RESEARCH ARTICLE | MARCH 24 2010

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AIP Conf. Proc. 1205, 160–167 (2010)
https://doi.org/10.1063/1.3382324

Entropy in the Present and Early Universe and Vacuum Energy

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Abstract. It is demonstrated that entropy and its density play a significant role in solving the problem of the vacuum energy density (cosmological constant) in the Universe and hence the dark energy problem. Taking this in mind, two most popular models for dark energy - Holographic Dark Energy Model and Agegraphic Dark Energy Model - are analyzed. It is shown that the fundamental quantities in the first of these models may be expressed in terms of a new small parameter. Besides, the results obtained on the uncertainty relation of the pair "cosmological constant - volume of space-time", where the cosmological constant is a dynamic quantity, are reconsidered and generalized up to the Generalized Uncertainty Relation (GUP).

Keywords: vacuum energy, fundamental length, generalized uncertainty relation, deformed density matrix
PACS: 03.65, 05.20

INTRODUCTION

The problem of vacuum energy is one of the key problems in a modern theoretical physics. This problem has attracted the attention of researchers fairly recently with the understanding that a cosmological constant determining the vacuum energy density is still nonzero despite its smallness. As is known, the cosmological constant Λ has been first introduced in the works of A.Einstein [1] who has used it as a antigravitational term to obtain solutions for the equations of a General Relativity (GR) in the stationary case. However, when A.Friedmann has found the solutions for GR in case of expanding Universe [2] and E.Hubble has derived an extension of the latter, A.Einstein refused from the cosmological constant considering its introduction to be erroneous [3].
But the situation was not so simple. In [4] it has been stated that any contribution into the vacuum energy acts exactly as the cosmological constant Λ and the Vacuum Energy Density is proportional to Λ. The principal problem of the cosmological constant resides in the fact that its experimental value is smaller by a factor of 10^{123} than that derived using a Quantum Field Theory (QFT) [5],[6].
And the theories actively developed at the present time (e.g., superstring theory, loop quantum gravity, etc.)offer a modified quantum theory including, in particular, the fundamental length at Planck's scale. The estimates of Λ obtained on the basis of these theories may be greatly differing from the initial ones derived from the standard QFT.
In this paper some of the properties of the Vacuum Energy Density are studied within the scope of a Quantum Field theory with UV cutoff (minimal length). Such a theory arises in the early Universe in all the models without exception since the fundamental length (probably on the order of Planck's but not necessarily) is acknowledged to be of a crucial importance in this case It is shown that for this case the experimental and theoretical values are close and may be expressed in terms of a new small parameter introduced in physics at Planck's scales. Additionally, within the scope of a dynamic approach to Λ, its behavior associated with the Generalized Uncertainty Principle is studied for the pair "cosmological constant - volume of space-time". In what follows, there is no differentiation between the notions of the cosmological constant Λ and Vacuum Energy Density ρ_{vac} Note that the Vacuum Energy Density is the main candidate for the solution of a more general problem – Dark Energy Problem [7]–[11].

VACUUM ENERGY DENSITY,HOLOGRAPHIC AND AGEGRAPHIC DARK ENERGY MODELS

As noted in Introduction, the Vacuum Energy is a major candidate for the Dark Energy. At the same time, due to the factor 10^{123} distinguishing the experimental value ρ_{vac}^{exp} [7] and the value ρ_{vac}^{QFT}, calculated using the standard QFT [5]– $\rho_{vac}^{QFT} \approx m_p^4$,

$$\frac{\rho_{vac}^{exp}}{\rho^{QFT}} \approx 10^{-123} \quad (1)$$

interpretation of Dark Energy as a Vacuum Energy presents great difficulties. But there are several methods enabling one to obviate the difficulties. We can name two most popular and acknowledged approaches.

CP1205, *The Sun, The Stars, The Universe and General Relativity*
edited by R. Ruffini and G. Vereshchagin

Holographic Dark Energy Models

The basic relation for this model is the "energy" inequality [12, 13, 14]

$$E_{\overline{\Lambda}} \leq E_{BH} \rightarrow l^3 \rho_{\overline{\Lambda}} \leq m_{\rm p}^2 l. \quad (2)$$

Here $\rho_{\overline{\Lambda}} = \overline{\Lambda}^4$ – vacuum energy density with the UV cutoff $\overline{\Lambda}$ and l is the length scale (IR cutoff) of the system. For the equality in (2) we have the **holographic energy density**

$$\rho_{\overline{\Lambda}} \sim \frac{m_{\rm p}^2}{l^2} \sim \frac{1}{(l_{\rm p} l)^2}. \quad (3)$$

Also, from (2) we can get the "entropic" inequality (entropy bound)

$$S_{\Lambda} \leq \left(m_{\rm p}^2 A\right)^{3/4}, \quad (4)$$

where $A = 4\pi l^2$ is the area of this system in the spherically symmetric case.
The number of works devoted to the Holographic Dark Energy Models, beginning from the first publication [12], is ever growing [15]to relieve us from citing the whole list.

Agegraphic Dark Energy Models

Agegraphic Dark Energy Models became the subject of study only two years ago [16]. These relations were based on the result of Károlyházy for quantum fluctuations of time [17, 18, 19]

$$\delta t = \lambda t_{\rm p}^{2/3} t^{1/3}. \quad (5)$$

Using the uncertainty relation of "energy-time" in the flat space

$$\Delta E \sim t^{-1}, \quad (6)$$

we can obtain the **agegraphic energy density** [20, 14]

$$\rho_{\rm T} \sim \frac{\Delta E}{(\delta t)^3} \sim \frac{m_{\rm p}^2}{t^2}. \quad (7)$$

The number of publications associated with models of this type is constantly increasing too [21]. This is caused by their relative simplicity and by a sufficiently good coincidence of the agegraphic energy density $\rho_{\rm T}$ with ρ_{vac}^{exp}.

DARK ENERGY PROBLEM AND QUANTUM THEORY WITH UV CUTOFF

By Holographic Dark Energy Models (explicitly) and by Agegraphic Dark Energy Models (implicitly) it is implied that QFT, where they are valid, is actually QFT with the UV cutoff or fundamental length.
As it has been repeatedly demonstrated earlier, a Quantum Mechanics of the Early Universe (Plank Scale) is a Quantum Mechanics with the Fundamental Length (QMFL)[22]. The principal approach to framing of QFT with UV cutoff is that associated with the Generalized Uncertainty Principle (GUP) [23, 24, 25] and with the corresponding Heisenberg algebra deformation produced by this principle [26]–[29].
Besides, QMFL has been framed first using the deformed density matrix and then it the produced corresponding Heisenberg algebra deformation [30]–[39], the density matrix deformation $\rho(\alpha)$ in QMFL being a starting object called the density pro-matrix and deformation parameter (additional parameter) $\alpha = l_{min}^2/x^2$, where x is the measuring scale and $l_{min} \sim l_p$. As has been indicated in this papers deformation parameter α is varying within the limits $0 < \alpha \leq 1/4$, moreover $\lim\limits_{\alpha \rightarrow 0} \rho(\alpha) = \rho$, where ρ being the density matrix in the well-known Quantum Mechanics (QM). The explicit form of the above-mentioned deformation gives an exponential ansatz:

$$\rho^*(\alpha) = exp(-\alpha) \sum_i \omega_i |i><i|, \quad (8)$$

where all $\omega_i > 0$ are independent of α and their sum is equal to 1.
The correspondent deformed quantum field theory is defined at the non-uniform lattice in hypercube $I_{1/4}^4$ with the side 1/4 in length and edge of $I_{1/4} = (0; 1/4]$ [35],[36]. It is designated as QFT^{α}. All the variables associated with the considered α - deformed quantum field theory are hereinafter denoted by the upper index $^{\alpha}$.
As follows from the holographic principle [40]–[42], the maximum entropy that can be stored within a bounded region $\mathfrak{R}$ in 3-space must be proportional to the value $A(\mathfrak{R})^{3/4}$, where $A(\mathfrak{R})$ is the surface area of $\mathfrak{R}$. Of course, this is associated with the case when the region $\mathfrak{R}$ is not an inner part of a particular black hole. Provided a physical system contained in $\mathfrak{R}$ is not bounded by the condition of stability to the gravitational collapse, i.e. this system is simply non-constrained gravitationally, then according to the conventional QFT $S_{\max}(\mathfrak{R}) \sim V(\mathfrak{R})$, where $V(\mathfrak{R})$ is the bulk of $\mathfrak{R}$. However in Holographic Principle case, as it has been demonstrated originally by G. 't Hooft [40] and later by other authors (for example R. V. Buniy and S. D. H. Hsu [43])

$$S_{\max}(\mathfrak{R}) \leq \frac{A(\mathfrak{R})^{3/4}}{l_p{}^2}. \quad (9)$$

In terms of the deformation parameter α the principal values of the Vacuum Energy Problem may be simply and clearly defined. Let us begin with the Schwarzschild black holes, whose semiclassical entropy is given by

$$S = \pi R_{Sch}^2/l_p^2 = \pi R_{Sch}^2 M_p^2 = \pi \alpha_{R_{Sch}}^{-1}, \quad (10)$$

with the assumption that in the formula for α $R_{Sch} = x$ is the measuring scale and $l_p = 1/M_p$. Here R_{Sch} is the adequate Schwarzschild radius, and $\alpha_{R_{Sch}}$ is the value of α associated with this radius. Then, as it has been pointed out in [44]), in case the Fischler- Susskind cosmic holographic conjecture [45] is valid, the entropy of the Universe is limited by its "surface" measured in Planck units [44]:

$$S \leq \frac{A}{4} M_P^2, \quad (11)$$

where the surface area $A = 4\pi R^2$ is defined in terms of the apparent (Hubble) horizon

$$R = \frac{1}{\sqrt{H^2 + k/a^2}}, \quad (12)$$

with curvature k and scale a factors.
Again, interpreting R from (12) as a measuring scale, we directly obtain(11) in terms of α:

$$S \leq \pi \alpha_R^{-1}, \quad (13)$$

where $\alpha_R = l_p^2/R^2$. Therefore, the average entropy density may be found as

$$\frac{S}{V} \leq \frac{\pi \alpha_R^{-1}}{V}. \quad (14)$$

Using further the reasoning line of [44] based on the results of the holographic thermodynamics, we can relate the entropy and energy of a holographic system [46, 47]. Similarly, in terms of the α parameter one can easily estimate the upper limit for the energy density of the Universe (denoted here by ρ_{hol}) [48]:

$$\rho_{hol} \leq \frac{3}{8\pi R^2} M_P^2 = \frac{3}{8\pi} \alpha_R M_P^4, \quad (15)$$

that is drastically differing from the one obtained with a known QFT

$$\rho_{QFT} \sim M_P^4. \quad (16)$$

Here by ρ_{QFT} we denote the energy vacuum density calculated from the known QFT (without UV cutoff) [5]. Obviously, as α_R for R determined by (12) is very small, actually approximating zero, ρ_{hol} is by several orders of magnitude smaller than the value expected in QFT - ρ_{QFT}.
In fact, the upper limit of the right-hand side of(15) is attainable, as it has been demonstrated in [49] and indicated in [44]. The "overestimation" value of r for the energy density ρ_{QFT}, compared to ρ_{hol}, may be determined as

$$r = \frac{\rho_{QFT}}{\rho_{hol}} = \frac{8\pi}{3} \alpha_{\mathbf{R}}^{\mathbf{-1}} = \frac{8\pi}{3} \frac{R^2}{L_P^2} = \frac{8\pi}{3} \frac{S}{S_P}, \quad (17)$$

where S_P is the entropy of the Plank mass and length for the Schwarzschild black hole. It is clear that due to smallness of α_R the value of α_R^{-1} is on the contrary too large. It may be easily calculated (e.g., see [44])

$$r = 5.44 \times 10^{122} \quad (18)$$

in a good agreement with the astrophysical data.
Naturally, on the assumption that the vacuum energy density ρ_{vac} is involved in ρ as a term

$$\rho = \rho_M + \rho_{vac}, \quad (19)$$

where ρ_M - average matter density, in case of ρ_{vac} we can arrive to the same upper limit (right-hand side of the formula (15)) as for ρ.

FISCHLER-SUSSKIND CONJECTURE AND GRAVITATIONAL HOLOGRAPHY

In this Section the arguments in support of the Fischler-Susskind cosmic holographic conjecture are given on the basis of the results obtained lately on Gravitational Holography.
Quite recently, T.Padmanabhan in a series of his papers [55]–[61] and some other works has convincingly demonstrated that Einstein equations may be derived from the surface term of the GR Lagrangian, in fact containing the same information as the bulk term.
And as Einstein-Hilbert's Lagrangian has the structure $L_{EH} \propto R \sim (\partial g)^2 + \partial^2 g$, in the usual approach the surface term arising from $L_{surf} \propto \partial^2 g$ has to be canceled to get Einstein equations from $L_{bulk} \propto (\partial g)^2$ [62]. But due to the relationship between L_{bulk} and L_{surf} [57]–[59],[62], we have

$$\sqrt{-g} L_{suf} = -\partial_a \left(g_{ij} \frac{\partial \sqrt{-g} L_{bulk}}{\partial(\partial_a g_{ij})} \right). \quad (20)$$

In such a manner one can suggest a holographic character of gravity in that the bulk and surface terms of the gravitational action contain identical information. However, there is a significant difference between the first case when variation of the metric g_{ab} in L_{bulk} leads to Einstein equations, and the second case associated with derivation of the GR field equations from the action principle using only the surface term and virtual displacements of horizons [54], whereas the metric is not treated

as a dynamical variable [62].
In the case under study, it is assumed from the beginning that we consider the spaces with horizon. It should be noted that in the Fischler-Susskind cosmic holographic conjecture it is implied that the Universe represents spherically symmetric space-time, on the one hand, and has a (Hubble) horizon (12), on the other hand. But proceeding from the results of [55]– [62], an entropy boundary is actually given by the surface of horizon measured in Planck's units of area [58]:

$$S=\frac{1}{4}\frac{A_H}{l_p^2}, \qquad (21)$$

where A_H is the horizon area.
Because of this, it should be noted that Einstein's equations may be obtained from the proportionality of the entropy and horizon area together with the fundamental thermodynamic relation connecting heat, entropy, and temperature [46]. In fact [55]– [62], this approach has been extended and complemented by the demonstration of holographicity for the gravitational action (see also [63]).
To sum it up, an assumption that space-time is spherically symmetric and has a horizon is the only natural assumption held in the Fischler-Susskind cosmic holographic conjecture to support its validity. Then there is a resemblance to thermodynamic systems, and one can associate the notions of temperature and entropy with them. In the case of Einstein-Hilbert gravity, it is possible to interpret Einstein's equations as the thermodynamic identity $TdS = dE + PdV$ [64].

SOME COMMENTS ON DYNAMICAL CHARACTER OF COSMOLOGICAL CONSTANT AND GUP

Generally speaking, Λ is referred to as a constant just because it is such in the equations, where it occurs: Einstein equations [1]. But in the last few years the dominating point of view has been that Λ is actually a dynamical quantity, now weakly dependent on(or practically independent of)time [65]–[67]. It is assumed therewith that, despite the present-day smallness of Λ or even its equality to zero, nothing points to the fact that this situation was characteristics for the early Universe as well. Some recent results [68]–[71] are rather important pointing to a potentially dynamic character of Λ. Specifically, of great interest is the Uncertainty Principle derived in these works for the pair of conjugate variables (Λ, V):

$$\Delta\Lambda\Delta V \sim \hbar, \qquad (22)$$

where Λ is the vacuum energy density (cosmological constant). It is a dynamical value fluctuating around zero; V is the space-time volume. Here the volume of space-time V results from the Einstein-Hilbert action [69]:

$$S_{EH} \supset \Lambda \int d^4x\sqrt{-g} = \Lambda V. \qquad (23)$$

In this case "the notion of conjugation is well-defined, but approximate, as implied by the expansion about the static Fubini–Study metric" (Section 6.1 of [68]). Unfortunately, in the proof per se (22), relying on the procedure with a non-linear and non-local Wheeler–de-Witt-like equation of the background independent Matrix theory, some unconvincing arguments are used, making it insufficiently rigorous (Appendix 3 of [68]). But, without doubt, this proof has a significant result, though failing to clear up the situation.
Let us attempt to explain (22)(certainly at an heuristic level) using simpler and more natural terms involved with the other, more well-known, conjugate pair (E,t) - "energy - time". We use the designations of [68],[69]. In this way a four-dimensional volume will be denoted, as previously, by V.
Just from the start, the Generalized Uncertainty Principle (GUP) is used. Then a change over to the Heisenberg Uncertainty Principle at low energies will be only natural. As is known, the Uncertainty Principle of Heisenberg at Planck's scales (energies) may be extended to the Generalized Uncertainty Principle. To illustrate, for the conjugate pair "momentum-coordinate" (p,x) this has been noted in many works [23]–[27]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_p^2 \frac{\triangle p}{\hbar}. \qquad (24)$$

In [32],[38] it is demonstrated that the corresponding Generalized Uncertainty Relation for the pair "energy - time" may be easily obtained from

$$\Delta t \geq \frac{\hbar}{\Delta E} + \alpha' t_p^2 \frac{\Delta E}{\hbar}, \qquad (25)$$

where l_p and t_p - Planck length and time, respectively.
Now we assume that in the space-time volume $\int d^4x\sqrt{-g} = V$ the temporal and spatial parts may be separated (factored out) in the explicit form:

$$V(t) \approx t\overline{V}(t), \qquad (26)$$

where $\overline{V}$ - spatial part V. For the expanding Universe such an assumption is quite natural. Then it is obvious that

$$\Delta V(t) = \Delta t\overline{V}(t) + t\Delta\overline{V}(t) + \Delta t\Delta\overline{V}(t). \qquad (27)$$

Now we recall that for the inflation Universe the scaling factor is $a(t) \sim e^{Ht}$. Consequently, $\Delta\overline{V}(t) \sim \Delta t^3 f(H)$, where $f(H)$ is a particular function of Hubble's constant. From (25) it follows that

$$\Delta t \geq t_{min} \sim t_p. \qquad (28)$$

However, it is suggested that, even though Δt is satisfying (28), its value is sufficiently small in order that ΔV be contributed significantly by the terms containing Δt to the power higher than the first. In this case the main contribution on the right-hand side of (27) is made by the first term $\Delta t\overline{V}(t)$ only. Then, multiplying the left- and right-hand sides of (25) by $\overline{V}$, we have

$$\Delta V \geq \frac{\hbar \overline{V}}{\Delta E}+\alpha' t_p^2\frac{\Delta E\overline{V}}{\hbar}=\frac{\hbar}{\Delta\Lambda}+\alpha' t_p^2\overline{V}^2\frac{\Delta\Lambda}{\hbar}. \qquad (29)$$

It is not surprising that a solution of the quadratic inequality (29) leads to a minimal volume of the space-time $V_{min}\sim V_p=l_p^3 t_p$ since (24) and (25) result in minimal length $l_{min}\sim l_p$ and minimal time $t_{min}\sim t_p$, respectively. (29) is of interest from the viewpoint of two limits:

1)IR - limit: $t\rightarrow\infty$

2)UV - limit: $t\rightarrow t_{min}$.

In the case of IR-limit we have large volumes $\overline{V}$ and V at low $\Delta\Lambda$. Because of this, the main contribution in the right-hand side of (29) is made by the first term as great $\overline{V}$ in the second term is damped by small t_p and $\Delta\Lambda$. Thus, we derive at

$$\lim_{t\rightarrow\infty}\Delta V\approx\frac{\hbar}{\Delta\Lambda} \qquad (30)$$

in accordance with (22) [68]. Here, similar to [68], Λ is a dynamical value fluctuating around zero.

And for the case (2) $\Delta\Lambda$ becomes significant

$$\lim_{t\rightarrow t_{min}}\overline{V}=\overline{V}_{min}\sim\overline{V}_p=l_p^3;\lim_{t\rightarrow t_{min}}V=V_{min}\sim V_p=l_p^3 t_p. \qquad (31)$$

As a result, we have

$$\lim_{t\rightarrow t_{min}}\Delta V=\frac{\hbar}{\Delta\Lambda}+\alpha_\Lambda V_p^2\frac{\Delta\Lambda}{\hbar}, \qquad (32)$$

where the parameter α_Λ absorbs all the above-mentioned proportionality coefficients.

For(32) $\Delta\Lambda\sim\Lambda_p\equiv\hbar/V_p=E_p/\overline{V}_p$.

It is easily seen that in this case $\Lambda\sim M_p^4$, in agreement with the value obtained using a naive (i.e. without supersymmetry and the like) quantum field theory [6],[5]. Despite the fact that Λ at Planck's scales (referred to as $\Lambda(UV)$) (32) is also a dynamical quantity, it is not directly related to well-known Λ (22),(30) (called $\Lambda(IR)$) because the latter, as opposed to the first one, is derived from Einstein's equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G_N\left(-\Lambda g_{\mu\nu}+T_{\mu\nu}\right). \qquad (33)$$

However, Einstein's equations (33) are not valid at the Planck scales and hence $\Lambda(UV)$ may be considered as some high-energy generalization of the conventional cosmological constant, leading to $\Lambda(IR)$ in the low-energy limit.

In conclusion, it should be noted that the right-hand side of (24),(25) in fact is a series. Of course, a similar statement is true for (32) as well.

Then, we obtain a system of the Generalized Uncertainty Relations for the Early Universe (Planck scales) in the symmetric form as follows:

$$\left\{\begin{array}{lcl}\Delta x & \geq & \frac{\hbar}{\Delta p}+\alpha'\left(\frac{\Delta p}{p_{pl}}\right)\frac{\hbar}{p_{pl}}+\ldots\\ \Delta t & \geq & \frac{\hbar}{\Delta E}+\alpha'\left(\frac{\Delta E}{E_p}\right)\frac{\hbar}{E_p}+\ldots\\ \Delta V & \geq & \frac{\hbar}{\Delta\Lambda}+\alpha_\Lambda\left(\frac{\Delta\Lambda}{\Lambda_p}\right)\frac{\hbar}{\Lambda_p}+\ldots\end{array}\right. \qquad (34)$$

The latter of the relations (34) may be important when finding the general form for $\Lambda(UV)$, low-energy limit $\Lambda(IR)$, and also may be a step in the process of constructing future quantum-gravity equations, the low-energy limit of which is represented by Einstein's equations (33).

FINAL COMMENTS AND CONCLUSION

I. As shown by numerous authors (to start with [28]), the Quantum Mechanics with the fundamental length (UV cutoff) generated by GUP is in line with the following deformation of Heisenberg algebra

$$[\vec{x},\vec{p}]=i\hbar(1+\beta^2\vec{p}^2+\ldots) \qquad (35)$$

and

$$\Delta x_{\min}\approx\hbar\sqrt{\beta}\sim l_p. \qquad (36)$$

In the recent works [72] it has been demonstrated that the Holographic Principle is an outcome of this approach, actually being integrated in the approach.

We can easily show that the deformation parameter β in (35),(36) may be expressed in terms of the deformation parameter α given in Section 3 of this work that is introduced in the approach associated with the density matrix deformation. Indeed, from (35),(36) it follows that $\beta\sim\mathbf{1/p^2}$, and for $x_{\min}\sim l_p$, β corresponding to $x_{\min}$ is nothing else but

$$\beta\sim 1/P_{pl}^2, \qquad (37)$$

where P_{pl} is Planck's momentum: $P_{pl}=\hbar/l_p$.

In this way β is changing over the following interval:

$$\lambda/P_{pl}^2\leq\beta<\infty, \qquad (38)$$

where λ is a numerical factor and the second member in (35)is accurately reproduced in momentum representation (up to the numerical factor) by $\alpha=l_{min}^2/l^2\sim l_p^2/l^2=$

p^2/P_{pl}^2:

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) = i\hbar(1+a_1\alpha+a_2\alpha^2+...). \quad (39)$$

The parameter α has one more interesting feature indirectly noted in Section 3:

$$\alpha_l^{-1} \sim l^2/l_p^2 \sim S_{BH}. \quad (40)$$

Here α_l is the parameter α corresponding to l, S_{BH} is the black hole entropy with the characteristic linear size l (for example, in the spherically symmetric case $l = R$-radius of the correspondent sphere with the surface area A), and

$$A = 4\pi l^2, S_{BH} = A/4l_p^2 = \pi\alpha_l^{-1}. \quad (41)$$

This note is devoted to the demonstration of the fact, that in case of the holographic principle validity in terms of the new deformation parameter α in QFT^{α}, considered above and introduced as early as 2002 [50]–[52], all the principal values associated with the Vacuum (Dark) Energy Problem may be defined simply and naturally. At the same time, there is no place for such a parameter in the well-known QFT, whereas in QFT with the fundamental length, specifically in QFT^{α} it is quite natural [30, 31, 33, 35, 36, 38].

II. It should be noted that the smallness of α_R (Section 3) results in a very great value of r in (17),(18). Besides, from (17) it follows that there exists some minimal entropy $S_{min} \sim S_P$, and this is possible only in QFT with the fundamental length.

III.This section is related to Section 3 in [53] as well as to Sections 3 and 6 in [54]. The constant L_Λ introduced in these works is such that in case under consideration $\Lambda \equiv l_\Lambda^{-2}$ is equivalent to R, i.e. $\alpha_R \approx \alpha_{l_\Lambda}$ with $\alpha_{l_\Lambda} = l_p^2/l_\Lambda^2$. Then expression in the right-hand side of (15) is the major term of the formula for ρ_{vac}, and its quantum corrections are nothing else as a series expansion in terms of α_{l_Λ} (or α_R):

$$\rho_{\rm vac} \sim \frac{1}{l_P^4}\left(\frac{l_P}{L_\Lambda}\right)^2 + \frac{1}{l_P^4}\left(\frac{l_P}{l_\Lambda}\right)^4 + \cdots = \alpha_{l_\Lambda} M_P^4 + ... \quad (42)$$

In the first variant presented in [53] and [54] the right-hand side (42) (formulas (12),(33) in [53] and [54], respectively)reveals an enormous additional term $M_P^4 \sim \rho_{QFT}$ for renormalization. As indicated in the previous Section, it may be, however, ignored because the gravity is described by a pure surface term. And in case under study, owing to the Holographic Principle, we may proceed directly to (42). Moreover, in QFT^{α} there is no need in renormalization as from the start we are concerned with the ultraviolet-finiteness.

Moreover, a series expansion of (42) in terms of α is a complete analog of the expansion in terms of the same parameter redetermining the measuring procedure in $QMFL^{\alpha}$ [31, 33, 35, 38]:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0'\alpha^2 + ... \quad (43)$$

As indicated in [39], the same expansion may be used to obtain quantum corrections to the semiclassical Bekenstein-Hawking formula (21) for the black hole entropy.

IV.Besides, the Heisenberg's algebra deformations are introduced due to the involvement of minimal length in quantum mechanics. These deformations are stable in the sense of [73]. But this is not true for the unified algebra of Heisenberg and Poincare. This algebra does not carry the indicated immunity. It is suggested that the Lie algebra for the interface of the gravitational and quantum realms is in its stabilized form. Now it is clear that such a stability should be raised to the status of a physical principle. In a very interesting work of Ahluwalia - Khalilova [73] it has been demonstrated that the stabilized form of the Poincare-Heisenberg algebra [74], [75] carries three additional parameters: "a length scale pertaining to the Planck/unification scale, a second length scale associated with cosmos, and a new dimensionless constant with the immediate implication that 'point particle' ceases to be a viable physical notion. It must be replaced by objects which carry a well-defined, representation space dependent, minimal spatiotemporal extent"

Thus, within the scope of a Quantum Field theory with the UV cutoff (fundamental length), closeness of the theoretical and experimental values for ρ_{vac} is adequately explained. In this case an important role is played by new parameters appearing in the corresponding Heisenberg Algebra deformation. Specifically, by the new small dimensionless parameter α, in terms of which one can adequately interpret both the smallness of ρ_{vac} and its modern experimental value. Besides, it is shown that the Generalized Uncertainty Principle (GUP) may be an instrument in studies of a dynamical character of the cosmological constant Λ.

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