Gamma-Ray Bursts and New Physics

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Gamma-Ray Bursts (GRBs) are the brightest explosions observed. After a serendipitous discovery and thirty years of search we begin to understand their origin (type Ic supernovae for long bursts and neutron star mergers for short ones) and the relativistic outflow that is at the basis of their operation. While the research on the nature of GRBs still goes on we turn to application of GRBs to study other phenomena. A lot have been said about the potential usage of GRBs to determine the star formation and the conditions at the early universe. GRBs have an enormous potential to explore new regimes of ultra-relativistic flow and of very high magnetic fields. Here we discuss two other applications of GRBs that involve, in the spirit of this conference, new physics — identification of GRBs as sources of gravitational radiation and using the arrival times of photons at different energies to set limits on Lorentz invariance violation that could arise from quantum gravity effects.

§1. Introduction

Gamma-Ray bursts (GRBs) are short (lasting a fraction of a second up to several hundred seconds) and intense (fluences of $10^{-8}$ to $10^{-4}$ ergs/cm$^2$) bursts of low energy (typically a few hundred keV) gamma-rays arriving from random directions in the sky. GRBs were discovered accidentally by the Vela defense satellites in the late sixties but the discovery was announced only in 1973.\(^1\)

Our understanding of GRBs has undergone several revolutions since their serendipitous discovery in the late sixties. In the early nineties the detector BATSE on board of the Compton-GRO satellite discovered that GRBs are cosmological. BATSE also discovered that there are two clear subgroups of short (lasting less than 2 seconds) and long (lasting more than 2 seconds) GRBs.\(^2\) In 1997 the Italian-Dutch satellite discovered GRB afterglow — long-lasting multiwavelength emission that follows the gamma-ray emission.\(^3,4\) This long lasting emission enabled exact localization of the origin of these burst and determination of their redshifts and host galaxies. The association of GRB 980425 with the supernova SN 1998bw\(^5\) was the first indication that the two phenomena are related. Bumps in several afterglow light curves\(^6\) as well as the fact that GRBs seemed to take place within the star forming regions\(^7,10\) supported this association. The final confirmation for the association of long duration GRBs and SN Ib,c came along in 2003. A clear type Ic supernova signature\(^11,12\) emerged from the afterglow of GRB 030329, a strong nearby ($z = 0.16$) burst that was localized by HETE II.

Until recently afterglow was detected only from long GRBs. Short bursts remained as mysterious as ever. During the last few months Swift and HETE II discovered afterglow from several short bursts. This enabled the localization and determination of host galaxies and redshifts. The locations (in some cases in elliptical
galaxies) and the redshift distribution provides evidence (even though it is somewhat inconclusive yet) that short GRBs arise from neutron star mergers.

These observational developments were accompanied by theoretical developments. In many cases the theoretical predictions preceded the observations. Long before BATSE while most researchers working in the field believed that GRBs are Galactic a minority suggested that GRBs are cosmological.\textsuperscript{13,14} Some of the basic building blocks of the current Fireball model (built for cosmological sources) were already suggested in the mid eighties\textsuperscript{15,16} and early nineties.\textsuperscript{17,18} Similarly the neutron star merger model which is currently the leading model for short GRBs was suggested already in 1989.\textsuperscript{19} Once it was established that GRBs are cosmological in the early nineties the basic theory of the current fireball model was developed very quickly\textsuperscript{20,21} and these ideas have lead to the predictions of the (i) afterglow\textsuperscript{22–25} (ii) breaks in the light curves\textsuperscript{26,27} commonly interpreted as jet breaks, (iii) prompt accompanying optical emission\textsuperscript{28} and (iv) polarization.\textsuperscript{29–31}

It is impossible to review GRB observations and theory in this short review. There are several recent detailed reviews\textsuperscript{32–40} that summarize both aspects. The focus of this meeting is future developments in physics and in particular developments in gravitational physics. We will examine, therefore, how GRBs can help us in this aspect. After a short review of the current internal-external shocks model and of the recent Swift discoveries we discuss the usage of GRBs to explore open questions in gravitational physics. In particular we discuss the implications of the observations of short bursts to the estimates of the rate of neutron stars mergers and the detection of gravitational radiation. Then we discuss the applications of GRBs to quantum gravity and in particular we examine the possibility and optimal strategy for measuring violation of Lorentz invariance (which would be reflected in different time of flight for photons of different energies) using GRBs.

\section{The internal-external shocks model.}

During the nineties the internal external shocks model emerged as the way to explain the observation both of the prompt gamma-ray emission that arises from internal shocks within a relativistic outflow and of the subsequent late emission, the afterglow that arises from external shocks between this outflow and the surrounding matter.

Figure 1 depicts an overall view of the internal-external shocks model. A compact source whose size is $\sim 10^6$ cm emits a collimated ultra-relativistic jet with a Lorentz factor of at least 100. For a long duration GRB this might take place within a collapsing star, as suggested by the collapsar model\textsuperscript{41} and confirmed by the association of long duration GRBs with type Ic supernovae.\textsuperscript{5,6,11,12} In this case the relativistic jet has to punch a hole in the stellar envelope, whose radius is $\sim 10^{10}$ cm. This envelope is not depicted in this picture. Such an envelope does not exist in short bursts, if indeed they are produced in neutron star merger events.

It should be stressed that while this picture has a reasonable observational support (some new puzzles arose with recent Swift observations that we discuss later) it is not clear what is the acceleration and collimation mechanisms that drive the
relativistic jet and how is it powered. A leading idea is accretion onto a newly formed black hole.\textsuperscript{42} Such a system would arise both in rapidly rotating collapse\textsuperscript{41} or in neutron star mergers.\textsuperscript{19,43} Accretion may be combined with the Blandford-Znaek mechanism\textsuperscript{44} from a magnetized black hole — accretion disk system\textsuperscript{45,46} or with magnetic reconnection within the disk.\textsuperscript{43} An alternative idea is to tap the energy of a rapidly rotating highly magnetized pulsar.\textsuperscript{47–49}

The question what is the acceleration and collimation mechanism is intimately related to the debate on the nature of the outflow. In the original fireball model it was suggested that it is baryonic. Later several authors\textsuperscript{46–49} suggested that the relativistic outflow is a Poynting flux or a combination of Poynting flux and baryonic flow. The dissipation arises, in this case, due to magnetic field reconnection or some other instability in this flow. Some models suggest a combination of protons, neutron and magnetic flux.\textsuperscript{50,51}

The kinetic energy of the wind is partially dissipated via internal shocks that take place around $10^{13} - 10^{15}$ cm. These shocks accelerate electrons to ultra-relativistic energies (the typical Lorentz factor of an electron in the local frame is $\sim 1000$). The electrons emit the observed prompt $\gamma$-rays via synchrotron radiation. The needed
magnetic field is carried from the inner engine or is generated and amplified by the shocks.

At a distance of $\sim 10^{16}$ cm the loading of the circum-burst material on the outgoing flow becomes effective and the ejecta begins to slow down. At first a two shocks system forms. A forward shock propagating into the circum-burst matter, a reverse shock propagating into the ejecta and a contact discontinuity between the two. This is a short lived phase during which the early afterglow arises.

After the reverse shock crosses the ejecta a rarefaction wave passes and the ejecta cools down adiabatically. At this stage, at distances of $10^{16} - 10^{18}$ cm from the center, the forward shock collects more and more circum-burst material. It expands adiabatically and approaches the self-similar Blandford-McKee solution.\(^{52}\) This is the ultra-relativistic analog of the well-known Sedov-Taylor Newtonian blast wave solution.\(^{52}\) As more material accumulates, the shock slows down with the Lorentz factor decreasing like $R^{-3/2}$ and $t^{-3/8}$.\(^{52}\)

Sometime during this phase the Lorentz factor $\Gamma$, drops below $\theta^{-1}$ the opening angle of the jetted outflow. At this stage we encounter a jet break in the afterglow light curve. For $\Gamma > \theta^{-1}$ most of the emitted radiation is beamed within $\theta$ but for $\Gamma < \theta^{-1}$ some of the radiation is lost sideways. Additionally for $\Gamma < \theta^{-1}$ the outflow expands sideways and if the expansion is rapid enough the additional collected matter causes a faster slowing down.

Eventually, around $10^{18}$ cm the blast wave collects sufficient external material to slow it down so that it reaches the Newtonian transition. This takes place a few months to a year after the burst. At this stage practically all the radiation is in the radio frequencies. Later the blast wave approaches the Taylor-Sedov solution.

2.1. Recent developments

The fast response Swift satellite\(^{53}\) was launched last year. One of its main objective is to detect the early afterglow and to explore the transition from the prompt emission to the afterglow phase. This is done by rapid slewing of a X-ray (XRT) and optical-UV (UVOT) telescopes towards the bursts. By now we have several complete light curves for this early phase. Some of the early X-ray afterglows depict an unexpected pattern of a rapid decay ($t^{-3}$ or steeper).\(^{54\text{-}56}\) This is followed by a slower decay phase ($t^0 - t^{-1/2}$) and then by a “regular” afterglow phase with a decay of $t^{-1.2}$. At late stage a “jet break” and a steepening of the light curve are seen.\(^{57\text{-}58}\)

While there are possible interpretations of this behavior within the current internal-external shocks model\(^{56\text{-}59\text{-}61}\) the full implications of this behavior is not clear. It might indicate a new ingredient that we have missed so far or a need for a full revision of the theory.

Swift detections have also lead to the identification of prompt or almost prompt optical emission from several bursts. In most cases these signals were not as bright as the 9th magnitude flash observed from GRB 990123\(^{62}\) and upper limits are lower than earlier predictions.\(^{63}\) An exception is the optical flash from GRB 050904\(^{64}\) the most distant burst detected so far at a redshift of 6.29.\(^{65}\) Still the observed values are within revised theoretical predictions.\(^{66\text{-}67}\)
§3. Short GRBs and neutron star mergers

Until recently no afterglow was detected from any short burst and those remained as mysterious as ever. This situation has changed with the detection of X-ray afterglow from several short bursts by Swift\(^{68,70}\) and HETE II\(^{71}\). In some cases optical\(^{72-80}\) and radio\(^{75,81}\) afterglow was detected as well. This has led to identification of host galaxies and to redshift measurements. While the current sample is very small, several features emerge. Unlike long GRBs that take place in galaxies with young stellar population short bursts take place also in elliptical galaxies in which the stellar population is older. In this they behave like type I Supernovae which arise mostly in older stellar population that lags after the star formation rate.

The interpretation of a lag between the star formation and the short bursts rate is consistent with the observation that the redshift and peak (isotropic equivalent) luminosity distributions of four short bursts detected by Swift and HETE II (see Table I) indicate that the observed short burst population is significantly nearer than the observed long burst population. This feature expected\(^{82-85}\) as \(\langle V/V_{\text{max}} \rangle = 0.39 \pm 0.02\) of the BATSE short burst population is significantly larger (and closer to the Euclidian value of 0.5) than the one of long bursts \(\langle V/V_{\text{max}} \rangle = 0.29 \pm 0.01\).\(^{86}\)

It is clear\(^{87}\) that the observed redshift distribution is incompatible with a model in which the short bursts follow the star formation rate (see Fig. 2). A Kolmogorov-Smirnov (KS) test between a burst model that follows the SFR and the observed sample rules out this model at a level of more than 99%. Similar conclusions have been reached independently using a similar analysis by Nakar et al.,\(^{88}\) and Nakar, Gal-Yam and Fox.\(^{90}\)

A distribution that follows the SFR with a constant logarithmic delay distribution, case (ii), is more interesting. A priori this was our favorite distribution as a logarithmic distribution of separations between the two members of a NS-binary (or a BH-NS binary) would lead to a logarithmic distribution of time delays.\(^{91}\) However, this distribution is somewhat inconsistent with the data. A KS test suggests that the probability that the observed data arises from this distribution is only 2% (see also Ref. 88)). The observed bursts are nearer (lower redshift) than expected from this distribution. While it is possible that the real distribution of time delays is not logarithmically constant there are several other possible explanations for this result. First we turn to the data and realize that selection effects that determine which short burst is detected and localized with sufficient accuracy to allow redshift determination are not clear. It is possible that we are dealing with small number statistics. It is also possible that the afterglows of these four localized bursts are brighter than the afterglow of a typical more distant bursts and this has influenced the sample. A second possibility is that the current data is a good sample of the

Table I. The Swift/HETE II current sample of short bursts with a known redshift.

<table>
<thead>
<tr>
<th>GRB</th>
<th>z</th>
<th>(L_{\gamma,iso}/10^{51}) erg/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>050509b</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td>050709</td>
<td>0.16</td>
<td>1.1</td>
</tr>
<tr>
<td>050724</td>
<td>0.257</td>
<td>0.17</td>
</tr>
<tr>
<td>0508132</td>
<td>0.722</td>
<td>1.9</td>
</tr>
</tbody>
</table>
Fig. 2. A comparison between the expected integrated observed redshift distributions of short bursts for model (i)–(iv) and the distribution of known redshifts of short bursts. The different models are: (i) A rate that follows the SFR. (ii) A rate that follows the SFR with a distribution of time lags: $P(\log(\tau))d\log(\tau) \sim \text{const.}$ It is expected that this reflects the NS-NS merger rate (Piran 1992). (ii$\sigma$) Same as (ii) but with the parameters that are one sigma from the best fit ones. (iii) A rate that follows the SFR with a delay distribution $P(\tau)d\tau \sim \text{const.}$ (iv) A constant rate (which is independent of redshift).

short burst distribution but the “best fit” parameters estimated using the BATSE short burst population are, due to a statistical fluctuation, slightly offset. For example, we have considered in case (ii$\sigma$) a distribution whose typical luminosity, $L^*$, is one $\sigma$ away from the maximal likelihood value. This distribution is consistent with the BATSE short burst sample and it is not ruled out by the current sample of short bursts with a known redshift. The $p$ value of the KS test is 0.1.

As an example for the flexibility of the data we have considered two other time delay distributions. Case (iii) in which the time delay distribution is uniform. and case (iv) in which the overall short burst rate is constant in $z$. Both cases are compatible with the BATSE short burst distribution and with the sample of short bursts with a known redshift (The KS $p$ values are 0.5 and 0.2 respectively.). This result is not surprising. The BATSE peak flux distribution depends on two unknown functions, the rate and the luminosity function. There is enough freedom to choose one function (the rate) and fit for the other. The sample of short bursts with known redshifts peaks at a rather low redshift and both time lag distributions considered
above push the intrinsic short burst rate to lower redshifts (as compared to the SFR).

The models that fit both the BATSE’s observed peak flux distribution (ii) and the Swift and HETE II sample (iii)–(iv) correspond to a rate of $\sim 8–30 \times h_{70}^3 \text{Gpc}^{-3} \text{yr}^{-1}$ and to a “typical” luminosity, $L^*$, ranging from 0.1 to $0.7 \times 10^{51}$ erg/sec. Similar results are obtained by Ref. 90) who considered lognormal time delay distributions. They find that a best fit of a typical time lag between the short burst rate and the SFR of 6 Gyr and that a short time delay of less than 3 Gyr can be ruled out. Their minimal rate is similar to the one obtained by Ref. 87) $\sim 10 \times h_{70}^3 \text{Gpc}^{-3} \text{yr}^{-1}$. However, they consider also the possibility that the true rate is larger by a factor of $10^4$ (we consider this possibility shortly).

Provided that the basic model is correct and we are not mislead by statistical (small numbers), observational (selection effects and threshold estimates) or intrinsic (two short burst population) factors, we can proceed and compare the inferred short burst rate with the observationally inferred rate of NS-NS mergers in our galaxy.92), 93) This rate was recently reevaluated with the discovery of PSR J0737-3039 to be $80^{+200}_{-66}$/Myr.94) If we assume that this rate is typical and that the number density of galaxies is $\sim 10^{-2}/\text{Mpc}^3$, we find a merger rate of $800^{+2000}_{-660}/\text{Gpc}^3/\text{yr}$. Recently Berger et al.75) derived a beaming factor of 30–50 for short bursts. This rate implies a total merger rate of $\sim 240–1500/\text{Gpc}^3/\text{yr}$. The agreement between the completely different estimates is surprising and could be completely coincidental as both estimates are based on very few events.

So far we considered a luminosity function that is limited to a factor of 100 above $L^*$ and a factor of 30 below.85), 87) As the luminosity function decreases rapidly at large luminosities, the upper limit is unimportant. However, the luminosity function increases (with decreasing luminosities) in the low luminosity regime. Guetta and Piran85), 87) have chosen a lower cutoff of 30 as weaker bursts are practically undetectable by current detectors. Thus the regime $L < L^*/30$ is not constrained by current observations. While the majority of the bursts could be in this region they do not contribute significantly to the observed populations. This region should be considered as “terra incognita” and should be eliminated from current estimates of event rates.

Naker, Gal Yam and Fox90) followed the suggestion of Tanvir et al. (2005) that there exists a local population of weak short bursts and considered a significantly smaller lower limit. As the luminosity function increases at lower luminosities, this can lead to a very high local event rate. In fact taking $10^{47}$ ergs/sec as a lower limit to the luminosity function lead them to a local rate of $\sim 10^5 \times h_{70}^3 \text{Gpc}^{-3} \text{yr}^{-1}$ — a factor of $10^4$ above current estimates for the rate of NS mergers! An independent indication for the possible existence of two populations of short bursts arises from the BATSE peak flux distribution.87) Figure 3 depicts a deep at an intermediate-high peak flux level, at $P = 6$ photons/cm$^2$/sec — about 6 times the minimal detectable flux. There is no observational reason why BATSE should have missed bursts at this flux level. Still a marginally significant deep is there. Is it possible that this deep reflects a real phenomenon and that there are two populations of short bursts? One that gives rise to the highest fluxes (at a level of $\sim 10$ photons/cm$^2$/sec) and
Fig. 3. The observed $n(C_{\text{max}}/C_{\text{min}})$ distribution taken from the BATSE catalog vs the predicted differential distribution, $n(P/P_{\text{lim}})$, vs for the best fit models (i)–(iv) and (ii $\sigma$). Note the minimum of the observed distribution around a flux level of 6 photons/cm$^2$/sec.

another that gives rise to the low flux ones.

Short GRBs can be used not only to determine the rate of possible neutron star mergers but also to enhance their possible detection. A coincidence between a short GRB and a gravitational radiation signal could certainly enhance the significance of the detection.$^{95}$–$^{98}$ This would be beyond the mere improvement in the statistical significance. However, because of beaming a typical merger will be a factor of $50^{1/3} \sim 4$ nearer to us than a typical merger that is accompanied by an observed burst. Alternatively one can look for a statistical correlation between gravitational radiation signals and location of GRBs.$^{99}$, $^{100}$

§4. GRBs, time of flight and quantum gravity

Amelino-Camelia et al.$^{101}$ (see also Ref. 102)) and Sato (this volume) pointed out that a comparison of time of arrival of photons at different energies from a GRB could be used to measure (or obtain a limit on) possible Lorentz invariance violation that leads to deviations from a constant speed of light at high photons energies. During the last few years Ellis et al.$^{103}$, $^{104}$) and Boggs et al.$^{105}$ have used GRBs to set limits on the minimal energy at which Lorentz violation takes place. These limits range from $0.0009m_{\text{pl}}$ to $0.015m_{\text{pl}}$. We consider here, following Rodrigues-Martinez and Piran$^{106}$ how GRBs can be used to performing these tests and consider the limitations and the best strategy for such measurements
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(see also Refs. 107 and 108) for recent reviews).

Several quantum gravity theories lead to an energy dependent speed of light. This possibility arises in quantum gravity models, ranging from noncommutative geometry to canonical quantum gravity. For a review of these models, which involve a breakdown or a modification of Lorentz invariance at high energies, see Ref. 109). We consider, here, a simple generic modification of the standard dispersion relation:

\[ E^2 - p^2 \simeq \epsilon E^2 \left( \frac{E}{\xi m_{pl}} \right)^n, \]  

(4.1)

where \( m_{pl} \) is the Planck Energy, \( \epsilon = \pm 1 \) and we use unit with \( c = 1 \). One expects that such a simple dispersion relation would be valid to a leading order at low energies in many models in which there is a violation or deformation of the Lorentz group. This dispersion relation leads to a modified speed of light for a photon with energy \( E \):

\[ v = \frac{1}{\sqrt{1 + \epsilon (E/\xi m_{pl})^n}}, \]  

(4.2)

where \( \xi m_{pl} \) is the Lorentz violation energy scale. In the rest of this section we will be interested in setting a lower limit on \( \xi \). For a source at \( z \) the energy dependent velocity leads to an energy dependent time delay:

\[ \Delta t \simeq \epsilon \frac{1 + n}{2H_0} \left( \frac{E}{\xi m_{pl}} \right)^n \sqrt{\Omega_m (1 + z)^3 + \Omega_A} \int_0^z \frac{(1 + z)^n dz}{\sqrt{\Omega_m (1 + z)^3 + \Omega_A}}, \]

\[ \approx \epsilon \frac{1 + n}{2H_0} \frac{z}{\sqrt{\Omega_m + \Omega_A}} \left( \frac{E}{\xi m_{pl}} \right)^n, \]  

(4.3)

where the first expression takes into account momentum changes along the cosmic flight of the photon and the second one is a low redshift approximation. For \( \xi = 1 \) a typical time delay is 60 msec for a GeV photon arriving from \( z = 1 \). Figure 4 depicts lines of constant time delay for \( n = 1 \). Notice that unlike the addition of mass to the photon, that leads to larger delays at low energies, here the delay is larger at higher energies (where presumably quantum gravity effects maybe important).

As the time delay increases with energy and distance one would expect that we have to search for high energy photons from very distant sources. However, the situation is not so simple. For best detection we need that the time delay \( \Delta t \) is larger than the resolution of the detector \( \Delta t_{det} \). The latter, in turn, depends on the intrinsic resolution of the detector, \( \Delta t_{int}^{det} \), and on the arriving photon flux:

\[ \Delta t_{det}(E_1, E_2) = \max[\Delta t_{int}^{det}, \Delta t_{ph} \equiv b/A_{det} N(E_1, E_2)], \]  

(4.4)

where is the intrinsic time resolution of the detector and \( \Delta t_{ph} = b/A_{det} N(E_1, E_2) \) is the theoretical limit on the temporal resolution. This limit is simply determined by the rate of arrival of photons to the detector: \( A_{det} N(E_1, E_2) \), where \( A_{det} \) is the detector’s area and \( N(E_1, E_2) \) is the photon flux in the detector’s energy window \( (E_1, E_2) \). The numerical factor, \( b \), determines the number of photons needed to identify a peak and is of order ten or so for an optimal quiet detector and a highly variable signal.
Fig. 4. Curves of constant time delay for $n=1$.

At high energies, above a peak energy, $E_p$ (which is of order 100–200 keV), the typical GRB spectra behaves like $N(E) \propto E^{-\beta}$ (typically $\beta \sim 2.5$). Using this we can estimate the theoretical resolution for a burst with a luminosity $\mathcal{L}$ at a redshift $z$ as:

$$\Delta t_{\text{res}} = \frac{4\pi d(z)^2 b}{A_{\text{det}} (\beta - 2)} \frac{(1+z)^\beta}{\mathcal{L}} \frac{E_0^{\beta - 2}}{E_1^{1-\beta} - E_2^{1-\beta}}.$$  \hspace{1cm} (4.5)

The condition for a detection of an energy dependent time delay, $\Delta t > \Delta t_{\text{det}}$ can be translated now to a condition on $\xi$, that measures the energy (in units of Planck energy) in which Lorentz violation becomes of order unity:

$$\xi^n < \frac{A_{\text{det}} H_0}{8\pi c^2 b} \frac{\beta - 2}{\beta - 1} E_0^{\beta - 2} \mathcal{L}_{\text{peak}} \left[ 1 - \left( \frac{E_2}{E_1} \right)^{1-\beta} \right] \frac{E_1^{n+1-\beta}}{m_{\text{pl}}^n} \mathcal{G}_n(z),$$  \hspace{1cm} (4.6)

where

$$\mathcal{G}_n(z) \equiv (1 + n) \sqrt{\Omega_m(1+z_0)^3 + \Omega_\Lambda} \int_0^{z_0} \frac{(1+z)^n dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \int_0^z \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}^{-2} (1+z)^{-\beta}.$$  \hspace{1cm} (4.7)

The behavior of $\mathcal{G}_n(z)$ as a function of the redshift is shown in Fig. 5 for $n = 1, 2$.

Several features arise from Eq. (4.6). First the energy dependence, $E_1^{n+1-\beta}[1 - (E_2/E_1)^{1-\beta}]$, implies that for $n + 1 < \beta$ the best bound can be achieved at the lowest possible energy. Since a typical value of $\beta$ is $\sim 2.5$ (note that for $\beta < 2$ the total energy diverges at high energies) this is the relevant case for $n = 1$. A typical lower limit on the relevant energy is $E_p$, below which the behavior $N(E) \propto E^{-\beta}$ breaks down. Other considerations, like the intrinsic energy dependent timing of the peak flux\(^{112}\) may force us to push this energy higher. This result is sort of
counter intuitive as one would have expected that the best limit can be obtained from the highest possible energy, where the delay is largest. However, the decrease in number of photons at higher energies, is more important than the increase in the time delay. From an observational point of view this means that GRB detectors that are operating in the 100 keV to MeV range are optimal for this search. The situation will be different and high energy detection will be preferred (for \( n = 1 \)) if \( \beta < 2 \). However, this is an atypical situation when the peak energy (peak of \( \nu F_\nu \)) is not in the 100 keV range but much higher. While such bursts exist (see Band 1993) they are very rare.

For \( n \geq 2 \), the power \( n + 1 - \beta \) is usually positive (Unless we have an atypical GRB with no high energy component.). In this case we should consider high energy detectors. However, here another limitation arises as at energies higher than \( \sim 10 \) GeV high energy photons are attenuated due to pair production with the IR background.

Intuitively one would expect that the best sources are the most distant ones. However an inspection of the \( z \) dependence of Eq. (4.6), which is included in the factor \( G_n(z) \) (see Fig. 5) shows that \( G_n(z) \) is maximal at low redshifts. The function \( G_n(z) \) peaks at the point where \( \Delta t_{\text{det}}^{\text{int}} = \Delta t_{\text{ph}} \). The flux increases when \( z \) decreases. As the subsequent improvement in the temporal resolution compensates over the smaller time delay, it is advantageous to look for low \( z \) bursts (which have a high flux simply because of geometric effects). Clearly, there is no point in decreasing \( \Delta t_{\text{ph}} \) below \( \delta t_{\text{det}}^{\text{int}} \). Hence the optimal value is obtained when \( \Delta t_{\text{det}}^{\text{int}} = \Delta t_{\text{ph}} \). As \( G_n \) depends only weakly on \( \beta \), this conclusion is quite general. Thus, we conclude that the best bursts for checking time delay are rather close ones. These are strong bursts located at \( z \sim 0.5 \). This will be important when we simulate\(^{106} \) the possibility to improve on current limits.

We rewrite Eq. (4.6) as:

\[
\xi > \sigma \left( \frac{L_{\text{peak}}}{L_*} \right)^{1/n} G_n^{1/n}(z),
\]  

(4.8)
where we have introduced the typical luminosity\textsuperscript{86}\( L_\ast = 6.3 \cdot 10^{51}\) erg/sec. The numerical constant \( \sigma \) is:

\[
\sigma = \left[ \frac{A_{\text{det}} H_0}{8 \pi m_{\text{Pl}}^n c^2 b} \left( \frac{\beta - 2}{\beta - 1} \right) L_\ast^{\beta - 2} E_1^{n+1-\beta} \left( 1 - \left( \frac{E_2}{E_1} \right)^{1-\beta} \right) \right]^{1/n} 
\]

\[
\approx 10^{(23-\beta)/n-22} \left( \frac{1}{3} \frac{\beta - 2}{\beta - 1} \right)^{1/n} 
\]

\[
\times \left( \frac{A_{\text{det}}}{2000 \text{ cm}^2} \right)^{1/n} \left( \frac{E_1}{\text{ MeV}} \right)^{1+1/n} \left( 1 - \left( \frac{E_2}{E_1} \right)^{1-\beta} \right)^{1/n}.
\]

(4.9)

For \( n = 1 \) we can expect limits of 0.1 from a typical but nearby burst. The limit can be ten time larger, that is, at a level of \( \xi = 1 \) (corresponding to a Lorentz violation taking place at \( E = m_{\text{Pl}} \)) from a strong and nearby burst. The limits for \( n = 2 \) should be around \( 10^{-12} \). It should be stressed that these limits are theoretical ones, namely the highest bound which could be set, were the best conditions achieved. The order of magnitude for typical bursts obtained in Eq. (4.8) agrees with actual limits found for specific bursts in the literature. Ellis et al.\textsuperscript{103} used a wavelet analysis to look for correlations between redshift and spectral time lags between the arrival times of flares at different energies, and obtained the bounds of \( \xi_1 > 5.6 \cdot 10^{-4} \) and \( \xi_2 > 2.4 \cdot 10^{-13} \) at a 95% of confidence level. The same authors improved later the limit, using a better statistical analysis, to \( \xi_1 > 0.9 \cdot 10^{-3} \). Boggs et al.\textsuperscript{105} used a single and extremely bright burst, GRB 021206, finding the bounds of \( \xi_1 > 0.015 \) and \( \xi_2 > 4.5 \cdot 10^{-12} \).

It is worthwhile to discuss this last limit, which is the strongest limit available so far. It was obtained for GRB 021206 that was observed by RHESSI at six energy bands spanning 0.2–17 MeV. The redshift of this burst is not known, but Boggs et al.\textsuperscript{105} estimated an approximated redshift \( z \approx 0.3 \) from the spectral and temporal properties. The observed flux of GRB 021206 was \( 1.6 \cdot 10^{-4} \) ergs/cm\(^2\) at the energy range of 25–100 keV.\textsuperscript{111} This puts GRB 021206 as one of the most powerful bursts ever detected. GRB 021206 showed also a very atypical photon spectrum at MeV. Instead of decreasing following a power law with \( \beta \approx -2.5 \), the photon flux is almost flat from 1 MeV up to 17 MeV (this implies that \( F_\nu \) increases with \( \nu \) in these energies). This flatness allowed\textsuperscript{105} to resolve a fast flare and to determine its peak time and uncertainty in several bands. For \( n = 1 \) and \( \beta > 2 \) it is advantageous to observe at low energies. However if \( \beta < 2 \), like in GRB 021206 in the MeV range, this conclusion does not hold and it is preferable to use the highest available energy band. Hence the higher limit!

A serious problem that arises in this procedure is the apparent intrinsic lack of simultaneity in the emission in different energy bands. Soft emission has an intrinsic time delay relative to high energy emission.\textsuperscript{112} While the reason for this phenomenon is not understood, in Ref. 107) an anti-correlation between the spectral evolution timescale and the peak luminosities has been found. One can try to decrease the intrinsic delay by choosing very luminous bursts and observing in MeV or higher, where the delay, if still exists, seems to be smaller. Alternatively Ellis et al.,\textsuperscript{104}
uses the fact that the delays produced by a violation of Lorentz symmetry increase with the redshift of the source, whereas intrinsic time delays are independent of the redshift. Using statistical techniques on a sample of 35 bursts with known redshifts, they establish a lower limit of $\xi_1 > 0.0009$.

We can turn now and ask what is the probability to improve on the current limit. It is clear that for $n = 1$ we need a strong nearby burst detected in two separate energy channels. One can\cite{106} compute, using a model for burst distribution and luminosity of GRBs\cite{86} the rate of such bursts. For $n = 1$ we\cite{106} find that it is difficult to improve significantly on the present best limits. It is likely to detect, within current (Swift) or even future (GLAST) detectors a burst that could yield a limit of $\xi = 0.1$ as compared with $\xi > 0.015$ from GRB 021206.\cite{105} However it is unlikely to reach $\xi = 1$. Bursts that are both powerful enough and nearby enough are quite rare. For $n = 2$ detection at high energies is advantageous. Here it is likely that GLAST whose main detector detects photons in the 25 MeV to 10 GeV range will improve the current limits of $\xi > 4.5 \times 10^{-12}$ significantly.

Acknowledgements

This research was supported by the US-Israel BSF and by the EU RTN “GRBs - Enigma and a Tool”.

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