

# Evolutionary multi-objective optimization of the design and operation of water distribution network: total cost vs. reliability vs. water quality

Raziyeh Farmani, Godfrey Walters and Dragan Savic

## ABSTRACT

An expanded rehabilitation of the hypothetical water distribution network of Anytown, USA is considered. As well as pipe rehabilitation decisions, tank sizing, tank siting and pump operation schedules are considered as design variables. Inclusion of pump operation schedules requires consideration of water system operation over the demand pattern period. Design of distribution storage facilities involves solving numerous issues and trade-offs such as locations, levels and volume. This paper investigates the application of multi-objective evolutionary algorithms in the identification of the pay-off characteristic between total cost, reliability and water quality of Anytown's water distribution system. A new approach is presented for formulation of the model. To provide flexibility, the network must be designed and operated under multiple loading conditions. The cost of the solution includes the capital costs of pipes and tanks as well as the present value of the energy consumed during a specified period. Optimization tends to reduce costs by reducing the diameter of, or completely eliminating, pipes, thus leaving the system with insufficient capacity to respond to pipe breaks or demands that exceed design values without violating required performance levels. Here a resilience index is considered as a second objective to increase the hydraulic reliability and the availability of water during pipe failures. Considering reliability as one of the objectives in the optimization process will decrease the level of vulnerability for the solutions and therefore will result in robust networks. However, oversized distribution mains and storage tanks will have adverse effects on water age with negative effects on water quality due to low flow velocity and little turnover, respectively. Therefore, another objective in the design and operation of distribution systems with storage facilities is the minimization of residence time, thus minimizing deterioration in water quality, which is directly associated with the age of water. Residence time must include not only the time in tanks but also the travel time before and after the water's entry into the storage facilities. The residence time of the water in the network is considered as a surrogate measure of water quality. Results are presented for the pay-off characteristics between total cost, reliability and water quality, for 24 h design and five loading conditions.

**Key words** | genetic algorithm, multi-objective, optimization, water distribution, water quality

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## INTRODUCTION

Aging of water distribution system infrastructure may lead to an increase in leaks and breaks, deterioration of water quality and reduction in hydraulic capacity. These problems are causing many water authorities to undertake costly

capital improvement programs that will result in cost-effective rehabilitation and replacement schemes for water distribution systems to respond to growth and regulatory requirements. Water distribution systems are usually

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designed on the basis of capital cost. Within this capital expenditure, there may be some optimization of the design in an attempt to reduce running costs. However, this approach pays no attention to the impact that marginal increases in capital cost might have on the reduction in running costs or on the improvement in benefits. Such a study is a highly complex optimization problem. The improvements may include upgrading the existing components of the network or addition of new pipes, tanks and pumps. These problems involve multiple measures of performance, or objectives that need to be optimized simultaneously. However, optimal performance according to one objective often implies low performance in one or more of the other objectives.

Multi-criteria decision-making (MCDM) methods evaluate the coupling between these design criteria and their impact on solutions. Traditional MCDM methods convert a multi-objective problem into a single objective optimization problem. In these methods only one Pareto optimal solution can be expected to be found in a single run. Furthermore, not all Pareto optimal solutions can be found by some algorithms in non-convex multi-objective optimization problems and all algorithms require some problem-specific knowledge such as suitable weights or target values for each objective. Despite the shortcomings, due to their simplicity and ease of implementation on a computer, these methods are the most common methods used in solving real world problems (Deb 2001).

However, multi-objective evolutionary algorithms (MOEAs) use a population-based evolutionary algorithm (EA), and offer a less subjective means of finding many Pareto optimal solutions in a single run. Depending on the preference of a decision-maker, the remaining task is to choose a group of solutions for more detailed analysis. Evolutionary multi-objective optimization methods can be classified into two groups of elitist and non-elitist methods (Deb 2001). Common methods of the non-elitist type are Vector Evaluated Genetic Algorithm (VEGA) (Schaffer 1985), Multi-objective Genetic Algorithm (MOGA) (Fonseca & Fleming 1993) and Non-dominated Sorting Genetic Algorithm (NSGA) (Srinivas & Deb 1994). Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler & Thiele 1998), Elitist Non-Dominated Sorting Genetic Algorithm (NSGAI) (Deb *et al.* 2000) and Pareto Archived Evolution

Strategy (PAES) (Knowles & Corne 2000) are the most common elitist methods.

A comparative study of the application of some of the MOEA techniques in water distribution systems (Farmani *et al.* 2003) showed that the elitist Non-dominated Sorting Genetic Algorithm method (NSGAI) (Deb *et al.* 2000) outperformed other techniques in satisfying both goals of Pareto multi-objective optimization (closeness to Pareto front and diversity among solutions in each front). Farmani *et al.* (2005a) studied the performance of NSGAI in comparison with the Strength Pareto Evolutionary Algorithm (SPEA2). They concluded that SPEA2 performed better than NSGAI in that it generated better pay-off curves for a number of benchmark problems. The only drawback of the SPEA2 is the lack of an operator to handle the constraints. In this work the application of the NSGAI method to the highly constrained problem of the Anytown water system (Walski *et al.* 1987) is presented.

Deb *et al.* (2000) suggested the elitist Non-Dominated Sorting-based multi-objective evolutionary algorithm (NSGAI) which alleviates some of the difficulties faced by non-elitist multi-objective evolutionary algorithms. In this method a fast non-dominated sorting approach with a selection operator is presented that creates a mating pool by combining the parent and offspring populations and selecting the best solutions (with respect to fitness and spread). The next generation is populated starting with the best non-dominated front, progressing through the rest until the population size is reached and, if in the final stage there are more individuals in the non-dominated front than available space, the crowded distance-based niching strategy is used to choose which individuals of that front are entered into the next population.

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## MULTI-OBJECTIVE OPTIMIZATION IN WATER DISTRIBUTION DESIGN AND OPERATION

DeNeufville *et al.* (1971) presented one of the first works which recognized that most water system designs are multi-objective. Water distribution network design usually involves conflicting objectives such as cost and benefit. Pareto multi-objective optimization has been applied

previously to the design of water networks. *Savic et al. (1997)* formulated a pump scheduling problem in water supply as a multi-objective optimization problem. The multi-objective approach used in their work was based on the concept of Pareto optimal ranking of *Goldberg (1989)*. They concluded that twin objective optimization of the energy cost and the pump switching criterion achieved very good solutions. *Halhal et al. (1997)* and *Walters et al. (1999)* developed a structured messy genetic algorithm (SMGA) for optimization of water network rehabilitation using capital cost and benefit as dual objectives. The progressive building up of solutions from simple elements developed for SMGA, combined with the multi-objective approach, which keeps a range of good solutions with varied costs throughout the process, proved very effective. The MOGA method (*Fonseca & Fleming 1993*) has been used in rehabilitation of water systems (*Dandy & Engelhardt 2001*), multi-objective design optimization of water distribution networks (*Savic 2002*) and optimal sampling design of water distribution model calibration (*Kapelan et al. 2003*). *Wu et al. (2002)* implemented a fast messy genetic algorithm method in solving multi-objective optimization of dual objectives (cost and optimum capacity) in water distribution networks. Recently, Pareto multi-objective optimization has also been used for problems where water quality is one of the objectives (*Doby et al. 2001*; *Rouhiainen et al. 2003*). *Rouhiainen et al. (2003)* presented two new Pareto-based multi-objective genetic algorithm techniques for determining the optimal schedule of chlorine dosing within a water distribution system with multiple conflicting objectives: primarily disinfection control and aesthetic control. Comparison of the two new techniques to a weighted sum model was given. *Cheung et al. (2003)* presented the application of the Strength Pareto Evolutionary Algorithm (SPEA) in the rehabilitation of a water distribution network. *Doby et al. (2001)* investigated a genetic algorithm-based method for determining the least cost design of looped networks while considering the residence time of the water in the network as a quality surrogate and various levels of redundancy. The conflict among the three objectives – cost, redundancy and water quality – was examined via multi-objective analysis. Their results showed that there are multiple solutions that are

equally viable from the standpoint of age, required pressure and cost.

In what follows, application of the evolutionary multi-objective optimization method to the optimum design and operation of Anytown's water distribution system is presented. The problem is posed as a multi-objective optimization problem with minimizing total cost, maximizing minimum resilience index and minimizing maximum water age as objectives. Results are assessed based on the final Pareto front and the diversity among the solutions in the pay-off curve. The effectiveness of the search method is examined in relation to the optimum sizing of pipes and tank volumes and locations, simultaneously with the optimization of the pump operation schedule, identification of the pay-off between the system expenditure, network resilience and water quality being the aim of the optimization.

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## THE ANYTOWN NETWORK

The Anytown water distribution system was set up by *Walski et al. (1987)* as a realistic benchmark on which to compare and test network optimization software, and has features and problems typical of those found in many real systems. The Anytown problem was originally tackled by participants at the Battle of the Network Models workshop, and has since been examined by *Murphy et al. (1994)* and *Walters et al. (1999)*. All participants in the original workshop used optimization models to size the piping system while manually choosing the location and size of tanks. The difference between the various methods originally used lies essentially in the pipe optimization models, which were based on linear programming, partial enumeration or non-linear programming techniques. No attempt was made to optimize the provision of tanks or pumps, except by use of expert judgement and experience. Most of the participants were able to develop a solution that would work at peak loading but would not have adequate capacity to fill tanks at off-peak loadings (*Walski et al. 1987*).

*Murphy et al. (1994)* later obtained a better solution to the problem using a Standard Genetic Algorithm (SGA), which was able to handle the tanks and pumps as additional design variables. To obtain a solution, network deficiencies

were incorporated into the objective function (cost) as penalties, using a set of weightings, one for each type of constraint violation (e.g. pressure deficiency, unbalanced tank flows). Selection of the most effective weighting required a number of trial runs.

Walters *et al.* (1999) introduced the Structured Messy Genetic Algorithm (SMGA) for solving the Anytown water system. The problem was posed as a multi-objective optimization problem for the first time, with minimum cost and maximum benefit as dual objectives. The design variables were the addition of new pipes, reconditioning and replacement of existing pipes, siting and sizing of new tanks and provision of new pumps. The method produced designs cheaper than any previously published. The progressive building up of solutions from simple elements developed for SMGA, combined with the multi-objective approach, which keeps a range of good solutions with varied costs throughout the process, proved very effective.

Murphy *et al.* (1994) and Walters *et al.* (1999) also developed solutions that would work at peak loading but would not have adequate capacity to fill tanks at off-peak loadings. In this study for the first time, as well as instantaneous peak flow and three different fire flow loading conditions, average day flow condition is considered, under which water must be supplied at a minimum pressure of at least 40 psi.

Although the Anytown water distribution network does not contain all of the features of real systems (e.g. multiple pressure zones, seasonal and local demand fluctuations, fiscal constraints, uncertainty of future demands and pipe roughness and complicated staging of construction) it serves as a challenging benchmark for optimization models that are able to consider many real system features, such as pump and tank sizing and location (Mays 2000).

Farmani *et al.* (2005b) investigated the multi-objective optimization of Anytown's water system under 5 loading conditions for the first time. A minimum pressure of at least 40 psi must be provided at all nodes at average day flow as well as instantaneous peak flow which is 1.8 times the average day flow. The system is also subject to fire flow under which it must supply water at a minimum pressure of at least 20 psi. Pay-off between the total cost and the resilience index was considered. The algorithm generated solutions along the pay-off curve that are fully feasible

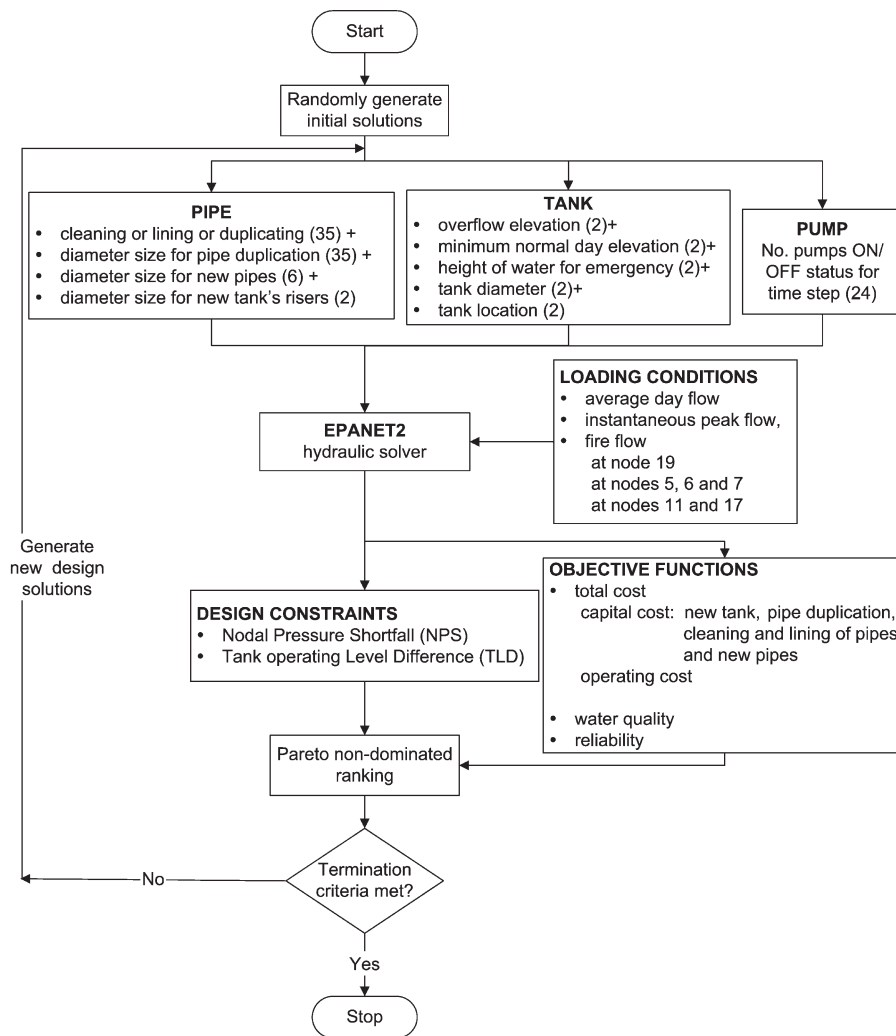
solutions operating under 5 loading conditions without violating any constraints. However, sensitivity analysis of the results on the pay-off curve showed that, although solutions performed well with respect to two objectives, the performance of the solutions regarding the water quality were poor. This led to the idea of considering all objectives simultaneously in the optimization.

In what follows, a summary of the problem formulation and a brief description of the cost function, the water age as a surrogate measure of water quality and the resilience index as a surrogate measure of reliability will be given. Figure 1 illustrates the general procedure for solving the optimization problem.

### Problem formulation

The multi-objective optimization of water distribution is investigated here through the analysis of the Anytown water supply system (Figure 2). The pipes in the central city are given as thick solid lines; the pipes represented by thin solid lines are in residential areas and new pipes are represented by dashed lines. The objective of the Anytown network problem was to determine the most economically effective design to reinforce the existing system to meet projected demands, taking into account pumping costs as well as capital expenditure. The town is formed around an old center situated to the south east of pipe 34, where excavations are more difficult to undertake and consequently are more expensive. There is a surrounding residential area, with some existing industries near node 17 and a projected new industrial park to be developed to the north. Options include duplication (in a range of possible diameters) of any pipe in the system, addition of new pipes, selecting the operation schedule of pumping stations and provision of new reservoir storage at any location. Water is pumped into the system from a water treatment works by means of three identical pumps connected in parallel. The link and node data, the average daily water use at each node for year 2005, the variation in water use throughout the day and different loading conditions are from CWS (2004).

The water in the treatment works is maintained at a fixed level of 10 ft. The two existing tanks are operated with water levels between elevations 225 ft and 250 ft, giving an



**Figure 1** | The optimization procedure.

effective capacity of 1,56,250 gal (US) for each tank. The volume of water below the level 225 ft and above 215 ft is retained for emergency needs, giving an emergency volume of 62,500 gal (US) for each tank. A minimum pressure of at least 40 psi must be provided at all nodes at average day flow as well as instantaneous peak flow, which is 1.8 times the average day flow. The system is also subject to fire flows under which it must supply water at a minimum pressure of at least 20 psi. The fire flow duration is two hours with tanks starting at their low operating levels and one pump being out of service and must be met while also supplying peak day flows, which are 1.3 times average day flow. It is assumed that during fire flow periods only the flow required for fires is supplied at the corresponding nodes.

The variation in water use throughout the day is given in [Table 1](#).

35 existing pipes are considered for duplication or cleaning and lining (giving 35 pipe options and 35 diameter variables). There are 6 additional new pipes. To achieve fully feasible designs the following approach is considered to model the problem. In this approach, up to two new cylindrical storage tanks can be considered. The location, overflow and minimum normal day elevation, diameter and bottom of tank from minimum normal day elevation are treated as independent design variables for the tanks, thereby defining completely the design of the storage. However, storage will initially not match the requirements of the network, and penalties are necessary to produce



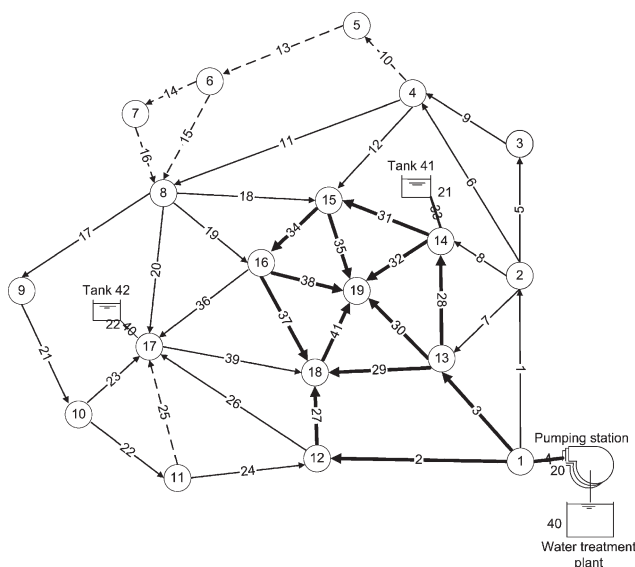


Figure 2 | The Anytown network.

feasible solutions. All nodes are considered as potential sites for new tanks, except those which are already connected directly to the existing tanks. Therefore the design variables for tanks are: 2 possible locations for the new tanks in the network; overflow and minimum normal day elevation, bottom of tank from minimum normal day elevation and diameter of each tank (8 variables). Tanks are connected to a node by a short pipe, known as a riser, of known length (101 ft) but of variable diameter, giving a total of 2 diameter variables for the risers (giving a total of 78 design variables

Table 1 | Daily water use pattern

Time of the day	Average day demand factor
00:00–03:00	0.7
03:00–06:00	0.6
06:00–09:00	1.2
09:00–12:00	1.3
12:00–15:00	1.2
15:00–18:00	1.1
18:00–21:00	1.0
21:00–24:00	0.9

for links in the network). Given a 24 h operation cycle and 1 h time step, the control variables for the three pumps give a further 24 design variables and a total of 112 design variables for pipes, tank volumes and pumps.

The range of each type of variable is given in Table 2. Pipe and riser sizes 0–10 correspond to the 10 available discrete pipe diameters (6, 8, 10, 12, 14, 16, 18, 20, 24, 30 inches) and one extra possible decision which is the “do nothing” option. The integer values for pipe status correspond to 0 for “leave”, 1 for “clean and line” and 2 for “duplicate” the existing pipes. A pipe which has been cleaned and lined has a Hazen–Williams coefficient of  $C = 125$  and for new pipes  $C = 130$  (Murphy et al. 1994). The number of pumps in operation during each hour is represented by integer values: 0, 1, 2 and 3 for the corresponding number of pumps. Every node in the network is a possible location for new tanks except those already connected to existing tanks (1, 17 and 19), therefore there are 16 possibilities. Minimum and maximum boundary values for decision variables of overflow and minimum normal day elevation, diameter and bottom of tank from minimum normal day elevation are also given in Table 2. The characteristic curve for the pumps, together with the corresponding wire-to-water efficiencies, are available from CWS (2004).

Supplying demand at an adequate pressure to consumers is the main constraint in the design of water distribution systems. The presence of tanks and pumps in the network adds extra difficulty to the optimization as these two are the most difficult components to model (Lansey & Mays 1989). Inclusion of water storage tanks in the network is to provide equalization and emergency storage. Where there is more than one tank, the problem of designing and operating the system becomes much more complicated. Numerous decisions must be made in the design of a storage tank, including size, location, type and expected operation (Walski 2000b).

All tanks should empty and fill over their operational ranges during the specified average demand day, leaving the specified emergency volumes untouched. The actual maximum and minimum water levels are then identified for each tank during a 24 h simulation (over a 24 h operation time). There will in general be a mismatch between the top and bottom water levels specified and those resulting from the

**Table 2** | Problem variables

Variable	Type	Lower bound	Upper bound
Pipe leave/clean and line/duplicate status(-)	Integer	0	2
Pipe size	Integer	0	10
No. of pumps in each period	Integer	0	3
Tank location	Integer	1	16
Cylinder tank diameter (ft)	Real	25	100
Overflow elevation (ft)	Real	200	250
Minimum normal day elevation (ft)	Real	180	240
Bottom of tank from minimum normal day elevation (ft)	Real	0	25

simulation, and also a mismatch between initial water levels and final water levels. The accumulated sum of the mismatch in levels is used as the Tank operating Level Difference (TLD) constraint (Table 3). Considering tank characteristics as explicit design variables will result in convergence of the algorithm to a region of feasible search space with less computational effort than using tank volume as the design variable. In the latter case adjusting the operational levels and tank diameters is a challenging task as they have secondary effects on the optimization process.

The performance of each candidate design solution is evaluated through simulation of the network flows. An extended period hydraulic network solver (EPANET2, Rossman 2000) is used to determine the head at all the system nodes for all design conditions, and the accumulated sum of the Nodal Pressure Shortfalls (NPS) is used as the head deficit constraint (Table 3).

### The optimization criteria

Walski (2000a) pointed out three competing goals for water distribution system operation: maximize reliability, which is achieved by keeping the maximum amount of water in storage in case of emergencies, such as pipe breaks and fires; minimize energy costs, which is achieved by operating pumps against as low a head as possible but near the best efficiency point for the pump; and meet water quality

standards, which involves minimizing the time the water is in the distribution system and storage tanks and is achieved by having storage-tank levels fluctuate as much as possible.

### Reliability measures of water distribution systems

The term “reliability” in water distribution systems mainly refers to the ability of the network to provide consumers with adequate and high quality supply, under normal and abnormal conditions. The reliability of water systems can be studied considering two types of failure: first, mechanical failure and second, hydraulic failure. Mechanical failure usually refers to failures of system components, such as pipe breakage or a pump being out of service. Hydraulic failures,

**Table 3** | Constraint functions

Constraint function	Constraint form	Constraint limit
Nodal Pressure Shortfall (NPS) at average day flow	$\geq$	40 psi
Nodal Pressure Shortfall (NPS) at instantaneous peak flow	$\geq$	40 psi
Nodal Pressure Shortfall (NPS) at fire flow condition	$\geq$	20 psi
Tank operating level difference (TLD) at average day flow	$=$	0

on the other hand, refer to uncertainties, such as in demand forecasting and cost estimation. Providing adequate service with reliability and a safety factor to allow for uncertainty and failures of system components is one of the main objectives in multi-objective water system design and operation. In this study a resilience index is considered as a surrogate measure to account for the reliability of the network.

The concept of resilience was introduced by [Todini \(2000\)](#) to account for the fact that water distribution networks are designed as looped systems in order to increase the hydraulic reliability and the availability of water during pipe failure. Water distribution networks are usually designed to deliver water at each node, satisfying the demand in terms of design flow and head. However, in the case of change in demand or a pipe failure the water flow will change and the original network is transformed into a new one with higher internal energy losses. This might make it impossible to deliver the desired flow rate at a minimum delivery pressure. Providing more power than that required at each node could be one of the possible ways to avoid this problem. This will add sufficient surplus to be dissipated internally in case of failures. This surplus has been used by [Todini \(2000\)](#) to characterize the resilience of the looped networks, although it does not involve statistical considerations on failures, its increase will lead to improved network reliability.

The resilience index of a looped network is defined as

$$I_r = 1 - \left( \frac{P_{\text{int}}}{P_{\text{max, int}}} \right) \quad (1)$$

where  $P_{\text{int}}$  is the amount of power dissipated in the network to satisfy the total demand and  $P_{\text{max, int}}$  is the maximum power that would be dissipated internally in order to satisfy demand  $q$  and head  $h_{\text{ava}}$  at junction nodes.

The resilience index can be written as

$$I_r = \frac{\sum_{i=1}^{\text{nnode}} q_i (h_{\text{ava}, i} - h_{\text{req}, i})}{\left( \sum_{j=1}^{\text{nreservoirs}} Q_j H_j + \sum_{k=1}^{\text{npump}} P_k / \gamma \right) - \sum_{i=1}^{\text{nnode}} q_i h_{\text{req}, i}} \quad (2)$$

where  $Q_j$  and  $H_j$  are the discharge and the head, respectively, at each reservoir,  $\gamma$  is the specific weight of water and  $P_k$  the power introduced into the network by the pumps.

A full description of the resilience index has been given in [Farmani \*et al.\* \(2005b\)](#).

### Total cost

The cost of the solution includes the capital costs of pipes and tanks as well as the present value of the energy consumed during a specified period. The unit costs for pipe laying, cleaning and lining are from [CWS \(2004\)](#). Pump station operating costs are based on a unit cost for energy, constant throughout the 24 h, equal to \$0.12/kWh. The present worth of energy costs is based on an interest rate of 12% and an amortization period of 20 yr. Tank costs are considered as a function of volume and are from [CWS \(2004\)](#). Intermediate tank sizes are considered in the proposed methods and the corresponding costs are interpolated linearly from standard sizes and costs.

### Water quality at network junctions and in storage tanks

Deterioration in water quality is associated with the age of the water. Therefore in design and operation of distribution system storage facilities, minimization of detention time is one of the main objectives.

The concept of average age ( $A_{ij}$ ) or average time of travel at node  $ij$  from all sources, under steady state condition, was introduced by [Males \*et al.\* \(1985\)](#). Steady-state water quality models proved to be useful tools for investigating the movement of a contaminant under constant conditions. However, the need for models that would represent the dynamics of contamination movement led to the development of models to simulate the movement under temporally varying conditions.

EPANET2's dynamic water quality simulator models the changes in water age over time throughout a network ([Rossman 2000](#)). It uses the flows from the hydraulic simulation to solve a conservation of mass equation for the travel time within each link connecting nodes  $i$  and  $j$ :

$$\frac{\Delta A_{ki}}{\Delta t} = - \frac{q_{ki}}{Area_{ki}} \frac{\Delta A_{ki}}{\Delta x_{ki}} + 1.0 \quad (3)$$

where  $A_{ki}$ , the age of water in pipe  $ki$ , is a function of distance  $x$  and time  $t$ .  $q_{ki}/Area_{ki}$  is the flow velocity in pipe



$ki$ . The addition of 1.0 indicates that with each unit of time the age increases by one unit.

The above equation must be solved with a known initial condition at time zero and the boundary condition at the beginning of the link as follows:

$$A_i(0, t) = \frac{\sum_k q_{ki} A_{ki}(L_{ki}, t) + Q_{\text{ext},i} A_{\text{ext},i}}{\sum_k q_{ki} + Q_{\text{ext},i}} \quad (4)$$

where  $A_i(0, t)$  is the water age at node  $i$ ,  $q_{ki}$  is the flow in all incoming links to node  $i$ ,  $A_{ki}(L_{ki}, t)$  is the water age of incoming links to node  $i$  with length  $L$  at time  $t$ ,  $Q_{\text{ext},i}$  is the external flow into node  $i$  and  $A_{\text{ext},i}$  is the age of the external water flow.

EPANET2 solves these equations by a numerical scheme called the Discrete Volume Element Method (DVEM).

It is assumed that the contents of storage facilities are mixed completely, especially in this work where the tanks are operating under fill and draw condition. Thus, the water age in the facility is a blend of the current age of water and any entering water:

$$\frac{\Delta V_s A_s}{\Delta t} = \sum_k q_{ks} A_{ks} - \sum_j q_{sj} A_s + 1.0 \quad (5)$$

where  $V_s$  and  $A_s$  are volume and water age in storage at time  $t$ ,  $q_{ks}$  and  $A_{ks}$  are flow and water age in inlet pipes at time  $t$  and  $q_{sj}$  is flow in outlet pipes at time  $t$ .

In this work the problem is set as a multi-objective optimization problem under multiple loading conditions. The multi-objective optimization of maximizing minimum resilience, minimizing maximum water age and minimizing total cost will lead to a set of non-dominated solutions. Considering five different loading conditions in the optimization provides additional robustness in the network. Reliability and water quality are optimized simultaneously with the total cost which is the sum of the pump operating costs, tank capital costs and pipe network capital cost. A five day water quality simulation period is considered as the time horizon to reach a steady state condition. The pay-off between the objectives will be analyzed in detail.

## SOLUTIONS AND ANALYSIS

### Performance of the algorithm in finding optimum pipe rehabilitation, tank size and siting and pump schedules

Figure 3 gives the pay-off characteristic between the total cost and the resilience for the Anytown network, extracted from the Pareto surface generated by the three-objective optimization.

Details of the design costs, resilience index and water age for all the solutions on the Pareto front are given in Table 4. It is evident that the main differences in the solution costs are from the pipe costs. This illustrates that, in order to provide higher reliability in the network, the network capacity has been increased to respond to pipe breaks or demands that exceed design values. Increase in resilience index did not have a huge effect on pump operation schedules and tank sizing. Pressure requirements were checked and were found to be satisfied for all the specified design loadings for all the solutions on the Pareto set every minute over the 24 h design period. Water age is primarily a function of water demand, system operation and system design. The water industry database (AWWA & AwwaRF 1992) indicates an average distribution system retention time of 1.3 d and a maximum retention time of 3 d. Water quality varies for different solutions on pay-off curve between minimum of 34.7 h (1.4 d) and maximum of 40.7 h (1.7 d), which is within the acceptable range of retention time for average distribution systems of 1.3 d. Detailed

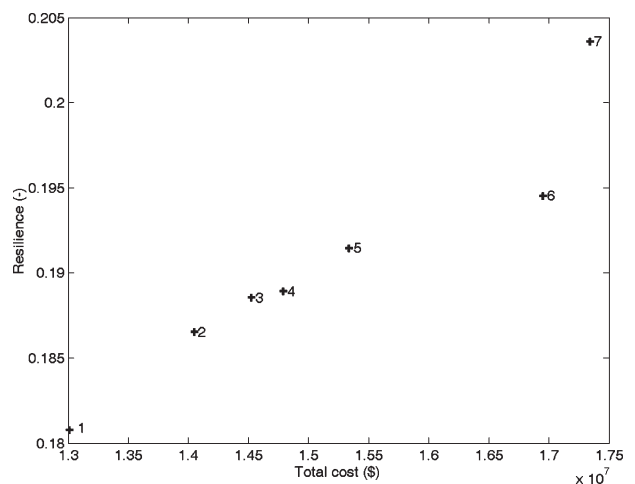


Figure 3 | Cost vs. resilience pay-off.

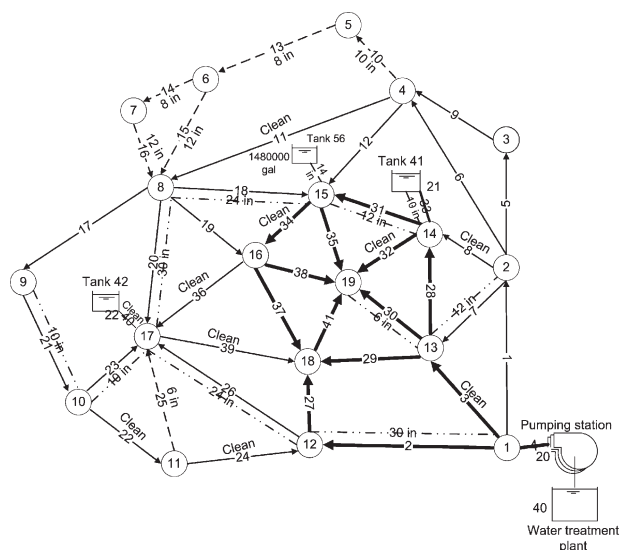
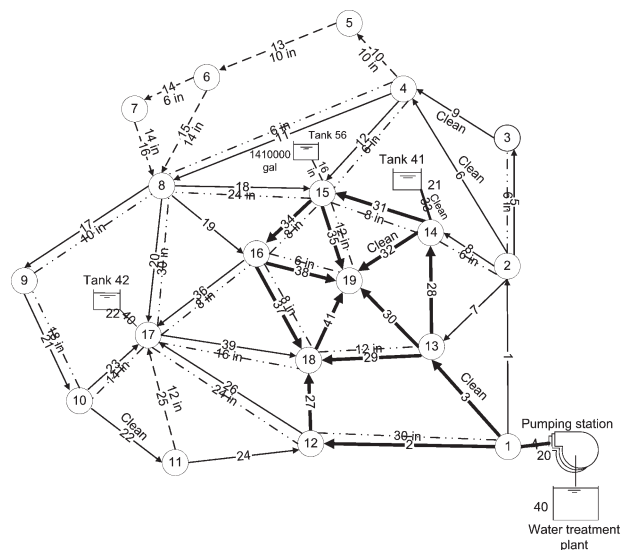
**Table 4** | Summary of design costs (\$million), resilience index and maximum water age ( $h$ ) (three-objective optimization)

Description	1	2	3	4	5	6	7
Pipes	6.200	7.267	7.749	7.952	8.488	10.220	10.566
Tanks	0.769	0.723	0.741	0.742	0.727	0.749	0.774
Pumps	6.044	6.060	6.035	6.098	6.124	5.981	6.002
Total	13.013	14.050	14.525	14.792	15.339	16.950	17.342
Resilience index	0.181	0.187	0.188	0.189	0.191	0.194	0.204
Maximum water age	34.895	40.664	36.864	38.222	37.401	37.336	34.658

inspection of the solutions along the pay-off curve shows that one new tank is included at node 15 with different volume sizes for solutions 1–5, one new tank for solution 6 at node 6 and one new tank for solution 7 at node 19. Changes in the location of the new tank is in response to competing objectives of reliability, water quality and cost.

Solutions 1, 7 and 4 representing solutions on the extremities and center of the pay-off curve respectively have been chosen for detailed study. Figure 4 shows the piping improvements and the tank location and capacities for solution 1 with a total cost of \$13,013,341, maximum water age value of 34.89 h and minimum resilience value of 0.181. One new tank is included at node 15 with an effective volume

of 1115,000 gal (US) and emergency volume of 365,000 gal (US), giving a storage cost of \$0.769 million. During the average to low demand period all three tanks are filled up, they stay full for a few hours at the early stages of the peak demand period, then release their contents towards the end of the peak demand period. The solution satisfies the pressure constraints for the average day flow as well as the different loading patterns with tanks assumed to be at their low operating levels for the fire flow condition. Figures 5 and 6 show the piping improvements and the tank location and capacities for solutions 4 and 7, respectively. The operation of reservoir balancing in the system, from a simulation over 24 one-hour time steps, for solutions 1,4 and 7 are shown in

**Figure 4** | Layout of solution 1.**Figure 5** | Layout of solution 4.

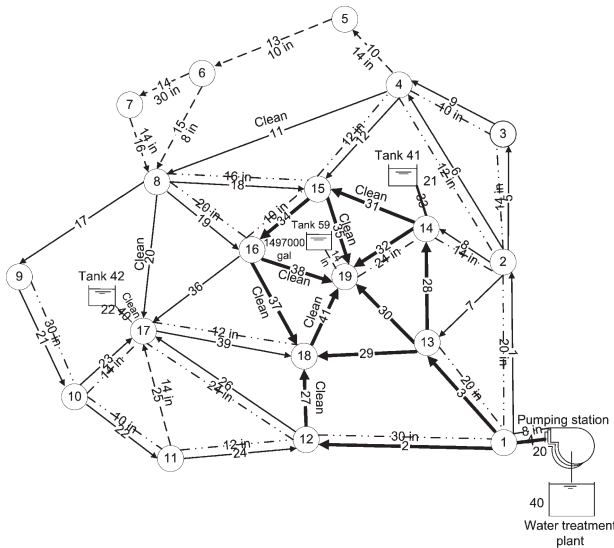


Figure 6 | Layout of solution 7.

Figures 7, 8 and 9 respectively. For solution 1 the tanks use all their operational volumes during the cycle, as shown in the figure.

Also the operations of reservoir balancing in the system, from a simulation over 24 one-hour time steps, are shown in Figures 8 and 9. Comparison of the operation of reservoir balancing in the system for solution 4 with those of 1 and 7

shows distinctive differences between the solutions from different parts of the pay-off curve. Figure 8 illustrates the importance of the operation schedules for the pumps. It is evident that the operation schedules are good if not optimum, as the number of pumps in use changes smoothly over the 24 h period, this being an important factor in prolonging useful pump life by reducing wear and tear. Solution 4 in the central part of the pay-off curve has a smoother operation as pumping follows the system demand pattern closely, filling up all three tanks; they stay full during the peak period, while pumps supply the demand in the network. This illustrates the level of reliability in the network for the solution, as in the case of pump breakdown all three tanks are full to supply the peak demand. However, that is not the case in solutions 1 (with the least total cost and the worst reliability) and 7 (with the highest total cost and the best reliability). Solution 1 has a higher level of fluctuation and also less retention time in the tanks compared to other solutions, thus contributing to better water quality and a poorer level of reliability. Solution 7 also has a reasonably high level of fluctuation which is in an attempt to improve the water quality. In order to achieve a high level of reliability one of the tanks stays full for the

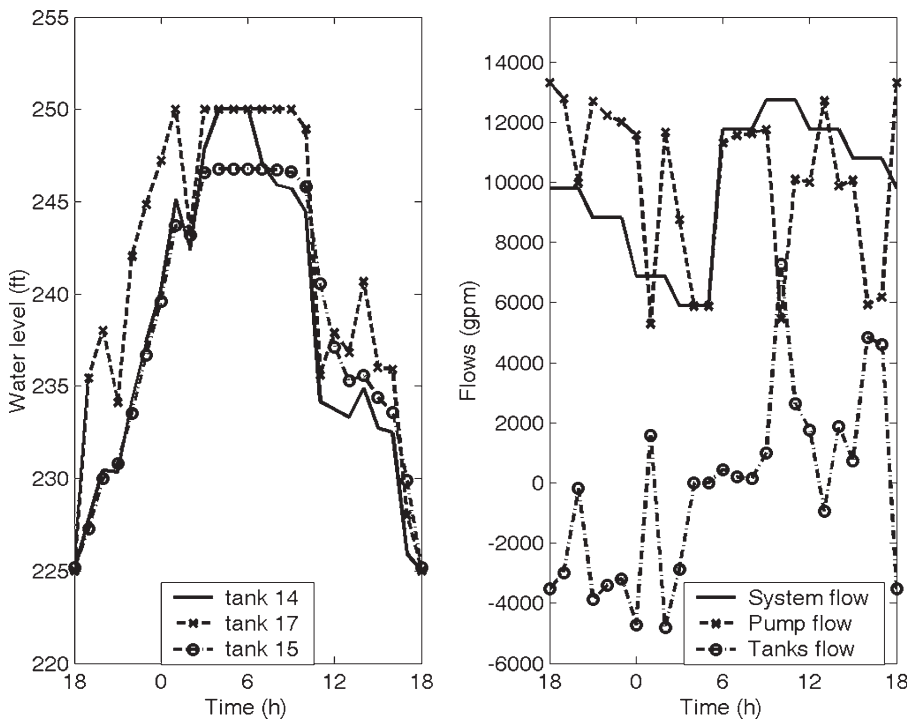


Figure 7 | Tank operating levels over a cycle of 24 h and different flow origins (solution 1).

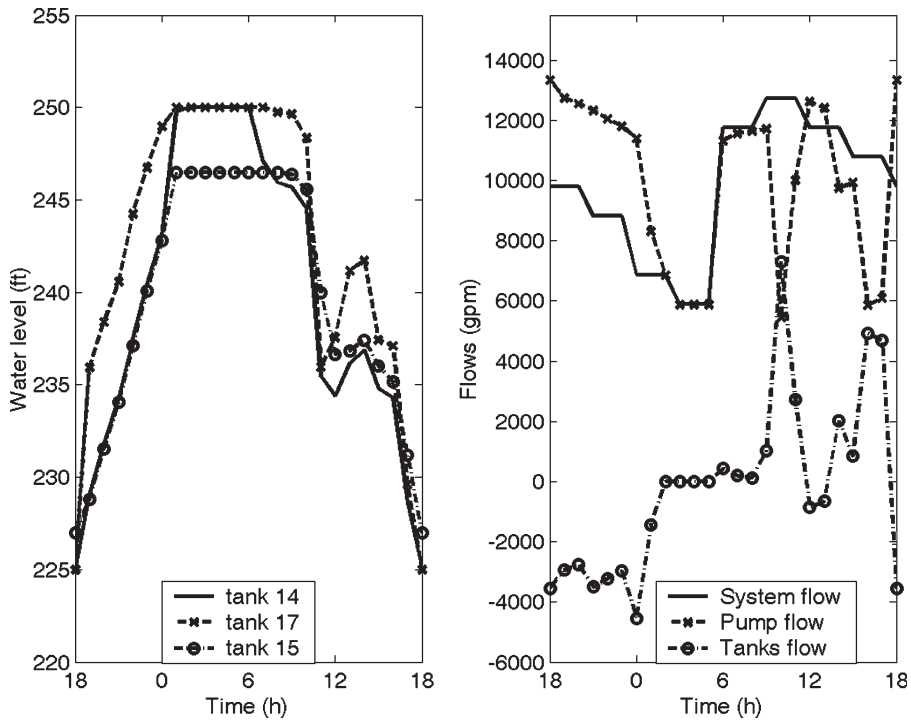


Figure 8 | Tank operating levels over a cycle of 24 h and different flow origins (solution 4).

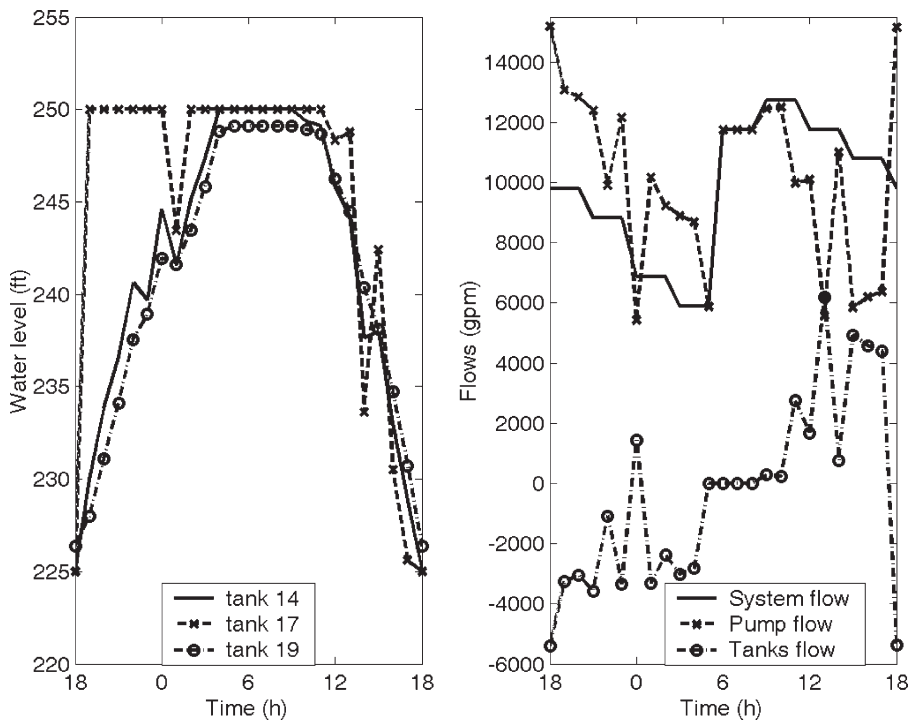


Figure 9 | Tank operating levels over a cycle of 24 h and different flow origins (solution 7).

**Table 5** | Summary of total costs (\$million), resilience index and maximum water age (*h*) (two-objective optimization)

Description	1	2	3	4	5	6	7	8
Total	13.348	13.511	13.522	14.830	16.035	16.810	17.146	18.476
Resilience index	0.182	0.183	0.184	0.198	0.200	0.203	0.208	0.214
Maximum water age	35.81	35.08	35.54	35.56	49.31	35.34	44.28	85.72

most part of the operating period which could contribute to water quality problems due to little turnover.

The search results indicate that the NSGAI has the potential to find Pareto optimal solutions for water distribution networks. In order to determine the pay-off characteristic between the total cost, the resilience index and water age, the multi-objective evolutionary algorithm was run with a population size of 100 sample solutions, and was allowed to run for 5,000 generations.

The solutions on the Pareto front are all fully feasible and satisfy all design and operational criteria in terms of tank sizing and siting and pump operational schedules. The main difference in the solution costs comes from the pipe costs. In order to understand the characteristics of trade-off between total cost and resilience index for solutions on the pay-off curve, the sensitivity analysis of the solutions under pipe failure can be considered. A full description of the way sensitivity analysis is conducted has been given in Farmani *et al.* (2005b).

### Comparison with other published results

Table 5 summarizes the fully feasible solutions for the Anytown water system reported by Farmani *et al.* (2005b). These solutions are fully feasible solutions that operate under 5 loading conditions without violating any constraints. These solutions represent pay-off curves generated by twin-objective optimization of total cost and reliability. Detailed inspection of the solutions shows poor water quality for some of them, as has been reported in Table 5. Solutions 5, 7 and 8 have quite large values for water age, especially the value of 85.72 h (3.6 d) for solution 8, which is not acceptable for the size of the system.

It is evident that solutions generated by the three-objective optimization of total cost, resilience and water age

have the advantage of better water quality in comparison with those from the two-objective optimization of total cost and resilience. The improvements are achieved at no extra cost with similar levels of robustness.

### CONCLUSION

This paper investigated the application of an evolutionary multi-objective optimization method in the search for a non-dominated set of solutions to the Anytown water distribution network problem. The problem was posed as a multi-objective optimization problem with total cost, water quality and reliability as the objectives. The design variables were the addition of new pipes, reconditioning and replacement of existing pipes, siting and sizing of new tanks and operation of existing pumps. The total cost of a solution is the sum of pump operating costs, tank capital costs and pipe network capital cost.

Inspection of the results indicates that the method is able to identify the pay-off surface characteristic between the total cost, water age (a surrogate measure for water quality) and the resilience index (a surrogate measure for reliability) for the network. The simultaneous optimization of the problem considering total cost, water quality and reliability as objective functions and design parameters and operation as decision variables resulted in high quality solution networks. Design characteristics of three solutions from the Pareto optimal set are presented. The solutions have the optimal pipe network and best siting arrangement for a new tank downstream of the main demand. This automatically minimizes the pipe sizes, provides reliability and also minimizes the water age in the network. Pump operation schedules are also optimum in that they fill all three tanks during the low demand period and stay full



during the peak period, during which pumps are required to supply the network demand. This adds to the reliability in the network, as in the case of pump breakdown all three tanks are full to supply the peak demand. Finally the solutions presented along the pay-off curve are fully feasible solutions that operate under 5 loading conditions without violating any constraints.

The location, overflow and minimum normal day elevation, diameter and bottom of tank from minimum normal day elevation were treated as independent design variables for the tanks. This approach resulted in convergence of the algorithm to a region of feasible search space with less computational effort than using tank volume as the design variable. The algorithm presented in this paper provides an efficient approach to design of tanks in water distribution networks.

It can be concluded that considering resilience index and water age as objectives, in addition to cost, in the optimum design and operation of Anytown's water distribution system resulted in networks with high reliability and water quality. These improvements were achieved at no extra cost in comparison to methods that consider only twin-objective optimization.

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