erroneous results. As the specific yield at the water table in most cases will amount to the absolutely dominating part of the long term storage (90% or more), the knowledge of the specific yield/long term storage should further provide information about the drawdowns at the free water table and vice versa. This could be extremely important in integrated hydrologic modelling, as the processes at the free water table constitutes the link between the ground water and surface water systems. Therefore the drawdowns of the free water table is an important factor in the description of changes in the surface water circulation caused by ground-water withdrawal.

As long term data however, is mostly not at hand in the planning phase, it would be an advantage if the long-term storage of the geohydrologic system could be reasonably estimated on the basis of, for instance a pumping test of limited duration, say 3 – 5 weeks (of course a regional value of the transmissivity should also be estimated, but this is generally a less difficult problem). This is easily done in the case of a rather shallow water table aquifer, as the drawdowns in this case will reach the water table significantly inside the mentioned time-limit, and thus allow determination of specific yield/long term storage. However in the artesian/ semiartesian aquifer/aquitard system, Fig. 1, a much larger pumping time is needed to get significant drawdowns in the aquifer beyond the pseudo-steady-state of leakage, in order to determine the late-time Theis-curve Fig. 1. Moreover these drawdowns will be so small, that they will probably disappear completely in the natural variations of the ground-water level. The same difficulty arises if the pumping takes place from the deep zone of an aquifer with considerable anisotropy.

A possible way to attack these difficulties is naturally to observe the drawdowns beyond the pseudo-steady-state at the place in the system where these drawdowns are generated, and therefore are most significant, namely at the water table. As the occurrence of the pseudo-steady-state shows, that the drawdowns have reached the free water table (the main storage in the system), it also shows, that the drawdowns here have commenced, as the storage here is now supplying the water pumped from the deeper part of the system (the long term situation). The long term drawdowns, mainly governed by the specific yield (and of course the transmissivity), thus start at the water table, and become significant here earlier than in any other part of the system.

In the following some previous published models, describing water table draw-

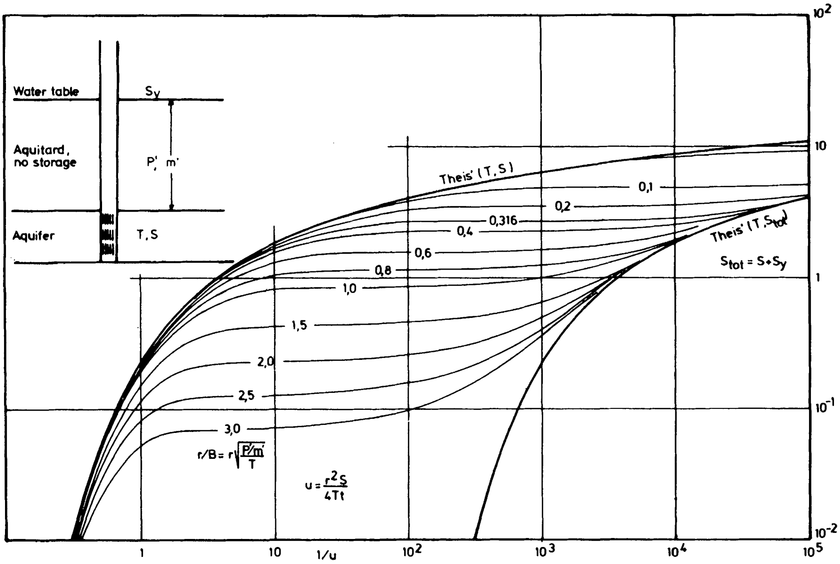

Fig. 1. The Boulton type-curves, showing the drawdowns in an artesian/semiartesian aquifer, overlain by a water table aquitard.

down in different systems are reviewed and commented, and the last section of the paper deals with some practical problems in registration of water-table drawdowns, experienced during some attempts to use these models in the determination of long term storage by the use of aquifer tests in regions with artesian or deep anisotropic aquifers. Finally two case stories with rather well documented drawdowns of the water table are briefly presented.

Review of Mathematical Models Describing Drawdowns at the Water Table due to Pumping of Deep Ground Water

To the authors knowledge, the first model of free water table response to pumping was published by Boulton in 1954. Later Barenblatt (1960) and Streltzova (1973) arrived at identical solutions, describing flow in fissured and porous media respectively. These solutions differ to some extent from the Boulton solution, especially close to the pumping well, because of different boundary conditions here (Streltzova 1973). The Boulton and Streltzova models are constructed mainly for the description of anisotropic, unconfined aquifers. However, they are most reliable in the aquifer/aquitard case (Fig. 1), as they both use a linear approximation to the upper boundary condition (the water table), involving an assumption of a »zone of

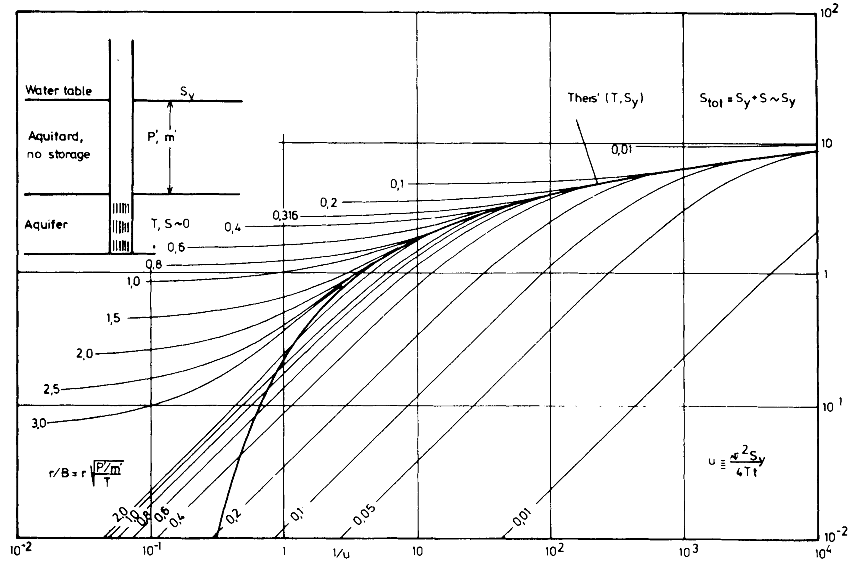

Fig. 2. Type-curves of water table drawdown in the aquifer/aquitard system, Barenblatt-Streltzova, in combination with the late-time Boulton-type-curves of the aquifer-drawdown.

vertical flow« of uniform thickness. This assumption is satisfactorily fulfilled in the aquifer/aquitard case if the permeability of the aquifer is more than about 100 times greater than that of the aquitard, but is generally not fulfilled in the case of a deep anisotropic water table aquifer. The Barenblatt-Streltzova model is shown in Fig. 2 in combination with the Boulton type-curves, describing the late-time drawdowns in the aquifer. It should be noted, that the curve-parameter *r*/*B* is the same for the water table and the aquifer.

It should be noted, that the water-table response in this model does not account for the elastic storage in the aquifer and aquitard. Therefore the drawdowns at the water table, according to the model, start momentarily as pumping commences. In nature a considerable delay will be observed in most cases, as the drawdowns take some time to reach the water table because of the elastic storage in the system. However, in both cases the drawdowns at the water table are linear in time for a long period (inclination *1* in the logarithmic diagram), in the model with a slope proportinal to $1/S_y$ and in nature to $1/S_{\text{total}}$(S_{total} = S_y + elastic storage in aquifer and aquitard). This implies that if the observed drawdowns are displaced backwards in time to start simultaneously with the pumping, an interpretation according to the model will give the value of S instead of S_y, which is almost an advantage; incidentally the two values will generally be almost equivalent as the elastic storage is small compared to S_y. This interpretation is illustrated in case story 1.

In the case of a deep anisotropic aquifer with impervious bottom, the Barenblatt-Streltzova model should not be used because of the above mentioned unsatisfactory approximation to the upper boundary condition. This approximation is also responsible for the semiempirical »delay index« in the r/B-parameter of the Boulton type-curves, when these are referred to an anisotropic water table aquifer instead of the aquifer/aquitard case, where the delay index α becomes equal to $P'/m'/S_y$. A satisfactory description of the anisotropic case can only be obtained through a model in three dimensions. This was first done by Stallman (1965) by the use of an electrical analog. He only considered the storage at the water table, S_y, the elastic storage was ignored, but he obtained a simulation of the drawdowns beyond the pseudo-steady-state in agreement with later published mathematical models. A three-dimensional mathematical model was first published by Dagan (1967) also accounting for water-table storage only. Finally Neumann (1974 and 1975) and Streltzova (1974) established mathematical models allowing for both elastic storage and water-table storage in three-dimensional anisotropic water table aquifers. A type curve set based on these models for fully penetrating pumping and observation wells, is very similar to Boultons of 1954, only the r/B-parameter containing the delay index is replaced by the parameter r^2K_z/b^2K_r, involving the anisotropy of the system. The relationship between the two parameters is given given by Neumann (1975).

Partial penetration of as well pumping as observation wells in a deep anisotropic aquifer involves too many parameters for a simple set of type-curves; these must be constructed in the particular case. In Fig. 3 is shown an example of drawdowns at the water table and the bottom of an anisotropic aquifer with no elastic storage, caused by pumping from the bottom zone of the system (pumping well screened in the bottom three tenths of the aquifer, Stallman (1965)). It may be seen that in this case, as in the aquifer/aquitard case, the late-time Theis-curve can be determined on the basis of water-table drawdowns, if the drawdowns in deeper parts of the system are not suited for this purpose.

It should be noted, that in the anisotropic case with partial penetration, the late time Theis-curve corresponds to a greater transmissivity than the early time Theis-curve, as the part of the aquifer taking part in the flow gradually increases from the screened part at the start of pumping, to the whole aquifer in the long-term situation.

Using hydraulic parameters inside reasonable ranges, based on practical experiences in the models in Figs. 2 and 3, it turns out that drawdowns at the water table in many cases in practice will amount ot about 10 cm or more inside the time-limit of 3-6 weeks of pumping in an aquifer test. Whether drawdowns of this size of order are significant in practice naturally depends on the circumstances under which they are observed, that is the amount and importance of various disturbing factors. In situations where such factors are of minor importance however, registration of the water-table drawdowns during a normal aquifer test should be a

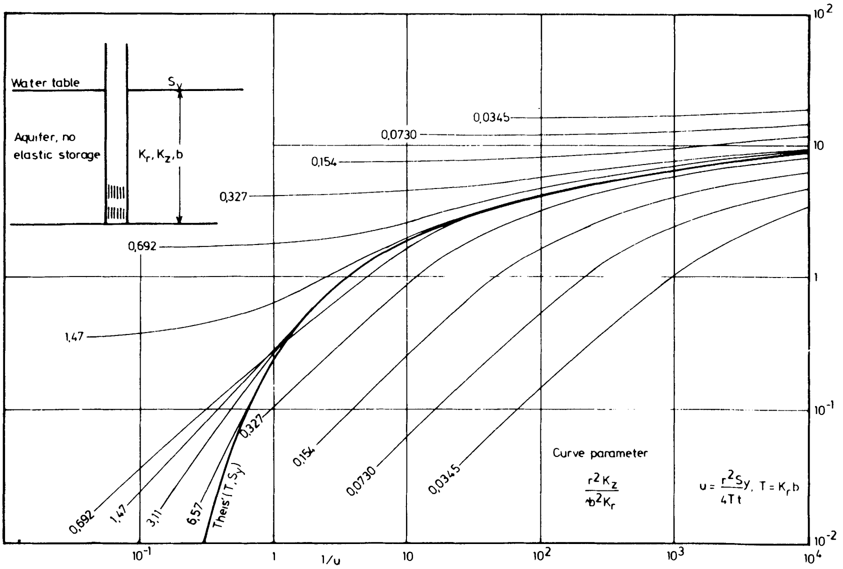

Fig. 3. Type-curves, showing drawdowns at the water table and the bottom of a deep anisotropic aquifer, caused by pumping from a well, screened in the bottom three tenths of the aquifer, Stallmann 1965.

possible way of determining reasonable values of the long term parameters of the system, in cases where this determination normally would necessitate long term drawdown data.

It should be noted, that the delayed yield from storage at the water table in fine grained sediments may result in determination of too small values of long-term storage. However the drawdown at the water table is generally very slow and therefore the delayed yield may be of minor importance. Furthermore the result will be on the safe side in practical use for drawdown forecasting, model work etc.

Practical Problems Case Stories

The observations of water-table drawdowns can be made in piezometers, screened in the water-table zone or from dug wells. The practical problems are more serious in the case of dug wells, and may be listed as follows:

1) The well is normally pumped daily, and it is therefore important to select wells with small reactions at daily use.

2) Wells, dug in fine grained sediments will often receive a great influx of surface water and/or interflow in wet periods. The aquifer test should therefore

preferably be carried out in dry periods (summer, or frost periods in winter) for the well water levels to represent the true free water table.

3) The drawdowns will normally be so small, that they are of the same size of order as the natural variations of the free water table in the area. It is therefore important to obtain data from wells, that are not influenced in the test period to be able to judge the natural variations of the water table inside the area of influence.

Generally the maximum possible amount of wells should be observed inside the investigation area in order to obtain the best possible results.

Case Story 1

Aquifer at Torkildstrup-Lillebrænde Waterworks, the northern of Falster, Denmark. Aquifer/aquitard case: White chalk aquifer overlain by drift clay aquitard, with a thickness of 25-50 m, not containing minor aquifers of any importance. The test was carried out in a frost period during the winter 78-79 with a rate of pumping of about 40 m^3/hour.

Drawdowns from 3 influenced dug wells are shown in the upper diagram of Fig. 4. The drawdowns continue after stop of pumping because of the delaying effect of the elastic storage in aquifer/aquitard. In the dates 2-3 of March the water level rises rapidly due to thaw, which spoils further observations. No significant natural variations of the water table have been registrated in the test period so far. All observation data are plotted s versus t/r^2 on logarithmic paper, Fig. 4. The linear drawdown expiration from the three dug wells are previously displaced to start simultaneously with the pumping. The interpretation is carried out as follows:

MATCH POINT 1: Theis' and Boultons type-curves, early period: $T = 1.3 \times 10^{-3}$ m^2/sec, $S \sim 1.2 \times 10^{-4}$ for the white chalk aquifer. P'/m' = ca. 2×10^{-10} sec^{-1} for the aquitard. On the basis of this value, r/B-values for the three dug wells are computed, and the model curves of Fig. 2 are used to determine the late time Theis-curve,

MATCH POINT 2:$T = 1.3 \times 10^{-3}$ m^2/sec. (as 1) and $S_{total} = 0.4\%$, a reasonable value for the aquitard material (drift clay). An estimate of total elastic storage in the aquitard (Hantush 1960) yields a size of order 3×10^{-4} m^{-1} (inhomogeneities in the aquifer makes this estimation rather uncertain), showing that the elastic storage in the system is probably less than about 10% of the total long term storage.

Case Story 2

Aquifer test at Karup Waterworks, western Jutland, Denmark. Anisotropic aquifer with saturated thickness of 50-60 m, the upper about 20-30 m consisting of fluvioglacial deposits of coarse and fine sands, the rest of miocene river deposits of sands, silt and thin local layers of clay. The bottom consists of a thick layer of clay from the miocene period.

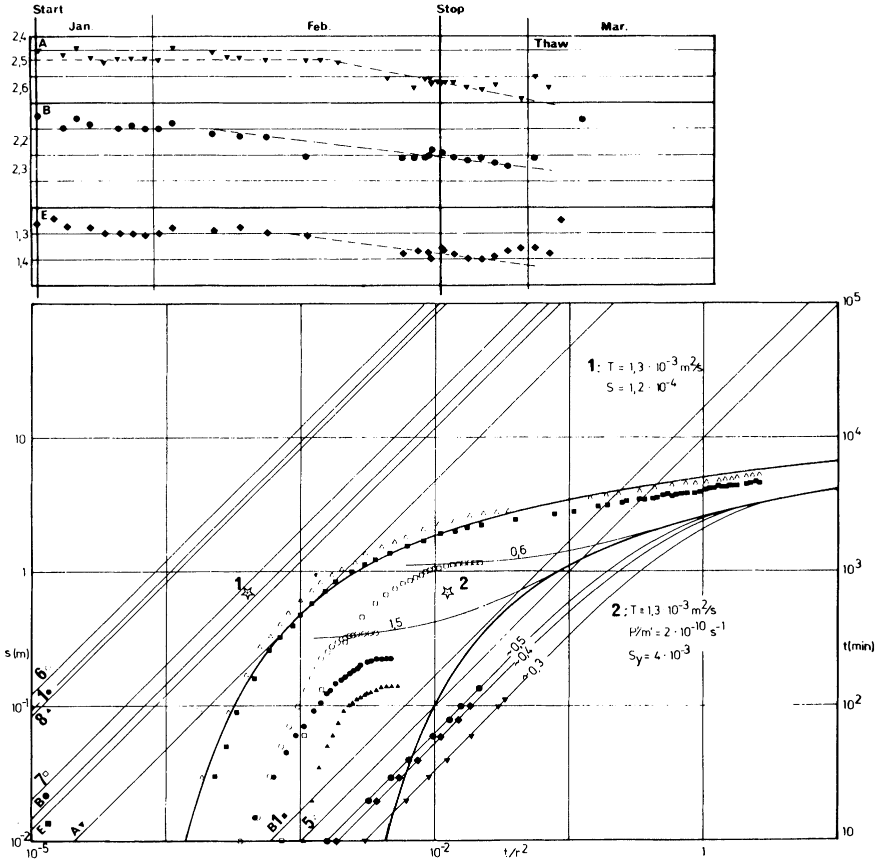

Fig. 4. Logarithmic plot of drawdown-date from observation wells during a pumping test at Torkildstrup-Lillebrænde Waterworks, the northern of Falster, Denmark.

12 dug wells and piezometers at the water table were observed during the test, 3 of them clearly influenced are plottet in Fig. 5, s versus t/r^2. The drawdowns are corrected for the natural variations, estimated on the basis of the other observations in the area. The drawdowns started very close to the start of pumping, meaning that the elastic storage in the system is rather small and the vertical permeability rather large, compared to case 1. The pumping well is screened in the bottom 10 m of the system and the observation well 2 from 15-21 m's above the bottom. The pumping rate was ca. 78 m³/hour for about 5 weeks. Corrected observation data are plotted logarithmic in Fig. 5.

MATCH POINT 1 is determined partly on the basis of pumping well data (not shown) giving a well defined initial transmissivity of 1.5×10^{-3} m²/sec. for the screened interval, partly on an interpretation of early data from 2 by a formula

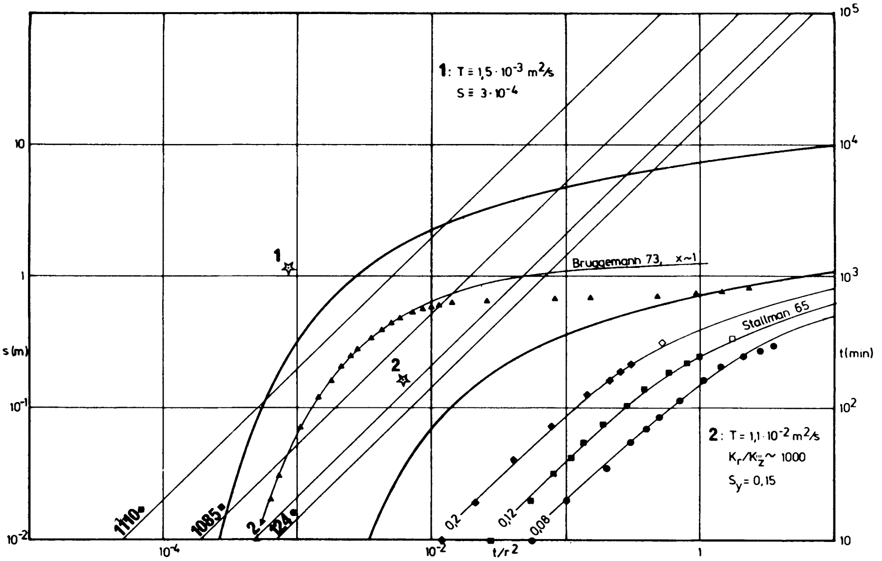

Fig. 5. Logarithmic plot of drawdown data from observation wells during a pumping test at Karup Waterworks, the western of Jutland, Denmark.

published by Bruggemann (1973), giving $S \sim 3 \times 10^{-4}$ (elastic storage in the screened zone of the aquifer).

MATCH POINT 2 is established by the model in Fig. 3 on the basis of the data from the dug wells (slightly erroneous as the model in Fig. 3 describes pumping from the bottom 0.30 of the aquifer, and the actual system is pumped from the bottom 0.15, the difference at the water table is insignificant though), giving $T_{\text{total}} = 1.1 \times 10^{-2}$ m²/sec. (in good agreement with late-time pumping well data) and $S_y = S_{\text{total}} = 15\%$. The relationship K_z/K_r is estimated to about 1/1000 in the best fit to the data.

Conclusions

It has been attempted to show a possibility of determining long-term parameters in artesian and semiartesian geohydrologic systems, based on drawdowns at the free water table during aquifer tests of rather normal duration (3-6 weeks). The normally observed drawdowns in deeper parts of such systems do not make this determination possible, and usually long-term drawdown data have been necessary for the computations of the long-term parameters.

Through the review of previously published models of water-table drawdowns in artesian and semiartesian systems a theoretical background has been established, showing that the water-table drawdown normally will be rather small in aquifer test situations.

However, two presented case stories demonstate, that the practical problems in collection of a sufficiently well documented observation material can often be solved and reasonable results obtained.

References

Barenblatt, G. I. et al. (1960) Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. *J. Appl. Math. Mech. 24(5),* 1286-1303.

Boulton, N. S. (1954) Unsteady radial flow to a pumped well allowing for delayed yield from storage. Intern. Assoc. Sci. Hydrology. General Assembly at Rome. v. 2. pub. 37.

Boulton, N. S. (1954) The drawdown of water table under nonsteady conditions near a pumped well in an unconfined formation. Proc. Inst. Civil. Engrs. (London) 3, Part III, 564-579.

Bruggemann, G. A. (1973) Mathematical analysis of steady and non-steady two- and three-dimensional flow in anisotropic aquifers. Mededelingen 73-1 (communication) of the Government Institute for Drinking Water Supply, The Hague, Holand.

Cooley, R. M., and Case, C. M. (1973). Effect of a water table aquitard on drawdown in an underlying pumped aquifer. *Water Res. Res, vol. 9, No. 2,* 434-447.

Dagan, G. A. (1967) A method of determining the permeability and effective porosity of an unconfined anisotropic aquifer. *Water Res. Res. Vol. 3 No. 4,* 1059.

Hantush, M. S. (1960). Modification of the theory of leaky aquifers. *Journ. Geophys. Res. vol. 65, No. 11,* 3713-3725.

Kærgaard, H. (1978) The Physics of long term drawdowns in pumping from artesian aquifers. *Nordic Hydrology, 9,* 249-262.

Neumann, S. P. (1974) Effect of partial penetration on flow in unconfined aquifers, considering delayed gravity response. *Water Res. Res. vol. 10, No. 2,* 303-312.

Neumann, S. P. (1975) Analysis of pumping test data from anisotropic unconfined aquifers. *Water Res. Res. vol. 11, No. 2,* 329-342.

Stallman, R. W. (1965) Effects of water table conditions on water level changes near pumping well. *Water Res. Res. vol. 1, No. 2,* 295-312.

Streltzova, T. D. (1973) Water table drawdown due to a pumped well in an unconfined aquifer. *Water Res. Res. vol. 9, No. 1.,* 236-242.

Streltzova, T. D. (1974) Drawdown in compressible unconfined aquifer. *Journ. Hydr. Div. nov. 1974, HV 11,* 1601-1616.

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