Multi-Affinity for Growing Rough Interfaces of Bacterial Colonies

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We have examined whether rough interfaces of bacterial colonies are multi-affine. We have used the bacterial species called *Bacillus subtilis*, which has been found to exhibit a variety of colony patterns when varying both the concentration of nutrient and solidity of agar medium. Consequently, we have found that the colony interface on a nutrient-rich, solid agar medium is multi-affine. On the other hand, the colony interface on a nutrient-rich, semi-solid agar medium is self-affine.

§1. Introduction

For the last few decades growing rough interfaces have been investigated extensively.1) In general, there exist concepts of fractal geometry for describing the behavior of growing rough interfaces.2), 3) Such models as Eden, ballistic deposition,3) Karder-Parisi-Zhang (KPZ)4) have been examined to confirm a standard self-affine behavior. Let us consider a rough interface which grows from the substrate of line seeds. And we analyze a rough interface grown from one-dimensional substrate in two-dimensional space. In general, the height-height correlation function $C(x, t)$ of the height of the interface from the substrate is given as $C(x, t) = \left( \langle |h(x', t') - h(x' + x, t + t)|^2 \rangle_{x', t'} \right)^{1/2}$, where $x$ is the width of a strip of substrate, $t$ the time, $h(x, t)$ the interface height at the substrate site $x$ and the time $t$. It has been known that various rough interfaces satisfy the relations

\begin{align}
C(x, t) &\sim t^{\beta}, \quad (t \ll t^*) \quad (1.1) \\
C(x, t) &\sim x^{\alpha}, \quad (t \gg t^*) \quad (1.2)
\end{align}

Here the scaling relation (1.1) means that the correlation function $C(x, t)$ grows with a power law of time in early stages of growth, while (1.2) means that $C(x, t)$ saturates to a time-independent value around some characteristic time $t^*$, and then it exhibits a power law of $x$ at later stages. This signifies that the growing rough interface is a self-affine fractal. The exponents $\alpha$ and $\beta$ are called the roughness and growth exponents, respectively.

Though the previous models, which are self-affine, satisfy the relation (1.1) and

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(1.2), the other numerical models and experiments, such as Zhang model\(^5\) and slow combustion of paper,\(^6\) do not only exhibit the standard self-affine interfaces but also multi-affine ones. The multi-affinity is defined by the \(q\)th order height-height correlation function as follows:

\[
C_q(x) = \langle |h(x') - h(x' + x)|^q \rangle_{x'}.
\]

(1.3)

The exponent \(\alpha_q\) is defined through the following relation:\(^7\)

\[
C_q(x) \sim x^{q\alpha_q}.
\]

(1.4)

If the value of \(\alpha_q\) is dependent of the degree \(q\), we decide that an interface is multi-affine one.

In this paper, we will examine whether rough interfaces of bacterial colonies are multi-affine. Throughout this experiment we used the bacterial species called Bacillus subtilis. Bacillus subtilis has been found to exhibit a variety of colony patterns when varying both the concentration of nutrient and solidity of agar medium (Fig. 1). We observed growing interfaces in the regions B and D. However, in the region D, we used a mutant strain which does not produce surfactant because the original, wild-type strain does not show any clear colony interfaces, but rather fuzzy ones. Note that there is no difference of patterns in the region B formed by a mutant and wild-type except the growth time. Therefore, we used a wild-type strain in the region B while we used a mutant strain in the region D. Wakita et al. have studied experimentally the self-affinity of Bacillus subtilis.\(^9\) They obtained the values of roughness exponent, \(\alpha \approx 0.78\) in the region B and \(\alpha \approx 0.50\) in the region D. In the region B, bacterial cells are inoculated on a nutrient-rich, solid agar medium. Then, bacterial cells do not move actively, and the colony interface consists of many chains of cells for multiplication. As a result, the colony interface is folded and moves with clear lateral correlations. On the other hand, in the region D, bacterial cells are inoculated on a nutrient-rich, semi-solid agar medium. Then, the interface consists of individual cells (cell length is about 2 \(\mu m\)) and moves locally through random pushes due to active movement of individual cells inside the colony. And the dynamics of interface growth in the region D agrees with numerical models such as Eden model. In fact, the value of roughness exponent of \((1+1)\)-d Eden model is 0.50, which is consistent with the result in the region D.

§2. Multi-scale analysis for bacterial colonies

The bacterial strain was line-inoculated on the agar plate surface (a diameter of 88 mm), which was inoculated at 35 °C and 90 % RH for designated time. Bacterial
colonies grew two-dimensionally on the agar plate surface. Then, we set a microscope (KEYENCE, Osaka) and used a personal computer to digitize the interface with high resolution of 1220 × 1620 pixels (1 pixel ≃ 1.7 µm in the region B and 0.13 µm in the region D, respectively). Figure 4 shows the $q$th order correlation functions for growing interfaces in the region B. There is a crossover length-scale up to which the different orders scale with $q$-dependent exponents $\alpha_q$. We conclude, therefore, that interfaces in the region B are multi-affine one. Similarly, Fig. 5 shows the $q$th order correlation functions for growing interfaces in the region D. In this region, we ignore the data of $x < 10$ pixels (i.e., 1.3 µm) because bacterial cell length is about 2 µm. From Fig. 5, the correlation functions are almost independent of order $q$, compared with those in the region B (Fig. 4). Hence we conclude that interfaces in the region D are mono-affine (self-affine) one.

§3. Discussion

We have examined multi-affinity of interface of bacterial colony. From the present results, we conclude that the interface in the region B is multi-affine, while
the interface in the region D is mono-affine. As mentioned above, the interface in the region B consists of many chains of cells. These elastic chains are deformed by local density-dependent pressures. This multi-scale property of the interface in the region B is caused by the competition between local density-dependent and the characteristic length of chain. Recently, we have obtained that growing rough interfaces of paper wetting and bacterial colony did not satisfy the normal dynamic scaling, but the extended dynamic scaling. The elucidation is an interesting future problem.

Furthermore, in numerical simulations, Barabási et al. investigated ballistic deposition model (this model has originally self-affinity) with power-law distributed noise. They found that interfaces produced by the model exhibit multi-affinity. Hence, the multi-affinity is closely related to noise properties. In fact, we have measured the distribution of fluctuation amplitudes \( \eta \equiv |\delta h(x', t) - \delta h(x', t + \tau)| \), where \( \delta h(x, t) \equiv h(x, t) - \langle h(x, t) \rangle_x \). Figure 6 shows the distribution \( P(\eta) \) against \( \eta \) for interfaces in the region B. In Fig. 6 the distribution \( P(\eta) \) has a power-law tail, which means that the interface of bacterial colony in the region B has an abundance of avalanche-like events. However, we must pay attention that the fluctuation amplitudes \( \eta \) in experiments cannot always be regarded as noise. In general, it is difficult to give meaning to the noise distribution in real phenomena. This is still another problem to make clear in the future.

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**References**