seen by comparing these two quantities. In making this comparison normal stress $\sigma^*$ should be reduced by a factor of 2 by virtue of the maximum-shear theory and by an additional factor between 2 and 3 to account for the difference in flow stress of a cone and a member in simple shear. The net result is that for the same size effect to be operative $\tau^*$ should be from 4 to 6 times $\tau^*$. As previously stated, the value of $\sigma^*$ should be expected to increase with increased load $N$, due to strain hardening and the mutual restraint of adjacent asperities. The manner in which $\sigma^*$ changes with $N$ in the foregoing example may be seen by dividing observed values of $N$ by calculated values of $A_1$ at each point in accordance with Equation [1]. When this is done, Fig. 6 results. It is evident that $\sigma^*$ is not constant as for ordinary sliders but increases with $N$.

![Fig. 6 Variation of Shear Hardness $\sigma^*$ With Normal Load $N$ for Example of Table 2](image)

The large increase in coefficient of friction ($\tau^*/\sigma^*$) with rake angle is due to the fact that $\tau^*$ remains essentially constant with $N$ (since even for light loads $\tau^*$ is at a much higher stress level than $\sigma^*$ is for relatively heavy loads and consequently $\tau^*$ is saturated with regard to strain hardening) while $\sigma^*$ increases with increased $N$. The reason for the increase in $N$ with decreased rake angle $\alpha$ is obviously due primarily to the decreased shear angle that accompanies a decrease in $\alpha$.

For ordinary sliders the value of $N/A$ usually amounts to 500 psi or less. If $A / A_1$ is computed from Equation [5] using the value of $B$ from the foregoing example, we obtain

$$A_1/A = 1 - e^{-0.0031 \times 1.2} = 0.00157$$

This is the order of magnitude of values of $A_1/A$ measured by Bowden and his associates (6) with ordinary steel sliders. Values of $\tau^*$ and $\sigma^*$ for such a slider may be taken approximately as follows from the foregoing example when the normal load $N$ is light

$$\tau^* = 413,000 \text{ psi}$$
$$\sigma^* = 318,000 \text{ psi}$$

In the discussion presented here the increase in $\sigma^*$ that is observed with increase in $N$ is considered to be due to strain hardening and mutual interaction of flowing asperities. As pointed out in a previous paper (2), part of this increase in $\sigma^*$ with increased $N$ could be due to a restraining action associated with the presence of the shear plane. By this latter mechanism, $\sigma^*$ would increase when the angle between shear plane and tool face decreased, and hence $\sigma^*$ would increase when $N$ increased as a result of an increase in $\alpha$. While it is not known how much of the increase in $\sigma^*$ with $N$ is due to strain hardening and how much is due to shear-plane restraint, it is now thought that the influence of strain hardening is the greater of the two.

The corresponding coefficient of friction would be 1.3 from Equation [3] if $K$ were taken equal to 1. While $K$ will approach 1 in cutting it undoubtedly will be less than 1 for an ordinary dry slider and the expected value of $f$ would be reduced to 0.9 to 1 for steel sliding on steel in dry air.

**BIBLIOGRAPHY**


**Discussion**

B. T. Chao and K. J. Trigger. The authors are to be congratulated for their new and useful treatment of the friction process, the calculation of the real area of contact, and the local stresses at the tool chip interface. This analysis clarifies certain aspects of metal-cutting phenomena which have heretofore been obscure. Our experience is in accord with the authors' contention that the subrupture dynamic flow stress along the shear zone is unaffected by the normal stress and, in a general way, we agree with the $\tau^*$ versus $\sigma^*$ relationship as depicted in Fig. 2.

In calculating the numerical values of $\tau^*$ and $\sigma^*$ (which are for small values of $N$) the authors assume that the length of contact along the tool face is twice the depth (feed) of layer removed. The writers have not found this to be the case in the machining of steel. Table 3 of this discussion compares the apparent tool-chip...
contact area as measured with that as calculated according to the
authors' assumption when annealed SAE 52100 steel is turned
with a steel-cutting-grade carbide tool. For HSS tools, the dis-
crepancy will be of the same order under comparable conditions.
It is evident that the measured apparent area of contact is
2 to 4 times the value used by the authors in the calculation of $\tau^*$
and $\sigma^*$. With HSS tools, even at such high rake angles as 30 or
40 deg, it seems very unlikely that the area could be estimated
from the assumed relation $A = 2b$. Consequently, the writers
believe that the values of $\tau^*$ and $\sigma^*$ as reported in this paper
are in error.

Also, from Table 3, a definite and noticeable decrease of the
apparent area of contact is observed when the rake angle is in-
creased. Hence the relationship between $\sigma^*$ and $N$, as shown
in Fig. 6 of the paper, should be modified accordingly. The trend is
to increase the relative magnitude of $\sigma^*$ at small values of $N$ or to
decrease $\sigma^*$ at high values of $N$.

Fig. 5 of the paper also should be modified for the same reason.
Evidently, when the variation of the apparent area of contact
with changes in tool rake is taken into account, the relationship
between $A_+$ and $N$ will tend to become linear, at least within the
range of the experimental data used by the authors.

Max Kronenberg.\(^9\) This paper has the particular merit of
offering plausible explanations for the conclusions in which the
authors and the writer concur with regard to the coefficient of
friction in metal cutting.

The two most significant sentences of the paper are as follows:
"A coefficient of friction is inadequate to characterize the sliding
between chip and tool."

"The coefficient of friction in metal cutting bears little relation-
ship to the ordinary friction process."

The writer came to the same conclusions several years ago\(^10\) and
suggested at that time going even a step further than the authors
do in this paper, by recommending to discontinue the use of the
concept of coefficient of friction in metal cutting.

The reasons for this recommendation are based not only on
scientific considerations but also on a practical point of view. In
the writer's opinion it is necessary to keep in mind the effect
which our findings in metal-cutting research may have on the
men in the shop and on productivity in general. A tool engineer
who is busy in his daily work has but little time to analyze the
reasons for a high or low value of the so-called coefficient of
friction and must come to entirely wrong conclusions unless we
discontinue the use of this concept. His conclusions may affect his
work substantially and also his actions, say, in the application of
cutting fluids, purchasing of tools, selection of work materials, and
other such items on which the efficiency of production depends.

Let us assume that our tool engineer learns about a new metal
and that he finds from cutting tests the coefficient of friction to be
higher when machining this new metal than in the case of his
older metal. What would be your conclusion? Supposedly, it
would be the same as that of the tool engineer. He would con-
tinue to use the older metal with the lower coefficient of friction,
assuming that he would get better conditions than with the new
metal with the higher coefficient of friction.

He would be wrong, as will be seen from the data of Fig. 7 of
this discussion, which represents an analysis of Table 2 presented
in the paper. Consider columns 2 and 3 for the friction force $F$
and the normal force $N$, respectively. You will notice, when
following from top to bottom, that they both decrease when the
true rake angle $\alpha$ in column 1 increases from 0 to $+50$ deg. The
friction force $F$ drops from 775 to 305 lb and the normal force
drops likewise, namely, from 885 to 165 lb. Such drop is cer-
tainly a desirable feature.

However, let us consider now what the coefficient of friction re-
veals. It will be seen from column 4 that the coefficient of friction
increases from top to bottom—in spite of the decrease of the two
forces, namely, from 0.88 to 1.85. Hence our tool engineer
comes to the conclusion that something undesirable has happened,
because the coefficient of friction has increased, although the
contrary is true. It follows that the concept of a coefficient of
friction is misleading in metal cutting and should be discarded.

Discarding the coefficient of friction, however, does not mean
that we should discontinue to investigate the friction process an-
together; to the contrary. Investigations as presented in this
paper should be continued because we can obtain from such re-
search useful data when using a different concept than the friction
coefficient.

It is suggested that the inverse value of the coefficient of friction
be used. This inverse value is the tangent of the angle of direction
of the resultant cutting force $R$, indicated by angle $\omega$ in Fig. 7
$tg\omega = N/F$.

This angle has been calculated from the data of the paper under
discussion and also the angle $\gamma$ which indicates the direction of the
resultant cutting force with respect to the flank of the tool.

The two angles are tabulated in columns 5 and 6. You may ask why
these two angles are of interest in metal cutting. If you follow
the change in angle $\gamma$ in column 6, you will see that the resultant
cutting force acts considerably closer to the tool flank when the
true rake is 80 deg than when it is 0 deg, and tension and com-

\(^9\) Consulting Engineer, Cincinnati, Ohio. Mem. ASME.

\(^10\) "On the Analysis of Cutting-Tool Temperatures," by E. G.
also Tool Engineer, October 1953, pp. 49-50.
expression in the tool face depend on angle $\omega$. It also will be noted from the two sketches at the bottom of Fig. 7, that the cutting forces in the left-hand sketch are large, and small in the right-hand sketch in spite of the inverse values for the so-called coefficient of friction. This graphic representation is a good indication for the misleading concept as discussed.

When recently publishing a book on the "Fundamentals of Metal Cutting Science" the writer came to the conclusion that the difference between a coefficient of friction in metal cutting and in ordinary friction results from the fact that the gliding body (chip) is plastically deformed in metal cutting, a deformation that does not occur in ordinary friction.

The authors differ somewhat in their explanation from that of the writer and attribute the difference in the behavior of a coefficient of friction in metal cutting to the change in the ratio of the real to the apparent area of contact between chip and tool that takes place as the normal force changes. This hypothesis is very interesting and the authors should be commended for developing it.

They indicate also that the number of capillaries would change with a change in the contact-area ratio, concluding that cutting fluids become less effective as the ratio approaches unity. Hence, in the case of negative rakes, cutting fluids would be less effective. In the writer’s opinion, however, cutting fluids also could be used in the case of negative rakes and carbides.

To explain this, Fig. 8 has been prepared to show the practical significance of the angles $\omega$ and $\gamma$ for the resultant cutting force in conjunction with the appearance of compression and tension in the tool face.

The angle $\omega$ is plotted at the left-hand scale, the so-called coefficient of friction of the right-hand scale, running in opposite direction.

Tension in the tool face is indicated by the field marked "Tension (Danger Zone)." compression in the tool face exists for combinations of $\alpha$ and $\omega$ represented by the lower field marked "Compression (Safety Zone)."

It will be seen that the compression zone increases as the true rake $\alpha$ becomes smaller and even more so as the true rake enters the negative field.

**Example.** When $\alpha = +20$ deg, compression in the tool face exists only for small angle $\omega$ (or for large coefficients of friction—greater than 1.2). Hence compression exists only when the resultant cutting force $R$ is relatively close to the tool face.

Using a cutting fluid and reducing the friction force would make the conditions worse, because we would reduce the so-called coefficient of friction to, say, 0.8 or increase angle $\omega$ and thereby cause tension (and perhaps cracks) in the tool face. (If we go still higher than the inclined line, conditions change, but this cannot be discussed here.)

However, when a negative true rake of $\alpha = -20$ deg is used it would be possible to employ a cutting fluid without entering the danger zone of tension, unless the angle of direction of the cutting force exceeds about $\omega = 65$ deg.

Cutting without a coolant is the rule in the case of carbides with positive rakes because a high coefficient of friction is required to keep out of the tension range. Fig. 8, however, indicates that new avenues for application of cutting fluids may be opened in connection with carbide tools when investigations on tension and compression produced in the tool face are carried out for various cutting fluids. Thus it may be realized why the paper presented by the authors is a valuable contribution toward explanation of these problems.

**ERNEST RABINOWICZ**. This interesting paper introduces two quantities that are relatively new to friction theory, namely, $A_r/A$ and $K$. The authors point out that for the case of sliding surfaces, $A_r/A$ will be small; however, this is not necessarily so. When concentrated contact occurs, e.g., a hemisphere on a flat surface, high normal pressures are produced, and we may estimate values of $A_r/A$ of $1/4$, using taper section methods of (cf. Moore) and $1/4$ using a comparison of friction and wear data. These values are of the same order of magnitude as those given by the authors.

In regard to $K$, the fraction of the surfaces that has formed welds, it should first be pointed out that Equation [2] given by the authors is not in the form customarily used in friction calculations, allowance being made in the latter for a friction force produced by the shearing of nonwelded junctions. In other respects, the constant $K$ is similar to that given by Archard, who calculated the fraction of junctions that produce metal-transfer fragments during sliding. Comparing Archard’s values with those in the paper, it is necessary to multiply his results by a factor of 3 for an assumed $A_r/A$ ratio of 1:3 and by a factor of 2, since metal transfer was measured in one direction only. The recalculated values of $K$ for unlike metal combinations range from 0.9 to 0.06 per cent. It is likely, but by no means certain, that $K$ during metal cutting is larger.

A complication that the authors have not considered is the effect of the difference in sliding conditions for points away from the cutting edge, which will lead to systematic variations in both $A_r/A$ and $K$.

It is felt that the stimulating treatment by the authors is definitely in the right direction but that an additional factor may have to be introduced to bring out fully the difference between the cutting and sliding situations.

**A. O. SCHMIDT**. The authors present again a thorough analysis of a significant sector in the metal-cutting field. The importance of their findings is expressed in their statements that the "coefficient of friction" in metal cutting is not the same as thought of in the ordinary friction process. The experimental data and
those given in Table 2 show the effect of different rake angles. This table could be extended into the range of negative rake angles and also include average chip temperatures as well as tool-tip interface temperatures.

Average chip temperatures have been determined by thermal balance from calorimeter measurements of heat in the chips during milling tests with SAE 1020; as 1280°F for \( \alpha = 90 \) deg negative and only 600°F for \( \alpha = 30 \) deg positive (cutting speeds, 200-1200 fpm; feed, 0.006 in. per tooth).

It can be stated that the relationship \( F/N \) is definitely influenced by temperatures occurring in and between chip, tool, and workpiece.

Another temperature value which can be obtained is that of the tool-chip interface which would vary in cuts of this type between a high temperature of 1650°F for the tool with 30-deg negative rake angle, to a low temperature of 800°F for the 30-deg positive rake angle.

Measuring and considering such temperature values will help to explain that \( F/N \) must vary because positive rake angles will entail lower temperatures in chips and at the tool-chip interface. There is such an intimate contact between the chip and tool that we can speak of a weld. This weld will be broken more easily at higher temperature, i.e., when using negative rake angles, and that will make the relation \( F/N < 1 \). However, it should be borne in mind that with negative rake angles there are always higher cutting forces first, which next bring about higher temperatures in the cutting region.

We hope that the authors will bring this investigation to a conclusion which will benefit all of us interested in these problems.

B. F. Y. Turkovich. The authors present another of their fine papers which definitely provide many fruitful thoughts. Some of their assumptions and conclusions deserve a rather thorough discussion. The mechanism of dry friction between metals has been a field of investigation for quite some time. Although several of its aspects are well known, the intimate process is still obscure in its essence. This is probably due to the fact that our knowledge in physics of metallic surfaces in intimate contacts is not sufficiently complete to permit a broad all-embracing theory of friction. It is therefore more than gratifying to study the paper of the authors because it takes so many factors into consideration, offering a plausible interpretation of the friction process in metal cutting.

Let us analyze Equation [1] of the paper. There is an implication of uniform distribution of the normal load. Although this may be true in the case of an ordinary slide, even with extremely high pressure it is questionable that it would also happen necessarily in the cutting of metals. If uniform load distribution is not the case, then the friction force would vary somehow along the chip-tool interface. Our measurements, however, would indicate only some sort of average value. It is also important to emphasize the influence of the temperature distribution as shown in a recent paper by Chao and Trigger.\(^{17}\)

**AUTHORS’ CLOSURE**

The authors wish to thank the several discussers for their valued comments.

Professors Chao and Trigger question the assumption that the area between chip and tool is given by \( A = 2t \) and present data in support of their comment. It would appear from these data that a better assumption might have been \( A = 7bt \) for the range of speed, rake angle, and undeformed chip thickness of Table 2.

Substitution of this relation would change none of the conclusions, but would result in the following changes in detail:

1. The value of \( A \) for the cutting tests considered would be 3.5 times the value given or 0.01103 sq in.
2. The value of \( r^* \) would be \( 1/1.5 \), the value given or 118,000 psi, which is still a very high value of flow shear stress for mild steel.
3. The value of \( \sigma^* \) would be \( 1/1.5 \), the value given or 91,000 psi.
4. The ordinates of Fig. 5 would be multiplied by 3.5, while those of Fig. 6 would be divided by 3.5.
5. The value of \( N \) in the slider discussion would be 3.5 times the value given and consequently the value of \( A/A \) for the slider would be 3.5 times the value given or 0.0055. The values of \( r^* \) and \( \sigma^* \) for the slider would each be divided by 3.5.

The measurement of the apparent area of contact between chip and tool is very difficult to make and, in view of the approximate nature of the results obtained, it does not seem justified to use an expression for this area more complex than the one used here. However, it would appear that the observations of Professors Chao and Trigger to the effect that the value of the constant \( 7 \) in the foregoing expression for area will decrease somewhat with increase in rake angle are correct. A hypothesis to this, this would tend to make the curves of Figs. 5 and 6 somewhat linear.

Dr. Kronenberg suggests that use of a coefficient of friction in cutting be discontinued since it can be misleading. While this is true when we observe what happens to the coefficient of friction when only the rake angle is changed, the coefficient of friction varies in the expected direction when only the cutting fluid is altered. That is, if the cutting fluid alone is changed, at constant rake angle, the coefficient of friction decreases as the fluid becomes a more effective boundary lubricant. In this connection it should be mentioned that Dr. Kronenberg’s \( \omega \) would not be a satisfactory measure of performance for cutting-fluid tests, since it would vary in an unexpected way just as the coefficient of friction does in the case of tests involving a variable rake angle.

The proposed variable \( \gamma \) does not have this difficulty.

Where it was stated that cutting fluids should be less useful for negative-rake-angle tools, since \( A \), will then be larger, it should be understood that we were speaking only of the boundary-lubrication action of the cutting fluid. Actually a cutting fluid can be even more important for negative-rake tools than for positive-rake tools, where the major function of the fluid is to act as a coolant. Dr. Kronenberg’s remarks concerning the reason for better performance of carbide tools having a positive rake angle when used dry are very interesting.

Professor Rabinowicz is correct in stating that \( A/A \) need not be small for ordinary slider experiments. Where ordinary sliders were referred to we had in mind a block sliding on a flat plate rather than a heavily loaded ball. The value of \( K \) in Archard’s treatment of wear is different from that used here. Archard’s \( K \) is the probability of a weld breaking in one surface or the other, while our \( K \) is the fraction of the real area that is actually welded. The stress to shear the unwelded portion of the area was not included in our discussion, since it was assumed to be relatively small. The variation in stress and area of contact along the tool face was not considered in this discussion. Actually it would appear that the real area is distributed more densely near the cutting point than near the region where the chip leaves the tool. While this refinement is apparently important in some calculations, as noted later, no attempt was made to include it in this paper.

As Dr. Schmidt points out, a change of chip-tool-interface
temperature will accompany a change in rake angle and this could be responsible for part of the change in coefficient of friction observed. This change in temperature would be in the right direction to cause a decrease in friction with decrease in rake angle. However, the main change in coefficient of friction is not thought to be due to this cause, since about the same change in coefficient of friction with rake angle is observed when cutting at the slowest of speeds where temperature could have no influence.

The reason the coefficient of friction is thought to change slowly with change of surface temperature lies in the fact that both \( \tau \) and \( \sigma \) for surface asperities are influenced by temperature in about the same way. The change in temperature with rake angle that is observed is due to the more superficial nature of \( \tau \) than \( \sigma \). A more refined analysis than that presented here should include the effect of chip-tool temperature on the coefficient of friction.

Mr. Turkovich raises the question of force distribution along the tool face, as did Professor Rabinowicz. It is only to gain a first approximation that we assume the normal stress to be uniformly distributed. Actually a better second approximation would involve a linear variation in normal and tangential force from a maximum at the tool point to zero at the point where the chip leaves the tool face. In the reference,\(^1\) cited by Mr. Turkovich, the authors compute the temperature variation based upon a uniform face distribution and find the chip-tool-interface temperature to be maximum not at the tool point, but at the point where the chip leaves contact with the tool. However, this is not where the crater first develops and we have reason to question this calculation on the same grounds on which Mr. Turkovich discusses the present calculation. We recently have repeated the temperature-distribution calculation with gradually varying forces instead of uniform ones and have found the maximum temperature now to come close to the center of the chip-tool contact length, which is also the point where the crater has its maximum depth. It would appear that the solutions to both the problem discussed here and the temperature-distribution problem will benefit from a nonuniform force distribution.