Nordic Hydrology, 8, 1977, 163-170

Routing of Floods in River Channels

Mohammad Akram Gill
Ahmadu Bello University, Zaria, Nigeria

An objective method of determining the constants of the Muskingum type linear equation is described. In situations where the relationship between storage and the weighted flow is nonlinear, an approximate method of analysis is proposed. These methods are explained by solving typical examples.

Introduction

Flood routing is a problem of great importance particularly in view of increasing urbanisation near river channels. In the unobstructed river channels, storage characteristics of the flood plain strongly influence the flood behaviour. The storage characteristics in turn depend on the detailed geometry of the flood plain. In many cases, the detailed topographical maps of the flood plain are not available so that the storage characteristics can not be directly determined. In such cases, it is attempted to establish a certain empirical relationship between the storage within the length of the river in which routing is to be performed, and the weighted flow determined from the inflow and outflow records. Such an empirical relationship is then used with the continuity equation to rout future floods.

In mathematical terms, the continuity equation is expressed as follows.

$$I - O = \frac{dS}{dt} \qquad (1)$$

where I is the rate of inflow into the channel reach at its upstream end, O is the outflow from the channel reach at its downstream end and dS/dt is the rate of change of the wedge storage with respect to time. In finite terms, Eq. (1) is written as

$$\frac{1}{2}(I_1+I_2) - \frac{1}{2}(O_1+O_2) = \frac{\Delta s}{\Delta t} = \frac{S_2-S_1}{\Delta t} \qquad (2)$$

where S_1 is the storage at the beginning of the time interval Δt and S_2 the storage at the end of the interval. Since the variation of I and O is generally not expressible in the form of algebraic equations, Eq. (2) is used instead of Eq. (1) for flood routing. The flood routing can be accomplished if S_1 and S_2 can be determined in relation to I and O. It has been found that storage at any given instant of time is function of both I and O. Such a functional relationship is commonly expressed as (Linsley et al. 1949)

$$S = \frac{b}{a}\left[xI^{m/n} + (1-x)\,O^{m/n}\right] \qquad (3)$$

where the constants a and n are measures of the stage discharge relations at the two ends of the channel reach and b and m are constants determined from the mean stage - storage relationship. The constant x is the weight factor. For purpose of flood routing in many practical situations, m/n is generally assumed to be unity whereby Eq. (3) is simplified to

$$S = K\left[xI + (1-x)\,O\right] \qquad (4)$$

It may however be mentioned here that m/n is not necessarily always equal to 1; where m/n is appreciably different from 1, use of Eq. (4) for routing may introduce large errors. Eq. (4) expresses a linear relationship between the wedge storage and the weighted flow (xI + (1-x)O). The values of K and x are determined using past records of I and O. These are then assumed to be the characteristic values for the channel reach in future flood routing provided the channel reach is not appreciably altered due to natural events or by man made structures in the mean time.

The procedure of determining the value of x which is commonly used is by trial and error. Assume a value of x, calculate the values of $xI + (1-x)\,O$ and plot them against the corresponding values of S. The value of x for which the width of the loop is reduced to minimum. Fig. 1, is taken as the correct value. Although using this method, a reasonable value of x may be determined in two or three trials, the method is subject to criticism due to its tentativeness. For instance, commenting on this trial and error method, Kulandiswamy et al. (1967) observed, »the values of x and K .... are determined by trial and error and for the same reach, two investigators may not get the same values for a given flood movement.« A better and a wholly objective method of determining x and K is by using the method of least squares which is explained here.

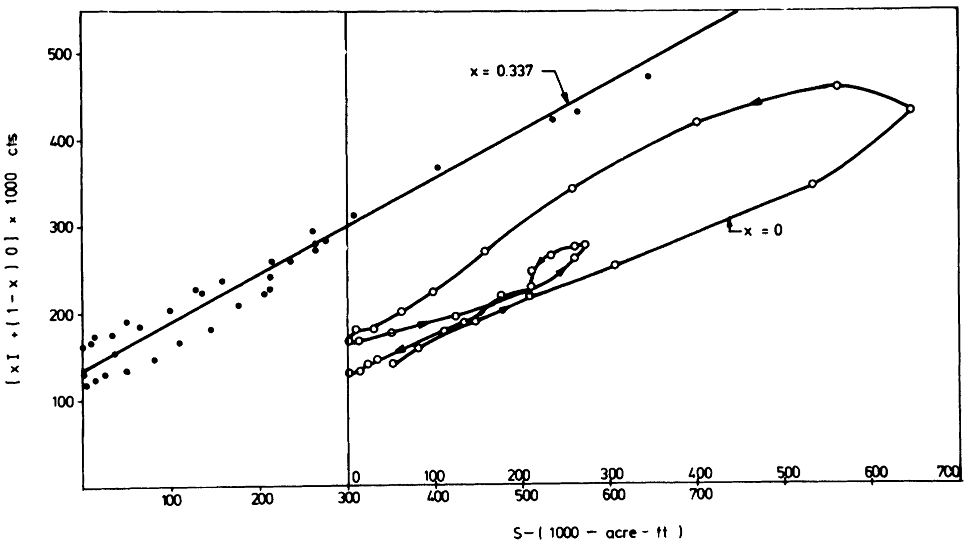

Fig. 1. Storage - flow relationship for the Sewickley - Wheeling reach of the Ohio river.

Method of Least Squares

It should be noted in Fig. 1 that the values of S are not absolute. The absolute values are not generally available. Therefore a straight line drawn through the plotted points will not generally pass through the origin. On the other hand, Eq. (4) is valid when the values of S are the absolute values since a line described by Eq. (4) always passes through the origin. To account for this discrepancy, Eq. (4) is modified for the purpose of using it in the method of least squares, as follows.

$$S = AI + BO + \sigma \qquad (5)$$

where S is the relative storage, σ is the difference between the absolute storage and the relative storage obtained from I and O records, $A = Kx$ and $B \equiv K(1-x)$. The problem is now reduced to determining the values of A, B, and σ which will fix the position of the straight line in a plot like Fig. 1 and minimize the width of the loop formed by the rising and the falling limbs. Described in this manner, the problem becomes very simple. Let the deviation between the predicted and the actual values of S be denoted by δ, so that

$$\delta_1 = S_1 - (AI_1+BO_1+\sigma) \; , \quad \delta_2 = S_2 - (AI_2+BO_2+\sigma)$$

and so on. It is sought to minimize

$$\sum_1^N \delta^2 = \delta_1^2 + \delta_2^2 + \ldots . \; \delta_N^2$$

where N is the number of available observations. Using the usual optimizing criteria, following normal equations are obtained from which the values of the constants can be determined.

$$\sum_1^N S - A\sum_1^N I - B\sum_1^N O - N\sigma = 0 \tag{6}$$

$$\sum_1^N OS - A\sum_1^N OI - B\sum_1^N O^2 - \sigma\sum_1^N O = 0 \tag{7}$$

$$\sum_1^N SI - A\sum_1^N I^2 - B\sum_1^N OI - \sigma\sum_1^N I = 0 \tag{8}$$

Once the values of A and B are determined, K and x are calculated using the following equations.

$$\frac{x}{1-x} = \frac{A}{B} \tag{9}$$

$$A + B = K \tag{10}$$

Although the values of absolute storage are not required for flood routing, it may however be recognised that the absolute storage is given by $(S-\sigma)$. The method is now illustrated by applying it to a specific case.

Linsley et al (1949) have tabulated the inflow and outflow data together with the calculated values of S for the Sewickley- Wheeling reach of the Ohio river. Their data are used here for determining x, K, and σ which give the best fit line on a plot between S and $xI + (1-x)O$, Fig. 1. There are 34 data items in the table and all of them are used here. The storage values are in units of 1000 acre- feet (1 acre- feet = 1233 m^3) and the weighted flow in units of 100,000 cfs (1 ft^3 = 0.028 m^3). The calculated optimum values of the constants are A = 0.610, B = 1.202, K = 1.812, x = 0.337, and σ = -243,734.

Nonlinear Storage - Flow Relationship

As noted earlier, the storage flow relationship is not always expressible by the linear formula, Eq. (4). If the element of nonlinearity is appreciable, Eq. (4) becomes inapplicable. In such cases, it is more appropriate to use Eq. (3). Eq. (3) presents several difficult problems. For instance, there does not seem to be any easy way of determining the numerical values of m/n and x for known values of I, O, and S. The task of determining the values of the constants can perhaps be performed on a digital computer by trying different combinations of m/n and x. Assuming these values are determined by a tedious trial and error method, the value of b/a can be determined easily.

With known values of the constants, routing can be done by solving Eqs. (2) and (3) simultaneously. Even this solution is not straightforward. Here too, trial and error is required for determining the values of the outflow at the end of a routing interval when the initial values of the inflow and outflow together with the inflow at the end of the interval are known. Fortunately, the method of trial and error for the simultaneous solution of Eqs. (2) and (3) is quite simple and can be easily handled.

In order to avoid the difficulties arising from the use of Eq. (3), a storage-weighted flow relationship of the following kind may be worthwhile.

$$S = K\,[\,Ix + (1-x)O\,]^{n} + \sigma \tag{11}$$

Although it may be argued that Eq. (11) is probably not of the same degree of reliability as Eq. (3), it is definitely more accurate than the linear relationship, Eq. (4). Eq. (11) has the advantage of being simpler than Eq. (3). Although it does not appear feasible to use the method of least squares with Eq. (11) to develop simple forms of normal equations, the value of x can nonetheless be determined in two or three trials such that the optimum value of x reduces the width of the loop considerably. Having determined such a value of x, evaluation of n, σ, and K is relatively a simple matter. Having determined K, x, and σ, Eq. (11) can then be used with Eq. (2) for routing of future floods. The method of determining the constants is now explained with reference to a typical example.

This example is taken from Wilson (1974). Using known data of I, 0, and S, Eq. (4) was first tried using the normal equations, Eqs. (6), (7), and (8). The values of the constants were determined as $x = 0.254$, $K = 4.611$, and $\sigma = -102.640$. The units of I and 0 are in m^3/s while those of S are (¼ day) m^3/s. The straight line fit is shown in Fig. 2. The value of x determined in three trials by Wilson is 0.25. An average line was then drawn arbitrarily by Wilson which gave a value of 6.0 for K.

However, it is apparent from Fig. 2 that the plotted points of the actual data appear to lie on a curve rather than a straight line. It was therefore tried to fit a nonlinear equation of the type of Eq. (11). Three characteristic points are selected such that they lie on the mean curve through the data points and cover the whole range of the data. Let these points be designated

$$\{S_1,(0.25\,I_1 + 0.75\,O_1)\}\ ,\ \{S_2,(0.25\,I_2 + 0.75_2 O\,)\}$$

and

$$\{S_3,(0.25\,I_3 + 0.75\,O_3)\}\ ;$$

Using these coordinates, write the following equations.

$$S_1 = K(0.25\,I_1 + 0.75\,O_1)^{n} - \sigma \tag{12}$$

$$S_2 = K(0.25\,I_2 + 0.75\,O_2)^{n} - \sigma \tag{13}$$

$$S_3 = K(0.25\,I_3 + 0.75\,O_3)^{n} - \sigma \tag{14}$$

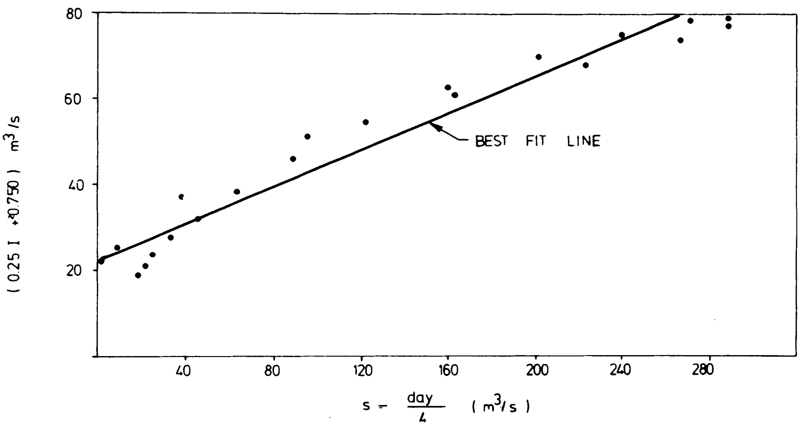

Fig. 2. Linear storage- flow relationship for the example from Wilson (1974).

The following results can now be obtained from Eqs. (12) to (14),

$$\frac{\log\left(\dfrac{S_3-\sigma}{S_2-\sigma}\right)}{\log\left(\dfrac{S_2-\sigma}{S_1-\sigma}\right)} = \frac{\log\left(\dfrac{0.25\,I_3+0.75\,O_3}{0.25\,I_2+0.75\,O_2}\right)}{\log\left(\dfrac{0.25\,I_2+0.75\,O_2}{0.25\,I_1+0.75\,O_1}\right)} \tag{15}$$

$$\log\left(\frac{S_3-\sigma}{S_2-\sigma}\right) = n\log\left(\frac{0.25\,I_3+0.75\,O_3}{0.25\,I_2+0.75\,O_2}\right) \tag{16}$$

From Eq. (15), σ can be determined by trial and error. With the calculated value of σ, n and K can be determined from Eq. (16) and any of the Eqs. (12), (13), and (14). Following this procedure, the values of σ, n, and K were found for the example considered here as -2, 2.347, and 0.010 respectively. The empirically fitted curve is shown with respect to the plotted positions of the data in Fig.3. The fit is quite satisfactory and definitely an improvement on the straight line fit of Fig. 2.

The routing equation, Eq. (2) can now be written in this example as

$$[(I_1+I_2) - (O_1+O_2)]\frac{\Delta t}{2} = 0.010\,(0.25\,I_2+0.75\,O_2)^{2.347} - 0.010\,(0.25\,I_1+0.75\,O_1)^{2.347} \tag{17}$$

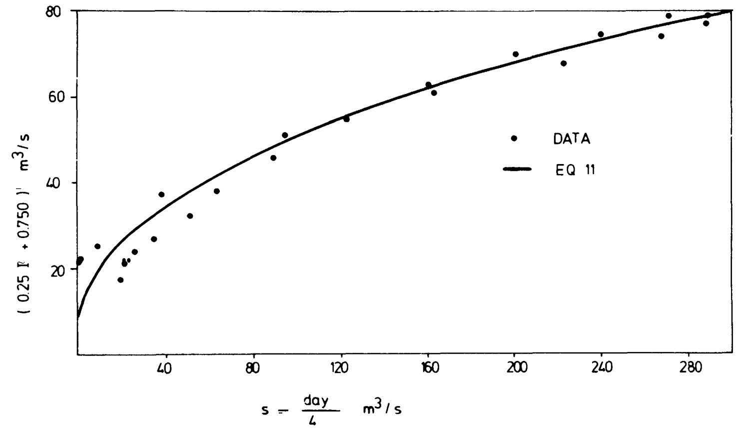

Fig. 3. Nonlinear storage- flow relationship for the example from Wilson (1974).

The initial values of I and 0 together with the values of I at the end of the time interval and the time interval $\triangle t$ itself are supposed to be known. The value of 0_2 can be calculated by trial and error. The calculated value of 0_2 becomes the initial value of 0 for the next time interval and the calculation is repeated to determine the next value of 0 and so on.

Summary and Conclusion

An objective method of determining the constants in the linear storage - flow equation, Eq. (4), is described. The application of the method is explained by a specific example. For the nonlinear storage versus weighted flow relationship, an approximate method of analysis is proposed.

Acknowledgment

The author is indebted to Professor S. Oleszkiewicz, Head of the Civil Engineering Department, Ahmadu Bello University, Zaria, Nigeria, for his continuous encouragement in research matters. The assistance of Mrs. J. Brezezina in preparing the diagrams used in this paper, is duly acknowledged.

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Received: 3 January, 1977

Address:
Department of Civil Engineering,
Ahmadu Bello University,
Zaria,
Nigeria.