An electrical model of the human eye

II. The model and the eye during tonography

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A basic electrical model of the eye can be distorted analogously to the distortion of an eye when a tonometer is placed upon it. It may then be caused to match a clinical tonogram by adjustment of facility of outflow and stress relaxation. The classical elastic concept of the eye is compared to the viscoelastic concept by the use of models. Stress relaxation is offered as an explanation of the "first minute effect." The degree of match of model to tonogram is illustrated by published normal tonograms. The electrical model does not provide very different quantitative values for facility of outflow, but it does strengthen the concept of viscoelasticity of the eye. The degree of fit of the model to published clinical tonograms and the data derived therefrom is illustrated and discussed.

The hydrodynamic and mechanical properties of the eye have been simulated by an electrical model of the eye. If this basic model is distorted analogously to the distortion occurring during tonography, then the dynamic electrical response of the model should match the dynamic pressure response of the eye. If matching occurs, there is at least partial validation of the concepts used to develop the model and the data used to quantitate those concepts.

This paper will show the degree of match between clinical tonograms and the electrical model incorporating within it different concepts and different quantitative data.

Materials and methods

Distortion of the model analogous to indentation tonography. Placing a tonometer upon an eye, and allowing it to remain there, is usually thought of as a single procedure. For illustrative purposes it may be divided into two parts: (1) the initial placement which distorts the eye and therefore increases the pressure, and (2) the subsequent effect of the tonometer on the eye.

The initial placement of the tonometer. This mechanism has been illustrated by stating that fluid is displaced from the anterior of the eye by the tonometer and causes a distension in the rest of the eye to accommodate the displaced fluid. Another explanation is that the tonometer changes the eye from nearly a sphere to an indented sphere, thus decreasing the volume-to-surface ratio (largest for a sphere), and the surface must stretch to maintain the same volume. The increase in tension in the outer coats is consequently reflected in a rise in pressure. This latter explanation would dictate that the model of the eye should have its volume-to-surface ratio decreased in order to analogize the action of the tonometer placement. A change of volume-to-surface ratio is difficult in the electrical model. An acceptable analogy would be to consider that the application of a tonometer to an eye is equivalent to an injection of a single volume of fluid, as the amount of fluid injected is just sufficient to raise

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the pressure to the pressure produced by the tonometer. It should be noted that placement of the tonometer is equivalent to a single injection, and that the removal of the tonometer is equivalent to the withdrawal of a single volume of fluid.

Although this single injection of fluid is not continuous for a long period of time, time is involved. We have two choices in applying this concept to the electrical model: we can discharge into the model a capacitor which will give a fixed amount of charge delivered in accordance with the time constant of the charging circuit, or we can apply a voltage from a battery for a short length of time. The latter method is more convenient at the present stage of development. The voltage increase, determined from the pressure increase shown on the tonogram (P at t = 0), may be preset. Assuming the tonometer is placed during one second, the voltage should be applied for about 1/60 second. The assumption of a 1 second placement can be roughly approximated by observing the initial slope of the recording pen as it moves, during the placement of the tonometer, from infinity to the reading at the start of the tonogram.

The initial placement of the tonometer upon the eye is analogized by applying a voltage equivalent to the pressure rise for 1/60 second (see Fig. 1).

The subsequent reaction of the tonometer. The tonometer does not maintain a constant amount of indentation, but as the pressure decreases the eye becomes softer and the tonometer indents the eye more. Both the footplate and the plunger indent more, but at a different rate. The net change in indentation will be used. This action of the tonometer (increasing indentation with decreasing pressure) must now be given an analogous counterpart in the electrical model. Since the plunger indents more and more as the pressure decreases, this means that the volume of indentation will increase as the pressure decreases.

The result of an indenting plunger is to hold the decay curve to a higher average value than would be the case if the tonometer were not present. This is analogized in the model by increasing the over-all time constant with a plunger capacitor (CAP_{pl}). This capacitor tends to hold the model's decay curve higher than without it (see Fig. 2).

The next step is to determine which size of capacitor, which is analogous to a tonometer with a 5.5 and 7.5 Gm. weight should be used. There are two sets of data bearing on this point: that of Friedenwald, and that of McBain. Both sets consist of tables relating pressure to volume of corneal indentation (P = Vc tables).

The relation of pressure (P) to volume of corneal indentation (Vc) may be approximated by several methods. To find the best linear fit, we have chosen the least squares method carried out on a computer.* This method yields four equations:

Friedenwald data
\[ V_c = 47.52 - 1.138P \] for the 5.5 Gm. weight.
\[ V_c = 43.44 - 0.781P \] for the 7.5 Gm. weight.

McBain data
\[ V_c = 79.75 - 2.060P \] for the 5.5 Gm. weight.
\[ V_c = 64.84 - 1.293P \] for the 7.5 Gm. weight.

Again, as in figuring the capacitor equivalent to the ocular rigidity, the ratio of the differential of the volume, with respect to the pressure, will yield the capacitance value:

Friedenwald:
\[ \frac{dV}{dP} = \frac{1.14}{0.78} \] for the 5.5 Gm. weight.
\[ \frac{dV}{dP} = \frac{2.06}{1.29} \] for the 7.5 Gm. weight.

The values are independent of pressure and are constant over the tonometer range: 4 < R < 10. There is a large difference between the values of the two investigators. This difference is much larger than any difference encountered among the data for immediate ocular rigidity obtained by the many investigators, including Friedenwald.

*University of California San Francisco Computer Center, San Francisco Calif. Supported in part by National Institutes of Health Grant No. FRO-0122. The observed data fit the computed line to an error of less than 5 per cent.
consequences of this difference will be pointed out below.

CAP will depend upon whether the 5.5 or the 7.5 Cm. weight was used in the tonogram. We are then left with the variables R_q, R_s, and R_out. In this work, R_q and R_s will not be changed independently, but will be changed while maintaining a ratio of approximately 1 to 35, respectively.

We have no way at the present time of determining whether or not the fast- and slow-relaxing components in the sclera are independent or dependent on one another. As a first approach we might say that they are dependent, that is, any change in the sclera due to disease or age will affect both components proportionately. If we fix them in the same ratio as was found in our work on scleral segments, we will reduce the number of variables and thwart no theoretical concepts. If, in the future, we find that this fixed ratio will not reproduce a number of tonograms, or that there is some experimental evidence for independent action, then we will consider other ways of handling these variables.

Having set the model to correspond to the parameters of the eye upon which the tonogram was performed, it is only necessary to vary R_q (with R_s in fixed ratio) and R_out in order to match the tonogram.

Specific adjustments of the model to correspond to clinical tonography.

Change of parameters of the eye from general to specific values. So far the electrical model has been developed for the average normal eye. Since we wish to have the model correspond to a specific eye during tonography, we can set some of the parameters of the model.

As seen in Fig. 3 there are ten variables in the electrical model; essentially all but three of these variables can be fixed or assumed.

At all times that the model is at rest, the P_s (pressure of secretion) is set to give an inflow which develops a P_0 corresponding to the applplanation pressure in the eye. P_0 can be determined from the tonogram and is equal to the pressure (P_s) to which the eye rises when the tonometer is initially applied. The P_s is equivalent to the P_r (episcleral venous pressure), which is assumed to be 10 mm. Hg. The capacitances (CAP, CAE, and CAP_s) are set according to the calculations described earlier. CAP is set for either the 5.5 or 7.5 Cm. weight. We have left only R_q, R_s, and R_out as variables for a specific eye, and of these three, the first two (R_q and R_s) have a fixed ratio.

Adjustment of axes. We now wish to place a clinical tonogram in the bed of an X-Y recorder and allow the model to draw a smooth curve through the mean (within ± 0.5 mm. Hg) of the pulse fluctuations. Since we have not built into our model any periodic fluctuations, only an "average" decay pressure will come out. We wish also to vary only R_q (and R_s in fixed ratio to it) and R_out in order to obtain the match, since the other variables have been fixed according to the rules above. In order to accomplish this we have to consider: (1) the time scale or X-axis, (2) the ordinate of the tonogram or Y-axis, and (3) calibration.

The time scale. From our first paper, the model was developed to act 60 times faster than the eye. This means that the distance on the clinical tonogram between time 0 and time 4 minutes must be traveled by the pen of the X-Y recorder in 4 seconds. This is accomplished by having a time sweep on the X-axis built into the model. The sweep is variable to accommodate those tonograms in which the distance between 0 and 4 minutes is different from the usual 8 inches.

The ordinate. The electrical model, referred to as P_r, indicates the pressure existing in the eye while the tonometer is in place during the tonographic run. The ordinate or vertical scale of the clinical tonogram is in scale units of the tonometer reading (R).° Thus the ordinate of the tonogram is not proportional, so far, to the ordinate of the output of the model. In order to correspond, we had to find a simple function which would ex-

**To avoid confusion, R when used alone or with numeric subscripts always refers to tonometer scale reading. An R with alphabetic subscripts refers to a resistance, either electrical or physiological. R_0 is only a numeric subscript and stands for the scale reading at time = zero.
press the relationship between R and P, and then change the output of the model by that same function. The Crescent tonometer has a linear scale on the face of its meter, while the Mueller tonometer has a nonlinear scale. Within certain limits of scale reading, the following relation between R and P, holds to the given limits of error for the 5.5 Cm. weight and similarly for the 7.5 Cm. weight:

\[
\text{Error (mm. limits Hg)}
\]

<table>
<thead>
<tr>
<th>Relation</th>
<th>Scale reading (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crescent</td>
<td>( \log P_t = c(R) )</td>
</tr>
<tr>
<td>Mueller</td>
<td>( \log P_t = c'(R) )</td>
</tr>
</tbody>
</table>

where c and c’ are constants to be discussed under calibration.

Thus if we convert the output \( (P_t) \) of the model to \( \log P_t \) and keep within the limits set, the ordinate of the clinical tonogram will be similar to the output of the model on the Y-axis.

**CALIBRATION.** We have shown above that the output of the model is similar to the clinical tonogram, and in order to make the two scales congruent it is necessary to find the value for c and c’ in the formula for the relation of \( \log P_t = (c \text{ or } c') (R) \) given above. This can easily be determined by adjusting the gain setting of the Y-axis of the X-Y recorder. In practice, two calibrating voltages are applied alternately to the model, and adjustment of the gain setting is made until the pen of the X-Y recorder rests alternately on \( R = 4 \), and \( R = 10 \) of the tonometer scale calibration accompanying the clinical tonogram in the bed of the X-Y recorder. Calibration voltages are the \( P_t \) values associated with \( R = 4 \) and \( R = 10 \). For the sake of consistency we have used McBain’s calibration scales which corresponds to \( P_t = 32.9 \) (\( R = 4 \)) and \( P_t = 22.1 \) (\( R = 10 \)) for the 5.5 Cm. weight. This calibration scale does not differ greatly from Friedenwald or Prijot.

The complete electrical model for analyzing Crescent or Mueller electronic tonograms.

So far, only simple diagrams have been used to illustrate the principles of the electrical model. The complete model is given in Fig. 4. The differences consist mainly in modifications for speed and ease of analysis. Batteries are replaced with voltage supplied from the ordinary 120 v., 60 cy. line. A meter is provided, calibrated in millimeters of mercury and micro liters per minute, so that the parameters of \( P_o \), \( P_h \), \( P_i \), flow, and \( P_s \) short may be set and read. \( P_s \) short is a device to return the model rapidly to steady state conditions.

The \( P_o \) voltage appears only across the equilibrium capacitor \( (C_{AF}) \) during the 3 seconds that the \( C_{AF} \) and \( C_{AFs} \) are shorted. The variable resistances and capacitors are digitally set with decade boxes. Because the pen is not fast enough to follow the electrical pulse (analogous to the initial placement of the plunger) it is held at its starting point by a separate power supply until the termination of the pulse (see Fig. 1). There are a series of relay switches in order that a single button times and actuates in sequence: the 3 second \( P_o \) short to assure steady state conditions; an application of a 17 msec. pulse to the sclera (initial \( P_t \)) with the pen independently positioned in writing position; start of a 4 second sweep; lifting of pen at about 5 seconds and return to starting position. Other switches handle calibration, readout, and adjustment on the meter of various parameters previously mentioned and change from 5.5 to 7.5 Cm. weight conditions.

Also incorporated in the computer are electrical components which simulate the distortion of the eye under conditions of scleral cup tonography. The conversion of the electrical model of the eye for scleral cup tonography will be discussed in a later paper.

**Results and discussion**

In order to show the matches obtained, it was decided to use published tonograms of normal eyes. Ordinarily the tonograms were photographed, enlarged for convenience, placed in the bed of the X-Y recorder, and analyzed. The trace which best fit the tonogram was inked over (white on dark background, black on white background), rephotographed, and reproduced in Figs. 5 to 9, 11, and 12. Values given in the original article or assumed from the tonogram are recorded on the figures. Also recorded are the \( P_v \) and \( C \) values obtained by digital computation (Computer Center) according to the data and methods of Friedenwald, Grant, and Moses and Becker from the original \( R_o \) and \( R_h \). Finally, the values obtained from the electrical model are given.

It is realized that some of the investigators may have published their tonograms more for illustrative purposes rather than as a thoughtfully considered average response curve but this series probably represents “normal” tonograms as judged by experts in the field.

Few of the tonograms contain full data, particularly applanation pressure, so that the ocular rigidity of the eye and the model can only be assumed to have normal ocular rigidity.
Fig. 4. Electrical schematics of model.
Grant7 (Fig. 5). This appears to be an excellent match with a low value for $R_0$.

**Becker and Friedenwald** (Fig. 6). The ordinate of this tonogram is given in millimeters of mercury of the undisturbed eye ($P_o$ values). Weinbaum10 has recently analyzed, by using a mathematical model, a tonogram appearing in the same paper. It might be noted that Weinbaum has assumed the ordinate of the original tonogram to be $P_t$ rather than $P_o$ values.

In order to arrive at appropriate $R$ values, Dr. Becker was consulted, and his best recollection was that $R_o = 5.5$ and $R_t = 8.75$, giving a $P_o$ of 16 and a $C$ value 0.29. Using these two points ($R_o$ and $R_t$) for calibration, the line was drawn by the model, as shown in Fig. 6, with values not much different from those of the author. The ordinate was in $P_o$ values but the calibration scale between $P_o$ and $R$ from this early work was not at hand and it seemed unnecessary for this illustration to ask the author to delve further into past records.

**Ballintine** (Fig. 7). This appears as a good match of model-drawn-line to tonogram but the $C$ value obtained from the model is considerably larger than the $C$ value calculated by the author (0.27). There is no ready explanation for this. The $C$ value obtained by digital computation is probably explained by the author's use of P-V tables before the 1955 revision.

**Becker and Shaffer** (Fig. 8). Here be-
begins the modern period, after the 1955 revision, and results appear to be more consistent. The ocular rigidity is within normal limits.

To illustrate the effect of using the ocular rigidity data of investigators other than McBain-Prijot average, the values of the three capacitors (CAP₀, CAP₆₀₀, and CAPₘ) were changed in the model to equal the ocular rigidity given by Langham for live human eyes and expressed in the form of the unifying formulation. The range used was 25 ± 5 mm. Hg. The values for the individual capacitances were calculated in the same way as were the values for the average of McBain and Prijot. The capacitances have these values: CAP₀ = 8.9, CAP₆₀₀ = 2.9, CAPₘ = 4.9. The rest of the model was unchanged. When matching of Becker and Shaffer's tonogram was accomplished, a line indistinguishable from the line shown in Fig. 8 was obtained with a change only in the value of Rₘ, which resulted in a change of C from 0.38 to 0.43. This shows that the use of current data on immediate ocular rigidity from investigators other than McBain-Prijot results in little change, compared to the large change between McBain and Friedenwald's P - Vc tables (volume of corneal indentation).

Moses (Fig. 9). This is an "ideal tonographic curve calculated from Grant's formula and Linner's correction for ΔPₗ." This shows the shape of the curve when the eye is considered to have simple elastic properties with no stress relaxation. The shape of the line does not correspond with the usual clinical tonogram, which shows
more curvature in the beginning. In order to make the shape of the model's curve fit the usual tonogram, it is necessary to introduce stress relaxation. We have fitted three curves to this calculated curve of Moses.

**CURVE I.** To produce this curve we used the Friedenwald model developed in the previous paper. This is an elastic model with a single capacitor representing the ocular rigidity of outer coats. The value assigned to this (0.60) is the reciprocal of the ocular rigidity determined by Friedenwald in the range of 20 to 30 mm. Hg. Fig. The value for the capacitor representing the "subsequent action of the tonometer" has been given above for the Friedenwald data for the 5.5 Gm. weight. All other values are set according to the rules given above. That the curve given by the model using Friedenwald data and the curve calculated by Moses should coincide so nicely is mutual confirmation of both the model's action and Grant's method and formula for calculating the action of the eye. It might be noted that the only difference between the model and Grant's formula is in the method of solving a single differential formula for C. The model solves it essentially by integrating the equation in time, thereby acting as an analog computer. Grant's formula solves it by expanding the differentials into finite quantities (i.e., $dx$ changed to $\Delta x$).

**CURVE II.** To produce this curve, McBain-Prijot immediate ocular rigidity data, plus St. Helen and McEwen stress relaxation data, were substituted for Friedenwald's ocular rigidity data (see Basic Model) and the value of the capacitance plunger is taken from McBain data above. The matching was accomplished by going through $R_0$ and $R_4$ with a value for $C$ (1 per $R_{out}$) closest to that calculated by Moses. This resulted in a $C$ of 0.30 and a stress relaxation of $R_0 = 10$. The curvature in the beginning of this curve seems greater than in Grant's curve with the same $R_0$ (see above). This is a consequence of the contracted scale of the latter.

**CURVE III.** This is the same as Curve II except the stress-relaxation resistance ($R_0$) was increased to $R_0 = 50$ necessitating a change in $R_{out}$ so that the curve would go through the initial and final points.

**First minute effect**

We can now see the difference in curves which we would expect from considering the eye as elastic and from considering the eye as viscoelastic. The value for the facility of outflow is not greatly changed, but the character of model's curve (visco-
elastic) is curved more in the first minute. This effect of the presence of a greater drop in the early part of tonograms is not predicted from Friedenwald and Grant's elastic model. It should be pointed out that extra large drops seen in the beginning of some tonograms are not caused by stress relaxation, but must be accounted for by such a mechanism as squeezing of the eyes prior to the placement of the tonometer and subsequent rapid drop upon relaxation after placement of the tonometer.

**Similarity of Grant’s elastic model to the viscoelastic model**

The question might be asked, “Why does the viscoelastic model, which differs radically in concept to the elastic model, yield data not surprisingly different in regard to facility of outflow?” It has been noticed that, in addition to the qualitative difference in concept of the two models, there is a quantitative difference. To illustrate both differences, the capacitors representing the ocular rigidity and the subsequent action of the plunger in the Friedenwald model may be rearranged into an exactly equivalent circuit.

Here again is shown the effect of the large difference between McBain and Friedenwald’s corneal indentation (P-Vc tables) values. The total capacitance values representing immediate ocular rigidity in the range 20 < P < 30 are: McBain-Prijot (1.21) and Friedenwald (0.80); but, when the corneal indentation capacitances are added, the totals become McBain-Prijot (1.21 + 2.0 = 3.21) and Friedenwald (0.80 + 1.14 = 1.94), a large absolute difference. The exactly equivalent Friedenwald model may be compared with the viscoelastic model in Fig. 10. The values given are taken directly from the models which gave Curves I and II above. The only difference now seen is that the Friedenwald model has an extra capacitor (CAPs) while the viscoelastic model has Rs instead. The effect of both of these elements is to cause a more rapid loss of the pressure after the initial distortion than would occur without them. They are not exactly equivalent, but in most eyes the Rout in both circuits, when adjusted so that the model’s curve goes through R0 and R4, are approximately the same. The Friedenwald elastic model will give only a slightly curved line, while the viscoelastic model is more capa-

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It is thus apparent that the Friedenwald data, to a certain extent, compensate quantitatively, but not qualitatively, for stress relaxation.

Prijot \textsuperscript{11} (Fig. 11). There is a fair correspondence here. The time scale is slightly short of four minutes.

Stepanik \textsuperscript{15} (Fig. 12). This appears as a rather steep curve, yet the facility of outflow is not large, illustrating, perhaps, the difficulty in judging the C value of a curve by inspection.

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REFERENCES