clarify some of the discrepancies in the reported values of entry lengths.

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Discussion

J. P. Hartnett.[4] The author is to be commended for having undertaken the formidable task of solving the problem of composite cylinders and also for pointing out how additional useful convection solutions may be obtained from the text of Carslaw and Jaeger (1).[5]

The solution of the energy equation in the case of the channel and of the pipe is obtained essentially by a finite-difference technique. What are the criteria to insure convergence of the "composite solution" to the solution of the energy equation? In particular, 12 slabs were selected for the channel case and excellent agreement with previous results are obtained—how was it decided to use 12 slabs and not 6 or 18? Has the analogous solution for the circular cylinder been obtained, and if so, how does it agree with previous analysis?

It is stated that the results may be used to determine the time lag in composite slabs or composite cylinders. Is such a determination equivalent to the determination of the thermal-entrance length? Was such a calculation carried out for either the slab or the cylinder?

The present thermal entrance-length prediction for high Prandtl-number fluids with a constant heat-rate boundary condition agrees well with the experimental results of reference (10) if the viscosity is evaluated at a temperature equal to the average of the wall and fluid temperature, at least for those cases above a Reynolds number of 20,000 based on this viscosity. When compared to the analysis of Deissler for the same case the present results yield somewhat longer thermal entry-length values, but agree with Deissler (6) in predicting a decrease of this length with increasing Reynolds number. There is disagreement, however, with Deissler's work in that Deissler predicts a decrease of the thermal-entrance length with increasing Prandtl number. The constant wall-temperature analysis of Berry (4) shows a marked increase with Reynolds number for Prandtl numbers of 100 but only a negligible influence of Prandtl numbers in this range. All of these analyses predict entrance lengths less than approximately 20 diameters for high Prandtl fluids over a Reynolds number range from 10^2 to 10^6.

The asymptotic Nusselt value predicted by Levy for the high Prandtl case appears questionable in that it is independent of the Prandtl modulus and in addition yields values which are considerably lower than obtained experimentally for Reynolds values of the order of 10^5.

Herbert Saunders.[6] The author has continued the trend of applying finite-difference methods to heat-transfer problems involving varied boundary conditions which either cannot be solved or solved with difficulty by analytical means. The important thing to be considered in all finite-difference approaches is the stability of the system equation. Many authors have given necessary solutions for stability and convergence of the heat-flow equation. In applying Jaeger's method, the author has not shown the stability limits of the solutions. This is extremely important because these limits convey the necessary information concerning the allowable mesh size and, in addition, the numerical errors in "rounding-off" procedure. Has the author done any further work on this problem concerning its stability?

Author's Closure

The author wishes to thank Professor Hartnett and Mr. Saunders for their comments. He agrees with their statement that the method presented is essentially one of finite difference and that stability of the equations needs further condideration. The complexity of the equations, however, eliminates the possibility of a simple stability criterion and a logical substitute would be to try various numbers of steps and ascertain that the choice is satisfactory by comparison with available results or between each set of calculations.

In order to determine the thermal-entry length it is necessary to find the zero of the determinant Δ_i. As Jaeger points out in

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[5] Numbers in parentheses refer to the references in the Bibliography at the end of the paper.

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his paper,[7] a crude approximation can be obtained to the smallest zero of Δ_i and its application shows that it is not accurate enough to yield a satisfactory answer to the thermal-entry-length problem.

Dr. Hartnett indicates that the asymptotic value of the Nusselt number should be dependent upon the Prandtl modulus. This is not true for the analogy solution as demonstrated by fig. 74 in the second edition of Heat Transmission by McAdams. This is a shortcoming of the analogy method which may carry over in the entrance-length solution. The author feels, however, that Deissler's work exhibits the same trend, namely, very little dependence of entry length with Prandtl number at very high values of the latter parameter.

[7] See reference (2) in paper.