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A Decade's Investigation of the Stability of Erodible Stream Beds

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This paper reviews work on the stability of a particulate stream bed to the erosive attack of a flow with a free surface. Attention is given to the development of the hydraulic, the potential-flow and the rotational-flow models, and to the roles of phase lags and transport laws. The relationship of the stability theory to the ultimate form of the stream bed is discussed, and the current level of understanding of bed features is examined. Some investigations relating to a wider range of erosive processes are noted, and an extensive bibliography is provided.

This survey was prepared for the Euromech 48 Symposium, 'Transport, erosion and deposition of sediment in turbulent streams', which took place in August, 1974, at the Technical University of Denmark. It is an attempt to put in perspective the considerable body of analytical work undertaken over the preceding decade. At the beginning of this period only a few fragments of analysis were available, but by the end of the decade several lines of attack were well established, and the analytical models had had considerable success in correlating observations relating to laboratory experiments and natural streams.

Fig. 1 defines the subject area to be examined, and shows how it is related to other problems concerned with the erosion of particulate materials under the action of wind or flowing water. We shall concentrate on attempts to predict the development of natural channels in which the flow of water is nominally steady, that is, on the stability of the system made up of the potentially interacting flow and stream bed. The

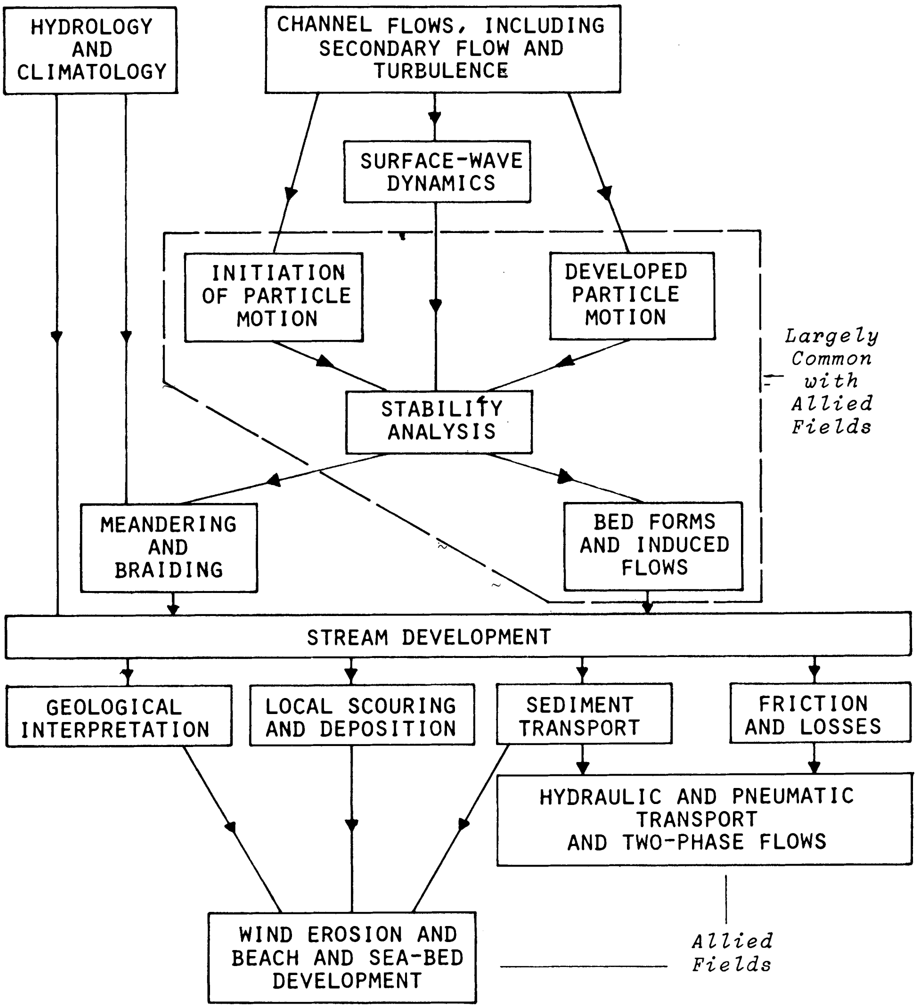

Fig. 1. The place of stability analysis in the general field of problems associated with the erosion and transport of particulate materials.

fundamental aspects of this problem have much in common with a number of allied topics: wind erosion, beach and sea-bed development, hydraulic and pneumatic transport, and other two-phase flows. The stability investigation has (or at any rate has been given here) a central place in the hierarchy of problems. It requires inputs of information concerning flow patterns and wave and particle dynamics; it supplies, we hope, explanations and predictions of bed forms and channel configurations.

In the absence of a comprehensive stability theory, the development of a stream can be predicted using empirical rules, and this is indicated in Fig. 1 by paths which by-pass this central topic. Why then is it necessary to undertake the stability analysis? Setting aside simple curiosity, perhaps a sufficient reason in itself, the justification lies in the difficulty which has been experienced in producing the empirical laws required to solve practical problems. So great are the difficulties of measurement and experiment that a supporting analytical structure is a vital adjunct in the planning and analysis of laboratory and field investigations. Even if the theory is wrong in detail, it serves to indicate which parameters are relevant and to suggest the magnitude of their contributions.

The situation that has most often been studied - and that to which our attention will be directed - is the development of erosion waves on an essentially plane bed over which flows an initially steady flow with a free surface. In this work the possibility that the entire channel be molded by erosion is set aside; thus we eliminate from the investigation the very largest of the bed features, those which alter the basic geometry of the stream. The body of analysis devoted to stream-bed stability will be discussed in terms of several themes running through it. These are defined by the available theoretical techniques and the simplifying artifices used in order that these techniques may be brought into play. At the end of the paper a few remarks will be made on recent studies of related topics; for example, detailed particle motions, bed forms and adjacent flows, meandering streams, and erosive processes in the marine environment. An important limitation of this survey should be noted: no serious attempt has been made to trace progress on erosion problems in the U.S.S.R., which in some ways parallels the developments mentioned here.

The final remark by way of introduction concerns the fundamental nature of the analytical models used to investigate bed stability. All the work to be considered assumes a sharp delineation between bed and fluid, the instability arising from an interaction between the two, subject to certain conservation and transfer laws. In this respect it differs from Liu's (1957) analysis, which concerned the stability of a continuous transition where the material behaved like a fluid of very high viscosity.

The Hydraulic Model

The first analysis of the stability of an erodible stream bed, undertaken by Exner (1925), used the one-dimensional flow model typical of classical hydraulics. Results obtained in this way are limited in application to processes which occur over streamwise distances many times greater than the flow depth: $kh << 1$, where $k = 2\pi/\lambda$ is the wave-number of a periodic bed wave, and h is the mean flow depth. On the other hand it is possible in the hydraulic model to take account of friction and non-linear accelerations, at least in a general way. Hence it is not surprising that even within the last decade some workers have adopted this model in investigating the behaviour of a fluid moving over an erodible surface.

We begin by mentioning two papers that do not deal explicitly with the stability of the bed, but discuss the phase shifts between a periodic bed wave and the shear stress and velocity variation within the fluid: Henderson (1962) and Raudkivi (1966). Essentially similar results were obtained (but not examined in detail) by Reynolds (1965), who went on to investigate the stability of the system, finding the combined effect of friction and bed slope to be of particular significance for near-critical flows ($F \simeq 1$).

In the work mentioned above, the quasi-steady approximation was adopted, the bed wave being assumed to move and develop so slowly that the fluid motion can be taken to be steady. Gradowczyk (1968) included non-steady terms in the hydraulic forms of the momentum and continuity equations, and thus obtained two 'dynamic' or surface-dominated waves in addition to the 'kinematic' wave dominated by the erosive processes at the bed. Special solutions were obtained for near-critical conditions, and it was concluded that stability could be achieved in such conditions (leading to a transition or flat-bed régime) without introducing the artifice of a phase shift between transport and velocity variation. However, the 'transition' obtained in this way is not extensive enough to represent the observed behaviour of erodible stream beds.

Another interesting result obtained by Gradowczyk (1968) was the prediction of instability of the surface waves at high Froude numbers, $F > 2$, approximately. This instability corresponds to roll waves in flow over a plane bed; it was supposed to correspond to the chutes-and-pools situation when bed waves are present.

The hydraulic model has usually been applied to situations in which the mean velocity vector is confined to a plane. Two attempts have been made to generalize the model to include 'three-dimensional' flows - strictly, these are two-dimensional in the hydraulic context where the vertical component of velocity is assumed to be zero for most purposes. Engelund and Hansen (1966) did this using a Boussinesq approximation, which improves on the simplest model by accounting for changes along the stream and thus extends the validity of the results to higher values of kh. This modification provides a phase shift which influences the stability, and an empirically determined 'delay distance' was also introduced into the analysis. Although this work has been introduced by referring to its discussion of three-dimensional fluid motions, the greater part is concerned with two-dimensional waves. The algebra involved in these discussions is complicated, and it is doubtful whether this kind of model (that is, the Boussinesq modification) will be of further interest. It lacks both the simplicity of the hydraulic pattern and the geometrical consistency of the models developed more recently.

The other three-dimensional hydraulic analysis is that of Callander (1969). Here secondary flows were neglected, the local bed stress and transport rate being assumed to have the same direction as the velocity vector, which is uniform from surface to bed in this model. Non-steady terms were included in the basic equations, as in the work of Gradowczyk, but their contributions were rejected in order that the bed wave could be studied in isolation.

Recently, Engelund and Fredsøe (1974) have used the hydraulic approximation in what is likely to be its permanently useful role - as a simple means of illustrating phenomena whose full discussion would require far more complicated techniques. They used this model to demonstrate the contrasting roles of the phase shift associated with losses within the flow, and of the lag associated with the settling of suspended particles.

Potential-flow Models

We shall severely limit the papers considered here, by relegating to other places those which apply the potential-flow model in more varied situations; for example, Kennedy (1964) dealing with ripples generated by the wind; Hampton (1972) considering turbidity currents, Barcilon and Lau (1973) dealing with transverse coastal bars, and Cartwright (1959) considering submarine sand waves.

Although the potential model was applied to bed-wave stability by Anderson (1953), it was Kennedy (1963) who first worked out its implications in detail. He sorted through a number of possible stability regimes defined by the mean-flow Froude number F, the relative wavelength kh, and the ratio $j = \delta / h$ of a lag distance δ to the mean depth of the flow. This lag distance is not tied very closely to actual transport processes, and therein lies both the strength and the weakness of this approach.

The most important result of the potential-flow analysis is the condition

$$kh\,F^2 = \tanh(kh) \qquad (1)$$

which is found to separate dunes and antidunes or, more generally, lower- and upper-range bed forms. This is a generalization of the critical condition $F = 1$ given by the hydraulic approximation, and recovered for $kh << 1$. A second predicition of Kennedy (1963) is

$$kh\,F^2 = 1 \qquad (2)$$

as the upper limit of (two-dimensional) antidunes, but this result did not prove to be in complete accord with the data he collected.

Reynolds (1965) revised the upper limit (2) to

$$kh\,F^2 = \coth(kh) \qquad (3)$$

a result in better agreement with measurements. This modified limit was adopted by Kennedy (1969) in making an even more elaborate investigation of the circumstances in which instability is possible. Reynolds also showed that waves corresponding to three-dimensional fluid motions - short-crested bed waves - lay beyond the limit (3), a fact that had been pointed out earlier by Kennedy.

The results he had obtained from the hydraulic model were generalized by

Gradowczyk (1970) by representing the flow using a velocity potential, rather than the hydraulic approximation. He showed that the 'kinematic' bed waves would develop and coallesce to form 'shocks' analogous to those in a compressible fluid. The forward faces of the waves steepen, and short waves overrun and are absorbed by longer waves.

A modification to the interpretation of the lag distance δ was introduced by Hayashi (1970), through the addition of a dependence of the local transport rate on the bed slope as well as the velocity, both reckoned at a point δ upstream. The outer limit of antidunes (3) is moved further out, and a flat-bed region appears even when $\delta = 0$. This work did show that the effects of gravity and perhaps of expansion losses are significant, though the technique used to account for them is not convincing.

One of the most useful results of these potential-flow analyses is the demonstration of the limitations of the hydraulic model. The results for finite values of kh are significantly different from those for $kh \to 0$; in particular, the critical condition is generalized to equation (1). However, the potential results are themselves severely limited: there is no bed shear stress, and the velocity variation with depth is unrealistic. These defects make it difficult to introduce more specific transport models to replace the hypothetical lag δ. Thus, while the potential-flow results (1, 3) outline the skeleton of the stability problem, they provide little guidance regarding its physical interpretation. They indicate what may happen, but not what will happen when account is taken of the additional constraints imposed by the actual mechanisms of sediment motion. For example, the linearized potential theory gives a singularity at the critical condition (1), from which we may infer that some interesting transition or series of transitions takes place in this vicinity. The nature of the transition becomes clear only if one can introduce a model of the sediment motion. Moreover, the existence of the singularity suggests that the model must be carefully constructed if meaningful predictions are to be obtained, and this proves to be the case.

These limitations were partially removed by Engelund and Fredsøe (1971). They introduced a particular mode of transport - suspension - thus limiting their results to the upper range where this contribution is important. For two-dimensional motion the consideration of a specific mechanism did not significantly alter the antidune limits (1,3) found by Reynolds (1965). But the critical condition (1) now emerges clearly as a stability boundary, and the results for short-crested bed waves differ from the potential-flow predictions; they indicate stability for long waves ($kh < 0.1$, say), presumably because $k\delta$ is small in such cases.

A more recent extension of the potential-flow analysis has been made by Shirasuna (1973). In some respects his work combines Liu's (1957) continuous model of the bed/flow transition with the more usual discontinuous model. Shirasuna respresents the interaction using a two-layer potential flow, the bed waves being internal waves at the interface between the layers. This procedure introduces an additional parameter into the theoretical model (the ratio of the mean velocities in the two layers); the generalization of the basic results of potential analysis is not unlike that achieved by Hayashi.

Rotational Models

The most important work in this area has been carried out at the Technical University of Denmark. Engelund (1970a) developed an analytical framework into which could be introduced the more important of the physical processes known to be relevant to bed stability. A stream function was used to specify the motion, the treatment thus being restricted to two-dimensional flows. With this single limitation, the model retained many of the advantages of the hydraulic models, accounting for friction and predicting bed shear, and of the potential-flow models, accounting for the role of the free surface for short waves. At the same time it allowed the introduction of specific representations of bed load and suspended load. The flow pattern was defined by a uniform eddy viscosity, with a slip velocity at the bed, and the distribution of suspended load was specified in a like manner.

When suspended load alone was considered, Engelund found that the upper-range instability (antidunes) was obtained, while the lower range remained stable. When bed load was introduced - that is, a component responding immediately to changes in the tractive stress - instability occurred in the lower range as well, taking the form of dunes. With coarse sediments the upper-range instability disappeared, but the antidune range was little affected with fine sediments, for which the bed load is less important. (It may be helpful to remind the reader that the term 'dunes' is commonly used for bed features whose variation in elevation is more-or-less out of phase with that of the surface above, while 'antidunes' are more-or-less in phase with the surface elevation. In a loss-free potential flow the phase relationships are exact.)

Smith (1970) adopted a flow model rather similar to Engelund's, but without a slip velocity at the bed, and with a restriction to low Froude numbers, at which only bed load is significant. He considered phase lags of the kind used by Kennedy (1963, 1969) and others, and paid particular attention to the tendency of the flow to separate in the lee of dunes and ripples.

Returning to the continuing investigations in Denmark, we note that Fredsøe (1974a) has extended Engelund's work in two respects. The inclusion of a component of submerged weight acting along the bed was found to have a significant influence on the bed-load variation and hence on the lower-range stability boundaries; in this work Fredsøe followed Hayashi (1970). Fredsøe also developed a non-linear model of the interaction, by expanding in terms of powers of a/h, the ratio of bed-wave amplitude to stream depth. While still limited to small values of this parameter, the theory does indicate the steepening of the downstream sides of dunes which is observed to occur in nature and is easily predicted using hydraulic models.

Finally, Engelund and Fredsøe (1974) have reviewed the position of the stability theory, pointing out that the subcritical régime requires an accurate specification of the phase shift between tractive stress and bed form, while the upper régime requires a knowledge of the phase shift between tractive stress and transport rate. Hence the transition region is strongly dependent on the ratio of bed load to suspended load. Consistent with these conclusions, they incorporated into the stability analysis a more precise way of determining the suspended load, one taking some account of the distribution of grain sizes within the bed material. The conditions under which dunes are formed are modified, but dunes are still found to be more likely when the sediment is coarse. The water temperature is found to have an important effect on stability, through the viscosity and fall velocity.

While bearing in mind these recent successes in stability analysis, we must remind ourselves that the results apply in the main to sinusoidal, long-crested bed waves. While these *may* be the bed forms that develop first, they are not very like the asymmetric, randomly placed, short-crested features that are usually found on stream beds. Nor has account been taken of the effect of the moving sediment on the mean velocity distribution, the effective fluid weight, and the fall velocity. It should not be difficult to incorporate corrections for these effects into the numerical procedures developed by Engelund and his co-workers. However, the basic geometrical constaints are much more intractable.

Phase Lags between Property Variations

The idea that the sediment load does not respond immediately to changes in velocity and tractive stress was perhaps first introduced by Bagnold (1941). It was later used, probably independently, by Cartwright (1959), Kennedy (1963) and Engelund and Hansen (1966). The phase lag or delay distance that has most often been considered is that between transport rate and velocity. This is not essentially different from the lag between transport rate and tractive stress, but is quite different from the phase shift between stress and bed form or between stress and velocity. Each of these two distinct kinds of phase shift has its own sphere of importance, and each acts as a stabilizing agent in the other's area of destabilization.

As has been noted already, Kennedy (1963) made extensive use of a lag distance δ. His most specific physical interpretation was in terms of the time taken for a sediment particle to settle to the bed once it had been displaced from it. On the other hand, when considering the conditions prevailing when the bed waves are fully developed, he argued that δ must take on particular values: 0 and λ, the wavelength, for dunes and antidunes, respectively. It is hard to reconcile these arguments with his earlier interpretation.

Later, Kennedy (1969) stated more clearly that the single distance δ combined the effect of a phase shift between bed form and tractive stress, and the effect of a relaxation time associated with sediment settling and less obvious adjustments of bed load.

Reynolds (1965) pointed out that the most obvious way of visualizing the lag δ is in terms of the phenomenon of saltation - in fact, more typical of particle motion in gas flows than in liquid flows - in which the particles are carried forward a fairly uniform distance having once been dislodged. He also distinguished between a lag distance independent of wavelength and the angular phase differences among the properties of a fluid passing over a bed wave. Kennedy (1969) took account of these two interpretations of δ, by carrying out stability analyses on the assumption that $k\delta$ was fixed and that δ was fixed.

A modification of the lag δ was made by Hayashi (1970). He introduced a dependence of bed load on bed slope, as well as velocity, in this way accounting in a general way for gravity and for other local factors that influence the phase of the transport relative to velocity and stress. When this is done, the lag δ is presumably a measure of the relaxation phenomenon alone. The role of gravity has since been taken into account in a more straightforward manner by Fredsøe (1974a).

Engelund and Hansen (1966) argued that δ was related to the average distance travelled by sediment particles, and also to the scales of the turbulence of the flow. They developed an empirical expression for the phase angle $k\delta$, as a function of mean-flow Froude number F. In later work, Engelund (1970a) did not make explicit use of the lag δ. Instead, he distinguished between: 1) bed load, the portion responding rapidly to changes in tractive stress, and 2) suspended load, for which the effective relaxation time was defined by a transport equation connecting upwards diffusion and settling under gravity. Thus the lag was no longer arbitrary, but was implicit in the laws describing the sediment motion. Engelund and Fredsøe (1974) followed Kennedy (1969) even more closely in distinguishing the phase shift between tractive stress and bed form (this is important for the lower-range instability associated with bed load) and the phase shift between tractive stress and transport rate (this is important for upper-range instability, associated with suspended load). Kennedy has in fact conceived of the two phase differences in a rather different way, arguing that dune formation was associated with suspended load, and only ripples with bed load. In this way he explained the joint occurrence of these two kinds of bed features.

Summarizing, we now have a good understanding of the lag δ, but no longer need to use it explicitly, since in recent models it is defined by the equations describing the sediment-transport mechanisms.

Sediment Transport Laws

Lacking more detailed information, students of erosion problems have invariably assumed that the laws governing overall transport could be used to predict local variations in the sediment-transport rate. There is little evidence either to refute or to support this assumption, and here we shall merely review the forms of the laws that have been adopted.

Kennedy (1963) took the transport to be proportional to an arbitrary power n of the velocity, the velocity of interest being that at the stream bed, but distance δ upstream. Hayashi (1970) followed Kennedy, but adopted a specific power ($n = 4$, implying a quadratic dependence on tractive stress), and introduced a measure of dependence on the local slope of the stream bed. Kennedy (1964, 1969) took the transport rate to be proportional to $(U - U_0)^n$, where U_0 is the velocity at which sediment motion is initiated.

Reynolds (1965) and Callander (1969) generalized Kennedy's approach slightly by considering an arbitrary function of velocity, and Gradowczyk (1968) went further by treating an arbitrary function of local velocity and depth. In doing this, he was guided by a consideration of formulae giving the sediment transport as proportional to $(\tau - \tau_0)^r$, where τ is the bed stress, τ_0 is the critical value at which motion begins, and r is an index (= 3/2 in the well-known formula of Meyer-Peter and Müller). Since $\tau = f(u, h)$, Gradowczyk took the transport rate to depend on both of these quantities.

In the work of Engelund and his co-workers, the bed load is reckoned using the Meyer-Peter formula. In recent work (Fredsøe, 1974a, Engelund and Fredsøe, 1974) a gravity term has been introduced into this formula to allow for variations in bed slope. The suspended load was at first related to the flow by an empirical result

$$\frac{q_s}{q} \propto \left(\frac{u_f}{w}\right)^4$$

with u_f the friction velocity and w the settling velocity. However, Engelund and Fredsøe (1974) adopted another method of determining the suspended load, based on a consideration of several fractions of small grains and the assumption that the total transport (suspended load plus bed load) is composed of material representative of the sediment as a whole. These changes in the sediment transfer laws have an important effect on the predictions of the stability analysis, as may be seen by comparing the results of Engelund (1970a) with those of Engelund and Fredsøe (1974).

Although the Meyer-Peter bed-load formula is that most commonly adopted in stability studies, Smith (1970) based his considerations on the formula proposed by Yalin; this also gives the transport rate as a function of actual tractive stress and the value for incipient motion.

Parameters defining the Bed Form

In the potential-flow models of Kennedy (1963, 1969) the kind of bed form that is established is conceived to be dependent upon F and $j = \delta / h$, the mean-flow Froude number and the ratio of lag distance to mean depth. The bed form is defined either as flat bed, or by a value of the parameter kh and a phase relationship between surface and bed waves. When this model was extended to three-dimensional motions by Reynolds (1965), Engelund and Hansen (1966) and Engelund and Fredsøe (1971), an additional parameter was necessary to determine the response, for example, k_2/k, the ratio of the wave-number components giving the cross-stream and streamwise periodicities.

Hayashi's (1970) modification of the concept of the lag distance gave rise to at second lag-specifying parameter

$$C = \alpha_s \frac{h}{\delta} F^2 = \frac{\alpha_s}{j F^2}$$

where α_s is the quantity specifying the influence of the local bed slope on the transport rate. The ratio $j = \delta / h$ can still play an independent part. As was noted earlier, Shirasuna's (1973) two-layer model introduces the ratio of the two mean velocities as an additional parameter, and this has a role like that of Hayashi's parameter C.

The hydraulic model of Reynolds (1965) throws up the parameter $k\,\delta$, an arbitrary measure of the lag, and

$$f = \frac{3\alpha}{kh\,(1 - F^2)}$$

which combines the roles of friction, Froude number and bed slope α. Henderson's (1962) and Raudkivi's (1966) analyses give results dependent on quantities essentially equivalent to the parameter f.

In the work of Engelund and his co-workers, the lag is implicit in the properties of the sediment and basic flow. Thus the bed configuration is dependent on F, U_a/u_f and u_f/w (or $u_f^2/(w F)$, a combination they have sometimes used), where U_a, u_f and w are the average velocity of the flow, the friction velocity, and the fall velocity of the sediment. In their most recent work (Engelund and Fredsøe 1974) the dispersion of size in the finer elements of the sediment is taken into account, and the temperature dependence of the fall velocity is acknowledged explicitly. The non-linear analysis of Fredsøe (1974a) also introduces some measure of dependence on a/h, the ratio of bed-wave amplitude to flow depth.

Not surprisingly, as the sophistication of the models has increased, the number of parameters required to define the bed form has also increased. What is more, these additional parameters have been found to have major effects on the predicted response. Examples are the significant roles of viscosity and sediment size - and the role of very fine suspended solids, the wash load, has not yet been investigated. This behaviour of the theoretical models is generally consistent with observations in the laboratory and in the field. The extensive experiments of Simons and his co-workers (reported, for example, in Simons et al. 1961, 1965 and Guy et al. 1966) have revealed a number of ways in which the bed-wave development depends on the detailed structure of the sediment. The distribution of particle sizes has a significant effect, and even the fine material which remains in suspension influences the larger elements through its modification of the apparent viscosity of the fluid. It seems unlikely that more thorough theoretical analysis will provide a simple, unified view of bed-wave development; it is more likely that further refinement will allow us to recognize the true variety of the natural phenomena.

Looking at the stream as a whole, we note that it has at least four characteristic dimensions:

1) breadth, which defines the scale of meanders and the largest bars,
2) depth, which defines the amplitudes and probably the other dimensions of dunes and antidunes,
3) boundary-layer thickness, which defines the scale of ripples, and
4) grain size(s), which defines the scale of certain small-scale striations and influences the development more generally through the fall velocity.

The linking of certain bed features to each of these scales has important implications for experimental technique, implying that small-scale experiments and experiments in relatively narrow, rigid-walled channels will not be representative of processes in larger natural channels.

Relationship to Developed Bed Forms

The simplest possible connection between the linearized stability theory and the ultimate form of the stream bed is a steady growth of nearly sinusoidal bed waves, without change of wavelength or crest shape until a limiting height is attained, perhaps defined by the water depth. Some such picture is implied by the plotting of data relating to fully developed beds together with curves given by the stability analysis; the plane F vs. kh has commonly been used to do this. In reality, observations show that the simplest model is incorrect in every respect, at least some of the time. A number of departures from the simple picture are noted below.

In the case of antidunes, growth is not steady, but intermittent, with surface-wave breaking obliterating the bed waves when they reach a certain height. Dunes and ripples do not retain a sinusoidal form, but rapidly develop steep downstream faces; however, antidunes do remain reasonably symmetrical. These observations give evidence of the dominance of bed load and suspended load in these two cases.

The characteristic 'wavelengths' of bed disturbances are seen to change - usually to lengthen - during development. Moreover, initially long-crested waves are observed to break up into a confused pattern of bed features, near which occur complex patterns of separating and reattaching flows. The maximum height is defined in a number of ways: by the mean water depth in the case of dunes, by surface-wave breaking for antidunes, and by the boundary-layer thickness for ripples. Finally, the initial disturbance does not always take the form of a uniform harmonic wave; rather, the developing wave field often propagates downstream into a flat-bed region. This suggests that the bed is in fact unstable to a disturbance of finite amplitude.

It has not proved possible to take all of these complications into account in relating the stability analysis to developed stream beds, but some useful ideas have been evolved in attempts to do so. Kennedy (1963) used the linear results themselves to investigate the conditions for maximum development, that is, for zero growth rate. He argued that the lag $\delta \rightarrow 0$ for fully developed antidunes, a rather unconvincing assumption since it is in this case that suspended load plays an important role. Later, Kennedy (1969) introduced the more reasonable proposal that $k\delta = \pi$ for maximum antidune development. In both cases he showed that the effect of gravity on particles moving up the forward face and down the rear face of the wave was consistent with his proposals, but again this does not account for the role of the suspended material.

For the case of dunes Kennedy (1963, 1969) proposed that $\delta = \lambda$, the dominant wavelength. He deduced this condition from the linear stability results, and was not able to specify the physical mechanisms with any precision. Arguing that the lag δ is increased by separation from the crests, he concluded that it would increase with amplitude, and thus justified the lengthening of the bed features. Kennedy (1963) also described the break-up of initially long-crested waves, as parts of the crests move more rapidly and ultimately break away. This behaviour might be ascribed to the development of cellular three-dimensional flows in the regions of separation in the lee of the crests.

The hydraulic model of bed-wave development used by Reynolds (1965) and Gradowczyk (1968, 1970) is able to predict the steepening of the downstream faces of dunes, a process that ultimately leads to the cascading of sediment down a slope defined reasonably well by the angle of repose of the material. Fredsøe's (1974a) more sophisticated non-linear analysis leads to the same conclusion. Gradowczyk (1970) also used kinematic-wave theory, as noted earlier, to describe the non-linear development, showing how faster-moving waves coallesce with the slower-moving. (In reality, it is observed that the smaller dunes are the more rapidly moving, since they require less sediment transfer to translate a given distance along the bed.) Gradowczyk interpreted the maximum-growth condition as that corresponding to the coincidence of crest and trough, as predicted by the kinematic-wave theory.

Smith (1970) emphasized the effect of the separation roller, and argued that the wavelength defined by the stability analysis would not persist, but would be replaced by one defined by the length of the separated flow, and therefore increasing with the bed-wave height.

Finally, Fredsøe (1974a) has pointed out that flow conditions change as the bed waves develop. Hence, assuming that the wavelength remains constant during development, one can determine the wave which would have developed in the undisturbed flow, and compare this with the observed bed forms. In essence, this argument makes the point that comparisons for fixed values of F and h are less relevant than those for fixed values of flow rate q and slope α. This argument suggests that comparisons on a plot of q^2k^3/g vs. α would be more appropriate than those in the F vs. kh plane.

Current Understanding

The best understood bed features are the least important - antidunes. These are of less importance because they are rather uncommon in nature and never appear under a flow lacking a free surface. We have a good understanding of the mechanism of instability - namely, the finite time required for the suspended sediment to adapt to changed conditions - and of the role of the free surface, both in providing the overall conditions for instability and in introducing an important mechanism of growth limitation. The appearance of antidunes corresponds quite well to the predictions of the linearized theories (Kennedy 1969) and the role of three-dimensional flows is understood in general terms (Reynolds 1965, Engelund and Fredsøe 1971).

A significant feature of antidunes is the adherence of the adjacent flow, even for quite large amplitudes, to the sinusoidal pattern assumed in the usual stability analyses. Dunes and ripples do not have this happy characteristic, the fluid motion downstream of each crest being dominated by separating and reattaching flows, in general of three-dimensional character. Although a good deal is known about these recirculating motions (see, for example, Raudkivi 1966 and Allen 1968) it has not been possible to introduce this information into the stability analysis in a convincing way. Moreover, the irregular character of these bed features is not closely approximated by the sinusoidal cross-stream variations which have been adopted in analytical models.

The general nature of the transition between dunes and antidunes is understood (Engelund and Fredsøe, 1974), but detailed predictions are not easy, since the stability boundaries are critically dependent on the balance between suspended load and bed load, among other factors. What is more, the nature of the bed in this region is often observed to vary in time.

It is widely conceded that ripples - dune-like features much smaller than the channel dimensions - are associated with the boundary layer on the perturbed channel bed, that their size is determined by the dimensions of the region of rapid velocity variation, and that the mechanism of instability presumably involves this variation. However, no analysis based on these ideas has been advanced. The case of wind ripples has been treated, it is true, but in the hydraulic environment the transport of particles is predominantly as bed load, unlike the aeolian situation where saltation is of great importance.

It is a privilege to have seen the subject of stream-bed stability develop within a period of little more than a decade. While we may hope that the expectation will prove to be incorrect, there is some reason to fear that progress will be less rapid in the future. The available mathematical techniques have revealed the general features of the interaction between flow and bed material. But it is apparent that our understanding decreases as the role of bed load becomes more dominant, because our insight into the interaction between the fluid and the particles on and near the bed is not sufficiently penetrating to allow the construction of simple, yet realistic models of the processes occurring there.

Peripheral Topics

The matters to be treated now are peripheral only in that they lie beyond the limited field on which our attention has been centred; each is of great interest in its own right. We consider first work done on topics that impinge directly on stability analysis (see Fig. 1) and then look even more briefly at subject areas further from the focus of attention of this paper.

Bed Forms and Adjacent Flows. A great deal of attention has been given to the fluid motions near the kinds of boundaries that develop spontaneously beneath a flowing stream. The most extensive investigations are those reported in Allen's (1968) monograph, but mention may also be made of the work of Robillard and Kennedy

(1967), Iwasa and Kennedy (1968), Allen (1969), Hsu and Kennedy (1971), Rifai and Smith (1971), Allen (1971), Mercer and Haque (1973), Ho and Gelhar (1973) and Fredsøe (1974c). Many observations have been made of actual bed waves and the flows near them: Raudkivi (1963), Znamenskaya (1963), Simons et al. (1965), Raichlen and Kennedy (1965), Nordin and Algert (1966), Brush et al. (1966), Guy et al. (1966), Hino (1968), Allen (1969), Hill et al. (1969), Chang and Simons (1970), Williams and Kemp (1971, 1972), Karcz (1972) and Engelund (1973).

Particle Motions. The mode of particle motion about which most is known is saltation, as is evidenced by the work of Bagnold (1941), Danel et al. (1953), Owen (1964) and Francis (1973). The details of other kinds of sediment motion have been considered by a few workers, among whom we may mention Owen (1960), Benedict et al. (1966), Einstein (1968), Raudkivi and Apperley (1969), Grass (1970), Engelund (1970b), Bagnold (1973) and Fernández Luque (1974). Others have considered the overall effects of sediment motion: Scheidegger (1961), Hino (1963), Chiu and McSparran (1966), Chiu (1967), Navntoft (1970), Hjelmfelt and Lenau (1970) and Chen (1971).

Meandering. The meandering and braiding of channels are particularly difficult to analyse, since gross alterations in channel form are involved. However, steady progress is being made, if only in revealing the true complexity of these problems, as is shown by the work of Leopold et al. (1964), Einstein and Shen (1964), Carlston (1965), Znamenskaya (1965), Toebes and Sooky (1967), Shen and Komura (1968), Callander (1969), Schumm (1969), Yalin (1971), Engelund and Skovgaard (1973) and Engelund (1974).

Allied Fields. A few notes will serve to suggest the areas which have been under active investigation over the past decade. The problems of wind ripples have been considered by Sharp (1963), Owen (1964), Kennedy (1964) and Kadib (1966), while the closely analogous problems of transport in closed channels have been discussed by Kennedy (1964), Owen (1969), Lysne (1969), Boothroyd (1971) and Fredsøe (1974b). Recently there have been a number of applications of ideas developed with reference to channel-bed stability to problems arising in the marine environment; for example, McCave (1971), Hampton (1972), Sonu (1973), and Barcilon and Lau (1973). A variety of geological applications are found in Pettijohn and Potter (1964), Middleton (1965) and Allen (1968), and more recently in papers such as Karcz (1972) and Hand et al. (1972).

There has been continuing interest in the practically important problems of predicting friction and sediment discharge in natural streams and in man-made channels. The level of activity can be indicated by mentioning some books published in the last few years that deal extensively with these problems: Leliavski (1959), Henderson (1966), Raudkivi (1966), Engelund and Hansen (1967), Blench (1969), Graf (1971) and Yalin (1972).

NOTATION

a	amplitude of bed wave, crest to mean level
C	parameter characterizing role of gravity in local transport
f	parameter characterizing role of friction and bed slope
F	Froude number, $U_a/(gh)^{1/2}$
h	mean depth of channel
j	ratio of lag distance to mean depth, δ/h
k	wave-number of bed disturbance
n	index in transport law
q	flow rate per unit breadth of channel
r	index in transport law
U_a	average or bulk velocity of flow in channel
u_f	fricton velocity, $(\tau/\rho)^{1/2}$ with ρ the mass density
w	fall velocity of sediment particles
α	mean slope of channel bed
δ	lag distance between two properties of the perturbed flow
λ	wavelength of bed disturbance
τ	shear stress at bed

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Characters indentifying major topics:

S	analysis of stability
A	allied field (that is, flow other than basically steady, unidirectional stream with a free surface)
B	bed forms and/or adjacent flows
G	more general work
M	meandering and braiding
P	particle motions

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