Chaotic Behavior in the Magneto-Resistance of Quantum Dot and Quantum Point Contact

Yuichi Ochiai,1 Yohei Ujiie,1 Noboru Yumoto,1 Shigeki Harada,1 Takahiro Morimoto,1 Nobuyuki Aoki,1 Jonathan P. Bird2 and David K. Ferry3

1 Department of Electronics and Mechanical Engineering, Chiba University, Chiba 263-8522, Japan
2 Department of Electrical Engineering, University at Buffalo, SUNY, Buffalo, NY14260-1900, USA
3 Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287-5706, USA

We have investigated quantum structures, such as narrow wires, single or coupled quantum dots (QDs), and quantum point contact (QPC), in order to clarify the origin of the chaotic and fractal behavior in their transport properties. Chaotic behavior in the electrical transport in QD and QPC has been studied in the low temperature magneto-resistance (MR), and by use of scanning gate microscopy (SGM). The quantum structures are fabricated using hetero-junction semiconductors. The low temperature MR has been measured using a 3He cryostat or dilution refrigerator with magnetic fields up to 8 T. We discuss the relation between the narrow constrictions in these structures and chaotic scatterings observed in the MR and SGM.

§1. Introduction

Chaotic behavior resulting from interference of electron waves has been frequently reported in quantum transport studies of quantum dots (QDs) and wires. As for fractal behavior in universal conductance fluctuations (UCF), the low-temperature magneto-resistance (MR) has been observed and studied by two kinds of analysis methods, exact and statistical self-similarity, so as to clarify the fractal transport in the quantum structures.1)−5) In a similar quantum structure, we also have found three- or four-fold self-similar structure in the zero field negative MR, weak localization (WL) peak, of a coupled dot system.6) Self similarity of the MR previously has been discovered and discussed in a classical physics in the framework of chaotic transport pictures. However, in a quantum transport picture, lead effects in the dot are more important for characterizing any chaotic transport behavior.7) Quantum-simulation studies show such lead effects clearly,8) so that a discussion based on a mixed phase in a Poincaré section is important even in QDs and wires. Recently, in the MR of a single quantum dot, by gradually opening the lead gate in the quantum confinement, we have observed the appearance of the alternative change of Lorentzian or cusp (linear) form in the WL peak of the negative MR.9) Such change in the localization peak has been explained to come from a corresponding switching between chaotic and regular transports in the quantum structures.10) In our QPC, when the conductance is kept at a constant quantized value (“at plateau regions”) the slope decay of the WL peak shows a cusp like linear form. However, in the transitions from one quantized value to another, the line shape in the slope decay shows
almost Lorentzian form. Further, a similar behavior is also observed in quantum wires.\textsuperscript{11)}

When we analyze chaotic behavior as either exact or statistical self-similar fractal behavior, a parameter, such as fractal dimension, can be expected to be strongly affected at the boundary between quantum and classical transport. This is primarily because chaotic behavior usually appears strongly in such a boundary region between the intensively different transport regimes. In the case of the two line-shapes, such a boundary should exist in a transition region of transport regime. We want to clarify whether our observations can really be explained with a similar mechanism such as chaos-regular transition, although there exists a clear change between Lorentzian and cusp forms in the WL peak. While discussing the influence of dynamical process which leads to Lorentzian decay in the chaos-regular transition, it should be possible to look for clear evidence of this transition. In order to understand this fully, lead effects which determine the phase coherence are very important and an enhancement of saddle-point-potential effect can affect to the transition between Lorentzian and cusp line-shape. In order to clarify the origin of the Lorentzian form, we have analyzed the change of the line-shape of the WL peak in detail. Therefore, in this paper, we focus on the relation between chaos-regular transition and scattering of electron waves in the quantum structures.

\section*{§2. Experiments}

\subsection*{2.1. Low temperature electrical transports}

Chaotic behavior in electrical transport of QDs and QPCs has been studied in the low temperature MR\textsuperscript{12)} and by scanning gate microscopy (SGM).\textsuperscript{13)} We have carefully performed precise measurement of the low temperature MR in the QD samples. Carrier concentration and each effective coherent area have been determined from Shubnikov de Haas (SdH) oscillations and quantum interferences of electron waves in the dot and wires. In those experiments, we have checked for any nonlinearity, which could arise from heating effects, by varying measurement currents when we determine the I-V characteristics. We perform extremely fine magnetic-field sweeps to obtain ultra-high resolution results, (field step $\sim$0.001 mT) in the MR measurements. The coupled QD and QPC that we study are fabricated by the split-gate method using a high-electron-mobility GaAs/AlGaAs hetero-junction wafer. The electron density and mobility of this wafer are $2.9 \times 10^{11}$ cm$^{-2}$ and 970,000 cm$^{2}$/Vs at 4.2 K, respectively. The QPC’s length and width, which are largely defined by the split gate geometry, are 600 and 350 nm, respectively. We measured the four terminal MR of the QD and QPC in perpendicular magnetic field in the range up to $\pm$8 T at temperature down to 0.08 K using a $^3$He-$^4$He dilution refrigerator or down to 0.3 K using a $^3$He cryostat, respectively.

\subsection*{2.2. SGM Measurements}

The QPC, used in our SGM experiments has been fabricated in a modulation-doped In$_{0.53}$Al$_{0.47}$As/ In$_{0.53}$Ga$_{0.47}$As/ In$_{0.53}$Al$_{0.47}$As hetero-structure. The two- di-
Fig. 1. (a) Topographic AFM image of the sample. The QPC is defined by trenches of 100 nm deep. (b) Corresponding SGM image taken at the base conductance of $\sim 2.7G_0$ at zero magnetic field and 0.28 K. The image is processed by high-pass filtering in order to emphasize the fluctuations.$^{13}$

mensional electron gas (2DEG) has two subbands occupied, with densities of 7.2 and $2.1 \times 10^{11} \text{ cm}^{-2}$, respectively, and mobility of $7.4 \times 10^{4} \text{ cm}^{2}/\text{Vs}$ has been obtained. This gives a mean free path of only 1.2 $\mu$m, which suggests that a degree of disorder scattering is present in the material. Using electron beam lithography, a pattern was opened within the photoresist to allow etching of trenches to define the QPC via in-plane gates, as shown in Fig. 1(a). The device was etched, using a solution of H$_3$PO$_4$:H$_2$O$_2$:H$_2$O, for 4 minutes at 15 °C to produce a 100 nm deep trench. The pattern has a minimum drawn QPC opening of about 0.6 $\mu$m and a radius of the trench of about 0.8 $\mu$m, as shown in the atomic force microscopy image in Fig. 1(a). The SGM measurements were performed at a system temperature of 280 mK with a piezolever whose tip was coated with 15 nm of PtIr. The tip was held 40 nm above the surface of the PMMA during the SGM measurement. The conductance of the QPC was stored in a PC synchronized to the tip position as the tip scanned the entire surface. In these experiments, the bias voltage applied to the tip was kept low enough so that the maximum change in conductance was $\sim 0.4G_0$ in a whole image. This was achieved with a bias of about $\sim 100 \text{ mV}$.

§3. Results of low temperature measurements

3.1. Experimental results on MR

Fractal behavior of the UCF in low temperature MR can be observed in the QD and the fractal dimension can be estimated. In our previous studies,$^{4,6}$ we scaled the dimension as a function of scale quality factor, $Q$, the ratio of the mean level spacing, $\delta E_n$, to the mean level broadening, $\delta E_B$. Figure 2 shows the estimated results of fractal dimension determined by two methods: self-similarity and box counting, for three kinds of coupled QD samples, noted as A, B and C as in Ref. 6). It is found that the cross-over between the two regions of classical and quantum transport is clear and the fractal behavior is strongest at the boundary $Q = 1$. Similar results have been reported for many kinds of QD structures.$^2$ It may be expected that chaotic behavior becomes strongest at $Q = 1$ near the semi-classical boundary.

There exists a similar critical boundary in the difference of the low-temperature MR line-shape; e.g., between the “chaotic stadium” and the “regular circle” ballistic microstructures. The chaotic stadium shows a Lorentzian line-shape, while the
Fig. 2. The figure shows estimated results of fractal dimension determined by two methods of self-similarity and box counting for three kinds of coupled quantum dot samples, named by A, B and C. Each averaged dimension value is showed as a function scaled quality factor, \( Q = \delta E_n / \delta E_B \). Only in A coupled dot, both methods are applied in the determination of the dimension.

regular circle indicates a cusp (linear) line-shape.\(^{10,14}\) Another study of Lorentzian line-shape has been reported in QD arrays.\(^{15}\) These phenomena are explained in terms of chaotic or regular transports in these microstructures. A similar line-shape change has been also observed in our QPCs. As the split gate voltage is varied, we see an alternating and systematic transition of the line-shape between Lorentzian and cusp line-shapes. At conductances corresponding to the transitions between two quantized conductance values, the line-shape is found to be Lorentzian, while the line-shape is cusp-like for values near the quantized plateaus. This transition is similar to the cases of our previous study of the single QD.\(^{7}\)

As demonstrated in Figs. 3 and 4, cusp like line-shapes are observed near the plateaus (or plateau regions).\(^{11}\) On the other hand, Lorentzian line-shapes are found between two plateaus (or transition regions). These alternative appearances have a conductance period of a quantized value \( G_0 = 2e^2/h \). The examples above two types of line-shapes are shown in Figs. 3 and 4, respectively, and are fitted with Lorentzian or cusp line-shapes.\(^{11}\) To show the systematic transition between Lorentzian and cusp, we performed Lorentzian fitting for the magnetic field from 0 to 0.5 T, and determined a set of error estimates. In the plateau regions, our error estimate has peak structures that correspond to the Lorentzian line-shapes.\(^{11}\) To verify the line-shape forms, we differentiate them with respect to the magnetic field. For the range from 0.1 to 0.4 T, the \( dR/dB \) is fit by linear-function so as to determine as a slope change in the linear fitting. In Fig. 5, the obtained slope is plotted as a function of the conductance. The slope shows minima and maxima that appear with a period of \( G_0 \). It is found that the line-shapes can be classified into Lorentzian or cusp...
Chaotic Behavior in the Magneto-Resistance of QD and QPC

Fig. 3. An example of Lorentzian like line-shape observed in the low field MR at the transition regions.\(^\text{11)}\)

Fig. 4. An example of cusp like line-shape observed in the low field MR at the plateau regions.\(^\text{11)}\)

Fig. 5. The slope has been determined by the linear function fitting and plotted as a function of the conductance. The peak structures are seen at the transition regions.\(^\text{11)}\)

Fig. 6. The zero field conductance as a function of the gate voltage. The variation of the line-shape is put on a step-like dependence on the conductance.\(^\text{11)}\)

line-shape as shown in Fig. 6. The variation of the line-shape is put on a step-like dependence on the conductance. In our previous study, the MR of the single QD has been measured\(^7\) and also exhibited an alternating transition between Lorentzian and cusp line-shapes. To quantitatively analyze the transition, those line-shapes were fit in the same way as the QPC above, so that we can classify into Lorentzian or cusp line-shape. In these previous results, the step-like behavior has been obtained depending on the conductance just as for the QPC’s case.

3.2. Experimental results on SGM

In the MR measured with the side gate at \(-3.8\) V, we find a WL peak at zero-magnetic field and magneto-conductance fluctuations within \(\pm 1.5\) T as shown in the inset of upper figure in Fig. 7. The center peak appears to have a full width at half maximum (FWHM) of 23 mT, corresponding to a coherent area of \(1.8 \times\)
Fig. 7. The upper figure shows results of a FFT analysis of the magnetoconductance fluctuations, shown in the inset, across the QPC at low magnetic field. The upper and lower horizontal axes show the magnetic frequency and the corresponding interference area, respectively. The lower figure shows results of a grain analysis of the fluctuations in the SGM image taken at 1.5 x 1.5 μm² region, shown in the inset, around the center of the QPC. The horizontal and the vertical axis show the area of the pattern and its counts, respectively. Both distributions are fit well by a GUE function.\cite{13}

10⁵ nm² (480 nm in diameter), which is close to the size of the constriction itself. When the tip is in the constriction region, its potential modifies the interference conditions created by fluctuations of the background potential within the QPC. The SGM images show a random but reproducible pattern of conductance fluctuations that have characteristic areas of 2 ∼ 3 x 10⁴ nm² around the channel region of the QPC as shown in Fig. 1(b). The area distribution shows a good agreement with the characteristic area obtained from the magneto-conductance fluctuations, and both distributions follow a Gaussian unitary ensemble (GUE) distribution.\cite{13}

§4. Discussions

4.1. Magneto-resistances

As shown in Fig. 2, the cross-over between classical and quantum is located at just in Q = 1 and the fractal dimension at the cross-over point takes the upper value estimated by theoretical study. It suggests that the boundary between the two regimes is important because tunneling and evanescent wave propagation and lead effects\cite{7} will become important beyond this point (as one moves into the quantum
Chaotic Behavior in the Magneto-Resistance of QD and QPC

regime, \( Q > 1 \)). It is also related to the notable problem the role of mixing and/or interference among such tunneling waves in the system. In this case, we can discuss their dynamics by introducing the ‘mixed phase space’ in the Poincaré section of the system. This leads to questions such as how does this affect chaotic behavior in the central peak slope of the MR in QPC?

In the QPC, the line-shapes cannot be completely fit with an ideal Lorentzian or cusp form by simple analysis. It can actually be classified as intermediate between the two forms with above differentiation analysis method.\(^{11}\) We define these intermediate forms as a “mixture” of two line-shapes (Lorentzian and cusp). Actually, we cannot distinguish between both forms from the error estimate in the fitting for Lorentzian, because our fitting method is highly sensitive to conductance fluctuations of the MR. In fact, these fluctuations affect the error estimate, and irregular oscillations appear in the error estimate. This effect is easily solved by the differentiation analysis method, since the slope as a function of the conductance shows the gradual and well-resolved evolution of the line-shape. Then, we can classify the line-shape into Lorentzian or cusp form. Mainly, cusp form can be observed at the plateau regions, while a Lorentzian seems to be observed in the transition regions.

We can explain the QPC’s cusp-function line-shape at plateau regions with van Houten’s idea that is based on geometrical backscattering caused by the finite width of the QPC.\(^{12}\) At conductance \( G_0 \) and \( 2G_0 \), when the channels numbers are distinct, the line-shapes are well fitted with their formula. Thus, the geometrical backscattering is the origin of the cusp line-shape at the plateau regions. At the transition regions, however, it is found that their formula is not valid for the Lorentzian line-shape. The reason is that their formula does not take into account lead effects between leads and channels. One of these lead effects is purely quantum mechanical tunneling between leads and channels, when a channel appears at a transition region. The lead effects cause another contribution to the conductance in addition to the geometrical backscattering related to real quantized states, and may cause mixed phase space where chaos and regular dynamics coexist, which may lead to the Lorentzian line-shape. Electron tunneling through the QPC is greatly reduced at plateau regions in which almost all electrons propagate only through real channels. Thus the line-shape in this region should be explained as only geometrical backscattering of the real states. In fact, the low temperature conductance of the QD also has a step-like dependence related to the quantized conductance. Therefore, we can observe Lorentzian line-shape in the transition regions, as well as the case of the QPC. This indicates a common origin of the line-shape change with independence of the geometry of microstructures. Since there is thermal smearing of the Fermi surface within a one-electron description, smearing of the Fermi surface can be observed at finite temperatures and the temperature increase smears the dip and peak structures as the slope of the error estimate becomes flat.\(^{11}\)

4.2. \textit{SGM in disordered QPC}

Transport of disordered QPC can be visualized by low temperature SGM.\(^{13}\) Features of SGM connect to conductive channel images and broad feature by disturbing the in-plane gates. We can obtain conductance fluctuations, similar to UCF,
by modulating the quantum-interference conditions and the characteristic peaks depend on the eigenmodes. How does affect disordered background potential for the transports? We note that there are no-fluctuations at high temperature. The analysis and image correlation with magnetic field dependence reflect the WL and the area distribution of the fluctuations is connected to UCF using the GUE, a distribution which applies when a system is chaotic.\textsuperscript{16,17}

The result of an FFT analysis of the quantum fluctuation components in the MR is shown in upper figure of Fig. 7. The upper and lower horizontal axis indicate the magnetic frequency of the oscillations and the corresponding interference area is converted by a relationship of $S = \Phi_0 / \Delta B$, respectively, where $S$ is the interference area, $\Phi_0$ is the magnetic flux quantum, and $1/\Delta B$ is the magnetic frequency. The peak of the distribution is located around $\sim 6 T^{-1}$, which corresponds to an area of $2 \sim 3 \times 10^4 \text{ nm}^2$. It shows good agreement with the peak of the distribution of the fluctuations in the SGM image shown in lower figure of Fig. 7. This indicates that the interference occurs within the same area probed by the SGM, although the source of the interferences may be different. One is due to the change of paths and another is due to the change of the phase (which results from those paths). Both distributions are well fit by GUE. Supposing that the electrons in the channel encounter scattering from a background potential determined by the random placement of impurities and the alloy potential, the confining walls and also from the tip induced potential, it is reasonable that the system could show chaotic behavior. However, GUE statistics are usually expected in a chaotic system specifically when the time reversal symmetry is broken.\textsuperscript{17,18} As such, it is unusual that GUE statistics should result from the SGM image without a magnetic field being applied. However, as shown in our zero field transport at the transition region in QPC, we can observe a chaotic behavior in confined electron path with lead effects due to tunneling waves. Also, a similar exceptional behavior has been reported previously.\textsuperscript{19}

The spreading of the fluctuations in the SGM images shrink towards the center region of the QPC when the magnetic field is increased more than 100 mT, which is consistent with the time reversal symmetry being broken. Simultaneously, the histogram shifts by a small amount toward the left. Furthermore, the fluctuations fade above 1.5 T, where the cyclotron radius becomes smaller than the opening and SdH oscillations appear in the magneto-conductance. The images taken at 6.6 T, in the quantum Hall regime, show a completely different appearance with no quantum interferences and are ascribed to the depopulation of the propagating edge channels across the QPC.\textsuperscript{20}

§5. Conclusions

In the QPC fabricated in a GaAs/AlGaAs hetero-structure, the low temperature MR line-shape has been studied and exhibits an alternating and systematic evolution between Lorentzian- and cusp-line-shapes. The mixed function has been classified into Lorentzian and cusp line-shapes using a differentiation analysis method. The result shows that the Lorentzian appears at the transition region between plateau parts in conductance quantization. On the other hand, the cusp appears at the
plateaus. It is also observed in our previous work on a single QD.\(^9\) The cusp can be explained in terms of geometrical back-scattering caused by a finite width of the QPC, while the Lorentzian must be related to a tunneling effect. At least, in the transition regions, lead effects arising from electron tunneling contribute together mixing and/or interference of evanescent electron waves. The lead effects must be important both in the QPC and the QD. While we cannot completely understand the dynamics, there may be a relationship between the evolution of the MR line-shape and chaotic transport. Therefore, the transition between both steady states should be very important for the origin of Lorentzian form in WL peak and come from a chaotic behavior in mixed states in connection with coherent tunneling near the entrance of the dot and QPC.

Also, we have studied quantum interference patterns observed in SGM images taken inside a QPC fabricated in an InAlAs/InGaAs/InAlAs hetero-structure. In the constriction region, the potential clearly modifies the interference conditions created by fluctuations of the background potential within the QPC. The SGM images show a random but reproducible pattern of conductance fluctuations around the channel region of the QPC. The area distribution shows a good agreement with the characteristic area obtained from the magneto-conductance fluctuations, and both distributions follow a GUE distribution. This agreement indicates a direct correspondence between the two experimental results and suggests the presence of a chaotic transport across the QPC and may indicate an existence of strongly fluctuated quantum states near the QPC. Furthermore, it is very important, since we can obtain a powerful tool for investigating chaotic transport in the nano-scale quantum systems.

References
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