Collapse-Driven Supernovae: When Do They Explode?

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I will give an overview of the current status of our understanding on the mechanism of collapse-driven supernovae and related phenomena, presenting some of our group’s recent results. Included among the issues discussed in this article are (1) 1D simulations with the most detailed neutrino transport, (2) the global asymmetry of supernovae and its implications for the explosion mechanism, (3) hydrodynamical instabilities after the shock stagnation, and (4) gravitational collapse of more massive stars.

§1. Introduction

The mechanism of collapse-driven supernovae has eluded our understanding for more than 40 years.\textsuperscript{1,2} The collapse-driven supernova is an explosive phenomenon which is supposed to occur at the end of the evolution of massive stars ($\gtrsim 8 \, M_\odot$). The highly energetic explosion with a typical explosion energy $E_{\text{exp}} \sim 10^{51}$ erg is triggered by the gravitational collapse of a central core after it exceeds a certain critical density ($\sim 10^{9.5} \, \text{g/cm}^3$) or temperature ($\sim 10^{9.7} \, \text{K}$). The event is followed by the formation of a neutron star or a black hole. The energy source of explosion is the gravitational binding energy liberated by the formation of a compact object, typically a neutron star. This enormous energy ($E_{\text{NS}} \sim 10^{53}$ erg) which is about two orders of magnitude larger than the observed explosion energy is largely transported by neutrinos copiously emitted by a nascent neutron star.

The difficulty in the modelling of collapse-driven supernova arises from the fact that it involves a rich variety of physics. Microphysics such as weak interactions and nuclear physics is supposed to dictate the macroscopic dynamics in a critical way. In particular, the understanding of the equation of state for the hot and dense nuclear matter and neutrino interactions therein are crucial.

On the other hand, it has become a common sense that the collapse-drive supernova is in general not spherically symmetric. Various convections are expected to occur at different places and times, and some of them inside the supernova core might play a major role in producing an energetic explosion and some in the mantle and envelope mix up heavy elements synthesized explosively, which fact was supported observationally in SN1987A. Recently, the instability of the standing shock wave against non-spherical perturbations, the so-called SASI (Standing Accretion Shock Instability),\textsuperscript{3} is attracting great interest of researchers not only as an important factor in producing explosion but also as an agent to generate a proper velocity, or a kick, as observed for young pulsars.

The stellar rotation is another ingredient which renders the explosion asymmetric. Recent HST images of SN1987A clearly show the asymmetrically expanding ejecta, whose major axis is almost aligned with the rotation axis inferred from the
ring. The global asymmetry may be quite common to the collapse-driven supernovae as suggested by compiled polarimetric observations. The rotation may be an indispensable factor for gamma-ray bursts and accompanying supernovae. The interest in the role of magnetic field in the supernova is also growing partly because of the existence of highly magnetized neutron stars, the so-called magnetars, and also in the expectation that the magnetic fields play a crucial role in the central engine of gamma-ray bursts.

The collapse-driven supernova occupies an important position in astrophysics. One of the reasons is that it is a major contributor to the chemical evolutions of universe. The supernova distributes heavy elements which are synthesized not only in the hydrostatic phase prior to the explosion but also during the explosion itself, thus increasing the metalicity of galaxies. The collapse-driven supernova is also expected to be a promising r-process site, since ejecta are in general neutron rich. A strong shock wave generated in the supernova is, on the other hand, thought to produce high energy cosmic rays. And last but not the least, the supernova is a promising source of the gravitational radiation, since the collapse of massive stars is in general non-spherical as mentioned above.

The supernova explosion is supposed to occur for massive stars in the mass range of $\sim 10 \cdots \sim 30 M_\odot$. Nowadays more massive stars are attracting much interest of researchers from beyond the supernova community. This is almost entirely due to the recognition that long gamma-ray bursts are somehow caused by the gravitational collapse of massive stars and subsequent black hole formation. Simultaneous detections of sometimes hyper-kinetic type Ic supernovae, which is often referred to as hypernova by supernova researchers, are also a factor that cannot be ignored. It should be also mentioned that the interest in the first generation stars, or population III stars (POP III stars), is also growing rapidly these days thanks to the results of WMAP as well as other observations of old stars. It is generally supposed that not a small fraction of POP III stars had a very high mass of $\sim 100 \cdots \sim 1000 M_\odot$. Their nucleosynthetic yields as well as contributions to the diffuse backgrounds of photons, neutrinos and gravitational waves are an important probe for the investigation of the past universe. The future supernova research, thus, should be defined in the greater perspective of the gravitational collapse of massive stars.

In the following sections, I will focus on the particular issues mentioned above in turn. The importance of these issues is, hopefully, already clear from the short introduction given above.

§2. Spherically symmetric models

First I will briefly outline the temporal evolution of the collapse-driven supernova. It commences with the gravitational collapse of a white-dwarf-like core due to the reduction of pressure either by the electron capture or by the photodisintegration of irons. The collapse does not halt until the central density exceeds the nuclear saturation density and matter gains enough pressure to prevent further collapse. During this collapsing phase, neutrinos emitted by the electron capture are trapped inside the core after the density becomes larger than about $10^{12} \text{ g/cm}^3$ and
matter becomes opaque even for neutrinos. As a result, the electron capture ceases because of the Fermi-blocking for neutrinos. At this point, the electron fraction is somewhat larger than 0.3. Since the entropy of the core remains low ∼ \(1k_B\) and the electron fraction is this large, nuclei survive just up to the saturation density, where matter becomes uniform. Incidentally, the collapsing core is divided into two parts; the inner part shrinking subsonically and self-similarly referred to as the inner core, and the outer part falling supersonically called the outer core. Their masses are one of the important factors for the subsequent dynamics.

When the pressure gets sufficiently large due to the nuclear forces, the inner core bounces and the shock wave is generated at the boundary of the inner and outer cores. The inner core serves as a piston to launch a shock wave outwards through the outer core. When the shock wave somehow reaches the surface of the core, we regard the explosion as successful, since the loosely bound envelope is hardly an obstacle for the shock wave. As mentioned above, heavy elements are synthesized explosively as the shock wave passes through the envelope, and when it breaks out the progenitor’s surface, the familiar optical display of the supernova emerges.

In the above sequence of events, what is most uncertain is how the shock wave manages to propagate the outer core up to the surface of the core. In fact, almost all elaborate simulations done so far showed that the initial shock energy is not large enough to push the shock beyond the core surface and the shock wave stalls somewhere inside the outer core, turning to an accretion shock there. As the shock propagates through the outer core, it loses its energy due to the endothermic photodisintegration of nuclei and to the neutrino emission. Thus, it is easy to understand that the prompt explosion becomes more difficult as the mass of iron core or the proportion of the outer core is increased.

Note that the gravitational energy of ∼ \(10^{53}\) ergs liberated by the collapse is stored as internal energy at first and is unavailable on the dynamical time scale. As mentioned, this large energy is later carried away by neutrinos diffusing out of the core on a much longer time scale. We plot in Fig. 1 the profiles of the enclosed energy for 15 \(M_\odot\) model, that is the difference between the gravitational mass and the baryon mass up to a particular radius from the center. The left panel shows the density profiles for different times. It is noted that we induced a prompt explosion artificially by suppressing the electron captures. The upper- and lower-right panels correspond to the stages (10) and (14), respectively. The upper panel corresponds to the bounce when the shock wave with positive velocity just starts to show up and the lower panel shows the situation after the shock wave has passed through the iron core.

While the enclosed energy of the inner core becomes large at first due to the compression and amounts to \(1.1 \times 10^{52}\) erg at \(M_B = 0.87 M_\odot\) in the upper-right panel, it decreases, in the lower-right panel, to \(5.7 \times 10^{51}\) erg as it approaches the final hydrostatic configuration due to decompression (see also the left panel). The difference of these energies is the initial shock energy of \(5.6 \times 10^{51}\) erg. The shock loses its energy due to the dissociation of irons when it passes through the outer iron core. The dissociation of irons between \(M_B = 0.87 M_\odot\) (the inner core mass) and \(M_B = 1.14 M_\odot\) (the remnant mass) requires the energy of \(4.3 \times 10^{51}\) erg. Due to
this energy loss, the shock energy is reduced to $1.3 \times 10^{51}$ erg, which corresponds to the final explosion energy transferred to the ejected material. In realistic simulations with electron captures, the inner core mass will be smaller than the present case in which the electron capture and the neutrino interaction are artificially suppressed. If the inner core mass were smaller by $\sim 0.1 M_\odot$ than the present case, the shock wave would lose more energy ($\sim 1.6 \times 10^{51}$ erg, corresponding to $\sim 0.1 M_\odot$) during the propagation and, therefore, the prompt explosion should become more difficult.

It should be no surprise that the theoretical research of the collapse-driven supernova has been done mainly in the context of the so-called delayed explosion, in which a stalled shock wave is re-energized by neutrinos copiously emitted out of the proto-neutron star. As mentioned already, the gravitational energy of the proto-neutron star, $E_{\text{PNS}} \sim 10^{53}$ erg, that is liberated by the collapse of a stellar core, is mainly transported by these neutrinos. However, the difficulty stems from the microphysical fact that neutrinos interact with matter only through the weak interactions and the deposition of their enormous amount of energy to matter is rather inefficient.

It is known that there is a critical neutrino luminosity for the shock revival.\textsuperscript{14,15} Hence the issue is whether this critical value is realized in the evolution or not. The evaluation, however, requires a quantitative numerical modelling with all the relevant neutrino physics taken into account. Such sophisticated simulations have been done by a couple of groups over the years.\textsuperscript{16–19} Although the employed numerical techniques are different among them, different EOS’s are tried and various minor interactions and corrections have been explored, the results are consistently negative, that is, the neutrino heating is not high enough to produce successful explosions.

For example, Fig. 2 shows our latest results.\textsuperscript{19} We solved the fully general relativistic hydrodynamical equations and Boltzmann equations for neutrinos simultaneously. The evolutions were followed for more than a second after the core bounce for two realistic nuclear EOS’s. The left panel shows the mass trajectories for Shen’s EOS.\textsuperscript{20} As mentioned, there is no hint of shock revival. In fact, the shock stalls...
Fig. 2. The radial trajectories of mass elements for Shen EOS (left panel) and the radial positions of the shock wave (right panel) as a function of time after bounce for 15 $M_\odot$ model. In the left panel, the dashed line indicates the location of the shock wave. In the right panel, the thick and thin lines represent the results for Shen EOS and Lattimer & Swesty EOS, respectively.

at around 200 km from the center and becomes an accretion shock and then starts to recede onto the proto-neutron star later on. The evolution is not qualitatively different for the other EOS by Lattimer & Swesty\textsuperscript{21} and no explosion was found in this case, either.

The right panel shows the evolutions of the shock wave for the two models. The difference is indeed quite minor for the first 200 ms after the bounce, but becomes clearer later. As Lattimer & Swesty EOS we employed here is softer than Shen EOS in the high density regime, the proto-neutron star is more compact for the former, and so is the shock radius. The difference is also reflected in the neutrino signals. These results suggest that the later evolutionary phase is more suited for the investigation of hot dense matter in the supernova core. This is even more so for more massive stars as will be shown later.

§3. Multi-dimensional models

Various multi-dimensional aspects of the supernova dynamics have been studied extensively. Here I pay main attention to the global non-sphericity as observed in the recent HST images of SN1987A. Before doing so, some words on convections are worth mentioning. For the last decade, it has been demonstrated by numerical simulations that the vigorous convections are in operation both in the heating region and inside the proto neutron star. The former is induced by the negative entropy gradient produced by the neutrino heating while the negative gradient of the lepton distribution is responsible for the latter. The controversy on how important these convections are for the shock revival has seemed to be settled now. The recent multi-dimensional simulations with an improved treatment of neutrino transport have consistently presented a negative answer.\textsuperscript{13,22} It is now thought unlikely that the convection alone can give a sufficient boost for the shock revival. These days, however, other numerical instabilities are gaining attention. This will be discussed later.
3.1. Rapid rotation of supernova core

As repeatedly mentioned above, the recent HST images of SN1987A clearly show asymmetrically expanding ejecta whose major axis is almost aligned with the rotation axis that is inferred from the ring.\textsuperscript{4} It is thus apparent that the rotation is responsible for this global asymmetry one way or another. The most naive idea is that the rotation is rapid enough prior to the collapse to make the ensuing dynamics non-spherical. Magnetic fields may be also conspiring in this. This issue will be addressed in the next section.

A caution should be necessary here. We know eventually nothing of the angular momentum distribution inside the star prior to the collapse observationally. We know just the surface of massive stars are rapidly rotating on the main sequence, which actually poses a problem of angular momentum excess. Hence it is clear that the angular momentum is transported outwards during the stellar evolution. The issue is then how much. Although the theoretical modelling of the evolutions of rotating stars has shown much progress these days,\textsuperscript{23} the prediction on the final state is highly uncertain. In my opinion, we had better at the moment consider both scenarios of rapid and slow rotations. In this section, we first consider the rapid rotation case.

The rotation of the core can affect the dynamics of supernova in a couple of ways. First of all, a successful prompt explosion becomes even more unlikely. This is simply because the centrifugal force prevents the core from shrinking and the core bounce occurs at a lower density.\textsuperscript{24,25} We showed that the shock energy is monotonically decreased as the angular momentum of the core is increased or the core rotates more differentially.\textsuperscript{25} Hence, in the following, we consider the rotation based on the neutrino heating scenario.

If a collapsing star rotates, the accretion flow through the stalled accretion shock onto the proto-neutron star becomes non-spherical due to the centrifugal forces. On the other hand, neutrinos are emitted anisotropically. A simple geometrical argument suggests that the neutrino flux is enhanced in the direction of the rotation axis. Then the neutrino heating and, as a result, the shock revival will be also affected. In order to see these effects qualitatively, we\textsuperscript{26} extended the approach by Burrows and Goshy.\textsuperscript{15} The latter authors approximated the dynamics after the shock stagnation by spherically symmetric, steady accretion flows through the standing shock wave. Varying the values of neutrino luminosity and mass accretion rate, which they assumed constant in time, they found that for a given mass accretion rate there is a critical neutrino luminosity, above which there exists no steady solution. Using this fact, they argued that the revival of stalled shock occurs when the neutrino luminosity exceeds this critical value. We solved steady rotational accretion flows and observed how the critical luminosity is changed by the rotation.\textsuperscript{26} The anisotropy of neutrino flux is also taken into account here.\textsuperscript{27}

As shown in Fig. 3, flows are accelerated near the rotation axis whereas they are decelerated near the equatorial plane. As a result, the critical luminosity is lowered as shown in the right panel of Fig. 3. This clearly demonstrates that the rotation assists the revival of stalled shock. According to our calculations, the critical
luminosity is $\sim 25\%$ lower for the mass accretion rate of $1 \, M_\odot/\text{sec}$ and the rotation frequency of 0.1 Hz at a radius of 1000 km than that of the spherically symmetric flow with the same mass accretion rate. What is more, we found that the condition of the flow velocity at the critical luminosity is first satisfied at the rotation axis. This suggests that the shock revival is triggered on the rotation axis and a jet-like explosion is likely to ensue.

In the above models, we assumed that neutrinos are emitted isotropically, which is no doubt unnatural. In fact, the centrifugal force renders the central core oblate, which also implies that the neutrino sphere becomes flattened. Then it is expected that neutrinos are emitted more preferentially in the direction of the rotation axis, which was indeed confirmed by recent simulations.\textsuperscript{28, 29} We are naturally inclined to ask what is the influence of this anisotropic irradiation of neutrinos for the critical luminosity. Still preliminary results\textsuperscript{27} suggest that this also tends to reduce the critical luminosity. In fact, we found that the rotation and anisotropic neutrino irradiation work additively to lower the critical luminosity as shown in Fig. 4. In these calculations we assumed that the neutrino temperature is given by $T_\nu \propto (1 + a \cos^2 \theta)$, where $\theta$ is a polar angle and $a = 0.1, 0.3$.

We have to wait for realistic multi-dimensional simulations before we can conclude if these effects are crucially important for the shock revival or not. Kotake et al.,\textsuperscript{28} for example, did 2D simulations of rotational collapse, varying the initial angular momentum and its distribution systematically. Although the neutrino transport was simplified, they found that the neutrino sphere becomes oblate in general around the shock stagnation as shown in Fig. 5. More recently, Walder et al.\textsuperscript{29} did 2D simulations of rotational collapse with the so-called multi-group flux-limited diffusion approximation for the neutrino transport. More importantly, they
Fig. 4. The critical luminosities for the anisotropic neutrino irradiations. The left panel represents the non-rotating case and the right panel shows the results with both the rotation and neutrino-anisotropy taken into account. The rotation frequency is 0.1 Hz at a radius of 1000 km.

Fig. 5. The color-coded contour of entropy (left panel) and the shapes of neutrino sphere (right panel). The initial ratio of the rotational energy to gravitational energy is 0.5% and the uniform rotation is assumed for this model.

continued computations for about 200 ms after the bounce, that is, up to the period where the neutrino heating is most efficient. They confirmed that the neutrino flux is enhanced in the direction of the rotation axis by a factor of $\sim 2$ for the rapidly rotating case ($\sim 1$ rad/s prior to the collapse). It is incidentally mentioned that the authors did not claim a successful explosion in this paper although it is premature to say anything conclusive about the shock revival at this point, since the employed approximation tends to underestimate the anisotropy of the neutrino flux and the implementation of the neutrino transport is still incomplete.

The convection in the core is also affected by the rotation. Generally speaking, the rotation tends to stabilize the convection as long as the specific angular momentum increases as the distance from the rotation axis becomes larger, which is usually the case. This was actually shown for the convection in the proto-neutron star by
Keil et al.\textsuperscript{30} In their two-dimensional simulations, the rapid rotation suppressed the convective motion near the rotation axis, where the rotation is most rapid. If this is really true, the anisotropic neutrino heating discussed above might be also affected. Similar results were obtained by Fryer et al.\textsuperscript{31} in his SPH simulations. Since the result is dependent on the distribution of the angular momentum, it is evidently necessary to do more consistent and systematic numerical experiments on this issue.

3.2. Magnetic fields

For the last few years we have seen a remarkable rise of interest in the possible role of magnetic fields in the supernova mechanism.\textsuperscript{7,32–34} We can count a couple of reasons for that. For example, the existence of strongly magnetized neutron stars, the so-called magnetars, is one reason. In order for the magnetic field to have an influence on the dynamics, it must exert a stress that is comparable to matter pressure, at least, locally. This is understandably a very strong magnetic field, much beyond the canonical field strength for radio pulsars, that is, $B \sim 10^{12–13} \text{ G}$.

The anomalous X-ray pulsars (AXP’s) and soft-gamma-repeaters (SGR’s) are likely to have such a very strong magnetic fields. This suggests that magnetic fields may have played some roles in the supernova explosions that produced them. The origin of the strong magnetic fields is not clear. It is noted, however, that they may be a fossil field that existed in the progenitors. In fact, some Ap stars and magnetic white dwarfs have a magnetic field large enough to produce magnetar fields if they were just compressed to the neutron star size.

Under such an assumption, some numerical studies have been done on the magneto-rotational collapse. For example, Sawai et al.\textsuperscript{35} did 2D MHD simulations of the collapse from the rapidly rotating ($|T/W| \sim 1\%$) and strongly magnetized ($|E_m/W| \sim 0.5\%$) core, where $T$, $W$, $E_m$ are rotational, gravitational and magnetic energies, respectively. They considered different configurations of initial magnetic fields and also varied the rotation velocity and strength of magnetic fields systematically.

Figure 6 shows their representative results. The initial magnetic fields parallel to the rotation axis produced jet-like prompt explosions in the direction of the rotation axis irrespective of the distribution of field strength. So did the quadrupole-type fields. Note that the model with rotation but not magnetic field failed to produce an explosion, and hence that the magnetic field played a critical role in producing explosions. Interestingly, the model with a purely toroidal field at the beginning also failed. Hence the poloidal component of the magnetic field is crucially important.

In fact, the poloidal magnetic fields are dragged and wrapped up by differential rotations to produce strong toroidal fields, which in turn produce explosion and collimate the expanding flows to the rotation axis. Note that the differential rotation is most remarkable around the boundary of the inner core and the outer core, since they are contracting differently, the former homologously and the latter like a free fall. The magnetic stress becomes comparable to matter pressure locally around the boundary of the two cores near the rotation axis, the fact which also helps launch the jet-like explosion. This is a sort of resurrection of the prompt explosion which has been long deserted.
The above models are meant for the magnetars. The natural question, however, is what happens to much weaker fields prior to the collapse. In this case, as mentioned above, one has to find a way to amplify magnetic fields much more efficiently if the magnetic fields were to affect the dynamics. The magneto-rotational instability (MRI) may be the key. Originally found by Balbus & Hawley\cite{36} and mainly discussed in the context of accretion disk, MRI was also suggested to be important in the supernova dynamics by Wheeler and his company.\cite{37}

MRI is an instability that occurs in differential rotations of magnetized matter. Applying the criterion obtained in the linear analysis to the post bounce configuration of the supernova core, Akiyama et al.\cite{37} argued that the magnetic fields will grow exponentially to the saturation level that is large enough to affect the dynamics. Fryer et al.\cite{38} claimed otherwise based on his own rotational collapse models without magnetic fields. In the simulations for the initially strong magnetic fields mentioned above,\cite{35} the operation of MRI was not very apparent since the fields are already very strong from the beginning.

It is obviously necessary to do detailed MHD simulations for initially weak magnetic fields, since little is known of the saturations of MRI in the non-linear regime as well as of the linear growth in the accretion flows. Bisnovatyi-Kogan and his collaborators have published\cite{34} some results that claim the confirmation of MRI in the
post-bounce core. Provided their crude approximation to the microphysics and the initial set up of the models, I think that it is premature to say something conclusive on the importance of MRI. It is incidentally mentioned that the strong magnetic fields produced by MRI should be somehow diminished by the time when young pulsars are observed to have a canonical dipole field strength if MRI were to be a key ingredient for ordinary supernovae.

3.3. Hydrodynamical instabilities

Recently, Blondin et al. demonstrated numerically that a hydrodynamical instability other than convection may be operating to drive non-spherical motions in the flow below the shock wave. The so-called standing accretion shock instability (SASI) is supposed to be a non-local hydrodynamical instability possibly caused by the cycle of the inward advection of velocity- and entropy-perturbations and outward propagation of acoustic waves, with fluctuations amplified after each cycle (see their latest paper for another interpretation). This mechanism of SASI was originally studied in linear analysis by Ref. in the context of accreting black holes. Adding small non-spherical perturbations to the spherically symmetric, isentropic, steady, post-shock accretion flows, Blondin et al. found in their numerical simulations that the perturbations grow exponentially up to the nonlinear regime with clear dominance of $\ell = 1$ mode at first and $\ell = 2$ mode later, leading to the global deformation of the shock wave. Here $\ell$ stands for the azimuthal index of the Legendre polynomials. One lesson to learn is that we should not impose the symmetry with respect to the equatorial plane in the simulations.

As mentioned already, since the large deviation from the spherical symmetry may have an important consequence to the explosion itself, the finding of Blondin et al. has since attracted much interest of other researchers. In their first paper, the neutrino heating and cooling are entirely ignored and the flow is assumed to be isentropic. In the recent paper, the authors took into account the cooling term of a simple analytic form, but no heating included yet. On the other hand, Scheck et al. demonstrated that similar asymmetric motions with no equatorial symmetry occur in their most realistic numerical models. Although their results show that the neutrino processes will not nullify SASI, the growths and saturations of individual modes under neutrino-irradiation are not clear, since they used highly complicated flows as an underlying model.

In the following, I will discuss the basic feature of SASI based on the results of our 2D numerical experiments. As an underlying unperturbed model, we utilize the spherically symmetric, steady, shocked accretion flows, which is stable against radial perturbations. Although this is certainly a crude approximation to what we found in the realistic simulations, it will enable us to do clear mode-analyses from the linear growths through the nonlinear couplings among various modes up to the eventual saturation of SASI. We employ a realistic equation of state and take into account the heating and cooling of matter via neutrino emissions and absorptions on nucleons. Due to the neutrino-heating, some initial models have a convectively unstable region in the classical sense and SASI is inevitably mixed with convection. By lowering the neutrino luminosity, however, we can also construct models with no
convectively unstable region. By comparing these models, we can assess the relative strength of these instabilities. We also discuss the implications that SASI might have for the shock revival.

Figure 7 shows in the meridian section the distributions of entropy (the left half of the panel) and density (the right half) for the models with $L_\nu = 5.5 \times 10^{52}$ erg/s after 1% of the $\ell = 1$ single-mode velocity perturbation is added. For both models, we observe the growth of the perturbations. In the case of $L_\nu = 5.5 \times 10^{52}$ erg/s, the shock surface is deformed at first by the increasing amplitude of the non-radial mode and then begins to oscillate with a large amplitude. In the case of $L_\nu = 6.0 \times 10^{52}$ erg/s (right panels), on the other hand, in addition to the oscillations of the shock surface, we observe the substantial increase of the average shock radius as the time passes. In fact, after $t = 400$ ms, the shock radius continues to increase and appears to produce an explosion. Since the model is stable against radial perturbations as mentioned above, the non-radial instability and the neutrino heating therein are responsible for the explosion. We think that this is a reconfirmation of the claim that the instability, whatever the cause, behind the shock is helpful for the shock revival.

Next we discuss the models with the random multi-mode velocity perturbations. In so doing, we also study the influence of the existence of a negative entropy-gradient. In Fig. 8, the temporal evolution of the spectrum of the spherical harmonics. The spherically symmetric component, the $\ell = 0$ mode, is omitted in the figure. The cases without and with a negative entropy-gradient are shown in the left and right panels, respectively. It is obvious that the modes with small $\ell$, especially those with $\ell = 1, 2$, grow rapidly in the linear regime ($t \lesssim 100$ ms). This is particularly the case for the model without a negative entropy-gradient ($L_\nu = 3.0 \times 10^{52}$ erg/s) and the growths of the modes with $\ell > 10$ are negligibly small. With a negative entropy-gradient ($L_\nu = 5.5 \times 10^{52}$ erg/s), the broadening of spectra to larger $\ell$ modes is observed although the dominance of smaller $\ell$ modes can be still found. The convective instability may enhance the growth of higher harmonics in the linear phase. The similarity of the two cases suggest again that SASI is dominant over the convection even when the latter is operating.

Recently Foglizzo et al.\textsuperscript{43} discussed the linear stability for convection in the accretion flows in the supernova core. They found that the classical criterion for convection, that is, the negative entropy-gradient is not sufficient for the accretion flows, since the limited time is available for growth. Although the classical convection has greater linear-growth rates for modes with larger wave numbers, they claimed that there are minimum ($k_{\text{min}}$) and maximum ($k_{\text{max}}$) wave numbers for unstable modes in the accretion flows, and that the growth rates are also modified. The important parameter is the ratio of the advection time through the gain region divided by the local timescale of buoyancy, $\chi$, given in Eq. (40) in their paper. Applying their formula to our models, we obtain $\chi = 4 \ldots 7$ for $L_\nu = 5.5 \ldots 6.5 \times 10^{52}$ erg/s, with larger values for greater luminosities. Hence, the initial configurations are unstable against convection for these models, since the criterion, $\chi > 3$ is satisfied. The minimum and maximum wave numbers estimated in our models are $k_{\text{min}} = 2 \ldots 6 \times 10^{-8}$ cm$^{-1}$ and $k_{\text{max}} = 1 \times 10^{-6}$ cm$^{-1}$, respectively. The smaller $k_{\text{min}}$ corresponds to larger luminosities. They roughly correspond to the minimum and maximum indices in
Fig. 7. Entropy- (the left half of each panel) and density- (the right half) distributions in the meridian section for 1% of the $\ell = 1$ single-mode velocity perturbation. $L_\nu = 5.5 \cdot 10^{52}$ erg/s is assumed for the left panels and $L_\nu = 6.0 \cdot 10^{52}$ erg/s is for the right panels.

the spherical harmonics, $\ell_{\text{min}} = 2 \ldots 4$ and $\ell_{\text{max}} = 70 \ldots 85$, respectively. The lower $\ell_{\text{min}}$ and larger $\ell_{\text{max}}$ are obtained for higher luminosities. These numbers appear to be consistent with the spectrum shown in Fig. 8 as also inferred from Fig. 5 in their paper.\(^{43}\) Since the classical growth rate of convection is comparable to that of SASI in our models and the true growth rate of convection will be much smaller than the classical estimation, we think that SASI is a dominant driving force for the non-radial motions we observed so far.

It is also interesting to note that the modes with $\ell = 1, 2$ are dominant in the nonlinear regime, which begins after $\sim 100$ ms. As clearly seen in the broadening
Fig. 8. Temporal evolutions of the spectra in the spherical harmonics decomposition for the models with $L_\nu = 3.0 \cdot 10^{52}$ erg/s (left panel) and with $L_\nu = 5.5 \cdot 10^{52}$ erg/s (right panel). The random multi-mode velocity perturbations are initially imprinted.

of the spectra in Fig. 8, various modes are amplified by nonlinear couplings with the dominant modes in this phase. The spectra are again broader for the model with a negative entropy-gradient. In both models, however, the dominance of the modes with $\ell = 1, 2$ is remarkable. This should correspond to the large deformations of shock wave found in the numerical simulations.\textsuperscript{3,41} In order to make clear the reason for the dominance of these modes in the nonlinear regime, it will be required to study the nonlinear couplings of various modes in more detail.

Finally, we discuss the influence of the instability on the heating of matter by neutrinos. Figure 9 shows the distributions of the net heating rates in the meridian section for the models with $L_\nu = 5.5 \cdot 10^{52}$ erg/s (left panel) and $L_\nu = 6.0 \cdot 10^{52}$ erg/s (right panel). In both cases, the shock wave is asymmetric with respect to the equator, the characteristics for SASI. For $L_\nu = 5.5 \cdot 10^{52}$ erg/s, however, the average shock radius is only slightly larger than that in the unperturbed state. As a result, both the heating and cooling regions are not much changed during the period from 100 ms to 200 ms. They are just oscillating. In the case of $L_\nu = 6.0 \cdot 10^{52}$ erg/s, on the other hand, not only the shock radius but also the heating region is getting larger as the time passes. It is further seen that the cooling region at 200 ms is smaller than that at 100 ms. This is caused by the non-radial flows which carry down colder matter more efficiently than in the spherical flows. The temperature near the inner boundary is lowered as a result. Since the cooling rates are roughly proportional to $T^6$, the net cooling region is shrunk. This mechanism has been discussed in the context of convections over the years. It is applicable to SASI as it is. This broadening of the heating region and shrink of the cooling region thrusts the shock wave outwards further, the positive feed back which eventually leads to the explosion as mentioned already.

We have to wait for realistic simulations before we judge if SASI can give enough boost for the shock revival. The numerical results\textsuperscript{13} obtained so far are not very encouraging. If that is really the case, we had better find something more to obtain
§4. Gravitational collapse of more massive stars

So far I have discussed theoretical researches on the collapse of $\sim 10 \cdots \sim 30 \, M_\odot$ stars, which are supposed to produce a supernova explosion eventually. As mentioned already, however, the fate of more massive stars ($\sim 30 \cdots \sim 100 \, M_\odot$), which will produce a black hole one way or another, is also attracting much interest these days. This is mainly because these massive stars are expected to produce long gamma ray bursts. On the other hand, the interest in POP III stars naturally motivates the study on the fate of very massive stars in the mass range of $\sim 100 \cdots \sim 1000 \, M_\odot$. In the following, I will show some of our recent results on these subjects.\textsuperscript{44, 45}

4.1. Black-hole-producing collapses

Stars more massive than $\sim 30 \, M_\odot$ have large iron cores and will be intrinsically too massive to have stellar explosion. Then, the outcome will be a formation of black hole though detailed scenarios are not well established lacking self-consistent simulations for whole dynamics.\textsuperscript{46} A number of such black-hole-forming supernovae can be a substantial fraction (20–40%) of supernovae depending on the mass range and the initial mass function. The detection of neutrinos is a clear and unique identification of such events and it is important to quantitatively predict the neutrino signals from the black hole formation.

The main purpose of this section is, therefore, to present the neutrino signals from non-rotating black-hole-forming massive stars. To predict the time profile of neutrino burst and the energy spectrum of neutrinos during the evolution, we perform elaborated simulations of $\nu$-radiation hydrodynamics. We start the simulations from the beginning of gravitational core-collapse to find when and how the black hole is formed from a massive star. We follow the evolution of shock wave and central core
with matter accretion for a long time scale (\(\sim 1\) s), which has not been explored before. We perform simulations until the formation of black hole triggered by re-collapse of massive proto-neutron star beyond the maximum mass.

Another purpose of this study is to probe dense matter at extremely high density and temperature in black-hole-forming massive stars. In order to assess the properties of dense matter, we adopt two sets of realistic EOS, which have been used in recent studies of ordinary supernovae\(^{19}\) for the current numerical simulations. We make comparisons of the evolutions of central core from the initial collapse up to the black hole formation for the first time. In so doing, we pay attention to differences of neutrino signals hoping to probe the properties of dense matter from terrestrial neutrino detection in future.

In Fig. 10 we show the trajectories of mass shells for the two models. In model LS, where the EOS by Lattimer & Swesty\(^{21}\) (LS-EOS) is employed, the shock wave recedes quickly down to \(\sim 20\) km. The central core contracts fast while its mass increases toward the maximum mass of stable lepton-rich hot configurations. At \(t_{\text{pb}} = 0.56\) s, a dynamical collapse finally sets in and the central core shrinks abruptly. By this time, the enclosed baryon mass and gravitational mass below the shock wave in model LS amount to \(2.10 M_\odot\) and \(1.99 M_\odot\), respectively. Within \(\sim 8\) ms, the central core becomes compact enough and the apparent horizon is formed at \(\sim 5\) km, which marks the formation of a black hole. The lapse function, \(e^\phi\), and the general relativistic Lorentz factor, \(\Gamma\), decrease to \(\sim 0.4\) at around the apparent horizon. The rapid termination of neutrino emission will be slightly delayed until the apparent horizon engulfs the neutrino sphere. Further evolution needs extra cares of time-slicing to avoid singularities and we stopped the simulations at the formation of apparent horizon.

In model SH, where the EOS by Shen et al.\(^{20}\) (SH-EOS), on the other hand, the shock wave recedes rather slowly over \(\sim 1\) s. The dynamical collapse starts when the enclosed baryon mass reaches \(2.66 M_\odot\) (gravitational mass \(2.38 M_\odot\)) at \(t_{\text{pb}} = 1.34\) s. The increase of baryon mass in time, which is determined by the accretion rate, is similar in two models, therefore, the maximum mass solely determines the final moment of the black hole formation. The accretion rate in the case of \(40 M_\odot\) is higher (\(\sim 1\) \(M_\odot\)/s at \(t_{\text{pb}} = 0.4\) s) than in the case of \(15 M_\odot\) (\(\sim 0.2\) \(M_\odot\)/s) and this fact results in relatively fast contraction of central cores in the former.

The difference in the evolutions can be also seen in the neutrino signals. The average energies and luminosities of neutrinos are shown as a function of time \((t_{\text{pb}})\) in Fig. 11. The end points in the figure correspond to the formations of apparent horizon, i.e. the births of black hole. The true termination of neutrino emission will occur slightly later when the neutrino sphere is swallowed by the horizon and will not be observed for another \(\sim 20\) ms by the observer located at \(\sim 6000\) km, during which the neutrinos outside the neutrino sphere will traverse the distance at the light velocity. It should be emphasized that the different timing of black hole formation \((t_{\text{pb}} = 0.57\) s for model LS and \(t_{\text{pb}} = 1.35\) s for model SH) is reflected in the different timing of the termination of neutrino emissions and will provide us with the information on EOS.

The time profile of luminosities right after bounce is similar to the ones in
ordinary supernovae having the neutronization burst of $\nu_e$ and the rise of $\bar{\nu}_e$, $\nu_\mu/\tau$ and $\bar{\nu}_\mu/\tau$. Luminosities afterward are dominated by the contributions from the accreted material, which is heated up by the shock wave and is cooled by neutrino emissions. As the proto-neutron star contracts quasi-statically, the luminosities become higher since the gravitational binding energy increases. While $\nu_\mu/\tau$ and $\bar{\nu}_\mu/\tau$ are emitted from the deepest region and are affected little by the accretion, $\nu_e$ and $\bar{\nu}_e$ originate from the accreted material.

The difference of radial position of neutrino sphere (hence, different temperatures) leads to form the hierarchy of average energies. The average energy of $\nu_\mu/\tau$ and $\bar{\nu}_\mu/\tau$ clearly reflects the difference of temperature profile in two models, having higher average energy in model LS at $t_{pb} \sim 0.5$ s, for example, due to fast contraction. It is remarkable that the luminosities and average energies increase by a factor of
two or more toward the formation of black hole. This increase will be used as a
signal of black hole formation.

The properties of EOS at high density and temperature beyond the range in
ordinary supernovae are crucial to predict the neutrino signals from the black hole
formation. Appearance of exotic phases can occur at a certain critical density and
temperature. Our simulations will provide the upper limit of the duration of neutrino
emission before the formation of black hole. If a new phase appears at a certain
density or temperature point, the softening of EOS immediately leads to a dynamical
collapse toward the black hole. In other words, the timing of the black hole formation
will become a probe of the exotic phases. For example, if a critical density is just
above the density at bounce, the re-collapse occurs immediately and the neutrino
signal diminishes soon after the initial rise of neutrino burst.

The current generation of neutrino detectors will detect thousands of neutrino
events from the next galactic supernova and afford to detect the above-mentioned
signal of the formation of black hole. Simultaneous detections of different neutrino
flavors at the SuperKamiokande, the Sudberry Neutrino Observatory and other fa-
cilities will be decisive to draw a conclusion for the black hole formation from the
initial burst of \( \nu_e \) and the hardening spectra of \( \nu_\mu \) toward the termination of neutrino
emission. In order to see the dependence of our results, we are currently doing sys-
tematic studies as a function of progenitor mass and metallicity. It is also necessary
to study the outcome for rapidly rotating massive stars to consider the other types
of black hole formation and associated phenomena.

4.2. Collapses of very massive stars

Population III (Pop III) stars are the first stars in the universe. They do not
contain metals and their formation and evolution may be different from that of
stars of later generations. In fact, according to the theory of star formation,\(^\text{12}\)
Pop III stars might have very massive components (\( \sim 100–10000 \ M_\odot \)). Here, we
computed the spherically symmetric gravitational collapse of these Pop III massive
stars. We solved the general relativistic hydrodynamics and neutrino transfer equa-
tions simultaneously, treating neutrino reactions in detail. Unlike supermassive stars
(\( \gtrsim 10^5 \ M_\odot \)), the stars of concern in this paper become opaque to neutrinos. We in-
vestigated 18 models covering the mass range of \( 300–10^4 \ M_\odot \), making this study the
most detailed numerical exploration of spherical gravitational collapse of Pop III
massive stars. This will also serve as an important foundation for multi-dimensional
investigations.

The collapse was simulated until after an apparent horizon is formed. We con-
firmed that the neutrino transfer plays a crucial role in the dynamics of gravitationa
collapse, and found also that the \( \beta \)-equilibration leads to a somewhat unfamiliar evolu-
tion of electron fraction. Contrary to the naive expectation, the neutrino spectrum
does not become harder for more massive stars as shown in Fig. 12. This is mainly
because the neutrino cooling is more efficient and the outer core is more massive
as the stellar mass increases. Here the outer core is the outer part of the iron core
falling supersonically.

We also evaluated the flux of relic neutrino from Pop III massive stars. We
adopted the IMF of Pop III stars proposed by Nakamura & Umemura. Their IMF is bimodal and the heavier component is given as

\[
\begin{align*}
\frac{dn}{dm} &= Bm^{-\beta} & \text{for } m \geq M_{\text{min}}, \\
\frac{dn}{dm} &= 0 & \text{for } m < M_{\text{min}},
\end{align*}
\]  

(4.1)

where \( \beta > 1 \) and \( B > 0 \) are independent of \( m \). Incidentally, the IMF given by Salpeter has \( \beta = 1.35 \) for the mass range, \( 0.4 \, M_\odot < m < 10 \, M_\odot \). According to the standard \( \Lambda \)CDM cosmology, the cosmological parameters are given as \( \Omega_m = 0.3, \Omega_A = 0.7 \) and \( H_0 = 71^{+4}_{-3} \, \text{km/s/Mpc} \). The Pop III star formation efficiency, one of the most uncertain factors, is suggested to be rather large, of the order of 10%, from the estimations of the contribution of Pop III stars to the cosmic infrared background. Here we assumed it to be 0.1. As for the star formation history, \( \psi(z) \), we employ the following formula:

\[
\psi(z) = \frac{1}{5\sqrt{2\pi}} \exp\left(-\frac{(z - 17)^2}{20}\right) H_0 (1 + z) \sqrt{\Omega_m (1 + z)^3 + \Omega_A},
\]

(4.2)

motivated by the observation by WMAP suggesting that the reionization occurred at redshift \( z = 17 \pm 5 \).

The result is shown in Fig. 13. As expected, the detection of these neutrinos is difficult for the currently operating detectors. To put it more precisely, the relic \( \nu_e \) fluxes from Pop III massive stars are overwhelmed by solar \( \nu_e \) below 18 MeV and relic neutrinos by ordinary supernova above \( \sim 10 \) MeV. As for \( \bar{\nu}_e \), the emissions from nuclear reactors are the main obstacle below 10 MeV. Thus the existing detectors can not distinguish Pop III relic neutrinos from others. However, because the solar and reactor neutrinos are not isotropic, removing them is possible at least in principle. For \( \bar{\nu}_e \), in particular, Pop III massive stars are the largest cosmological sources. In the future, we may be able to discuss the Pop III star formation history with these diffuse neutrino fluxes.
§5. Summary

In this talk I briefly wrapped up the current status of our understanding on the collapse of massive stars. For the mass range of $\sim 10 \cdots \sim 30 M_\odot$, we expect the supernova explosion will result. As I explained, we are not yet able to claim what is the key element for successful explosions. Spherical symmetry seems to be too strong a restriction. On the other hand, convection, rotation, anisotropic neutrino emission and SASI are all shown to be helpful for shock revival in the neutrino heating mechanism. So far, however, numerical results have not supported that they are a sufficient boost for explosions. Probably we have to wait for more realistic multi-dimensional simulations to make a final judgement on this. In the mean time, other mechanisms than the neutrino heating are worth further investigations. For example, the magnetic fields may have a significant leverage for dynamics if MRI is really operating in the core and magnetic fields are amplified from pretty small seed fields. Very recently, on the other hand, Burrows and his company proposed a mechanism based on the energy deposition by sound waves induced by $g$-modes in the proto-neutron star.\textsuperscript{47} In this scenario, they claim, neutrinos play only a minor role. This is an interesting proposal and will be studied intensively in the years to come.

For more massive stars, we expect a black hole formation one way or another at the end of evolutions. We showed that the neutrino signals from the collapse of $40 M_\odot$ star, in which accretions continue after core bounce up to the black hole formation, have a unique feature distinguishable from those for ordinary supernovae, and can be used as a new probe for the investigation into the dense matter properties. Diffuse neutrinos from the collapse of very massive POP III stars, on the other hand, encode an important information of the star formation history and the initial mass function in the past universe. Hence not only supernova-producing but also black-hole-producing gravitational collapse will provide us with valuable informations. We are now required to consider the physics of gravitational collapse of massive stars in this wide perspective.
Collapse-Driven Supernovae

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