4 Hollow-ended rollers with bore reliefs in accordance with the "thick-rimmed" configuration operate with bore rim stresses 15 to 25 percent of the maximum combined orthogonal shear stress found near the roller surface.

5 L_{10} lives of hollow-ended rollers under severe misalignment are significantly greater than the lives of solid rollers under the same operating conditions.

References

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2 Goodelle, R. A., Derner, W. J., and Root, L. E., "A Practical Method For Determining Contact Stresses In Elastically Loaded Line Contacts," *ASLE Transactions*, Vol. 13, 1970, pp. 269–277.

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APPENDIX

Finite Element Method

The development of a three-dimensional model from that of a two-dimensional one is relatively straight forward. One of the earlier applications of the latter was the dimensional analysis of planar structures, essentially an approximation of two-dimensional elasticity theory [6]. In the "stiffness" or "displacement" formulation, the body lying in the X-Y plane is divided into many finite triangular elements. Within each element, the displacement is assumed to be a linear function of X and Y leading to a constant state of strain (and stress) in the element. Each edge remains straight during deformation. The "stiffness" matrix of such an element may then be calculated by appropriate superposing of all the element stiffness matrices. The elastic properties of the total structure is obtained, and the stresses and displacements corresponding to prescribed load and boundary conditions may then be calculated [7]. These processes are performed by a digital computer program.

Having considered a stress analysis in the X-Y plane, extension to axisymmetric loading of bodies of revolution is a natural extension. The X and Y coordinates are placed by cylindrical coordinates R and Z, Fig. 21, and the finite element becomes a torus of triangular cross section, i.e., a revolved "triangle" or triangular ring element, Fig. 22. As yet, this is still a two-dimensional problem since the nodes (vertices of triangular elements) are allowed to displace only in the radial (R) and axial (Z) directions, hence, the displacements are the same for any circumferential coordinate "θ."

This is not adequate to describe the deformation of a roller subjected to axially distributed line loads on opposite sides. However, an assumption is made that the nodes can also move in the circumferential direction and that the amplitude varies sinusoidally with the coordinate "θ." The three displacements, Fig. 23(a), at each node would be in the form:

$$u_\theta(\theta) = U_{\theta m} \sin m\theta$$

$$u_R(\theta) = U_{Rm} \cos m\theta$$

$$u_Z(\theta) = U_{Zm} \cos m\theta$$

where "m" is any integer and the sine or cosine terms are those appropriate to physical symmetry about the plane $\theta = 0$ deg. It can be shown that the solution for the "mth" harmonic is independent of each other harmonic, and that the stresses, Fig. 23(b), have the same behavior.

$$\sigma_\theta,\ \sigma_R,\ \sigma_Z,\ \tau_{RZ} \rightarrow \sigma_m \cos m\theta$$

$$\tau_{\theta R},\ \tau_{\theta Z} \rightarrow \sigma_m \sin m\theta$$

The reader unfamiliar with three-dimensional elasticity theory in cylindrical coordinates will find reference [8] helpful.

The various harmonics can be superposed so that solutions are obtainable for any loading condition expressed as a Fourier series in "θ." For example, the pressure at the outside diameter is a variable function of "θ" where:

$$p(\theta) = -\sigma_R(\theta) = \sum_{m=0,\,2,\,4}^{\infty} p_m \cos m\theta \quad \text{(psi)}$$

In the present case of roller loading, a line load at the O.D. varying with Z is considered. Since this load varies in the circumferential coordinate (symmetric about the $\theta = 0$ deg plane) and could be applied at any node, the foregoing procedure permits three-dimensional analysis of arbitrary bodies of revolution under the influence of a symmetric line load in the "θ" coordinate.

DISCUSSION

C. A. Moyer[3]

Mr. Derner and his colleagues pose an interesting solution to the line contact problem whenever misalignment or excessive loading causes nonuniform line contact stress to occur. There are numerous solutions to this problem; Kubinek [9][4] summarized these some time ago including the one considered by the authors. Kubinek further stated that the dynamic capacity of the recessed rollers derived both by theory and experiment matched that achieved with more conventionally crowned rollers. The problems of manufacture of the recessed ends, however, make roller modifications by either end taper or end contour combined with a center uncrowned portion or by a full curvature crown attractive from a practical standpoint.

Considering the range of profiles that can be put on a roller, from completely uncrowned to a full radius crown, it is common only to consider the extremes and the elastic deformation relationship for each, i.e., that the exponent for truncated line contact deformation is 1.0 and 1.5 for point contact [10]. The actual load-deformation relationship is important for fully understand-

[3] The Timken Co., Canton, Ohio.

[4] Numbers in brackets designate Additional References at end of discussion.

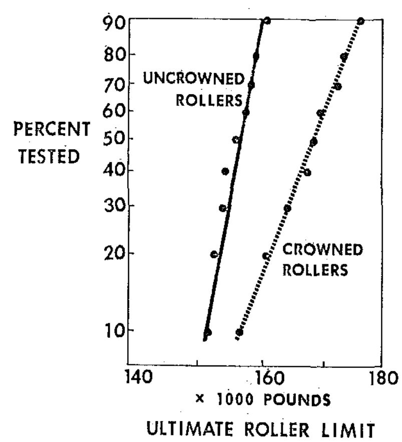

Fig. 24 Results of a test in which rollers were statically loaded until each roller reached its ultimate crushing limit, that is first evidence of cracking

ing load distribution among rollers in a radially loaded bearing. Have the authors studies included such a deformation investigation or an indirect indication of the internal load distribution by their footprint technique? It would also be of interest if their stress determinations for the hollow ended roller can be extrapolated into the plastic range, i.e., is there reason to believe the static limit for such rollers would be as high as a fully crowned conventional roller? Static tests on fully crowned rollers have shown that the favorable stresses within the contact region persist even to the ultimate limit so that their advantage over uncrowned rollers is as illustrated in Fig. 24 [11]. What behavior might be expected to the hollow ended roller as it approaches this ultimate limit?

Additional References

9 Kubinek, M., "New Design Trends in Antifriction Bearings," *Engineers Digest*, Vol. 24, No. 8, Aug. 1963, pp. 63-65.

10 Zantopulos, H., "The Effect of Misalignment on the Fatigue Life of Tapered Roller Bearings," ASME Paper No. 71-Lub-6 presented at ASLE-ASME Joint Lubrication Conference, Pittsburgh, Pa., Oct. 6, 1971.

11 Haager, P. L., and Moyer, C. A., "The Tapered Roller Bearing for Enclosed Gear Drive Applications," presented at the AGMA Semi-Annual Meeting, Chicago, Ill., Oct. 25, 1964.

Authors' Closure

The authors are appreciative of Mr. Moyer's thoughtful comments which as always, serve to stimulate a useful and rigorous discussion.

Particularly interesting is Moyer's reference to the work of Kubinek—which the authors had not been aware of—where it is stated that, "The dynamic load capacity achieved with such rollers (Edit. recessed ends) is the same as curved rollers (Edit. conventionally solid, crowned/rollers). . . . ." Kubinek goes on to say, ". . . but problems of manufacture limit the use of these rollers to special purpose bearings for the time being." Moyer infers agreement with Kubinek on these two statements of opinion thereby suggesting that no great advantage is achieved by employing hollow-ended rollers in bearings. The authors point out that fatigue test data show a substantial advantage in life for hollow-ended rollers in comparison to the solid type under conditions of heavy load and misalignment. More recent results of additional fatigue testing at our facilities, employing various radial bearing sizes, show a significant improvement in life at different alignments and load levels. These test results are fully supported by the finite element stress analysis as to failure mode, stress level, and location. Most important, the analysis forecasts a lower maximum subsurface shear stress level in the hollow-ended rollers than in the solid, which may be properly associated with an increased dynamic capacity.

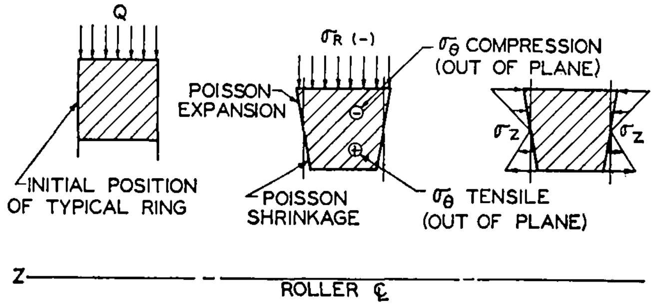

Fig. 25 Typical ring section of rim with resultant stresses

As to manufacturing the recessed ends, this is not the problem that it appeared to Kubinek. The author's company is currently mass producing hollow-ended rollers for production purposes.

Concerning Mr. Moyer's suggestion that it is common only to consider extremes of roller crowning (i.e., deflection exponents from 1.0 for truncated line contact and 1.5 for point contact), we appreciate his point but would suggest that this oversimplifies the real situation for modified line contact. As an example, consider the following roller-to-outer race contacts for a 1 × 1 in. roller in a bearing with a pitch diameter of approximately 8.33 in.:

(*A*) Uncrowned cylindrical roller against uncrowned cylindrical outer ID.

(*B*) Fully crowned roller (large radius of crown) against uncrowned cylindrical outer ID.

(*C*) Fully crowned roller (large rad.) against similarly crowned outer ID.

(*D*) Fully crowned roller (small radius of crown) against uncrowned cylindrical outer ID.

(*E*) Fully crowned roller (small rad.) against similarly crowned outer ID.

(*F*) Barrel-type roller (crown radius commensurate with spherical radius of outer) against spherically crowned outer ID., with:

(i) Line contact considered

(ii) Point contact considered

(*G*) 1 in. dia ball on same 8.33 in. pitch diameter against conventional outer ball race (54 percent curvature taken).

The following table summarizes the pertinent data governing comparison of the foregoing defined combinations:

Contact description	Relative conformity factor[a]	Theoretically determined Half-contact length, a	Half-contact width, b	Ratio a/b	Remarks
(*A*)	2.240	0.450	0.013	34.6	
(*B*)	2.233	0.450	0.013	34.6	
(*C*)	2.227	0.450	0.013	34.6	
(*D*)	2.179	0.450	0.013	34.6	
(*E*)	2.121	0.450	0.013	34.6	
(*F*)	[i] 2.235[b]	0.450	0.013	34.6	
	[ii] 83.8	0.579	0.014	41.4	Point contact not valid
(*G*)	9.62	0.141	0.028	5.0	Valid point contact

(a) For line contact—$C_L = \dfrac{4}{d\Sigma P}$

For point contact—$C_P = (\mu\nu)^3 \left(\dfrac{4}{d\Sigma\rho}\right)^2$

(b) Assuming 98 percent osculation

With respect to (F) [ii] in the foregoing, though point contact formulas were used to calculate a and b, we have to conclude that it was improper to do this in keeping with accepted criterion for line contact, as set forth by LUNDBERG-PALMGREN. The latter states that if the major axis of contact ellipse is approximately equal to roller length, then classical "point contact" is transformed into "line contact." All of the results, in the foreing table, (A) through (F) show that the load-deflection exponent should not vary and should be about 1.0 for any of the varied combination of: (i) uncrowned cylindrical roller; (ii) moderately crowned cylindrical roller on either an uncrowned or crowned race; (iii) heavily crowned cylindrical roller on either an uncrowned or crowned race; (iv) barrel roller on spherical race.

In addition, the authors have earlier investigated the practicality of using a higher value (than 1.1) for load-deflection exponent to explain observed individual roll-body loads on whole bearings.[5] This exponent study did not prove conclusive. Also, in this same referenced work, mention was made of increasing evidence that elastic deformation for mutually contacting roll bodies is less than that predicted from the theoretical work of Lundberg.[6]

Regarding Mr. Moyer's query about extrapolation of stress level for the hollow-ended roller, into the plastic realm, we have conducted very limited static fracturing load tests on this type of roller. Summarizing work completed on two different, medium size rollers (between 1/2 and 1 in. dia) the average roller fracturing load for the hollow-ended roller varied from 34 to 39 percent of the fracturing load for the same size solid roller.

The discusser has presented an interesting graph (Fig. 24) depicting the improvement in static fracturing load of fully crowned, (presumably) tapered rollers, over similar uncrowned rollers. Rough scaling of Fig. 24, indicates that:

$$\left[\frac{\text{Crowned}}{\text{Uncrowned}}\right]_{B\text{-50-Fract.}} = 1.07$$

$$\left[\frac{\text{Crowned}}{\text{Uncrowned}}\right]_{B\text{-10-Fract.}} = 1.03$$

The authors' company has similarly found an improvement in static fracturing load for partially crowned cylindrical rollers over the same size uncrowned rollers. In particular, examining such comparative tests we find that:

$$\left[\frac{\text{Crowned}}{\text{Uncrowned}}\right]_{B\text{-50-Fract.}} = 1.06$$

$$\left[\frac{\text{Crowned}}{\text{Uncrowned}}\right]_{B\text{-10-Fract.}} = 1.14$$

Interestingly enough, the authors' most recent work in static fracturing load has indicated that the most propitious crown for maximum static fracturing load, or ultimate limit, is shy of a full crown $\left(\text{viz. } \left[\frac{\text{crown length}}{\text{roller length}}\right] = 0.5\right)$, being rather with a ratio $\left[\frac{\text{crown length}}{\text{roller length}}\right] = 0.29$.

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[5] Goodelle, R. A., Derner, W. J., and Root, L. E., "Determinations of Static Load Distributions From Elastic Contacts in Rolling Element Bearings," *ASLE Transactions*, Vol. 14, No., 4, 1971, pp. 275–291.

[6] Lundberg, G., "Cylinder Compressed Between Two Plane Bodies," S.K.F. Reg. 413-4, S.K.F. Industries, Feb. 1949.